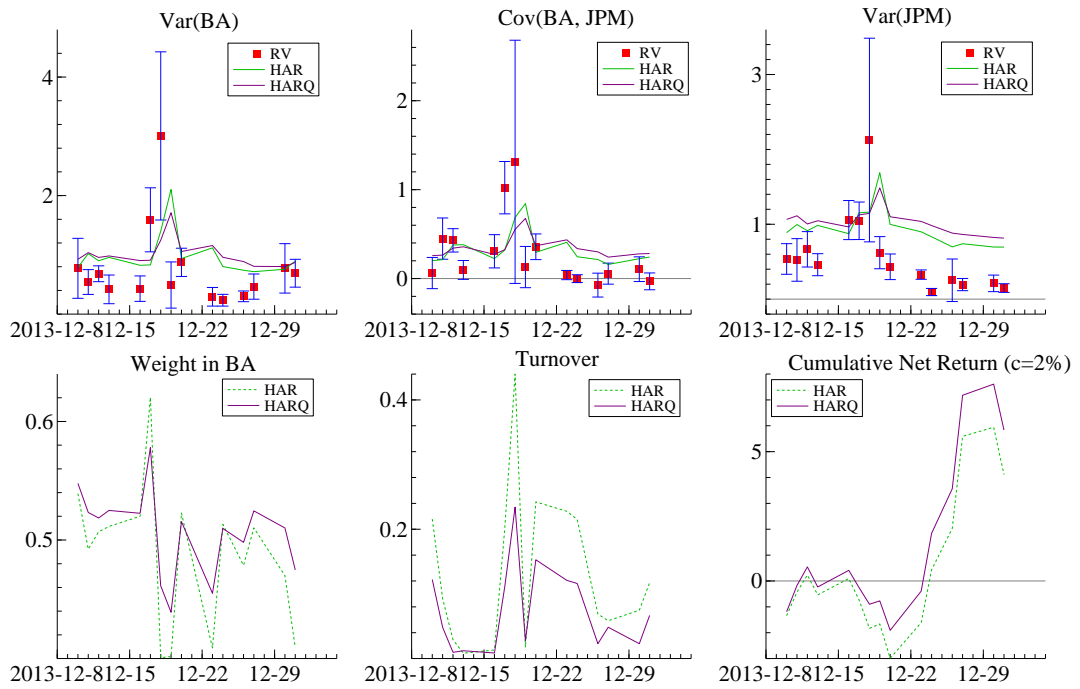


Not-for-Publication Supplemental Appendix to:  
Modeling and Forecasting (Un)Reliable Realized Covariances  
for More Reliable Financial Decisions

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Figure A.1: Dynamic Shrinkage



*Note:* The top row of panels shows the estimated (co)variance for BA and JPM, along with the forecasted values from the HAR and HARQ models. The bottom left panel shows the GMV weights for BA based on the HAR and HARQ models, the middle panel shows the turnover, and the right panel the cumulative portfolio return net of transaction costs.

Table A.1: Ex-Post Shrinkage HARQ-DRD Forecasts

		HARQ-DRD	Shrunk HARQ-DRD			
			Factor	Equicorr	Identity	No Short
TO		0.339	0.322	0.316	0.260	0.280
CO		0.497	0.490	0.489	0.438	0.471
SP		-0.082	-0.072	-0.069	-0.045	0.000
Mean Ret		4.371	4.546	4.605	4.088	4.633
Std Ret		14.566	14.531	14.520	14.601	14.855
$c = 0\%$	Sharpe	0.300	0.313	0.317	0.280	0.312
	$\Delta_1$		-18.0	-24.1	28.9	-17.8
	$\Delta_{10}$		-22.7	-30.2*	33.4*	-22.4
$c = 1\%$	Sharpe	0.241	0.257	0.262	0.235	0.264
	$\Delta_1$		-22.2	-29.8*	9.0	-32.4*
	$\Delta_{10}$		-26.9	-35.9*	13.5	-36.7*
$c = 2\%$	Sharpe	0.183	0.201	0.207	0.190	0.217
	$\Delta_1$		-26.4	-35.5*	-10.9	-51.2*
	$\Delta_{10}$		-31.0	-41.6*	-6.4	-47.4*

*Note:* The table reports global minimum variance portfolios formed based on exogenously shrunk HARQ-DRD forecasts. The top panel shows turnover (TO), portfolio concentration (CO), short position (SP), as well as the average annualized portfolio volatility. Each bottom panel shows the economic gains of switching from the alternatives to the HARQ-DRD model forecasts in annual basis points,  $\Delta_\gamma$ , for various transaction cost levels  $c$  and risk aversion coefficients  $\gamma$ . Asterisks denote  $\Delta_\gamma$  significantly different from zero at the 5% level.

Table A.2: Ex-Post Shrinkage RW Forecasts

		HARQ-DRD	RW	Shrunk RW			
				Factor	Equicorr	Identity	No Short
TO		0.339	2.506	1.383	1.245	0.892	1.139
CO		0.497	0.867	0.620	0.596	0.453	0.572
SP		-0.082	-0.547	-0.175	-0.136	-0.095	0.000
Mean Ret		4.371	3.649	4.497	4.600	1.437	2.742
StDev Ret		14.566	18.861	16.277	16.169	16.056	16.576
$c = 0\%$	Sharpe	0.300	0.193	0.276	0.284	0.090	0.165
	$\Delta_1$		144.0*	13.8*	1.7	316.2*	194.2*
	$\Delta_{10}$		790.1*	251.6*	223.8*	521.5*	476.0*
$c = 1\%$	Sharpe	0.241	-0.141	0.062	0.090	-0.050	-0.008
	$\Delta_1$		690.0*	276.8*	230.1*	455.5*	395.8*
	$\Delta_{10}$		1334.5*	514.2*	451.9*	660.6*	677.3*
$c = 2\%$	Sharpe	0.183	-0.476	-0.152	-0.104	-0.190	-0.181
	$\Delta_1$		1236.1*	539.7*	458.6*	594.7*	597.4*
	$\Delta_{10}$		1879.1*	776.7*	680.0*	799.7*	878.6*

*Note:* The table reports the results for different shrinkage procedures based on the asymptotic distribution of MKernel. The top panel shows turnover (TO), portfolio concentration (CO), short positions (SP), as well as the average annualized return and volatility. The bottom panel shows the economic gains of switching from the different alternatives to the HARQ-DRD-based forecasts in annual basis points,  $\Delta_\gamma$ , for various transaction cost levels  $c$  and risk aversion coefficients  $\gamma$ . Asterisks denote  $\Delta_\gamma$  significantly different from zero at the 5% level.

Table A.3: Zero Mean Minimum Variance Portfolios

		HAR	HARQ	HAR- DRD	HARQ- DRD	EWMA	EWMAQ	HEAVY	HEAVYQ
TO		0.522	0.385	0.391	0.339	0.135	0.096	0.172	0.122
CO		0.513	0.517	0.487	0.497	0.505	0.506	0.497	0.497
SP		-0.105	-0.109	-0.070	-0.082	-0.095	-0.096	-0.089	-0.089
Mean Ret		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
StDev Ret		14.912	14.781	15.016	<b>14.566</b>	14.972	<b>14.637</b>	14.922	<b>14.614</b>
$c = 0\%$	Sharpe	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	$\Delta_1$		1.9		6.7		5.0		4.6
	$\Delta_{10}$		19.5		66.6*		49.7*		45.5*
$c = 1\%$	Sharpe	-0.088	-0.066	-0.066	-0.059	-0.023	-0.017	-0.029	-0.021
	$\Delta_1$		36.5*		19.7		14.7		17.3
	$\Delta_{10}$		54.0*		79.6*		59.4*		58.2*
$c = 2\%$	Sharpe	-0.176	-0.131	-0.131	-0.117	-0.045	-0.033	-0.058	-0.042
	$\Delta_1$		71.0*		32.7*		24.5		30.0*
	$\Delta_{10}$		88.6*		92.7*		69.2*		70.9*

*Note:* The table shows the results for the global minimum variance portfolio (GMV) with ex-post demeaned returns. We report turnover (TO), portfolio concentration (CO), and short positions (SP), as well as the average annualized return and standard deviation. Standard deviations in bold indicate the models that belong to the 90% model confidence set (MCS) of lowest ex-post daily volatility. The table also reports the economic gains of switching from the conventional model to the Q-model in annual basis points,  $\Delta_\gamma$ , for various transaction cost levels  $c$  and risk aversion coefficients  $\gamma$ . Asterisks denote  $\Delta_\gamma$  significantly different from zero at the 5% level.

Table A.4: Zero Mean Ex-Post Shrinkage HAR-DRD Forecasts

		HARQ-DRD	HAR-DRD	Shrunk HAR-DRD				1/N
				Factor	Equicorr	Identity	No Short	
TO		0.339	0.391	0.369	0.361	0.303	0.322	0.009
CO		0.497	0.487	0.481	0.480	0.433	0.466	0.316
SP		-0.082	-0.070	-0.061	-0.060	-0.039	0.000	0.000
Mean Ret		0.000	0.000	0.000	0.000	0.000	0.000	0.000
Std Ret		14.566	15.016	14.985	14.978	15.074	15.296	18.579
$c = 0\%$	Sharpe	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	$\Delta_1$		6.7	6.2	6.1	7.5	11.3	66.5*
	$\Delta_{10}$		66.8*	61.8*	60.9*	75.3*	112.6*	664.2*
$c = 1\%$	Sharpe	-0.059	-0.066	-0.062	-0.061	-0.051	-0.051	-0.001
	$\Delta_1$		19.8*	13.9	11.7	-1.5	3.9	-16.5
	$\Delta_{10}$		79.9*	69.5*	66.5*	66.2*	105.2*	581.1*
$c = 2\%$	Sharpe	-0.117	-0.131	-0.124	-0.122	-0.101	-0.102	-0.003
	$\Delta_1$		32.8*	21.5*	17.3	-10.6	-3.5	-99.6*
	$\Delta_{10}$		93.0*	77.2*	72.2*	57.2*	97.9*	498.1*

*Note:* The table reports global minimum variance portfolios formed based on exogenously shrunk HAR-DRD forecasts, with ex-post demeaned returns. The top panel shows turnover (TO), portfolio concentration (CO), short position (SP), as well as the average annualized portfolio volatility. Each bottom panel shows the economic gains of switching from the alternatives to the HARQ-DRD model forecasts in annual basis points,  $\Delta_\gamma$ , for various transaction cost levels  $c$  and risk aversion coefficients  $\gamma$ . Asterisks denote  $\Delta_\gamma$  significantly different from zero at the 5% level.

Table A.5: No Short-Sale Minimum Tracking Error Portfolios

	HAR	HARQ	HAR- DRD	HARQ- DRD	EWMA	EWMAQ	HEAVY	HEAVYQ
TO	0.172	0.102	0.134	0.114	0.063	0.045	0.080	0.056
CO	0.339	0.337	0.339	0.342	0.340	0.340	0.338	0.338
StDev TE	6.610	<b>6.450</b>	6.609	<b>6.489</b>	6.630	<b>6.470</b>	6.630	<b>6.463</b>
$c = 0\%$	$\Delta_1$	27.8		38.0		31.7		39.0
	$\Delta_{10}$	37.2		45.1*		41.1		47.0
$c = 1\%$	$\Delta_1$	45.4*		43.0*		36.3		36.8
	$\Delta_{10}$	54.9*		51.0*		45.7		46.7*
$c = 2\%$	$\Delta_1$	63.1*		48.0*		48.0*		42.8*
	$\Delta_{10}$	72.6*		55.1*		53.0*		52.7*

*Note:* The table reports the results for minimum tracking error portfolios that do not allow for short positions. The top panel shows the portfolio turnover (TO), portfolio concentration (CO), and annualized tracking error volatility. Standard deviations in bold indicate models that belong to the 90% model confidence set (MCS) of lowest ex-post daily volatility. The bottom panel reports the economic gains of switching from the conventional model to the Q-model in annual basis points,  $\Delta_\gamma$ , for various transaction cost levels  $c$  and risk aversion coefficients  $\gamma$ . Asterisks denote  $\Delta_\gamma$  significantly different from zero at the 5% level.

Table A.6: Longer Horizon Tracking Portfolios

		Weekly		Monthly	
		HAR-DRD	HARQ-DRD	HAR-DRD	HARQ-DRD
TO		0.041	0.033	0.012	0.011
CO		0.339	0.342	0.339	0.342
SP		0.000	0.000	0.000	0.000
StDev	TE	6.639	6.577	6.794	6.746
$c = 0\%$	$\Delta_1$		23.6*		7.0
	$\Delta_{10}$		27.3*		9.9*
$c = 1\%$	$\Delta_1$		25.5*		7.0
	$\Delta_{10}$		29.2*		9.9*
$c = 2\%$	$\Delta_1$		27.3*		7.1
	$\Delta_{10}$		31.0*		10.0*

*Note:* The table reports the long-horizon minimum tracking error portfolio results. The portfolios are re-balanced weekly or monthly based on the relevant weekly and monthly covariance matrix forecasts. The top panel shows turnover (TO), portfolio concentration (CO), short positions (SP), and annualized tracking error volatility. The bottom panel reports the economic gains of switching from the standard HAR-DRD model to the HARQ-DRD model in annual basis points,  $\Delta_\gamma$ , for various transaction cost levels  $c$  and risk aversion coefficients  $\gamma$ . Asterisks denote  $\Delta_\gamma$  significantly different from zero at the 5% level.



Table A.7: Weekly and Monthly versus Daily Tracking Portfolios

		Weekly		Monthly	
		HAR-DRD	HARQ-DRD	HAR-DRD	HARQ-DRD
c=0%	$\Delta_1$	9.0	46.9*	46.1*	84.0*
	$\Delta_{10}$	7.0	52.0*	55.1*	100.1*
c=1%	$\Delta_1$	-16.5	26.4*	15.2	58.1*
	$\Delta_{10}$	-18.5	31.5*	24.2	74.2*
c=2%	$\Delta_1$	-41.9*	5.9	-15.7	37.7*
	$\Delta_{10}$	-43.9*	11.1*	-6.7	48.3*

*Note:* The table reports the economic gains of switching from weekly or monthly re-balanced minimum tracking error portfolios based on HARQ-DRD model forecasts to a daily strategy based on HAR(Q)-DRD forecasts in annual basis points,  $\Delta_\gamma$ , for various transaction cost levels  $c$  and risk aversion coefficients  $\gamma$ . Asterisks denote  $\Delta_\gamma$  significantly different from zero at the 5% level.

Table A.8: Ex-Post Shrinkage HAR-DRD Forecasts - Tracking Portfolios

		HARQ-DRD	HAR-DRD	Shrunk HAR-DRD			1/N	
				Factor Model	Equi-correlation	Identity	No Short-Sale	
TO		0.114	0.134	0.129	0.128	0.105	0.134	0.009
CO		0.342	0.339	0.338	0.338	0.331	0.339	0.316
SP		0.000	0.000	0.000	0.000	0.000	0.000	0.000
Std Ret		6.521	6.610	6.636	6.491	6.589	6.609	7.146
$c = 0\%$	$\Delta_1$		27.8*	19.9*	-20.0*	52.1*	27.7*	36.9*
	$\Delta_{10}$		35.0*	16.9*	-14.8	45.8*	33.0*	85.4*
$c = 1\%$	$\Delta_1$		42.8*	23.8*	-16.5*	49.9*	32.8*	20.5*
	$\Delta_{10}$		50.0*	20.8*	-11.3	43.6*	38.1*	59.1*
$c = 2\%$	$\Delta_1$		47.8*	27.7*	-13.1	47.6*	37.9*	-5.8
	$\Delta_{10}$		55.0*	12.3	12.7	38.1*	43.2*	32.7*

*Note:* The table reports minimum tracking error portfolios formed based on exogenously shrunk HAR-DRD forecasts. The top panel shows turnover (TO), portfolio concentration (CO), short position (SP), as well as the average annualized tracking error volatility. Each bottom panel shows the economic gains of switching from the alternatives to the HARQ-DRD model forecasts in annual basis points,  $\Delta_\gamma$ , for various transaction cost levels  $c$  and risk aversion coefficients  $\gamma$ . Asterisks denote  $\Delta_\gamma$  significantly different from zero at the 5% level.

Table A.9: Ex-Post Shrinkage HARQ-DRD Forecasts - Tracking Portfolios

HARQ-DRD		Shrunk HARQ-DRD			
		Factor Model	Equicorrelation	Identity	No Short Sale
TO	0.114	0.094	0.093	0.079	0.097
CO	0.342	0.338	0.338	0.331	0.339
SP	0.000	0.000	0.000	0.000	0.000
Std Ret	6.521	6.521	6.526	6.520	6.489
$c = 0\%$	$\Delta_1$	13.4	15.6	6.0	8.4
	$\Delta_{10}$	13.4	15.9	5.9	8.9
$c = 1\%$	$\Delta_1$	8.3	10.2	-2.9	3.9
	$\Delta_{10}$	8.3	10.5	-2.9	4.5
$c = 2\%$	$\Delta_1$	3.1	4.8	-11.7	-0.5
	$\Delta_{10}$	3.1	5.1	-11.8	0.1

*Note:* The table reports minimum tracking error portfolios formed based on exogenously shrunk HARQ-DRD forecasts. The top panel shows turnover (TO), portfolio concentration (CO), short position (SP), as well as the average annualized tracking error volatility. Each bottom panel shows the economic gains of switching from the alternatives to the HARQ-DRD model forecasts in annual basis points,  $\Delta_\gamma$ , for various transaction cost levels  $c$  and risk aversion coefficients  $\gamma$ . Asterisks denote  $\Delta_\gamma$  significantly different from zero at the 5% level.

Table A.10: Ex-Post Shrinkage RW Forecasts - Tracking Portfolios

		HARQ-DRD	RW	Shrunk RW			1/N	
				Factor Model	Equi-correlation	Identity	No Short-Sale	
TO		0.114	1.128	0.618	0.555	0.415	0.884	0.009
CO		0.342	0.461	0.380	0.375	0.344	0.416	0.316
SP		0.000	-0.087	-0.004	-0.001	-0.004	0.000	0.000
StDev TE		6.521	8.577	7.243	7.085	7.060	7.714	7.146
$c = 0\%$	$\Delta_1$		254.5*	41.9*	-60.4*	152.4*	165.2*	36.9*
	$\Delta_{10}$		394.3*	86.7*	-25.8	185.5*	241.7*	85.4*
$c = 1\%$	$\Delta_1$		510.1*	168.9*	50.7*	228.2*	359.1*	20.5*
	$\Delta_{10}$		649.8*	213.6*	85.2*	261.3*	435.6*	59.1*
$c = 2\%$	$\Delta_1$		765.7*	295.9*	161.8*	304.0*	553.1*	-5.8
	$\Delta_{10}$		905.3*	340.6*	196.3*	337.1*	629.5*	32.7*

*Note:* The table reports minimum tracking error portfolios formed based on exogenously shrunk forecasts. The top panel shows turnover (TO), portfolio concentration (CO), short position (SP), as well as the average annualized tracking error volatility. The bottom panel shows the economic gains of switching from the alternatives to the HARQ-DRD model forecasts in annual basis points,  $\Delta_\gamma$ , for various transaction cost levels  $c$  and risk aversion coefficients  $\gamma$ . Asterisks denote  $\Delta_\gamma$  significantly different from zero at the 5% level.

Table A.11: Zero Mean Minimum Tracking Error Portfolios

	HAR	HARQ	HAR- DRD	HARQ- DRD	EWMA	EWMAQ	HEAVY	HEAVYQ
TO	0.173	0.102	0.134	0.114	0.063	0.045	0.080	0.057
CO	0.339	0.338	0.339	0.342	0.340	0.340	0.338	0.338
SP	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
StDev TE	6.618	<b>6.489</b>	6.610	<b>6.521</b>	6.636	<b>6.513</b>	6.637	<b>6.506</b>
$c = 0\%$	$\Delta_1$	1.1		0.8		1.1		1.1
	$\Delta_{10}$	11.1		7.9		10.8		11.4
$c = 1\%$	$\Delta_1$	18.8*		5.8		5.7		7.1
	$\Delta_{10}$	28.8*		12.9*		15.4*		17.3*
$c = 2\%$	$\Delta_1$	36.5*		10.7*		10.2		13.1
	$\Delta_{10}$	46.5*		17.9*		20.0*		23.3*

*Note:* The table shows the results for the minimum tracking error portfolio with ex-post demeaned returns. We report turnover (TO), portfolio concentration (CO), and short positions (SP), as well as the standard deviation of the tracking error (TE). Standard deviations in bold indicate the models that belong to the 90% model confidence set (MCS) of lowest ex-post daily volatility. The table also reports the economic gains of switching from the conventional model to the Q-model in annual basis points,  $\Delta_\gamma$ , for various transaction cost levels  $c$  and risk aversion coefficients  $\gamma$ . Asterisks denote  $\Delta_\gamma$  significantly different from zero at the 5% level.

Table A.12: Zero Mean Ex-Post Shrinkage HAR-DRD Forecasts - Tracking Portfolios

		HARQ-DRD	HAR-DRD	Shrunk HAR-DRD			1/N	
				Factor Model	Equi-correlation	Identity	No Short-Sale	
TO		0.114	0.134	0.129	0.128	0.105	0.134	0.009
CO		0.342	0.339	0.338	0.338	0.331	0.339	0.316
SP		0.000	0.000	0.000	0.000	0.000	0.000	0.000
Std Ret		6.521	6.610	6.636	6.491	6.589	6.609	7.146
$c = 0\%$	$\Delta_1$		0.6	0.8	-0.2	0.4	0.6	4.3
	$\Delta_{10}$		5.9	7.6	-2.0	4.5	5.8	42.7*
$c = 1\%$	$\Delta_1$		5.7	3.9	2.5	-8.3	3.3	-22.1*
	$\Delta_{10}$		11.0	10.8	0.8	-4.3	8.5	16.4*
$c = 2\%$	$\Delta_1$		10.8*	7.1	5.3	-17.1*	6.0	-48.4*
	$\Delta_{10}$		16.1*	14.0*	3.5	-13.1	11.2	-9.9

*Note:* The table reports minimum tracking error portfolios formed based on exogenously shrunk HAR-DRD forecasts. The top panel shows turnover (TO), portfolio concentration (CO), short position (SP), as well as the average annualized tracking error volatility. Each bottom panel shows the economic gains of switching from the alternatives to the HARQ-DRD model forecasts in annual basis points,  $\Delta_\gamma$ , for various transaction cost levels  $c$  and risk aversion coefficients  $\gamma$ . Asterisks denote  $\Delta_\gamma$  significantly different from zero at the 5% level.