

Supplemental Appendix to:
Exploiting the Errors:
A Simple Approach for Improved Volatility Forecasting
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Appendix B. Out-of-sample forecast results

Table B.1: MSE Ratios - Rolling Window

	AR	HAR	HAR-J	CHAR	SHAR	ARQ	HARQ
MSFT	1.1934	1.0000	1.0071	1.0205	1.0182	1.0488	0.9878
KO	1.1780	1.0000	1.0665	1.0228	1.0045	0.9393	0.8889
DD	1.1730	1.0000	1.0299	1.0166	0.9867	1.0175	0.9652
XOM	1.0300	1.0000	1.3184	1.0558	0.9921	1.0212	0.9792
GE	1.0379	1.0000	1.0941	1.1415	1.1253	0.9958	0.9564
IBM	1.2765	1.0000	1.0171	1.0357	0.9868	0.9681	0.9204
CVX	1.2671	1.0000	1.0023	1.0370	0.9478	0.9160	0.8825
UTX	1.1472	1.0000	1.0815	1.0395	0.9853	0.9094	0.9159
PG	1.1036	1.0000	0.9494	0.9489	1.0258	1.0175	1.0098
CAT	0.9671	1.0000	0.9488	0.9081	1.0047	0.9888	0.9571
BA	1.1498	1.0000	0.9663	0.9624	0.9796	1.0112	0.9456
PFE	1.1789	1.0000	1.0103	0.9995	1.0263	0.9896	0.9418
JNJ	0.9323	1.0000	0.9926	1.0158	1.0041	0.8428	0.9990
MMM	1.2289	1.0000	0.9328	1.0820	0.9370	0.8540	0.8706
MRK	1.0721	1.0000	0.7373	0.7634	0.7480	0.7277	0.7378
DIS	1.0038	1.0000	0.9019	0.9735	1.0704	0.9586	0.9093
MCD	1.1102	1.0000	1.1189	1.1168	1.1028	0.9412	0.9388
JPM	1.0389	1.0000	1.0733	1.0047	0.9924	0.8247	0.8342
WMT	1.2096	1.0000	1.0236	0.9895	1.0020	1.0502	1.0403
NKE	1.2602	1.0000	1.0376	1.0401	1.0049	1.0804	0.9614
AXP	1.3382	1.0000	1.0115	1.0219	0.9872	0.8759	0.8572
INTC	1.2063	1.0000	1.0007	0.9971	0.9917	0.9953	0.9400
TRV	1.1022	1.0000	1.0188	1.0107	1.2486	1.1935	1.1221
VZ	1.2448	1.0000	1.0043	0.9909	1.0488	0.9570	0.8631
HD	1.2732	1.0000	1.0560	1.0458	1.0018	0.9263	0.8758
CSCO	1.1999	1.0000	0.9865	0.9784	1.0070	1.0415	0.9662
UNH	1.1402	1.0000	1.0193	0.9974	0.9952	0.9864	0.9750

Note: The table reports the loss ratios of the HARQ to HAR models for each of the individual stocks.

Table B.2: MSE Ratios - Increasing Window

	AR	HAR	HAR-J	CHAR	SHAR	ARQ	HARQ
MSFT	1.2165	1.0000	1.0082	1.0125	1.0129	1.0941	0.9690
KO	1.2337	1.0000	0.9761	0.9644	1.0311	1.0015	0.9209
DD	1.1998	1.0000	1.0028	0.9968	0.9805	1.0962	0.9910
XOM	1.1875	1.0000	1.0012	1.0519	0.9550	1.0270	0.9278
GE	1.1065	1.0000	1.0047	0.9914	0.9519	0.9615	0.9484
IBM	1.2835	1.0000	1.0038	1.0165	0.9919	1.0701	0.9447
CVX	1.2589	1.0000	1.0048	1.0401	0.9352	0.9666	1.0317
UTX	1.1922	1.0000	1.0082	1.0145	0.9692	1.0396	0.9525
PG	1.2092	1.0000	1.0423	1.0081	1.0028	1.0724	0.9794
CAT	1.1902	1.0000	0.9739	0.9523	1.0003	1.1058	1.0036
BA	1.2571	1.0000	0.9623	0.9620	0.9936	1.0872	0.9538
PFE	1.2161	1.0000	0.9989	0.9942	1.0060	1.0197	0.9399
JNJ	1.2026	1.0000	0.9949	0.9905	1.0042	1.0207	0.9478
MMM	1.2182	1.0000	1.0123	1.0233	0.9556	0.9522	0.9242
MRK	1.1207	1.0000	1.0925	1.0811	1.0656	0.8663	0.8485
DIS	1.1535	1.0000	0.9985	0.9625	1.0765	1.0879	0.9793
MCD	1.1813	1.0000	0.9901	0.9797	0.9908	1.0236	0.9515
JPM	1.0496	1.0000	1.0289	1.0115	0.9605	0.8818	0.8537
WMT	1.3765	1.0000	1.0061	0.9901	0.9768	1.1509	0.9851
NKE	1.3517	1.0000	1.0008	1.0012	0.9969	1.1293	0.9525
AXP	1.3136	1.0000	0.9892	1.0010	0.9758	0.9586	0.8794
INTC	1.2275	1.0000	0.9896	0.9923	0.9872	1.0323	0.9541
TRV	1.1002	1.0000	1.0242	1.0111	1.0283	1.0517	1.0134
VZ	1.2547	1.0000	0.9911	0.9831	1.0055	1.0928	0.9614
HD	1.2470	1.0000	1.0134	1.0142	1.0012	0.9756	0.9006
CSCO	1.2340	1.0000	0.9802	0.9745	1.0051	1.0820	0.9755
UNH	1.1690	1.0000	1.0096	1.0130	0.9968	1.1536	1.0282

Note: The table reports the loss ratios of the HARQ to HAR models for each of the individual stocks.

Table B.3: QLIKE Ratios - Rolling Window

	AR	HAR	HAR-J	CHAR	SHAR	ARQ	HARQ
MSFT	1.3690	1.0000	0.9984	1.0264	0.9970	1.1894	0.9900
KO	1.4975	1.0000	0.9762	0.9595	1.0113	1.1143	0.9389
DD	1.3252	1.0000	1.0087	1.0014	0.9816	1.1994	1.0444
XOM	1.3568	1.0000	0.9866	1.0049	1.0019	1.1434	0.9828
GE	1.4602	1.0000	1.0552	1.0113	0.9565	0.9702	0.8334
IBM	1.5559	1.0000	0.9860	1.0025	0.9824	1.2305	0.9789
CVX	1.3802	1.0000	0.9858	1.0209	1.0142	1.2134	0.9971
UTX	1.3484	1.0000	0.9836	1.0017	0.9645	1.2397	1.0662
PG	1.4715	1.0000	0.9601	0.9572	0.9817	1.0758	0.9863
CAT	1.3267	1.0000	0.9976	1.0071	0.9939	1.1781	1.0224
BA	1.2353	1.0000	1.0031	0.9945	1.0001	1.2240	1.0497
PFE	1.2521	1.0000	0.9896	0.9868	0.9995	1.0744	0.9881
JNJ	1.4044	1.0000	1.0330	1.0052	1.0061	1.1279	1.0389
MMM	1.3493	1.0000	1.0068	1.0282	0.9527	1.0553	0.9599
MRK	1.3383	1.0000	1.0325	0.9855	0.9887	1.0005	0.9210
DIS	1.3685	1.0000	1.0082	1.0527	0.9955	1.1561	1.0147
MCD	1.4891	1.0000	0.9418	0.9317	0.9677	1.0341	0.9128
JPM	1.4688	1.0000	0.9926	0.9803	0.9977	1.1916	1.1107
WMT	1.4935	1.0000	0.9929	0.9831	0.9830	1.2255	1.0203
NKE	1.4670	1.0000	0.9960	1.0068	0.9870	1.2282	1.0004
AXP	1.7802	1.0000	1.0762	0.9868	0.9511	1.2099	0.9435
INTC	1.4305	1.0000	0.9966	1.0004	0.9884	1.1133	0.9698
TRV	1.4883	1.0000	1.0036	1.0308	1.0194	1.1947	0.9276
VZ	1.3952	1.0000	0.9908	0.9771	0.9941	1.1832	1.0224
HD	1.5055	1.0000	1.0014	1.0184	1.0057	1.1130	1.0177
CSCO	1.4370	1.0000	1.0445	1.0144	1.0107	1.2513	1.0054
UNH	1.3574	1.0000	0.9996	1.0218	1.0033	1.1082	0.9916

Note: The table reports the loss ratios of the HARQ to HAR models for each of the individual stocks.

Table B.4: QLIKE Ratios - Increasing Window

	AR	HAR	HAR-J	CHAR	SHAR	ARQ	HARQ
MSFT	1.5419	1.0000	1.0111	1.0391	0.9929	1.2737	0.9714
KO	1.6397	1.0000	0.9845	0.9978	0.9985	1.2566	0.9441
DD	1.4166	1.0000	0.9964	1.0080	0.9806	1.1544	0.9726
XOM	1.4475	1.0000	1.0030	1.0316	0.9857	1.0781	0.9550
GE	1.6703	1.0000	0.9929	1.0281	0.9694	1.1592	0.9028
IBM	1.7969	1.0000	0.9780	0.9881	0.9441	1.3047	0.9039
CVX	1.3868	1.0000	0.9898	1.0409	1.0019	1.1856	0.9888
UTX	1.4787	1.0000	0.9827	1.0111	0.9621	1.1716	0.9606
PG	1.7407	1.0000	0.9609	0.9711	0.9559	1.2534	0.9059
CAT	1.3894	1.0000	1.0001	1.0261	0.9934	1.2128	1.0264
BA	1.4558	1.0000	0.9959	1.0100	0.9915	1.1569	0.9550
PFE	1.4837	1.0000	0.9842	0.9961	1.0007	1.1430	0.9318
JNJ	1.5782	1.0000	1.0068	1.0565	0.9795	1.1359	0.9241
MMM	1.4569	1.0000	0.9996	1.0338	0.9574	1.1669	0.9694
MRK	1.3740	1.0000	0.9918	0.9918	0.9965	1.0626	0.9217
DIS	1.5515	1.0000	0.9902	0.9716	0.9997	1.2901	0.9654
MCD	1.8009	1.0000	0.9773	0.9931	0.9767	1.3586	0.9087
JPM	1.5565	1.0000	1.0022	1.0200	0.9887	1.1505	1.0101
WMT	1.9259	1.0000	0.9587	0.9298	0.9338	1.3100	0.8977
NKE	1.8064	1.0000	0.9922	1.0166	0.9901	1.4343	0.9302
AXP	1.7946	1.0000	1.0097	1.0065	0.9608	1.1732	0.8869
INTC	1.5868	1.0000	0.9987	1.0163	0.9916	1.1464	0.9922
TRV	1.4615	1.0000	1.0124	1.1004	1.0198	1.2069	0.9632
VZ	1.5604	1.0000	0.9843	0.9972	0.9948	1.2401	0.9584
HD	1.6611	1.0000	1.0026	1.0447	0.9845	1.0779	0.9732
CSCO	1.7259	1.0000	1.0046	1.0384	0.9791	1.1942	0.9193
UNH	1.3785	1.0000	1.0009	1.0362	1.0077	1.1677	0.9767

Note: The table reports the loss ratios of the HARQ to HAR models for each of the individual stocks.

Appendix C. Individual stock in-sample estimates

Table C.1: Average Parameter Estimates Across Stocks

	AR	HAR	AR-HL	ARQ	HARQ	HARQ-F
β_0	1.1252 (0.3612)	0.3050 (0.1011)		0.4600 (0.2617)	0.0960 (0.1155)	-0.0295 (0.1161)
β_1	0.6263 (0.0844)	0.2661 (0.0908)	0.9350 (0.0271)	0.8874 (0.0974)	0.5233 (0.1142)	0.4710 (0.1188)
β_2		0.3629 (0.1205)			0.2929 (0.1188)	0.4386 (0.1325)
β_3		0.2686 (0.0852)			0.1764 (0.0893)	0.1381 (0.1090)
β_{1Q}				-0.2324 (0.1371)	-0.1504 (0.0929)	-0.1300 (0.0930)
β_{2Q}						-0.1142 (0.1301)
β_{3Q}						0.0250 (0.1466)
	HAR-J	HARQ-J	CHAR	CHARQ	SHAR	SHARQ
β_0	0.3184 (0.1062)	0.1102 (0.1171)	0.3704 (0.1181)	0.1631 (0.1077)	0.2741 (0.1090)	0.0388 (0.1198)
β_1	0.3284 (0.1001)	0.5392 (0.1090)	0.2996 (0.1036)	0.5700 (0.1118)		
β_2	0.3477 (0.1227)	0.2915 (0.1191)	0.3980 (0.1282)	0.3257 (0.1244)	0.3434 (0.1161)	0.2845 (0.1183)
β_3	0.2575 (0.0844)	0.1746 (0.0883)	0.2659 (0.0932)	0.1738 (0.0863)	0.2572 (0.0820)	0.1608 (0.0863)
β_J	-0.3813 (0.3489)	-0.1637 (0.3005)				
β_1^+					-0.0052 (0.2559)	0.1758 (0.2367)
β_1^-					0.6307 (0.3179)	0.9797 (0.3125)
β_{1Q}		-0.1463 (0.0878)		-0.2262 (0.1510)		
β_{1Q}^+						-0.3539 (0.3446)
β_{1Q}^-						-0.0473 (0.2052)

Note: This table reports the average parameter estimates across the individual stocks. The standard deviations of the estimates obtained across the stocks are reported in parentheses.

Appendix D. Models including $RV^{(all)}$

Table D.1: HAR(Q) Models Including $RV^{(all)}$

Baseline Model		HARQ	HAR	HAR	HARQ	HARQ	
Added Variable			$RV^{(All)1/2} \cdot RV$	$RV^{(All)1/2}$	$RV^{(All)1/2} \cdot RV$	$RV^{(All)1/2}$	
S&P500							
MSE	RW	0.7953	0.8175	0.8396	0.8188	0.8079	
	IW	0.8857	0.9269	1.0049	0.8904	0.8886	
QLIKE	RW	0.9975	0.9251	1.0246	1.0710	1.2329	
	IW	0.8705	0.8694	1.0120	0.8579	0.9091	
Individual Stocks							
MSE	RW	Average	0.9333	0.9403	0.9964	0.9807	0.9678
		Median	0.9409	0.9451	0.9888	0.9353	0.9560
	IW	Average	0.9496	0.9611	1.0054	0.9832	0.9751
		Median	0.9525	0.9646	1.0042	0.9593	0.9752
QLIKE	RW	Average	0.9901	1.0081	1.0197	1.1126	1.0350
		Median	0.9936	1.0057	1.0122	0.9995	1.0151
	IW	Average	0.9477	0.9671	1.0127	0.9647	0.9721
		Median	0.9550	0.9596	0.9964	0.9647	0.9485

Note: The table reports the loss ratios for each of the different models relative to the HAR model. The augmented HAR(Q) models include $RV^{(All)1/2}$ or $RV^{(All)1/2} \cdot RV$ as an additional explanatory variable, as indicated in the table. $RV^{(all)}$ is estimated with 1-minute returns. The results for the S&P500 are based on the SPY ETF contract.

Appendix E. Forecasting alternative IV measures

Table E.1: HARQ Ratios with the Same Left- and Right-Hand-Side Volatility Measure

			SS-RV	TS-RV	RK	PA-RV	BPV
S&P500							
MSE	RW		1.0714	1.2985	0.8753	0.9384	0.8864
	IW		0.8174	0.9864	0.9204	0.7892	0.8843
QLIKE	RW		1.0793	0.9706	1.0185	1.0163	1.0019
	IW		0.8720	0.9583	0.9205	0.9747	0.9010
Individual Stocks							
MSE	RW	Average	0.9504	0.9722	0.9825	0.8075	0.9542
		Median	0.9703	0.9765	0.9904	0.7765	0.9694
	IW	Average	0.9463	0.9846	1.0052	0.7709	0.9463
		Median	0.9612	0.9825	0.9867	0.7404	0.9524
QLIKE	RW	Average	0.9346	0.9874	0.9894	0.9392	0.9651
		Median	0.9535	0.9928	0.9821	0.9448	0.9660
	IW	Average	0.9079	0.9719	0.9644	0.9416	0.9350
		Median	0.9168	0.9740	0.9619	0.9339	0.9384

Note: The table reports the loss ratios of the HARQ model to the HAR model where the same IV estimator is used on the left- and right-hand side. This table contrasts with Table 10, where RV is always on the left-hand side.