Production-Based Term Structure of Equity Returns

Hengjie Ai, Mariano M. Croce, Anthony M. Diercks and Kai Li*

Abstract

We study the term structure of equity in economies featuring different production and risk structures. Accounting for heterogeneous exposure to aggregate productivity shocks across capital vintages creates a V-shaped term structure, consistent with the empirical findings of Binsbergen et al. (2012a). Adding an endogenous intangible stock of growth options reconciles the duration and the long-run risk evidence on the cross-section of returns.

Keywords: Term Structure of Equity, Vintage Capital, Growth Options

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1 Introduction

This paper studies the term structure of equity returns across production economies with different risk and technology structures. We show that standard real business cycle models not only produce a very small equity risk premium, but also imply an upward sloping term structure of equity returns, in contrast to recent empirical findings by Binsbergen et al. (2012a), Binsbergen et al. (2012b), and Boguth et al. (2012). This result generates an anomaly that we resolve by proposing a novel way to think about capital accumulation of both tangible and intangible assets.

Consistent with prior work (Ai et al. 2012), we consider a production economy with recursive preferences as in Epstein and Zin (1989) and heterogenous risk exposure of capital vintages. We show that this model generates a V-shaped term structure of equity returns. Specifically, risk premia of zero-coupon equities tend to decline over a maturity horizon of 10 years, as in Binsbergen et al. (2012b). On the other hand, expected zero-coupon equity returns increase over longer maturity horizons, as in standard long-run risk models (Bansal and Yaron 2004).

In addition, we introduce an intangible stock of growth options in the production structure of our economy, making it consistent with the empirical relationship between the cross-section of expected returns and cash-flow duration. In our model, high book-to-market ratio (value) stocks have both a higher long-run risk exposure than low book-to-market ratio (growth) stocks (Bansal et al. 2005, Bansal et al. 2007, Hansen et al. 2008) and a shorter cash flow duration (Dechow et al. 2004 and Da 2009).

The reconciliation of the duration and the long-run risk literature as well as the findings of Binsbergen et al. (2012a) is particularly relevant because it has been a challenge for several leading asset pricing models, such as the habit model of Campbell and Cochrane (1999), the long-run risk model of Bansal and Yaron (2004), and the rare disaster model of Reitz (1988), Barro (2006) and Gabaix (2009). As shown in Binsbergen et al. (2012a), these models typically imply either an upward sloping or a flat term structure of equity returns. In contrast to our model, all the aforementioned endowment-based asset pricing economies are based on a stylized and exogenous stochastic process for dividends. Whether the empirical term structure on equity returns is consistent with any endogenous production-based dividend processes is an open question. We contribute to the literature providing a positive answer.
We start our analysis with a production economy with long-run productivity shocks (Croce 2008) and convex investment adjustment costs (Jermann 1998). We show that (i) the term structure of equity returns in this model is upward sloping, and (ii) the risk premium on the very short term dividend strips is strongly negative. To better understand the latter result, note that our dividends equal capital income minus total investment, i.e. net aggregate payout to the household. With a Cobb-Douglas production technology, capital share is constant at about 33% of total GDP. On the other hand, investment accounts for roughly 30% of GDP and is highly volatile and pro-cyclical. This makes dividends on average very small and hence very sensitive to even moderate investment fluctuations, as they represent a very long position in capital income and a very short position in investment.

Due to the high volatility of investment, short-term dividends become a powerful hedge against contemporaneous productivity shocks and require a negative premium at the equilibrium. In other words, the short-end of the term structure of equity is strongly negative. This observation is relevant as it is shows that the well-known difficulty in generating sizeable risk premia in a production economy (Rouwenhorst 1995) is driven by the behavior of short-term aggregate dividends.

Turning our attention to the long-end of the term structure of equities, it is important to notice that along the balanced growth path, the long-run properties of investment, capital income, and aggregate consumption are similar. For this reason, long-term dividends are as risky as zero-coupon equities on aggregate consumption. In order to resolve the equity premium puzzle in a production economy, attention has to be devoted to the short-end of the term structure, i.e., on short-duration dividends.

Incorporating heterogenous exposure to aggregate productivity risk across capital vintages dramatically changes the shape of the term structure and lets short-term dividends be as risky as their long term counterparts. Specifically, consistent with the empirical evidence in Ai et al. (2012), we assume that young investments have zero exposure to aggregate productivity risk whereas all older vintages have an exposure of one. In this setting, aggregate productivity shocks refer to the productivity of the old capital vintages. In particular, good aggregate productivity shocks are relatively bad news for young investments, as they pick up this productivity shock only with a delay as they age.

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1In our investigation, we abstract away from both government expenditure and net-exports. For this reason our investment-output ratio is higher than usually reported.
We show that this feature of the model only marginally affects the behavior of the economy with respect to short-run shocks. The relevance of this aspect is twofold. First, this implies that our model inherits the success of standard production economies on the quantity side. Second, the resolution of the Binsbergen et al. (2012a) anomaly is fully driven by the fact that long-run shocks command higher risk premia for short-term dividends.

When young vintages are not immediately exposed to long-run shocks, good long-run news actually produces a drop in aggregate investment, consistent with the empirical literature on macroeconomic news (Barsky and Sims 2011). Investors, indeed, find it optimal to delay their investment to the future when the productivity gap between young and old investments declines. Meanwhile, investors anticipate higher future income and consume more. Part of this increase in consumption is financed by an increase in net payouts from the representative firm to the household. As investment drops, dividends increase sharply. The positive payout comes at a time when the discount rate is low since good news for the long-run lets the marginal utility decline substantially. Long-run productivity shocks turn into a source of short-term dividends risk. Equivalently, note that aggregate dividends always represent a long position in capital income and a short position in investment. Under our capital vintages setting, the positive exposure of capital income and the negative exposure of investment with respect to news shocks reinforce each other to produce high long-run risk exposure of short term dividends.

Since the response of investment with respect to news shocks dissipates over time, the risk exposure of dividends becomes identical to that of aggregate consumption over the long-run. This is due to the cointegration restrictions imposed by our general equilibrium model. As a result, over longer horizons the term structure of equity returns slopes up, as it does in standard long-run risk models.

In the final step of our analysis, we introduce an intangible stock of growth options in our production structure and examine its implications on cash-flow duration, cross-section of equity returns, and aggregate term structure of equities. We model tangible capital as a stock of assets in place that can be expanded only by exercising real options. Unexercised real options do not produce any cash-flow. Options are necessary only to create new assets in place through option exercise, i.e., through new tangible investment. In our setting, growth stocks are option-intensive, whereas value stocks are tangible capital-intensive. This implies that our cross-sectional risk heterogeneity can be tied back to risk differences
across tangible and intangible capital, in the spirit of Hansen et al. (2005) and Li (2009).

We allow growth options to be produced and then either immediately exercised or stored. Thanks to storability, upon the arrival of good news for the long-run, growth option exercise is delayed in our model with tangible capital vintages. This is because upon the realizations of good news for the long-run, investment in the young capital vintage declines, implying a low demand of growth options as well. In this state of the world, the market value of the growth options falls and generates lower returns. As a result, our growth options provide a substantial hedge against long-run shocks and are less risky than assets in place, consistent with the long-run risk empirical evidence of Kiku (2006), Bansal et al. (2005) and Hansen et al. (2008).

Studying this two-shock-two-capital setting is interesting for several reasons. First of all, as shown by Ai et al. (2012), this economy simultaneously accounts for the large equity risk premium, large value premium, the sizeable volatility of tangible aggregate investment, and the empirical responses of macroeconomic quantities to new shocks about future productivity. Furthermore, this setting replicates the failure of the CAPM and is consistent with the duration evidence. Indeed, since the cash-flow duration of a growth option is always longer than the duration of the underlying asset in place, our growth stocks have longer duration than value stocks. We show that our model preserves the V-shaped term structure of equity produced with vintage capital alone. By doing so, our setting reconciles simultaneously the duration, the long-run risk and the Binsbergen et al. (2012a) view. This is our main contribution.

Using an affine quadratic discount rate, Lettau and Wachter (2007, 2011) are the first ones to explore correlation structures consistent with the negative relationship between cash flow duration and expected returns in the cross section. Santos and Veronesi (2010) study the impact of cash flow risk on the aggregate term structure of equity returns in a habit-based model. Croce et al. (2007) study the term structure of equities in a learning model with long-run risks. Belo et al. (2012) emphasize the role of financial leverage as a device to reallocate risk from the long-end to the short-end of the term structure of net equity payout. All these studies produce a downward sloping term structure of equity returns. We differ from these papers in at least two respects. First, we study dividends that are a direct outcome of investment and production decisions, whereas previous studies focus on exogenously specified cash-flows. This allows us to tie the asset pricing implications of our model to the dynamics of several macroeconomic quantities.
More broadly, our production economy framework enables us to impose joint structural restrictions on cash-flows and the pricing kernel using data about both macroeconomic aggregates and cross-sectional returns. Second, our results suggest that the term structure of equity may be V-shaped, as opposed to being monotonic and decreasing over maturities.

Similarly to us, Gomes et al. (2003) propose a general equilibrium model with both tangible assets and growth options. We differ from Gomes et al. (2003) in several dimensions. First, in our setting, the ability to produce and store growth options makes them an insurance factor, as options are less risky than assets in place. In the Gomes et al. (2003)’s economy, growth options are not storable and represent an additional risk source. In Gomes et al. (2003), options are a device to shift the whole term structure of equity returns upward and obtain higher equity premium. In our setting, adding options to the model with tangible capital shifts the term structure of dividends downward. Since heterogenous exposure to producibility shocks across capital vintages makes short-term dividends very risky, the average level of our term structure of dividends is consistent with the equity premium observed in the data.

Most importantly, in Gomes et al. (2003) the term structure of equity is upward sloping, implying that value stocks have longer duration than growth stocks, in contrast to the duration evidence. This is because in Gomes et al. (2003) the term structure of equity returns is mainly a reflection of their upward sloping term structure of real bonds. In our model, the term structure of real bonds is downward sloping and the equity term structure is V-shaped.

Papanikolaou (2011), and Kogan and Papanikolaou (2009, 2010, 2012) focus on investment specific shocks to explain cross-sectional returns in production-based general equilibrium models. A broader review of the production based asset-pricing literature is provided by Kogan and Papanikolaou (2011). This class of models can produce a downward sloping term structure, consistent with the Binsbergen et al. (2012a) findings. We complement Kogan and Papanikolaou (2012)’s analysis and add two novel insights. First, we abstract away from investment-specific shocks and highlight the role of the intertemporal composition of productivity risk, i.e. long- and short-run risks, to explain cross-sectional returns. Second, our model suggests that the term structure of equity may change slope at different maturity horizons.

Hansen and Scheinkman (2010), Jaroslav et al. (2011) and Hansen (2011) develop novel methods to quantify the time profile of risk exposure of cash flows to macroeconomic shocks. Borovička and Hansen
provide novel tools to analyze both risk exposure and risk compensation of cash flows at different horizons in nonlinear models. Specifically, Borovička and Hansen (2011) apply their methods to study the risk profile of tangible and intangible capital in the context of the Ai et al. (2012) model. We differ from them because of our focus on the term structure of zero-coupon equity returns, i.e., the closest model counterpart of what is measured in Binsbergen et al. (2012a), Binsbergen et al. (2012b), and Boguth et al. (2012).

The rest of the paper is organized as follows. In the next section we detail our model. Section 3 reports no-arbitrage conditions required to price zero-coupon equities. In section 4, we study the term structure of equity returns in a standard production economy with convex adjustment costs. In section 5, we introduce capital vintages. In section 6, we consider intangible capital. Section 7 concludes.

2 General Formulation of Our Model

Across all the economies that we consider, equilibrium quantities in the decentralized economy coincide with those obtained under the central planner’s problem. For this reason, in this section we describe only the key elements of the Pareto problem of our economies. Under the assumption that markets are complete, asset prices are recovered from the planner’s shadow valuations.

Preferences. Time is discrete and infinite, \( t = 1, 2, 3, \ldots \). The representative agent has Kreps and Porteus (1978) preferences, as in Epstein and Zin (1989):

\[
V_t = \left\{ (1 - \beta) \left[ u(C_t, N_t) \right]^{1/\psi} + \beta \left( E_t \left[ V_{t+1}^{1-\gamma} \right] \right)^{1/(1-\gamma)} \right\}^{1/1-\psi},
\]

where \( C_t \) and \( N_t \) denote, respectively, the total consumption and total hours worked at time \( t \). For simplicity, we consider a Cobb-Douglas aggregator for consumption and leisure:

\[
u(C_t, N_t) = C_t^\rho (1 - N_t)^{1-\rho},\]
and set \( N_t = 1 \) when \( o \to 1 \), i.e., when we consider a fixed supply of labor. When, instead, \( o < 1 \), the optimal allocation of labor is determined by the following intratemporal condition:

\[
\frac{1 - o}{o} \frac{C_t}{1 - N_t} = MPN_t,
\]

where \( MPN_t \) denotes the marginal product of labor.

**Productivity risk structure.** Productivity is specified as in Croce (2008) and captures both short-run, \( \varepsilon_{a,t+1} \), and long-run, \( x_t \), productivity risks:

\[
\log \frac{A_{t+1}}{A_t} \equiv \Delta a_{t+1} = \mu + x_t + \sigma_a \varepsilon_{a,t+1},
\]

\[
x_{t+1} = \rho x_t + \sigma_x \varepsilon_{x,t+1},
\]

\[
\begin{bmatrix}
\varepsilon_{a,t+1} \\
\varepsilon_{x,t+1}
\end{bmatrix}
\sim i.i.d. N\left(\begin{bmatrix}
0 \\
0
\end{bmatrix}, \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}\right), \quad t = 0, 1, 2, \cdots
\]

**Production function and resource constraint.** Across all models, output, \( Y_t \), is a neoclassical Cobb-Douglas aggregation of tangible capital and labor services. Output is either consumed or used to promote investment:

\[
Y_t = K_t^\alpha (A_t N_t)^{1-\alpha} = C_t + I_t^{Tot}
\]

Total investment coincides with tangible investment in the economy featuring only tangible capital. When we introduce intangible capital, total investment embodies intangible investments as well.

**Accumulation of tangible capital.** Following Ai et al. (2012), we allow investments in different vintages of tangible capital to have heterogenous exposure to aggregate productivity shocks. The productivity processes are specified as follows. First, we assume that the log growth rate of the productivity process for the initial generation of production units, \( \Delta a_{t+1} \), is given by equation (2).

Second, we impose that the growth rate of the productivity of capital vintage of age \( j = 0, 1, \ldots, t - 1 \)
is given by

\[ \frac{A_{t+1}^{t-j}}{A_t^{t-j}} = e^{\mu + \phi_j (\Delta \alpha_{t+1} - \mu)}. \] (4)

Under the above specification, production units of all generations have the same unconditional expected growth rate. We also set \( A_t^t = A_t \) to ensure that new production units are on average as productive as older ones. Heterogeneity hence is driven solely by differences in exposure to aggregate productivity risk, \( \phi_j \).

The empirical findings in Ai et al. (2012) suggests that \( \phi_j \) is increasing in \( j \), that is, older production units are more exposed to aggregate productivity shocks than younger ones. To capture this empirical fact, we adopt a parsimonious specification of the \( \phi_j \) function as follows:

\[ \phi_j = \begin{cases} 
0 & j = 0 \\
1 & j = 1, 2, \ldots 
\end{cases} . \]

That is, new production units are not exposed to aggregate productivity shocks in the initial period of their life, and afterwards their exposure to aggregate productivity shocks is identical to that of all other existing generations.

This specification of \( \phi_j \) is consistent with the learning model of Pástor and Veronesi (2009). In Pástor and Veronesi (2009)’s economy, young firms are subject to substantial idiosyncratic risks but have very little exposure to aggregate shocks. The reason is that young firms are embedded with new technologies, which are highly uncertain. It is not optimal to operate these new technologies on a large scale until the uncertainty is reduced with learning. As a result, shocks to young firms have little impact on aggregate quantities. Over time, as young firms age, their productivity becomes more correlated with aggregate output because their technologies are adopted on a larger scale.

As proven in Ai et al. (2012), aggregate tangible capital expressed in terms of old vintage capital, \( K_t \),
evolves as follows:

\[ K_1 = M_0 \]  
\[ K_{t+1} = (1 - \delta_K) K_t + \varpi_{t+1} M_t, \quad t = 1, 2, \ldots \]
\[ \varpi_{t+1} = \left( \frac{A_{t+1}^l}{A_{t+1}} \right)^{\frac{1}{1-\alpha}} = e^{-\frac{1}{1-\alpha} (x_t + \sigma_a \epsilon_{a,t+1}) (1 - \phi_0)} \quad \forall t. \]  

\( M_t \) measures the mass of new vintage capital. The process \( \varpi_{t+1} \) is endogenous and takes into account the productivity gap between the youngest capital vintage and all the other older vintages. Note that when \( \phi_0 = 1 \), the young capital vintage has the same exposure to aggregate productivity as the older capital. In this case, the process \( \varpi_{t+1} \) simplifies to one and plays no role.

**Neoclassical creation of new tangible capital.** In a standard neoclassical RBC model, new tangible capital is formed through tangible investment:

\[ I_{t}^{Tot} = I_t \]  
\[ M_t = \left[ \frac{\alpha_1}{1 - \frac{1}{\tau}} \left( \frac{I_t}{K_t} \right)^{1-\frac{1}{\tau}} + \alpha_0 \right] K_t. \]

This specification allows for convex adjustment costs as in Jermann (1998). When the parameter \( \tau \) tends to infinity, there is no adjustment cost.

**Real option-based intangible capital.** Following Ai et al. (2012), we consider the following two-capital stock set up:

\[ I_{t}^{Tot} = I_t + J_t \]  
\[ M_t = \left( \nu I_t^{1-\frac{1}{\eta}} + (1 - \nu) S_t^{1-\frac{1}{\eta}} \right)^{\frac{1}{1-\eta}} \]
\[ S_{t+1} = [S_t - M_t] (1 - \delta_S) + H(J_t, K_t) \]
\[ H(J_t, K_t) = \left[ \frac{a_1}{1 - 1/\xi} \left( \frac{J_t}{K_t} \right)^{1-1/\xi} + a_2 \right] K_t \]
where $S_t$ and $J_t$ denote the intangible capital and intangible investment, respectively. In this setting, intangible capital is a set of blueprints, i.e., growth options that can be produced, stored and exercised over time.

Specifically, at time $t$ the agent has available a mass $S_t$ of growth options. If the agent finds it optimal to create $M_t$ units of new vintage tangible capital to promote production, a mass $M_t$ of growth options has to be exercised. Each growth option can be exercised only once and in each period it faces a probability $\delta_S$ of becoming obsolete. This implies that the stock of growth options available in the next period drops to $[S_t - M_t](1 - \delta_S)$ if no intangible production takes place. The agent can replenish the stock of growth options by promoting intangible investment, $J_t$. As in Ai et al. (2012), we allow for a concave production technology in the intangible sector, $H$, so that intangible investment features decreasing marginal returns in terms of creation of growth options.

Exercising $M_t$ growth options requires proceeding with tangible investments, $I_t$, necessary to promote new production activity. Following Ai (2007), we assume that growth options have heterogeneous profitability solely because they feature different exercise costs, i.e., all growth options create an additional unit of tangible capital if exercised, but they require different initial tangible investments. As shown in Ai (2007), under the optimal option exercise the most profitable options are exercised first. This implies that tangible investment has decreasing marginal returns in terms of creation of new tangible capital, $M_t$. Furthermore, under the optimal option exercise strategy, the new capital mass, $M_t$, is a concave and homogenous of degree one CES aggregation of tangible investment, $I_t$, and intangible capital, $S_t$.

### 3 Asset Pricing Recursions for the Term Structure

Consider a cash-flow stream $\{CF_t\}_{t=1}^\infty$. Let $P_{n,t}$ denote the price of a zero-coupon equity paying the possibly risky payoff $CF_{t+n}$. It is possible to price the zero-coupon equities adopting the following recursion:

$$
\begin{align*}
    P_{0,t} &= CF_t \\
    P_{n,t} &= E_t[A_{t+1}P_{n-1,t+1}] \quad n = 1, 2, \ldots ,
\end{align*}
$$

(9)
where $\Lambda_{t+1}$ is the stochastic discount factor in the economy:

$$
\Lambda_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-1} \left( \frac{u_{t+1}}{u_t} \right)^{1-\frac{1}{\psi}} \left( \frac{V_{t+1}}{E_t \left[ V_{t+1}^{1-\gamma} \right]^{1-\gamma}} \right)^{\frac{1}{\psi}-\gamma}.
$$

The one-period excess return of a zero-coupon equity with maturity $n$ is computed as follows:

$$
r_{CF}^{n,t+1} - r_f^t = \frac{P_{n-1,t+1}}{P_{n,t}} - r_f^t,
$$

where $r_f^t = \frac{1}{E_t[{\Lambda}_{t+1}]}$ is the short-term real risk-free rate. In our analysis, we assume that markets are complete and hence we can characterize the term structure of any cash-flow in the economy. In particular, we focus on the term structure of the following cash-flows:

$$
CF_t = \begin{cases}
C_t & \text{Consumption} \\
\alpha Y_t - I_{t}^{Tot} & \text{Dividends} \\
I_t & \text{Investment} \\
1 & \text{Real Bond}.
\end{cases}
$$

We study both the term structure of consumption and aggregate dividends. Note that in our setting, dividends equal total net corporate payout and can be replicated by a long position in a fraction of GDP and a short position in investment. Given this consideration, to better understand the composition of dividends risk, we also study the term structure of a claim to investment. Finally, we also report the implications for the term structure of real interest rates so that we can better disentangle dividends risk premia from bonds risk premia over different horizons.

4 Classical RBC Models

In this section, we focus on an economy where the only available saving device consists of tangible capital accumulation subject to adjustment costs, consistent with our system of equations (1)-(7). We compare different risk structures and capital accumulation formulations and look at their implications for the shape
Table 1: RBC with Tangible Capital Only

<table>
<thead>
<tr>
<th>Data Models</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
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<tr>
<td>$\sigma(\Delta c)$</td>
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<td>0.276</td>
<td>0.322</td>
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<tr>
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<td>16.40</td>
<td>10.05</td>
<td>05.58</td>
<td>06.15</td>
<td>13.87</td>
</tr>
<tr>
<td>$E[r_f]$</td>
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<td>02.47</td>
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<td>00.60</td>
<td>02.90</td>
<td>04.00</td>
<td>04.20</td>
</tr>
</tbody>
</table>

All entries for the models are obtained from repetitions of small samples. Data refer to the U.S. and include pre-World War II observations (1930–2007). Model (3) is calibrated at an annual frequency as follows: $\gamma = 10; \psi = 2; \alpha = 0.33; \delta = 0.1; \tau = 3; \mu = 0.02; \sigma = 4.5; \sigma_x = 0.15 \cdot \sigma; \rho = 0.92; o = 1; \text{ and } \phi_0 = 1$. Model (2) differs from model (3) because we eliminate long-run risk by setting $\sigma_x = 0$. Model (1) differs from model (2) because it features no adjustment cost, i.e., $\tau = \infty$. Model (4) differs from model (3) because it features heterogenous vintage capital, $\phi_0 = 0$, and null investment adjustment cost, $\tau = \infty$. Model (5) is calibrated as model (4), with two exceptions: (i) we set $\sigma < 1$ to match the share of hours worked; and (ii) we set effective labor-adjusted risk aversion, $\gamma \cdot o$, to 10. Excess returns are levered by a factor of three, consistent with García-Feijo and Jorgensen’s (2010).

of the term structure of dividends. We show that a standard RBC model with convex adjustment costs features an upward sloping term structure of equity. This result holds even when productivity growth is an i.i.d. process and makes the Binsbergen et al. (2012a)’s finding an anomaly for standard RBC models.

In columns 3–5 of Table 1, we summarize the main statistics produced by the three formulations of the RBC that we consider in this section, i.e., models (1)–(3). All three of these models produce counterfactual implications either on the volatility of investment or on the equity premium. These inconsistencies are resolved when we introduce heterogeneity across capital vintages, models (4)–(5).

For the time being, we keep the labor supply constant. Considering time varying labor would not change our results about the existence of a term structure anomaly in the context of a standard production economy. We relax this assumption when introducing capital vintages. In what follows, we discuss the implications of the neoclassical RBC model for the term structures.
This figure shows annual excess returns of zero coupon equities for different maturities associated with different cash-flows. Excess returns are multiplied by 100. These results are based on the calibration used for model 1, as reported in Table 1.

**Short-run risk only.** For model (1) in Table 1, we consider the RBC model in its most essential form, i.e. without any investment frictions and subject only to short-run shocks. Even though this model produces counterfactual implications for asset prices, it is a useful starting point to understand key features of the term structures in a production economy.

Focusing on Figure 1, we can see that all macroeconomic quantity cash-flows have the same risk premium over the long-horizon. The long-run level of the term structure of quantities, such as consumption and investment, is entirely driven by productivity risk. This is simply the result of the fact that in an RBC model, all long-term dynamics are driven only by productivity, as all endogenous variables tend to grow with productivity over the long-run. The faster the mean reversion of the endogenous quantities, the faster all term structures converge to their common long-run level. Under the calibration used for model (1), convergence is reached after about 25 years.

To better understand the behavior of the term structures over the short-horizon, it is convenient to
study the impulse response function of key variables upon the realization of a short-run shock. In figure 2, we depict responses for both macroeconomic quantities (left column) and asset prices (right column). As expected, consumption and investment move together with the productivity shock, although their responses feature very different magnitudes. Because of consumption smoothing, consumption is less volatile than the original productivity shock, while investment adjusts substantially.

As well known in the literature, dividends are small in this setting, as the capital income share is very close to the investment-output share in this class of models. This makes dividends mechanically volatile. The counter-cyclicality of aggregate dividends is due to the fact that aggregate investment is much more volatile than output, and is consistent with consumption smoothing. When a temporary positive shock to productivity materializes, the household finds it optimal to increase savings and finance an expansion of the representative firm. In the aggregate, the total net payout of the firm declines and reduces capital income. The reduction in capital income makes it optimal to keep consumption growth below output growth.

Note that upon the realization of the shock there is an immediate drop in dividends. Hence, the dividends zero-coupon-equity with maturity one is a great insurance device, as it pays a low cash-flow in good times. This explains the substantially low intercept of the term structure of dividends depicted in the bottom right panel of Figure 3 (solid line). This panel, therefore, shows that the equity premium puzzle in a production economy is mainly a problem of dividends behavior over the short maturities.

In the simplest version of the RBC model, the term structure of equity returns is upward sloping, even when the productivity risk structure features only an i.i.d. shock. This observation is already telling us that the slope of the term structure cannot be immediately interpreted as a statement about sources of risk, as it crucially depends on technology and investment dynamics. In model (1), for example, the term structure is upward sloping because investment is used to smooth consumption and reduce dividends risk in the short-run. The monotonicity of the term structure of dividends is a reflection of the monotonic mean reversion of dividends rather than a statement about long-run productivity risk in the sense of Bansal and Yaron (2004).

Another way to analyze the term structure of dividends is to think about its two subcomponents: a position long in capital income, $\alpha Y_t$, and short in investment, $I_t$. Since both capital income and
investment move together upon the realization of long run shocks, what determines the final cash-flow adjustment is the relative volatility of these two components. Since investment fluctuations are sizeable in the short-run, they dominate.

As shown in the bottom-left panel of Figure 3, the term structure of investment cash-flow is downward sloping, as investment is very volatile and pro-cyclical over the short horizon. Since the dividends’ loading on investment is negative, one can think about the term structure of dividends as the mirror image of the term structure of investment.

Turning our attention to the term structure of consumption, we see that it is slightly upward sloping
Fig. 3: Term Structures for Model (1) and (2): the Role of Adjustment Costs.
This figure shows annual excess returns of zero coupon equities for different maturities associated with different cash-flows. Excess returns are multiplied by 100. Dividends returns are levered. “AC” denotes positive adjustment costs. The solid and dashed lines refer to model (1) and (2), respectively. All the parameters are calibrated to the values reported in Table 1.

even in the case in which productivity follows a random walk. In an RBC model, indeed, investment absorbs a substantial portion of risk over the short horizon so that short-term consumption volatility is reduced. Finally, the top left panel shows a small negative spread between long-term and short-term real bonds. This is just a reflection of the fact that when the term structure of consumption is upward sloping, long-term real bonds provide more valuable insurance.

The role of adjustment costs. A common way to improve the asset pricing implications of a production economy is to introduce convex adjustment costs, as they allow for fluctuations in the marginal
value of capital, $q_t$. As documented by Croce (2008), this class of adjustment costs is unsatisfactory in the long-run risk framework because it poses a strong trade-off between the equity premium and investment volatility. In model (2), for example, we calibrate the adjustment costs to achieve a levered equity premium of almost 3%, but we obtain a volatility of investment growth three times smaller than in the data. In this section, we document that convex adjustment costs are also unsatisfactory for their inability to produce a downward sloping term structure of equity returns.

Specifically when we introduce adjustment costs, as shown in Figure 2, consumption responds more to productivity shocks and investment adjusts by less, as any investment adjustment is costly. The less pronounced reaction of investment is responsible for a smaller decline of aggregate dividends upon the realization of good productivity shocks.

Consistent with this observation, in the bottom-right panel of Figure 3 we show that after introducing adjustment costs the intercept of the term structure of dividends increases, i.e., short-duration zero-coupon equities offer less insurance than before. Since the short-front of the term structure of dividends increases, adjustment costs allow a higher aggregate risk premium, but they are not sufficient to get a downward sloping term structure. Adjustment costs make all term structures flatter, but do not alter the results obtained with short-run shocks alone.

The role of long-run risk. Turning our attention to model (3), we can examine the relevance of adding long-run shocks to the economy studied so far. In Figure 4, we show the impulse response of quantities and prices to a positive long-run productivity shock. In Figure 5, we depict the implied term structure of our relevant cash-flows and contrast them to those obtained in model (2) so that we can isolate the contribution of long-run risk.

When a positive long-run shock materializes and the IES is high enough, the substitution effect dominates the income effect, i.e., the agent responds by decreasing consumption and investing more. Because of the adjustment costs, the marginal value of capital ($\Delta q > 0$) also increases and a positive market excess return is realized when the agent has low marginal utility.

This mechanism explains why the overall equity premium increases to 4% under model (3). It is important, however, to notice that adding long-run risk enhances the degree of insurance of a short-term
Fig. 4: Long-Run Shock Impulse Response Functions for Model (3).

This figure shows annual log-deviations from the steady state upon the realization of a long-run shock. Returns are not levered. “AC” denotes positive adjustment costs. All the parameters are calibrated to the values reported in Table 1 for model (3).

zero-coupon equity. This is because positive long-run news gives an incentive to immediately invest more even though short-term capital income remains constant. In this setting, dividends need to fall in the short-run and by doing so they provide insurance against long-run news. This is also true for the consumption cash-flow.

These considerations explain the reason why the intercept of both the consumption and the dividends term structure are lower than under model (2). The right tail of these term structures, instead, is much higher than under model (2), as it is driven by long-run productivity risk. Overall, adding long-run risk to a standard RBC model makes the Binsbergen et al. (2012a)’s findings even more puzzling. In the next...
section, we propose a resolution of the equity term structure anomaly.

5 RBC with Heterogenous Capital Vintages

Capital vintages and optimal investment. In this section, we remove convex adjustment costs ($\tau = \infty$) and take seriously the empirical observation that young investments are less exposed to aggregate productivity risk than old capital vintages ($\phi_0 = 0$). As shown in equation (5), our vintage capital structure introduces uncertainty on the contribution of new capital to the aggregate stock of tangible
assets through the $\omega_{t+1}$ process. Since this uncertainty is resolved after the investment decision, it is a key determinant of investment in the first place.

Specifically, let $q_{k,t}$ and $p_{k,t}$ denote the ex- and cum-dividends price of aggregate capital, respectively. Capital prices satisfy the following equations:

$$
p_{k,t} = \frac{\alpha Y_t}{K_t} + (1 - \delta)q_{k,t},
$$

$$
q_{k,t} = E_t[\Lambda_{t+1}p_{k,t+1}],
$$

implying that the returns of capital are:

$$
1 + r_{k,t+1} = \frac{p_{k,t+1}}{q_{k,t}}.
$$

Optimal investment satisfies the following condition

$$
1 = E_t[\Lambda_{t+1}p_{k,t+1}\omega_{t+1}],
$$

and takes into account uncertainty on both the future value of aggregate capital, $p_{k,t+1}$, and the productivity-adjusted quantity of tangible assets added to the economy, $\omega_{t+1}$.

**Vintage capital and short-run shocks.** In Figure 6, we depict the response of both asset prices and quantities to short-run shocks for model (4) with and without heterogenous capital vintages. The main goal of this figure is to show that heterogenous exposure to aggregate productivity alters the equilibrium marginally with respect to short-run shocks. This is indeed a positive aspect of our friction, as we know that a frictionless production economy explains several key features of macroeconomic quantities. Indeed, as shown in Table 1, our model inherits the success of the standard RBC model on the quantity side and reproduces the right amount of consumption and investment volatility.

Figure 6 also suggests that our friction has no impact on the risk premium through the short-run shock. In order to understand the higher equity risk premium reported in Table 1 for model (4), we need to turn our attention to the interaction between long-run shocks and the heterogeneous productivity of
This figure shows annual log-deviations from the steady state over time upon the realization of a positive short-run shock. Returns are not levered. Dashed (Solid) lines refer to model (4), i.e, RBC with (without) overlapping generations of vintage capital, denoted as OLG. All the parameters are calibrated to the values reported in Table 1.

**Vintage capital and long-run shocks.** As shown in Figure 7, the impulse responses to long-run shocks are significantly different with and without heterogenous capital vintages. With a one-standard-deviation change in the long-run productivity shock, the excess return on physical capital in model (4) increases by about 1.5%, whereas the change in the frictionless model is null.

To explain the behavior of $r_K$, we focus our attention on the ex-dividend price of physical capital, $q_{k,t}$ (see Figure 7, bottom-right column). Iterating equation (11) forward, we can express $q_{k,t}$ as the present...
This figure shows annual log-deviations from the steady state over time upon the realization of a positive long-run shock. Returns are not levered. Solid (Dashed) lines refer to model (4), i.e, RBC with (without) overlapping generations of vintage capital, denoted as OLG. All the parameters are calibrated to the values reported in Table 1.

value of the infinite sum of all future payoffs:

\[
q_{k,t} = \sum_{j=1}^{\infty} (1 - \delta K)^j E_t \left[ \Lambda_{t,t+j} \alpha \left( \frac{K_{t+j}}{A_{t+j}} \right)^{\alpha-1} \right],
\]  

Equation (13) implies that the price of physical capital, \( q_{k,t} \), is the present value of the marginal product of physical capital in all future periods. This equation holds regardless of heterogenous exposure to aggregate productivity. A positive innovation in the long-run productivity component \( x_t \) has two effects on the future marginal product of physical capital. The first is a direct effect: keeping everything
else constant, an increase in $x_t$ raises the marginal product of physical capital by increasing all future $A_{t+j}$ for $j = 1, 2, \cdots$. The second effect comes from the general equilibrium. An increase in the marginal productivity of capital also triggers more investment, which augments $K_{t+j}$ in all future periods. Because of the decreasing returns to scale ($\alpha < 1$), an increase in $K_{t+j}$ mitigates the direct effect.

In the model without heterogeneous capital vintages, investment responds elastically to long-run shocks. Hence, the return on physical capital responds little to long-run productivity shocks because the direct effect of long-run productivity on the price of physical capital is offset by movements in investment (the general equilibrium effect).

In our model (4), however, investment rises after a long-run productivity shock, but only after a delay, whereas the return on physical capital increases immediately and sharply. Because of the initial drop in investment, the normalized capital stock, $K_t/A_t$, initially drops and then starts to rise, always staying below the level obtained in the model without heterogenous capital vintages. Because the marginal product of capital, $\alpha(K_t/A_t)^{-\alpha}$, is a decreasing function of normalized capital stock, in the benchmark model the marginal product of physical capital remains almost permanently above that observed in the model without vintage capital, producing a strong increase in $q_{K,t}$.

In this case, the direct and general equilibrium effects of long-run productivity shocks affect $q_{k,t}$ in the same way, thereby reinforcing each other. The marginal product of capital increases both because a positive shock in $x_t$ increases $A_{t+j}$ in all future periods and because the sluggish response of investment to long-run shocks results in a nearly permanent reduction of physical capital stock relative to that in model 1.

To understand the lagged response of investment to long-run news in model (4), note that a long-run shock increases the productivity of all existing vintages of capital almost permanently but affects the productivity of the new production units only after a delay. This generates an incentive to postpone investments. As a result, a long-run productivity shock immediately produces a strong income effect (the agent anticipates a persistent increase in the productivity of all existing vintages of capital and prefers to consume more) without generating a significant substitution effect (the return on new physical investment is unaffected by long-run productivity shocks for an extended period of time). At time 1, when a positive long-run shock materializes, the net effect is an immediate increase in consumption and a decrease in
Fig. 8: Term Structures for Model (4): the Role of Vintage Capital.

This figure shows annual excess returns of zero coupon equities for different maturities associated with different cash-flows. Excess returns are multiplied by 100. Dividend returns are levered. Solid lines refer to model (4), i.e., the economy with vintage capital. Dashed lines refer to model (4) without overlapping generation of vintage capital (OLG), i.e., with $\phi_0 = 1$. All the parameters are calibrated to the values reported in Table 1.

investment. This feature of the model is consistent with recent empirical findings of Barsky and Sims (2011) and Kurmann and Otrok (2010).

**Vintage capital and V-shaped term structure of dividends.** As shown in Figure 8, these different responses of investment to long-run shocks have substantially different implications for the short-end of the term structures. Specifically, the drop in investment upon the realization of good news for the long-run makes one-period ahead investment a great insurance device. This explains why the intercept of the investment term structure becomes negative.
In this setting, dividends increase upon the realization of good news to support more immediate consumption (see Figure 7, bottom-left panel). The intercepts of both the consumption and the dividends term structures increase because of the co-movement of these cash-flows with long-run shocks.

In the case of dividends, the increase in riskiness is amplified by the fact that the total net payout is small on average. At the equilibrium, the zero coupon equity with maturity one is approximately as risky as the zero coupon equity paying long-term dividends. As the investment dynamics converge over time to the trajectory observed without vintage capital, the term structure of dividends slopes up over longer maturities. Taken together, these facts originate a V-shaped term structure. In other words, the high equity premium produced by our model is due to its ability to increase the riskiness of short-term dividends, consistent with Binsbergen et al. (2012a)’s findings.

Also in this case, the shape of the term structure of dividends reveals very little about the risk structure in the economy. In our model, short-term dividends are risky because they immediately respond to long-run news and adjust in the opposite direction of household’s marginal utility. The short-term riskiness of dividends is fully originated by the interaction of long-run shocks and our vintage capital structure. This point is relevant because it shows that the Binsbergen et al. (2012a)’s findings are not a challenge for the long-run risk model in the context of a production economy.

**Vintage capital in the long-run.** Over the long-horizon, all term structures exhibit similar behavior regardless of whether or not young investments are exposed to short-run productivity. This is because over the long-run all capital vintages share the same productivity growth rate and short-term productivity gaps play no significant role. The slight difference in the long-term behavior of the term-structure of real bonds is entirely driven by differences in the very short-term risk-free rate. Since we plot returns in excess of the short-term interest rate, the term structure of bonds always starts from zero. If we were to consider differences in the short-term risk-free rate, the term structure of the real excess return would be higher in the model without vintage capital. This is because without vintage capital, consumption growth is less risky, as shown in the top-right panel of Figure 8.
This figure shows annual excess returns of zero coupon equities for different maturities associated with different cash-flows. Excess returns are multiplied by 100. Dividend returns are levered. Dashed lines refer to model (4), i.e., the economy with vintage capital and fixed labor supply. Solid lines refer to model (5), i.e., model (4) with endogenous labor. All the parameters are calibrated to the values reported in Table 1.

**Endogenous labor.** We conclude this section by exploring the role of endogenous labor. In a production economy in which labor is endogenous, all the main results obtained so far are preserved and actually enhanced. As shown in Figure 10, the possibility of gradually adjusting the labor margin at the equilibrium produces a smoother term structure of dividends, except for the very first maturity. With endogenous labor, in fact, the first maturity zero-coupon equity has an excess return of 60%, four times greater than before.

This number seems too high with respect to the Binsbergen et al. (2012a) results, but it is not a
main concern for two reasons. First, the calibration employed for model (5) is the same as that of model
(4) except for the labor disutility parameter $o$. We have chosen to keep the same calibration to better
isolate the additional role of labor. If we were to recalibrate the model, we could obtain a less excessive
intercept of the term structure of dividends.

Second, in the next section we present our benchmark model and show that the short-end of the term
structure of equity is quantitatively close to the Binsbergen et al. (2012a)'s findings. This is another good
reason to study an economy with intangible capital modeled as a stock of growth options.

6 Growth Option-Based Intangible Capital

Asset prices and optimality. In this section we study the term structure of dividends in a model
characterized by equations (1)–(8). In this setup, the cum-dividend price of tangible and intangible
capital, $p_K$ and $p_S$, evolve as follows:

\begin{align}
  p_{K,t} &= \alpha K_t^{\alpha-1} (A_t N_t)^{1-\alpha} + H_{K,t} q_{S,t} + (1 - \delta_K) q_{K,t} \\
  p_{S,t} &= \frac{1 - \nu}{\nu} \left( \frac{I_t}{S_t} \right)^{\frac{1}{\eta}} + (1 - \delta_S) q_{S,t},
\end{align}

where $H_J$ and $H_K$ denote the partial derivative of $H$ with respect to $J$ and $K$, respectively. Ex-dividend
prices are computed using the equilibrium discount factor:

\begin{align}
  q_{K,t} &= E_t[\Lambda_{t+1} p_{K,t+1}] \\
  q_{S,t} &= E_t[\Lambda_{t+1} p_{S,t+1}],
\end{align}

and optimal investment rules prescribe what follows:

\begin{align}
  E_t [\Lambda_{t,t+1} \omega_{t,t+1} p_{K,t+1}] &= (1 - \delta_S) E_t [\Lambda_{t,t+1} p_{S,t+1}] + \frac{1}{G_{I,t}}, \quad (16) \\
  q_{S,t} &= 1/H_{J,t}. \quad (17)
\end{align}
The right-hand side of equation (16) measures the opportunity cost of a marginal unit of assets in place. In order to get extra tangible capital, the agent has to invest $1/G_I$ units of general output and exercise a growth option. Since exercised options expire immediately, the opportunity cost of new tangible capital has to also include the loss of value of the growth option component, $(1 - \delta_S) E_t [A_{t,t+1}p_{S,t+1}]$. Equation (17) prescribes that intangible investment has to be set so the ex-dividend value of a marginal growth option equals its own marginal production cost. All other equations are unchanged in the economy with only tangible capital.

**Growth option less risky than assets in place.** Consistent with Ai et al. (2012), equation (15) suggests that the return of an option increases in the investment-induced demand of options, $I_t$, and decreases with the pre-determined supply of options, $S_t$. In the model with vintage capital, the ratio $I_t/S_t$ declines upon the realization of good news for the long-run, as $S_t$ is pre-determined and there is an incentive to postpone investment in new technologies, i.e., $I_t$ falls. This implies that the options return declines as well, exactly when the marginal utility of the agent is low. At the equilibrium hence options provide insurance against long-run shocks and require lower returns than tangible assets, consistent with Hansen et al. (2008) and Bansal et al. (2005). On the other hand, the behavior of the returns of tangible capital in the Ai et al. (2012) model is very similar to that explored in section 5 of this article. For this reason, we do not discuss it further.

**Results.** For the sake of brevity, we only discuss the main statistics obtained under our benchmark model. All figures are reported in Table 2. The model’s performance with aggregate quantities is strong. The model captures the right amount of consumption and investment volatility. In particular, we also match the properties of tangible and intangible investments. The volatility and co-movements of labor with consumption growth is consistent with the data as well. Although unreported, both the 1-year and the 10-year correlation between consumption and tangible investment are also consistent with U.S. data.

On the asset pricing side, our model produces a substantial spread between tangible and intangible capital. Since growth firms are growth option-intensive whereas value firms are tangible capital-intensive

\[ \text{Our measure of intangible investment is discussed in detail in Ai et al. (2012). Under our benchmark calibration, } E[I/J]=1, \text{ consistent with Corrado et al. (2006)’s methodology.} \]
Table 2: RBC with both Tangibles and Intangibles

<table>
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<th>$\sigma_{DC}$</th>
<th>$\sigma_{DI}/\sigma_{DC}$</th>
<th>$\sigma_{DI}/\sigma_{DI}$</th>
<th>$\sigma_{n}$</th>
<th>$\rho_{DC,\Delta n}$</th>
<th>$E[r_{M}^{L,ex}]$</th>
<th>$E[r_{K}^{L} - r_{S}^{L}]$</th>
<th>$\alpha_{K} - \alpha_{S}$</th>
<th>$\Delta_{K} - \Delta_{S}$</th>
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All entries for the models are obtained from repetitions of small samples. Data refer to the U.S. and include pre-World War II observations (1930–2007). Numbers in parentheses are GMM Newey-West adjusted standard errors. $E[r_{K}^{L} - r_{S}^{L}]$ and $E[r_{M}^{L,ex}]$ measure the levered spread between tangible and intangible capital returns, and the market premium, respectively. The leverage coefficient is 3 (Garca-Feijo and Jorgensen (2010)). The difference in the intercept of the CAPM regression for tangible and intangible returns is denoted by $\alpha_{K} - \alpha_{S}$. The difference in the duration of cash flows of tangible and intangible capital is denoted by $\Delta_{K} - \Delta_{S}$ and is expressed in number of years. The empirical duration spread is from Dechow et al. (2004). The model is calibrated at an annual frequency as follows: $\beta = 0.986$, $\psi = 2; \alpha = 0.30; \delta = 0.11; \tau = +\infty; \mu = 0.02; \sigma = 4.4\%; \sigma_{x} = 0.15 \cdot \sigma; \rho = 0.925; \phi_{0} = 1, \nu = 0.81; \eta = 3.8; \xi = 5$. We set $\phi < 1$ to match the share of hours worked, and assume an effective labor-adjusted risk aversion, $\gamma \cdot \phi$, of 10. Excess returns are levered by a factor of three, consistent with Garca-Feijo and Jorgensen’s (2010).

(see, among others, Kogan and Papanikolaou 2010), our model explains a substantial share of the observed value premium. Furthermore, since we are working with a two-shock productivity process, we are able to reproduce the failure of the CAPM. Last but not least, this model generates the right difference in duration between growth and value stocks and reproduces the negative relation between duration and risk premia observed in the cross-section.

In Figure 10, we compare the term structure of equity obtained with vintage capital and no intangible capital, to the case in which there is growth option accumulation. In both cases, labor is endogenously time-varying. We make three final remarks. First, our benchmark model features a V-shaped term structure of aggregate equity returns. Second, the term structure in the benchmark model is lower than the term structure without intangible capital. This is because our growth options are less risky than assets in place, and hence intangible capital reduces the overall riskiness of aggregate dividends at all horizons.

Third, under the benchmark model, the term structure is very smooth at all maturities. The log average excess return of the first zero-coupon equity is about 5%. Since this excess return is also quite
7 Conclusion

In this paper, we show that the empirical findings in the duration literature (see, among others, Dechow et al. 2004 and Da 2009) and in recent work by Binsbergen et al. (2012a) represents an anomaly for standard neoclassical production economies. We resolve this anomaly in an equilibrium model featuring long-run risk, recursive preferences and vintage capital.
Further, considering a broader setting with intangible capital formed by growth options allows us to simultaneously reconcile: (1) the duration evidence, (2) the Binsbergen et al. (2012a)’s results on aggregate dividends, (3) both the equity and value premium, and (4) the failure of the CAPM in pricing the cross-section of returns.

By doing so, we propose a novel way to think about production activity, dividends riskiness and capital accumulation. We believe that our framework should be expanded further to include endogenous corporate leverage in the spirit of what is done in the endowment economy by Belo et al. (2012). We also believe that future research should focus on the impact fiscal policy could have on the shape of the term structure of equities.

Our model also provides a novel way to think about both the dynamics and the term structure of the natural level of output, a concept extremely important for monetary policy. Even though we abstract from price stickiness, nominal rigidities could easily be included in our economy in order to study inflation risk and the alteration of the term structure of real bonds and dividends.

In this work, we take long-run productivity risk as an exogenous process. Future research should look at term structure of equities in economies with endogenous long-term risks, in the spirit of Panageas et al. (2012) and Kung and Schmid (2012).
References


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