

ECONOMICS 690 — SPRING 2015
TOPICS IN ECONOMIC THEORY: CONTINUOUS TIME METHODS

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Textbook: *Brownian Motion Calculus*, Ubbo Wiersema, Wiley; *Stochastic Calculus for Finance II*, Steven Shreve, Springer; *The Economics of Inaction*, Nancy Stokey, Harvard.

Where: TBD

Course Content

The course consists of two modules. The goal of the course is to provide more than a passing familiarity with stochastic calculus so that you are able to read research papers and use these techniques in your own research. The first module will provide us with basic techniques in stochastic calculus, while the second module will look at applications, from finance, operations research, and macroeconomics.

Prerequisites

This course will *not* assume familiarity with measure theory. It will also not aim to teach you any. Instead, we will use intuitive probabilistic arguments, and bypass measure-theoretic questions as and when we can. With that being said, it is essential that you are familiar with the following:

- Normal and other continuous distributions.
- Conditional Expectations for discrete and continuous distributions.
- ODEs to the extent that you can solve first order ODEs and know where and how to find solutions to second order ODEs.
- Basic analysis, which includes being able to work with \limsup and \liminf , among other things.

Module 1

The course will begin with a quick review of discrete time martingales. Pride of place will be the random walk, which will serve as a source of inspiration as well as

examples. We will then cover the basics of continuous time stochastic processes that have continuous paths. This will include a study of Brownian motion, continuous time martingales, stopping times and related properties, a construction of the Ito stochastic integral, Ito's formula, the PDE connection, Girsanov's Theorem, the Martingale Representation Theorem, and connections to the heat equation and the Black-Scholes formula, as well as infinitesimal generators for Markov processes (diffusions), and Kolmogorov's equations.

This will be groundwork for the Module 2.

Module 2

I will assume that you have taken Continuous Time Methods, Module 1. We will primarily be interested in the control and regulation of continuous time, continuous path stochastic processes. Topics include the HJB equation, stopping problems, impulse control problems, inventory problems, asset pricing problems (eg, Black-Scholes), and other problems that we might think interesting.

Economic applications will include papers on corporate finance and macroeconomics; information acquisition and experimentation; equilibrium models at the interface of finance and macro. As note above, we will also look at inventory and storage models, which are very useful in the macro literature. A text for the second half is Stokey's *The Economics of Inaction*.

Below is a small sample of the papers we will read. We will try and understand these papers very well. The last couple of weeks of the course will involve student presentations of other papers.

Papers

- P. DeMarzo and Y. Sannikov. Optimal security design and dynamic capital structure in a continuous-time agency model. *Journal of Finance*, 61(6):2681–2724, 2006.
- B. Biais, T. Mariotti, G. Plantin, and J.-C. Rochet. Dynamic security design: Convergence to continuous time and asset pricing implications. *Review of Economic Studies*, 74(2):345–390, 2007.
- P. Bolton and C. Harris. Strategic experimentation. *Econometrica*, 67:349–374, 1999.
- Y. Sannikov. A continuous-time version of the principal-agent problem. *Review of Economic Studies*, 75(3):957–984, 2008.