Realized Semibetas: Signs of Things to Come

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Abstract

We propose a new decomposition of the traditional market beta into four *semi*betas that depend on the signed covariation between the market and individual asset returns. We show that semibetas stemming from negative market and negative asset return covariation predict significantly higher future returns, while semibetas attributable to negative market and positive asset return covariation predict significantly lower future returns. The two semibetas associated with positive market return variation do not appear to be priced. The results are consistent with the pricing implications from a mean-*semi*variance framework combined with arbitrage risk driving a wedge between the risk premiums for long and short positions. We conclude that rather than betting against the traditional market beta, it is better to bet on *and* against the "right" semibetas.

Keywords: Cross-sectional return variation; downside risk; semicovariances; semibetas.

JEL: G11, G12, C58

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1. Introduction

The Capital Asset Pricing Model (CAPM) reigns supreme as the most widely-studied and practically-used model for valuing speculative assets. In its basic form the model predicts a simple linear relationship between the expected excess return on an asset and the beta of that asset with respect to the aggregate market portfolio. While early empirical evidence largely corroborated this prediction (e.g., Fama, Fisher, Jensen and Roll, 1969; Blume, 1970), an extensive subsequent literature has called into question the ability of the standard market beta to satisfactorily explain the cross-sectional variation in returns, with the estimated risk premiums being too low, often insignificant, and sometimes even negative (e.g., Roll, 1977; Bhandari, 1988; Fama and French, 1992). Numerous explanations have been put forth to explain these findings, ranging from measurement errors (e.g., Shanken, 1992; Hollstein, Prokopczuk and Simen, 2019), to agency problems (Baker, Bradley and Wurgler, 2011), to the need for separate betas associated with cash-flow and discount rate news (Campbell and Vuolteenaho, 2004), to leverage constraints (Frazzini and Pedersen, 2014) and the need for separate liquidity and fundamental betas (Acharya and Pedersen, 2005), to name but a few.

These "rescue attempts" notwithstanding, another strand of literature, tracing back to the early work by Roy (1952), Markowitz (1959), Hogan and Warren (1972, 1974) and Bawa and Lindenberg (1977), posits that the mean-variance, or quadratic utility, framework underlying the basic CAPM and the resulting security market line and linear beta pricing relationship is too simplistic. If investors are averse to volatility only when it leads to losses, not gains, then the relevant measure of risk is not (total) variance but rather the *semivariance* of negative returns.[1] Intuitively, if investors only care about downside variation, then the covariation associated with a positive aggregate market return should not be priced in equilibrium. These same pricing implications also arise in

[1]This same basic idea also underlies the notion of loss aversion and the prospect theory pioneered by Kahneman and Tversky (1979), as supported by an extensive subsequent experimental literature and other empirical evidence.

a setting with disappointment aversion preferences as in Gul (1991), and its generalization in Routledge and Zin (2010), recently explored by Farago and Tedongap (2018).[2]

Consistent with these ideas, Ang, Chen and Xing (2006a) find that the downside beta version of the CAPM does a better job than the traditional CAPM in terms of explaining the cross-sectional variation in U.S. equity returns. The study by Post and van Vliet (2004) reaches the same conclusion, and Lettau, Maggiori and Weber (2014) similarly finds that a downside beta version of the CAPM better explains the variation in the returns across other asset classes. In contrast, recent work by Atilgan, Bali, Demirtas and Gunaydin (2018) has called into question the ability of downside betas to satisfactorily explain the cross-sectional variation in more recent U.S. and international equity returns. Levi and Welch (2020) also concludes that downside betas do not provide superior cross-sectional return predictions compared to the predictability afforded by traditional betas.

Set against this background, we propose a new four-way decomposition of the traditional market beta into four *semi*betas. Our decomposition relies on the newly-developed *semi*covariance concept of Bollerslev, Li, Patton and Quaedvlieg (2020a). Letting r and f denote the returns on some risky asset and the aggregate market portfolio, respectively, the four semibetas are then defined as

$$\beta \equiv \frac{Cov(r,f)}{Var(f)} = \frac{\mathcal{N} + \mathcal{P} + \mathcal{M}^{+} + \mathcal{M}^{-}}{Var(f)} \equiv \beta^{\mathcal{N}} + \beta^{\mathcal{P}} - \beta^{\mathcal{M}^{+}} - \beta^{\mathcal{M}^{-}}. \qquad (1)$$

The $\mathcal{N}$, $\mathcal{P}$, $\mathcal{M}^{+}$ and $\mathcal{M}^{-}$ semicovariance components refer to the respective portions of total covariation $Cov(r,f)$ defined by both returns being positive (the "P" state), both returns being negative ("N"), mixed sign with positive market return ("M^{+}"), and mixed sign with negative market return ("M^{-}"). Since the mixed-sign semicovariances are always weakly negative numbers, with lower values indicating stronger covariation, to ease the interpretation of the risk premium estimates in our empirical analyses, we purposely define the mixed-sign semibetas as $\beta^{\mathcal{M}^{+}} \equiv -\mathcal{M}^{+}/Var(f)$ and $\beta^{\mathcal{M}^{-}} \equiv -\mathcal{M}^{-}/Var(f)$.

[2] As shown by Anthonisz (2012), they may also be cast in a more traditional stochastic discount factor pricing framework assuming a "kinked" pricing kernel.

The traditional CAPM, of course, does not differentiate between *any* of the four covariation components ($\mathcal{N}$, $\mathcal{P}$, $\mathcal{M}^+$ and $\mathcal{M}^-$), combining them into a single market β and a single risk premium. The downside version of the CAPM, investigated in the aforementioned studies, effectively combines the pricing of the two negative-market-return covariation components ($\mathcal{N}$ and $\mathcal{M}^-$) into a single downside beta and the two positive-market-return covariation components ($\mathcal{P}$ and $\mathcal{M}^+$) into a single upside beta, each with their own individual risk premiums.[3] Anticipating our empirical results, we strongly reject these pricing restrictions in the data.

In a frictionless financial market, the risks associated with $\mathcal{N}$ and $\mathcal{M}^-$ ($\mathcal{P}$ and $\mathcal{M}^+$) should be priced the same, as a short position in an asset simply switches the signs of the corresponding semicovariation components. However, as forcefully argued by Pontiff (1996) and Schleifer and Vishny (1997), legal constraints and charters impede many institutional investors from short-selling, and many individual investors are simply reluctant to sell short, effectively creating limits-to-arbitrage and arbitrage risk (see also the discussion in Hong and Sraer, 2016). This arbitrage risk in turn induces a wedge between the pricing of the $\mathcal{N}$ and $\mathcal{M}^-$ ($\mathcal{P}$ and $\mathcal{M}^+$) semicovariation components, and the risk premiums associated with the $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ ($\beta^{\mathcal{P}}$ and $\beta^{\mathcal{M}^+}$) semibetas. Intuitively, assets that covary positively with the market when the market is performing poorly will exacerbate downside return variation, while assets that covary negatively with the market when the market is performing poorly help mitigate downside risk. Correspondingly, we find that the former types of assets command higher risk premiums.

True betas and semibetas, of course, are not directly observable. However, as demonstrated in the burgeoning realized volatility literature, the advent of high-frequency intraday data allows for the construction of much more accurate risk measures compared with measures calculated using daily or monthly data. In particular, as formally shown by Barndorff-Nielsen and Shephard (2004), the traditional market beta of an asset may

[3]As discussed further below, the scaling of the up and downside betas employed in some of the aforementioned empirical studies differ from the scaling of the semibetas employed here. However, the resulting cross-sectional fits are not affected by these differences in the scaling.

be consistently (for increasingly finer sampled data) estimated by the so-called realized beta, defined as the ratio between the realized covariance of the asset and the market and the realized variance of the market.[4] Similarly, relying on the infill asymptotic results pertaining to realized semicovariances in Bollerslev, Li, Patton and Quaedvlieg (2020a), the new semibetas defined here may be consistently estimated by their corresponding realized semibeta counterparts, defined as the ratios of the relevant realized semicovariance components and the realized market variance. We discuss this further in Section 2.

Building on these new measures, we offer three main empirical contributions. Our first empirical investigations use daily realized semibetas based on high-frequency intraday data for all of the S&P 500 constituent stocks over the 1993-2014 sample period. The estimated semibetas clearly reveal the existence of asymmetric dependencies between the individual stocks and the market beyond those of the linear dependencies captured by the traditional market beta. More importantly, our results strongly support the hypothesis that these non-linear dependencies are priced differently: stocks with higher $\beta^{\mathcal{N}}$ are associated with significantly higher subsequent daily returns; stocks with higher $\beta^{\mathcal{M}^-}$ are associated with significantly lower subsequent daily returns; while neither $\beta^{\mathcal{P}}$ nor $\beta^{\mathcal{M}^+}$ appear to carry a significant risk premium. Corroborating the thesis that the difference in the risk premiums for $\beta^{\mathcal{N}}$ and $-\beta^{\mathcal{M}^-}$ may be attributed to market frictions and limits-to-arbitrage, we show that the rejection of the hypothesis that the two risk premiums are identical is stronger for portfolios made up of stocks with higher arbitrage risk, as proxied by the level of idiosyncratic volatility (e.g., Pontiff, 1996; Stambaugh, Yu and Yuan, 2015), and stocks that are more difficult to value, as proxied by the rate of turnover (e.g., Harris and Raviv, 1993; Blume, Easley and O'Hara, 1994; Kumar, 2009).

Further underscoring the significance of this difference in the pricing of the semibetas, the two-way decomposition of the traditional market beta into separate up and downside betas previously explored in the literature is also strongly rejected against the four-way

[4]For additional discussion of the realized beta concept along with empirical applications, see also Andersen, Bollerslev, Diebold and Wu (2006) and Patton and Verardo (2012).

semibeta decomposition proposed here. These same findings for the daily realized semibetas and future daily returns carry over to longer weekly and monthly return horizons. They also remain robust to the inclusion of a long list of other return predictor variables previously analyzed in the literature.

This prima facie evidence notwithstanding, the requirement of intraday high-frequency data for accurately estimating the realized semibetas limits the time span and number of stocks underlying our analyses. To expand the scope of our analyses, our second empirical contribution constructs monthly semibetas from daily returns for a much broader cross-section of stocks over a longer 1963-2017 sample period. Using this broader and longer sample we arrive at the same conclusions: $\beta^{\mathcal{N}}$ and $-\beta^{\mathcal{M}^-}$ are priced differently, with estimated annualized risk premiums of 10.43% and 6.42% respectively, while the estimated risk premiums for $\beta^{\mathcal{P}}$ and $\beta^{\mathcal{M}^+}$ are both statistically insignificant at conventional levels. By comparison, the estimated risk premium for the traditional market beta is 4.10%.

Finally, we investigate whether these statistically significant differences in the compensation for the different semibetas also translate into "economically significant" differences in the performance of simple portfolio strategies. We find that a long-short semibeta strategy generates average annual excess returns of 9.8%, and an annualized Sharpe ratio of 1.05. By comparison, similar portfolio strategies based on the standard CAPM betas and the Ang, Chen and Xing (2006a) downside betas generate excess returns of 5.0% and 7.5%, respectively, with Sharpe ratios of only 0.30 and 0.48. Using the four- and five-factor models of Carhart (1997) and Fama and French (1993, 2015) to assess the risk-adjusted performance, we find annualized alphas of 8.4% and 9.7% respectively, and overwhelmingly significant t-statistics. By comparison, the traditional beta and the downside beta portfolios produce much smaller and at best only borderline significant alphas. Hence, adding to the recent literature and debate about betting on or against beta (see, e.g., Frazzini and Pedersen, 2014; Cederburgh and O'Doherty, 2016; Bali, Brown, Murray and Tang, 2017; Novy-Marx and Velikov, 2018; Schneider, Wagner and Zechner, 2020), we conclude that it is better to bet on *and* against the "right" semibetas.

In addition to the previous studies on downside risk noted above, our empirical findings are also related to the vast existing literature on asymmetric dependencies in stock returns, including among others Longin and Solnik (2001), Ang and Chen (2002), Patton (2004), Hong, Tu and Zhou (2006), Elkamhi and Stefanova (2014) and Engle and Mistry (2014). They are also related to the more recent and rapidly growing literature on the pricing of downside tail, or crash, risk, including Bali, Demirtas and Levy (2009), Bollerslev and Todorov (2011), Kelly and Jiang (2014), Cremers, Halling and Weinbaum (2015) Bollerslev, Li and Todorov (2016), Chabi-Yo, Ruenzi and Weigert (2018), Farago and Tedongap (2018), Barunik and Nevrla (2019), Bondarenko and Bernard (2020), Chabi-Yo, Huggenberger and Weigert (2019), Lu and Murray (2019) and Orlowski, Schneider and Trojani (2019). In contrast to all of these studies, however, which rely on the use of options and/or non-linear procedures for assessing the asymmetric joint tail dependencies and the pricing thereof, we maintain a simple linear pricing relationship together with a simple-to-implement additive decomposition of the traditional market beta into the four semibeta components. Our new semibeta measures are also distinctly different from, and much simpler to implement than, the entropy approach of Jiang, Wu and Zhou (2018) designed to measure asymmetries in up and downside comovements.

The semibetas, and the joint dependencies captured by them, are also related to the notion of coskewness originally proposed by Kraus and Litzenberger (1976), and the corresponding notion of cokurtosis, as investigated empirically by Harvey and Siddique (2000), Dittmar (2002), Conrad, Dittmar and Ghysels (2013), Langlois (2020) and Schneider, Wagner and Zechner (2020), among others. We find that the semibetas remain highly significant for explaining the cross-sectional variation controlling for coskewness and cokurtosis, while both of these co-dependency measures are rendered insignificant by the inclusion of the proposed semibeta measures. Our reliance on the new semicovariance concept for decomposing the systematic market risk and defining the semibetas also sets our analysis apart from other recent studies based on the semivariance concept for defining and empirically investigating asset specific "good" and "bad" volatility measures and the separate pricing thereof, as in, e.g., Feunou, Jahan-Parver and Okou (2018), Bollerslev, Li and Zhao (2020b) and Feunou and Okou (2019).

The remainder of the paper is structured as follows. We begin in Section 2 by discussing our construction of the daily realized semibetas and the theory underpinning their consistent estimation, along with a brief summary of their empirical distributional features. Section 3 presents our key empirical findings related to the pricing of the daily realized semibetas based on firm level cross-sectional regressions. Section 4 discusses our results based on monthly semibetas estimated from daily data across a much broader cross-section of stocks and over a longer time period. Section 5 considers the performance of simple semibeta-based portfolio strategies, along with comparisons to other similarly constructed beta-based portfolios. Section 6 concludes. Additional empirical results and robustness checks are detailed in a Supplemental Appendix.

2. Realized Semibetas

We begin by formally defining realized semibetas. We then briefly discuss the high-frequency data that we use in our main empirical investigations, followed by a summary of the salient distributional features of the resulting daily realized semibeta estimates.

2.1. Definitions

Let $r_{t,k,i}$ denote the return on asset i over the k^{th} intradaily time interval on day t, with the concurrent return for the aggregate market denoted by $f_{t,k}$. Define the signed high-frequency asset returns by $r^{+}_{t,k,i} \equiv \max(r_{t,k,i}, 0)$ and $r^{-}_{t,k,i} \equiv \min(r_{t,k,i}, 0)$, with the signed high-frequency market returns defined analogously. The realized semibetas are then defined by:

$$\begin{aligned} \widehat{\beta}^{\mathcal{N}}_{t,i} &\equiv \frac{\sum_{k=1}^{m} r^{-}_{t,k,i} f^{-}_{t,k}}{\sum_{k=1}^{m} f^{2}_{t,k}}, & \widehat{\beta}^{\mathcal{P}}_{t,i} &\equiv \frac{\sum_{k=1}^{m} r^{+}_{t,k,i} f^{+}_{t,k}}{\sum_{k=1}^{m} f^{2}_{t,k}}, \\ \widehat{\beta}^{\mathcal{M}^-}_{t,i} &\equiv \frac{-\sum_{k=1}^{m} r^{+}_{t,k,i} f^{-}_{t,k}}{\sum_{k=1}^{m} f^{2}_{t,k}}, & \widehat{\beta}^{\mathcal{M}^+}_{t,i} &\equiv \frac{-\sum_{k=1}^{m} r^{-}_{t,k,i} f^{+}_{t,k}}{\sum_{k=1}^{m} f^{2}_{t,k}}, \end{aligned} \tag{2}$$

where m denotes the number of high-frequency return intervals each day. The semibetas provide an exact four-way decomposition of the traditional realized market beta:

$$\widehat{\beta}_{t,i} \equiv \frac{\sum_{k=1}^{m} r_{t,k,i} f_{t,k}}{\sum_{k=1}^{m} f_{t,k}^2} = \widehat{\beta}_{t,i}^{\mathcal{N}} + \widehat{\beta}_{t,i}^{\mathcal{P}} - \widehat{\beta}_{t,i}^{\mathcal{M}^+} - \widehat{\beta}_{t,i}^{\mathcal{M}^-}. \tag{3}$$

As previously noted, we purposely change the sign on the two mixed semibetas, to make them positive, thereby allowing for an easier interpretation of the correspondingly decomposed risk premium estimates.

Let $\mathcal{RV}_t$ and $\mathcal{COV}_{t,i}$ denote the latent true daily variation of the return on the market and the covariation between the market return and the return on the individual asset i, with the corresponding true semicovariation measures denoted by $\mathcal{P}_{t,i}$, $\mathcal{N}_{t,i}$, $\mathcal{M}_{t,i}^+$ and $\mathcal{M}_{t,i}^-$, respectively. Barndorff-Nielsen and Shephard (2004) show that, for increasingly finely-sampled high-frequency returns, or $m \to \infty$, realized betas consistently estimate the true latent betas:

$$\widehat{\beta}_{t,i} \xrightarrow{p} \frac{\mathcal{COV}_{t,i}}{\mathcal{RV}_t}. \tag{4}$$

Similarly, the in-fill asymptotic theory in Bollerslev, Li, Patton and Quaedvlieg (2020a) pertaining to realized semicovariances imply that the realized semibetas consistently estimate the true semibetas:

$$\widehat{\beta}_{t,i}^{\mathcal{N}} \xrightarrow{p} \frac{\mathcal{N}_{t,i}}{\mathcal{RV}_t}, \qquad \widehat{\beta}_{t,i}^{\mathcal{P}} \xrightarrow{p} \frac{\mathcal{P}_{t,i}}{\mathcal{RV}_t}, \qquad \widehat{\beta}_{t,i}^{\mathcal{M}^+} \xrightarrow{p} \frac{-\mathcal{M}_{t,i}^+}{\mathcal{RV}_t}, \qquad \widehat{\beta}_{t,i}^{\mathcal{M}^-} \xrightarrow{p} \frac{-\mathcal{M}_{t,i}^-}{\mathcal{RV}_t}. \tag{5}$$

For ease of notation, in the remainder when not necessary we will drop the subscripts and hats, and refer to these realized (semi)beta measures simply as β, $\beta^{\mathcal{N}}$, etc.

If the market and individual asset returns were jointly Normally distributed, the four semibetas would convey no new information over and above the conventional market beta. In particular, utilizing the distributional results in Bollerslev, Li, Patton and Quaedvlieg

(2020a) it follows that under joint Normality:

$$\beta^{\mathcal{N}} = \beta^{\mathcal{P}} = \frac{1}{2\pi}\left(\sqrt{\frac{\sigma_r^2}{\sigma_f^2} - \beta^2} + \beta \arccos\left(-\frac{\sigma_f}{\sigma_r}\beta \right) \right),$$
$$\beta^{\mathcal{M}^+} = \beta^{\mathcal{M}^-} = \frac{1}{2\pi}\left(\sqrt{\frac{\sigma_r^2}{\sigma_f^2} - \beta^2} - \beta \arccos\left(\frac{\sigma_f}{\sigma_r}\beta \right) \right).$$

If the market and individual asset returns are *not* Normally distributed, the concordant semibetas ($\beta^{\mathcal{N}}$ and $\beta^{\mathcal{P}}$) and the disconcordant semibetas ($\beta^{\mathcal{M}^+}$ and $\beta^{\mathcal{M}^-}$) will generally differ, and each of the four semibetas may convey additional useful information to that of the standard market beta. As such, each of the semibetas may also be priced differently.

2.2. Data and Summary Statistics

Our primary empirical investigations rely on high-frequency data obtained from the Trades and Quotes (TAQ) database. We include all of the S&P 500 constituent stocks during the January 1993 to December 2014 sample period, resulting in a total of 5,541 trading days and 1,049 unique securities. We adopt a 15-minute sampling scheme, or $m = 26$ return observations per day, in our calculations of the realized semibeta measures. This choice strikes a judicious balance between biases induced by market microstructure effects when sampling too finely versus the theoretical continuous-time arguments underlying the consistency of the realized semicovariance measures that formally hinges on increasingly finer sampled intraday returns.[5]

We further match the intraday TAQ data and sample of stocks to the Center for Research in Securities Prices (CRSP) database to obtain the full-day returns for each of the stocks. All of our subsequent asset pricing investigations are based on these full-day

[5] Although a finer 5-minute sampling frequency has often been used in the realized volatility literature for the calculation of univariate realized volatility measures (see, e.g., Liu, Patton and Sheppard, 2015, and the many references therein), market microstructure effects are further compounded in a multivariate setting by the so-called Epps (1979) effect, which leads to a downward bias in realized covariation measures stemming from asynchronous prices. Correspondingly, we resort to a coarser 15-minute sampling frequency, also used by Bollerslev, Li, Patton and Quaedvlieg (2020a) in their analysis of realized semicovariances for a similar sample of individual stocks.

Table 1: **Summary Statistics.** The top panel reports the time series averages of the cross-sectional means, medians and standard deviations of the daily realized semibetas constructed from fifteen minutes intraday returns. The bottom panel reports the time series averages of the cross-sectional correlations. The sample consists of all S&P 500 constituent stocks from January 1993 to December 2014.

	β	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{P}}$	$\beta^{\mathcal{M}^+}$	$\beta^{\mathcal{M}^-}$
Mean	0.92	0.68	0.72	0.27	0.25
Median	0.83	0.57	0.61	0.16	0.15
St.Dev.	1.06	0.47	0.49	0.36	0.34
β	1.00	0.66	0.67	-0.33	-0.33
$\beta^{\mathcal{N}}$		1.00	0.44	0.19	0.06
$\beta^{\mathcal{P}}$			1.00	0.06	0.18
$\beta^{\mathcal{M}^+}$				1.00	0.38
$\beta^{\mathcal{M}^-}$					1.00

and resulting longer weekly and monthly returns.[6] We also rely on the daily market capitalization for each of the individual stocks from the CRSP database in our construction of the high-frequency value-weighted market index.

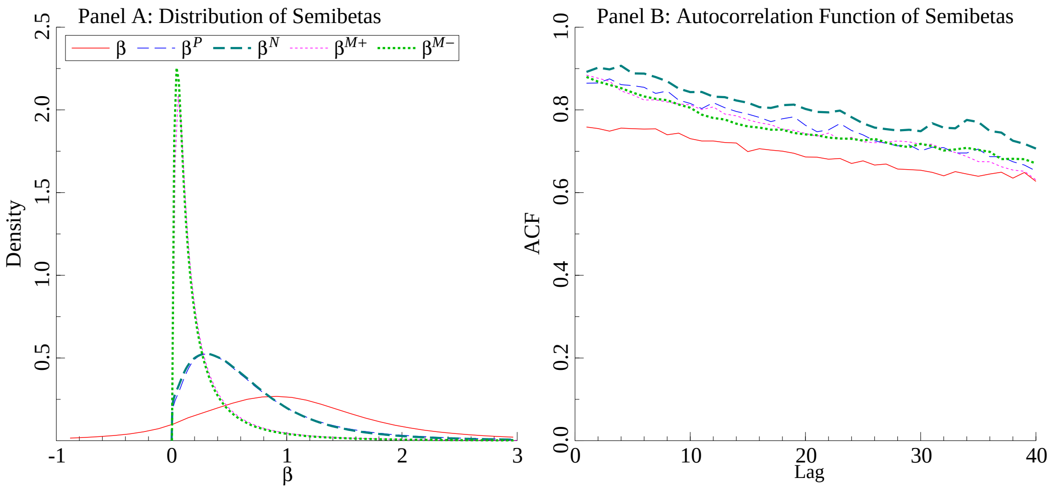
Turning to the resulting daily realized (semi)beta estimates, the top panel of Table 1 reports the time series averages of the cross-sectional means, medians and standard deviations averaged across all of the stocks in the sample. The bottom panel gives the time series averages of the cross-sectional correlations. Consistent with on average positive dependencies between the market and each of the individual stocks, the two concordant semibetas ($\beta^{\mathcal{P}}$ and $\beta^{\mathcal{N}}$) on average far exceed the two discordant semibetas ($\beta^{\mathcal{M}^+}$ and $\beta^{\mathcal{M}^-}$). The two concordant semibetas also correlated more strongly with the traditional market beta (β), and more so than with each other. Nonetheless, the correlations with the traditional beta are still far below unity, suggesting that the semibetas do convey different, and potentially useful, information over and above that of the traditional market beta.[7]

To help visualize the differences in the betas, Panel A of Figure 1 depicts the unconditional distributions of each of the daily realized betas and semibetas across all of the days

[6]As discussed in Bollerslev, Li and Zhao (2020b), this matching of the TAQ intraday data with the daily returns from CRSP also ensures proper handling of stock splits and dividends.

[7]To further highlight the additional information about asymmetric dependencies conveyed by the semibetas, Appendix A compares the realized semibeta estimates to the limiting values that would obtain if the individual stock and market returns were jointly Normally distributed.

Figure 1: **Unconditional Distributions and Autocorrelations.** Panel A displays kernel density estimates of the unconditional distribution of the daily realized beta and semibetas averaged across time and stocks. Panel B reports the average autocorrelation functions for the daily realized beta and semibetas averaged across stocks. The sample consists of all of the S&P 500 constituent stocks from January 1993 to December 2014.



and stocks in the sample. The distribution of the conventional betas is centered around one, as expected, and appears close to symmetric. Meanwhile, the realized semibetas are all weakly positive by construction, and thus unsurprisingly their distributions are all right-skewed. Further echoing the summary statistics in Table 1, the semibeta distributions are all centered below unity. Also, the unconditional distributions of the two concordant semibetas ($\beta^{\mathcal{P}}$ and $\beta^{\mathcal{N}}$) are almost indistinguishable, as are the distributions of the two discordant semibetas ($\beta^{\mathcal{M}^+}$ and $\beta^{\mathcal{M}^-}$).

The average autocorrelation functions shown in Panel B of Figure 1 indicate a strong degree of persistence for all of the semibetas, with the autocorrelations remaining in excess of 0.6 even at the 40^{th} lag.[8] Underpinning the cross-sectional return predictability regressions that we rely on in our asset pricing investigations, the high first-order autocorrelations of around 0.9 for each of the semibetas, also imply that today's realized semibetas for a given stock provide accurate predictions of tomorrow's semibetas for that same stock.

[8] We rely on the instrumental variable approach of Hansen and Lunde (2014), using lags 4 through 10 as instruments, to adjust for measurement errors in the realized betas, thereby allowing for more meaningful comparisons of the autocorrelation functions across the different betas.

We turn next to our main empirical analysis pertaining to the pricing of the non-linear dependencies encoded in these new realized semibeta measures.

3. Semibetas and the Cross-Section of Expected Returns

We begin our empirical investigations by presenting the results from standard Fama and MacBeth (1973) type cross-sectional predictive regressions. These regressions conveniently allow for the simultaneous estimation of separate risk premiums for each of the semibetas. In particular, for each day $t = 1, ..., T-1$, and all of the stocks $i = 1, ..., N_t$, available on day t and $t+1$, we first estimate the day $t+1$ lambdas from the cross-sectional regression:

$$r_{t+1,i} \;=\; \lambda_{0,t+1} + \lambda_{t+1}^{\mathcal{N}} \hat{\beta}_{t,i}^{\mathcal{N}} + \lambda_{t+1}^{\mathcal{P}} \hat{\beta}_{t,i}^{\mathcal{P}} + \lambda_{t+1}^{\mathcal{M}^+} \hat{\beta}_{t,i}^{\mathcal{M}^+} + \lambda_{t+1}^{\mathcal{M}^-} \hat{\beta}_{t,i}^{\mathcal{M}^-} + \; \epsilon_{t+1,i}. \qquad (6)$$

Based on these $T-1$ cross-sectional estimates, we then estimate the risk premiums associated with each of the semibetas by the time series averages of the lambdas over all of the days in the sample:

$$\hat{\lambda}^j \;=\; \frac{1}{T-1} \sum_{t=2}^{T} \hat{\lambda}_t^j \;, \qquad j = \mathcal{N}, \mathcal{P}, \mathcal{M}^+, \mathcal{M}^-. \qquad (7)$$

The resulting annualized estimates, along with their t-statistics based on Newey-West robust standard errors (using 21 lags), together with the time-series average of the R^2s from the first-stage cross-sectional regressions in equation (6), are reported in the second row of Table 2. As a benchmark, the first row of the table reports the estimated risk premium for the traditional realized beta. Consistent with the basic mean-variance framework, the traditional beta carries a statistically significant risk premium of 4.58% per year. This estimated risk premium is somewhat below the average annual equity risk premium of 8.56% observed over the sample, corroborating the basic intuition underlying the "betting against beta" idea (Frazzini and Pedersen, 2014).

Table 2: **Fama-Macbeth Regressions on Semibetas** The table reports the estimated annualized risk premia and Newey-West robust t-statistics from daily Fama-MacBeth cross-sectional predictive regressions. The daily semibetas are calculated from fifteen-minute intraday data. All of the control variables are measured prior to the daily returns, as detailed in Appendix B. The estimates are based on all of the S&P 500 constituent stocks and days in the January 1993 to December 2014 sample.

β	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{P}}$	$\beta^{\mathcal{M}^+}$	$\beta^{\mathcal{M}^-}$	ME	BM	MOM	REV	IVOL	ILLIQ	R^2
4.58											2.70
3.04											
	22.54	-1.58	-4.29	-8.48							5.43
	5.62	-0.52	-0.86	-2.02							
	22.47	-5.67	-2.90	-12.20	-2.23	-1.77	0.11				8.23
	5.75	-2.02	-0.65	-3.14	-3.83	-1.95	3.47				
	20.36	-2.91	1.68	-6.15	-7.42	-1.65	0.09	-0.55	-3.07	-4.88	10.32
	5.44	-1.08	0.41	-1.68	-7.71	-1.87	2.55	-5.82	-3.56	-6.42	

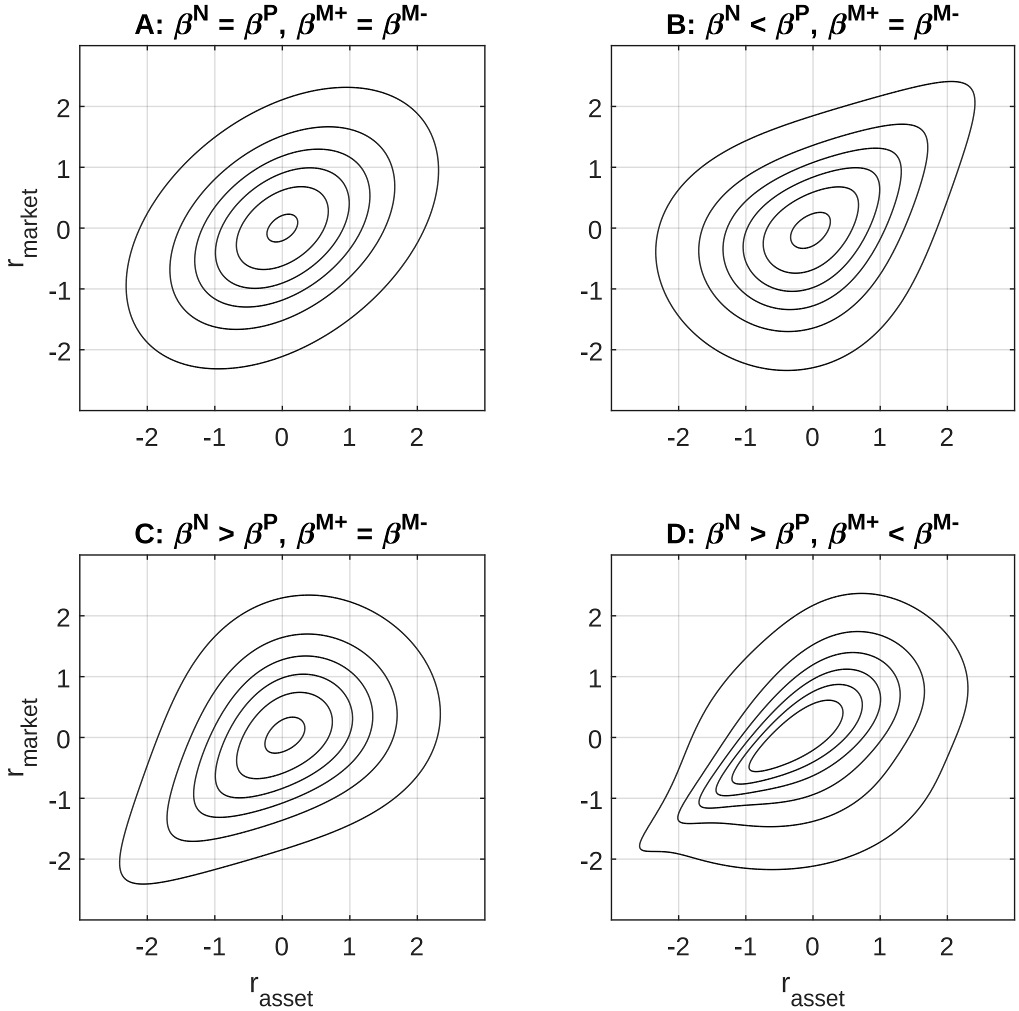
Meanwhile, the cross-sectional fit, reported in the final column, rises from 2.70% when using the CAPM beta to 5.43% when using semibetas. We can formally test whether this gain in R^2 is statistically significant by noting that the semibeta-based pricing model reduces to the traditional CAPM model if the semibeta risk premiums satisfy:

$$H_{0,t}^{CAPM}: \; \lambda_t^{\mathcal{N}} = \lambda_t^{\mathcal{P}} = -\lambda_t^{\mathcal{M}^+} = -\lambda_t^{\mathcal{M}^-}. \tag{8}$$

We reject this restriction at the 5% level for 68% of the 5,541 days in our sample, representing a strong rejection of the traditional one-beta model in favor of a model that exploits the additional information contained in the semibetas.

The risk premium estimates reported in the second row of Table 2 highlight the richer pricing implications of the mean-semivariance framework: $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ are both associated with large and statistically significant risk premiums, while $\beta^{\mathcal{P}}$ and $\beta^{\mathcal{M}^+}$ do not appear to be associated with any significant differences in returns across stocks. Underscoring not just the statistical significance of the estimated risk premiums, but also the economic significance, a one standard deviation increase in $\beta^{\mathcal{N}}$ relative to its cross-sectional mean is associated with an increase in the expected annual return of 10.59%.

Figure 2: **Hypothetical Return Distributions.** The figure presents isoprobability contours of the bivariate PDFs for four hypothetical return distributions, all of which have standard Normal marginal distributions and all of which imply a CAPM beta of one.



Meanwhile a one standard deviation increase in $\beta^{\mathcal{M}^-}$ relative to its' cross-sectional mean lowers the expected return by 2.88%. Both of these changes are economically significant given the average market return is 8.56% over our sample period.

To help further intuit the estimated risk premiums associated with different semi-betas, Figure 2 presents bivariate contour plots for the returns on the market and four hypothetical assets, each of which have traditional CAPM beta equal to one.[9] Since

[9]The contours are generated using standard Normal marginal distributions with dependence between the two variables captured by either a Normal copula or one of three different "Clayton" copulas, see

the CAPM betas are the same, the CAPM predicts identical expected returns of 8.34% for all four assets. Meanwhile, consider the asset in Panel A, which is jointly Normally distributed with the market, and the asset in Panel B, which (contrary to most equity returns) has less correlation during market downturns and greater correlation during market upturns ($\beta^{\mathcal{N}} < \beta^{\mathcal{P}}$). The semibeta model estimates in Table 2 imply an annual expected excess return for asset A of 7.95%, and only 4.82% for asset B, a finding that is consistent with investors being particularly averse to downside risk, and thus willing to accept lower expected returns for an asset displaying the desirable dependence featured in Panel B. On the other hand, the asset depicted in Panel C, which is more strongly correlated with the market during downturns than upturns ($\beta^{\mathcal{N}} > \beta^{\mathcal{P}}$) and as such less desirable from a mean-semivariance perspective, has an annual expected excess return of 10.86%. This represents an increase in expected return of nearly 3% relative to asset A, and over 6% relative to asset B, two assets with the exact same market beta as asset C, highlighting the economic significance of the differences in the estimated semibeta risk premiums. Finally, consider the asset in Panel D. Similar to asset C, asset D is more strongly correlated with the market during downturns than upturns ($\beta^{\mathcal{N}} > \beta^{\mathcal{P}}$), but its asymmetric mixed semicovariation ($\beta^{\mathcal{M}^-} > \beta^{\mathcal{M}^+}$) imbue asset D with superior hedging benefits relative to asset C, and thus a lower expected excess return of 9.90%.

3.1. Standard Risk Factors and Controls

A plethora of other risk factors and firm characteristics constructed from lower frequency daily or monthly data have, of course, been put forth in the literature as significant drivers of the cross-sectional variation in equity returns; see, e.g., the recent account by Harvey, Liu and Zhu (2016). We focus on a subset of the more prominent variables that have received the most attention in the literature, namely size (ME) (Banz, 1981), book-to-market (BM) (Fama and French, 1993), momentum (MOM) (Jegadeesh and Titman, 1993), return reversals (REV) (Jegadeesh, 1990), idiosyncratic volatility (IVOL) (Ang,

Nelsen (2006). Using values for market volatility and average firm volatility from our data, 0.92% and 2.26% respectively, a beta of 1 implies a linear correlation of 0.41, which is used in all four panels.

Hodrick, Xing and Zhang, 2006b), and illiquidity (ILLIQ) (Amihud, 2002); further details concerning the construction of each of these variables are given in Appendix B.

The third row of Table 2 reports the average risk premium estimates from the cross-sectional regressions that in addition to the semibetas include ME, BM and MOM, mimicking the popular Fama-French-Carhart four factor (FFC4) model. Consistent with the extant literature, the estimated risk premiums for ME and MOM are both strongly significant, while the premium for BM is only marginally significant at conventional levels. Correspondingly, the inclusion of the three additional risk factors also increases the average cross-sectional R^2 from 5.43% for the regressions based solely on the four semibetas to 8.23% for the semibeta+FFC-based model. Importantly, the risk premiums associated with $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ remain highly statistically significant.

The bottom row of Table 2 further incorporates REV, IVOL and ILLIQ as controls, which increases the average cross-sectional R^2 to 10.32%. Again, the inclusion of the additional controls does not meaningfully alter the large and highly significant t-statistic associated with $\beta^{\mathcal{N}}$. Also, even though the t-statistic for $\beta^{\mathcal{M}^-}$ is somewhat diminished compared to some of the earlier regressions, the risk premium estimates for $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ are both remarkably similar to the estimates obtained without the inclusion of any controls reported in the second row, underscoring the robustness of the semibeta pricing.

3.2. Arbitrage Risk and Semibeta Pricing

The semibeta risk premium estimates discussed above are based on the traditional Fama-MacBeth cross-sectional regression approach involving the returns on long positions in each of the individual stocks. It follows readily from the definition of the semibetas in equation (2) that $\hat{\beta}^{\mathcal{N}}_{t,i}$ ($\hat{\beta}^{\mathcal{P}}_{t,i}$) for a long position in stock i equals $-\hat{\beta}^{\mathcal{M}^-}_{t,i}$ ($-\hat{\beta}^{\mathcal{M}^+}_{t,i}$) for a short position in that same stock i. Hence, in a frictionless market, in which the expected return on a short position is equal to the negative of the expected return on a long position, the risk premium associated with $\hat{\beta}^{\mathcal{N}}_{t,i}$ ($\hat{\beta}^{\mathcal{P}}_{t,i}$) should be equal to the negative of the risk premium associated with $\hat{\beta}^{\mathcal{M}^-}_{t,i}$ ($\hat{\beta}^{\mathcal{M}^+}_{t,i}$). To facilitate a test of each of these restrictions, it is useful to reparameterize the cross-sectional regression in (6) as:

$$r_{t+1,i} \;=\; \lambda_{0,t+1} + \lambda^{\mathcal{N}}_{t+1}(\hat{\beta}^{\mathcal{N}}_{t,i} - \hat{\beta}^{\mathcal{M}^-}_{t,i}) + \lambda^{\mathcal{P}}_{t+1}(\hat{\beta}^{\mathcal{P}}_{t,i} - \hat{\beta}^{\mathcal{M}^+}_{t,i}) + \delta^{\mathcal{M}^+}_{t+1}\hat{\beta}^{\mathcal{M}^+}_{t,i} + \delta^{\mathcal{M}^-}_{t+1}\hat{\beta}^{\mathcal{M}^-}_{t,i} + \epsilon_{t+1,i}. \quad (9)$$

This reparameterization does not change the fit of the regression, nor the estimates for $\lambda^{\mathcal{N}}_{t+1}$ and $\lambda^{\mathcal{P}}_{t+1}$. However, it allows for the construction of a simple t-test for the hypothesis that the risk premiums for $\beta^{\mathcal{N}}$ and $-\beta^{\mathcal{M}-}$ ($\beta^{\mathcal{P}}$ and $-\beta^{\mathcal{M}+}$) are the same based on the time series average of the $T-1$ delta estimates:

$$\hat{\delta}^j \;=\; \frac{1}{T-1}\sum_{t=2}^{T}\hat{\delta}^j_t \;, \qquad j \in \{\mathcal{M}^+, \mathcal{M}^-\}. \quad (10)$$

The resulting t-statistic for testing $\delta^{\mathcal{M}^-}_{t+1} = 0$, reported in the top row in Table 3, reveals that $\beta^{\mathcal{N}}$ and $-\beta^{\mathcal{M}-}$ risks are priced differently in the cross-section. Also, consistent with the finding that neither $\beta^{\mathcal{P}}$ nor $-\beta^{\mathcal{M}+}$ risk appear to be priced, the t-statistic on $\delta^{\mathcal{M}^+} = 0$ is insignificant. This naturally raises the question of what causes the risk premiums for the two downside semibetas to differ?

Most large cap stocks, like the S&P 500 constituents underlying our risk premium estimates, can be easily and cheaply borrowed, see, e.g., the detailed analysis in D'Avolio (2002) considering both the level of institutional ownership and the direct lending fees associated with shorting, as well as the more recent analysis in Henderson, Jostova and Philipov (2019). Hence, the difference in the risk premiums cannot simply be attributed to "hard" short-sales constraints. Instead, as argued by Pontiff (1996) and Schleifer and Vishny (1997), with legal restrictions and charters impeding many institutional investors from short-selling, and many individual investors simply reluctant to sell short, this effectively creates "soft" limits-to-arbitrage and related arbitrage risks (see also the discussion in Hong and Sraer, 2016). This arbitrage risk in turn may cause systematic risks associated with long and short positions to be priced differently.[10]

[10]Flights to liquidity and accompanying downward liquidity spirals, as discussed by Brunnermeier

To corroborate this conjecture, we follow the literature (e.g., Pontiff, 1996; Stambaugh, Yu and Yuan, 2015) in using idiosyncratic volatility (IVOL) as a proxy for arbitrage risk. Intuitively, if arbitrageurs are able to neutralize their exposure to benchmark risks, then IVOL, as opposed to total volatility, can be used as a measure of arbitrage risk, with higher IVOL implying greater impediment to price-correcting arbitrage. We follow Ang, Hodrick, Xing and Zhang (2006b) and calculate the daily IVOL for each of the stocks in the sample as the residual standard deviation from rolling three factor Fama-French regressions based on daily returns over the past twenty days; see Appendix B for a more detailed discussion. Each day, using the IVOL estimates for the previous day, we then split the cross-section into two separate groups comprised of the stocks with the 50% highest and lowest IVOLs, respectively. The resulting semibeta risk premium estimates for each of the two cross-sections are reported in Panel B of Table 3. Consistent with the thesis that the different risk premiums for $\beta^{\mathcal{N}}$ and $-\beta^{\mathcal{M}-}$ may be attributed to arbitrage risk, the t-statistic on $\delta^{\mathcal{M}^-}$ is 4.73 for the high IVOL group, strongly rejecting the null, compared with an insignificant t-statistic of 0.72 for the low IVOL group.[11]

To further buttress the role played by arbitrage risk and valuation uncertainty, we also consider grouped estimates based on turnover. Turnover tends to be higher for stocks that are more difficult to value and subject to greater investor disagreement (e.g., Harris and Raviv, 1993; Blume, Easley and O'Hara, 1994), and thus exhibit greater arbitrage price discrepancies (e.g, Kumar, 2009, among others). We calculate the monthly daily volume turnover ratio (TO) for each of the individual stocks as the ratio of the number of shares traded during the day divided by the number of shares outstanding. We then split the cross-section into two groups comprised of the 50% of the stocks with highest and lowest turnover ratios, respectively, relying on the previous day's TO estimates for

and Pedersen (2009) and Anthonisz and Putnins (2017), might also further exacerbate these pricing differences.

[11]The coefficients on $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{P}}$ in Panel A of Table 3 are identical to those in the second row of Table 2, but the parameter $\delta^{\mathcal{M}-}$ ($\delta^{\mathcal{M}+}$) differs slightly from the simple sum of the coefficients on $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}-}$ ($\beta^{\mathcal{P}}$ and $\beta^{\mathcal{M}+}$) due to the geometric compounding used in annualizing the daily risk premium estimates. This is only a reporting issue; the fits of the models are identical.

Table 3: **Semibeta Pricing and Arbitrage Risk** The table reports the estimated annualized risk premia and Newey-West robust t-statistics from daily Fama-MacBeth cross-sectional predictive regressions for firms with below and above median arbitrage risk, proxied by Idiosyncratic Volatility (IVOL) and Turnover (TO). The daily semibetas are calculated from fifteen-minute intraday data. The control variables are measured prior to the daily returns, as detailed in Appendix B. The estimates are based on all of the S&P 500 constituent stocks and days in the January 1993 to December 2014 sample.

	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{P}}$	$\delta^{\mathcal{M}-}$	$\delta^{\mathcal{M}+}$	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{P}}$	$\delta^{\mathcal{M}-}$	$\delta^{\mathcal{M}+}$	R^2
	Panel A: Full-Sample Estimates								
	22.54	-1.58	12.16	-5.81					5.43
	5.62	-0.52	2.22	-0.93					
	Panel B: Sorting on Arbitrage Risk								
	Below Median				Above Median				
IVOL	22.29	-0.10	3.99	-3.41	17.43	-4.63	34.81	10.19	6.62
	5.45	-0.03	0.72	-0.52	4.03	-1.52	4.73	1.33	
TO	22.23	-12.64	-2.10	-9.54	19.57	1.69	8.06	-5.13	6.93
	5.42	-4.40	-0.34	-1.37	4.66	0.50	2.27	-0.76	

the group assignments. The resulting estimates, reported in Panel B of in Table 3, tell the same story as the IVOL groupings: the t-statistic on $\delta^{\mathcal{M}^-}$ is 2.27 for the high TO group comprised of stocks that are more difficult to value, compared to -0.34 for the low TO group of stocks subject to less arbitrage risk.

3.3. Upside and Downside Betas

In addition to the standard set of predictor variables included in Table 2, other beta decompositions have previously been found to improve upon the traditional CAPM. Most closely related to the present analysis are the up and downside betas advocated in the widely-cited study by Ang, Chen and Xing (2006a). High-frequency versions of the upside and downside betas advocated in that study are naturally defined as:

$$\hat{\beta}^{+}_{t,i} \equiv \frac{\sum_{k=1}^{m} r_{t,k,i} f^{+}_{t,k}}{\sum_{k=1}^{m} (f^{+}_{t,k})^2}, \qquad \hat{\beta}^{-}_{t,i} \equiv \frac{\sum_{k=1}^{m} r_{t,k,i} f^{-}_{t,k}}{\sum_{k=1}^{m} (f^{-}_{t,k})^2}. \tag{11}$$

In contrast to the semibetas proposed here, which account for joint asymmetric dependencies by conditioning the covariation on *both* the signed market and individual asset returns, the upside and downside betas condition only on the sign of the market return.

For ease of comparison, the first row in Table 4 repeats the baseline results for semibetas from Table 2. The second row in Table 4 reports the estimated average risk premiums associated with the upside and downside betas. The results are broadly consistent with the previous findings of Ang, Chen and Xing (2006a) in that only β^{-} carries a significant risk premium. The results are also in line with the estimated risk premiums for the semibetas presented in the top row, which show that only $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$, which account for negative market comovements, are associated with significant risk premiums.

To more directly compare and contrast the pricing of the semibetas with the pricing of the up and downside betas, the third row in Table 4 reports the estimates obtained by including *all* of the six betas in the same cross-sectional regressions. Despite the relatively high correlation between the semibetas and the up/downside betas,[12] the estimated risk premium for $\beta^{\mathcal{N}}$ clearly stands out as the most significant with a t-statistic of 3.86, followed by the premium for $\beta^{\mathcal{M}^-}$ with a t-statistic of -1.79. Meanwhile, the risk premium for β^{-} has a t-statistic of only 0.82, suggesting that the information contained in semibetas effectively subsumes the information in the downside beta in terms of explaining the cross-sectional variation in the returns. A joint test that all coefficients on semibetas are zero, leaving only the up and downside betas with nonzero coefficients, also rejects the null with a p-value of less than 0.01. In contrast, a joint test that both coefficients on up and downside betas are zero, leaving only the semibetas with nonzero coefficients, fails to reject the null, with a p-value of 0.13.

To facilitate a more direct test of whether the semibetas provide superior cross-sectional pricing predictions compared to the up and downside betas, notice that the

[12]Correlations between all of the semibetas and the up/downside betas, along with the other controls, are presented in Appendix C. The time series averages of the cross-sectional correlations between β^{+} and $\beta^{\mathcal{P}}$, and β^{-} and $\beta^{\mathcal{N}}$, in particular, are as high as 0.81, thus hindering a precise estimation of each of the individual risk premiums.

latter can be obtained as a weighted sum of the former:

$$\hat{\beta}^{+}_{t,i} = (\widehat{\beta}^{\mathcal{P}}_{t,i} - \widehat{\beta}^{\mathcal{M}^+}_{t,i}) \frac{\sum_{k=1}^{m} f^2_{t,k}}{\sum_{k=1}^{m} (f^{+}_{t,k})^2}, \tag{12}$$

$$\hat{\beta}^{-}_{t,i} = (\widehat{\beta}^{\mathcal{N}}_{t,i} - \widehat{\beta}^{\mathcal{M}^-}_{t,i}) \frac{\sum_{k=1}^{m} f^2_{t,k}}{\sum_{k=1}^{m} (f^{-}_{t,k})^2}. \tag{13}$$

Since the weights on the semibetas only involve functions of market returns, they do not vary in the cross-section. Accordingly, the semibeta model proposed here reduces to the up and downside beta model of Ang, Chen and Xing (2006a) if the following restrictions hold on a per period basis:

$$H^{UP+DOWN}_{0,t} : \; \lambda^{\mathcal{N}}_t = -\lambda^{\mathcal{M}^-}_t \; \cap \; \lambda^{\mathcal{P}}_t = -\lambda^{\mathcal{M}^+}_t. \tag{14}$$

We find that this hypothesis is rejected at the 5% level for 58% of the 5,541 daily cross-sectional regressions (recall that the stricter CAPM restrictions in (8) were rejected at the 5% level for 68% of the days in the sample). Going one step further, we can also test the stronger hypothesis that only downside beta risk is priced:

$$H^{DOWN}_{0,t} : \; \lambda^{\mathcal{N}}_t = -\lambda^{\mathcal{M}^-}_t \; \cap \; \lambda^{\mathcal{P}}_t = -\lambda^{\mathcal{M}^+}_t = 0, \tag{15}$$

and we find this is rejected at the 5% level for an impressive 70% of days in the sample.

The period-by-period restrictions in (14) and (15) obviously imply that the same restrictions must hold on average. The t-statistic for testing the hypothesis that $\lambda^{\mathcal{N}} = -\lambda^{\mathcal{M}^-}$ based on the $\delta^{\mathcal{M}^-}$ estimate from equations (9) and (10) discussed in Section 3.2 above already rejected this weaker hypothesis. The additional empirical results discussed in that section also pointed to arbitrage risk as the likely culprit behind the rejection. In other words, the presence of market frictions and limits-to-arbitrage implies that considering only downside betas entails a significant loss of information relative to a model based on downside semibetas.

Table 4: **Fama-Macbeth Regressions on Other Measures** The table reports the estimated annualized risk premia and Newey-West robust t-statistics from daily Fama-MacBeth cross-sectional predictive regressions. The daily semibetas, up and downside betas, and coskewness and cokurtosis measures are calculated from fifteen-minute intraday data based on all of the S&P 500 constituent stocks and days in the January 1993 to December 2014 sample.

$\beta^{\mathcal{N}}$	$\beta^{\mathcal{P}}$	$\beta^{\mathcal{M}^+}$	$\beta^{\mathcal{M}^-}$	β^+	β^-	CSK	CKT	R^2
22.54	-1.58	-4.29	-8.48					5.43
5.62	-0.52	-0.86	-2.02					
				-1.17	6.88			3.70
				-1.11	5.54			
17.31	-8.10	-12.66	-3.86	-2.40	7.90			6.61
3.86	-0.13	0.03	-1.79	-1.67	0.82			
						-4.40	0.81	1.52
						-1.55	0.76	
30.92	-3.79	-3.89	-16.33			10.09	-3.59	6.26
6.20	-1.12	-0.76	-3.69			2.66	-3.22	

3.4. Coskewness and Cokurtosis

The semibetas account for non-Normally distributed systematic risks by conditioning on the signed returns. A number of other measures have been explored in the literature as a way to capture non-Normal asymmetric joint return dependencies and the possible pricing thereof, most notably the notion of coskewness originally proposed by Kraus and Litzenberger (1976), and analyzed more extensively by Harvey and Siddique (2000) and Christoffersen, Honarvar and Ornthanalai (2017) among others. Other studies have similarly argued that cokurtosis appears to be priced in the cross-section; see, e.g., Dittmar (2002) and Ang, Chen and Xing (2006a). Directly following these studies, we define the daily coskewness and cokurtosis measures for stock i by:

$$CSK_{t,i} = \frac{\frac{1}{m}\sum_{k=1}^{m}(r_{t,k,i}-\bar{r}_{t,i})(f_{t,k}-\bar{f}_t)^2}{\sqrt{\frac{1}{m}\sum_{k=1}^{m}(r_{t,k,i}-\bar{r}_{t,i})^2}\ \frac{1}{m}\sum_{j=1}^{m}(f_{t,k}-\bar{f}_t)^2}, \quad (16)$$

$$CKT_{t,i} = \frac{\frac{1}{m}\sum_{k=1}^{m}(r_{t,k,i}-\bar{r}_{t,i})(f_{t,k}-\bar{f}_t)^3}{\sqrt{\frac{1}{m}\sum_{k=1}^{m}(r_{t,k,i}-\bar{r}_{t,i})^2}\ \left(\frac{1}{m}\sum_{k=1}^{m}(f_{t,k}-\bar{f}_t)^2\right)^{3/2}}, \quad (17)$$

where $\bar{f}_t$ and $\bar{r}_{t,i}$ denote the average daily return on the market and stock i respectively.

Comparing the top row with the penultimate row of Table 4 we see that the semibeta model fits the data much better than the coskewness/cokurtosis model, with an average R^2 of 5.43% compared to 1.52%. Combining all six measures in a single model, as in the bottom row of Table 4, we see the fit improves slightly, to 6.26%. The relatively high contemporaneous correlation between the realized semibetas and the CSK and CKT measures (see Appendix C) makes precise estimation of the magnitudes of the risk premiums associated with each of the individual measures challenging. Nonetheless, the regression reported in the last row of Table 4, which incorporates all six measures, shows that the t-statistic associated with $\beta^{\mathcal{N}}$ is by far the largest, followed by that of $\beta^{\mathcal{M}^-}$, supporting the idea that the priced non-Normal systematic risks is best captured by these two semibetas.

Interestingly, joint tests that the semibeta coefficients, or the coskewness/cokurtosis coefficients, are zero can be rejected at the 5% level in both cases. This indicates that while coskewness and cokurtosis have substantially less explanatory power than the semibetas, as evidenced by the R^2 values in the first and fourth rows of Table 4, they do contain additional information not accounted for by the semibetas. This is perhaps unsurprising, as coskewness and cokurtosis primarily capture information about the tails, and several recent studies have argued that systematic tail risks appear to be priced differently from more "normal" risks (see, e.g., Kelly and Jiang, 2014; Bollerslev, Li and Todorov, 2016; Orlowski, Schneider and Trojani, 2019). By contrast, the semibetas rely on a simple decomposition of the standard covariation with the market and "normal" systematic risks.

3.5. Longer Investment Horizons

The strong relationship between the daily realized semibetas and the cross-sectional variation in the subsequent daily returns naturally raises the question of whether this same predictive relationship carry over to longer investment horizons. To investigate this, we rely on the identical day t realized semibetas and cross-sectional regression in (6) in which we replace the left-hand-side daily returns with the cumulative returns from day $t + 1$ to day $t + h$ with $h = 5$ and $h = 20$, corresponding to a "week" and a "month," respectively.[13] This effectively amounts to using the daily semibetas to predict multiple daily returns, and the sum thereof, further into the future, and as such one might naturally expect the longer horizon results to be weaker than the one-day-ahead return predictions.

Despite this, the results reported in Table 5 are consistent with the daily findings reported above: the estimated risk premiums for $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ are both highly statistically significant, while neither $\beta^{\mathcal{P}}$ nor $\beta^{\mathcal{M}^+}$ appear to be priced. When we test the restrictions implied by the CAPM, given as $H_{0,t}^{CAPM}$ in equation (8), we are able to reject the null at the 5% significance level for 65% and 62% of the weekly and monthly specifications respectively. Thus even when aggregating returns to the monthly frequency, we are able to reject this restriction for nearly two-thirds of the months in the sample. Testing the symmetric pricing restriction, given as $H_{0,t}^{SYM}$ in equation (14), as also implied by the up and downside beta pricing framework of Ang, Chen and Xing (2006a), we are able to reject the null at the 5% level for 56.0% and 53.6% of the weekly and monthly specifications. This is strong evidence in favor of the proposed semibeta model. Moreover, the joint hypothesis that the risk premiums for $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ are the same and that the risk premiums for $\beta^{\mathcal{P}}$ and $\beta^{\mathcal{M}^+}$ are both equal to zero, as stipulated by the $H_{0,t}^{DOWN}$ hypothesis in equation (15), is rejected at the 5% level for 66% and 64% of the weekly and monthly regressions, respectively.

Further corroborating the previous findings based on a shorter daily investment horizon, both the weekly and the monthly $\lambda^{\mathcal{N}}$ and $\lambda^{\mathcal{M}^-}$ estimates remain statistically significant after including the same control variables as in Table 2. At the same time, comparing the magnitudes of the risk premium estimates, the (annualized) monthly estimates are naturally smaller than the (annualized) weekly estimates, as the strength of the predictability afforded by the daily semibetas diminishes with the return horizon.[14]

[13] We purposely rely on overlapping return windows and appropriately adjusted standard errors and t-statistics to enhance the efficiency of our inference, but qualitatively similar findings are obtained with non-overlapping return windows.

[14] Conversely, the pairwise correlations between the concordant ($\beta^{\mathcal{N}}$ and $\beta^{\mathcal{P}}$) and discordant ($\beta^{\mathcal{M}^+}$

Table 5: **Weekly and Monthly Investment Horizons.** The table reports the estimated annualized risk premia and Newey-West robust t-statistics from daily Fama-MacBeth cross-sectional regressions for predicting the future weekly (5-days) and monthly (20-days) returns. The daily semibetas are calculated from fifteen-minute intraday data on the last day preceding the return window. All of the control variables are measured prior to the daily returns. The estimates are based on all of the S&P 500 constituent stocks and days in the January 1993 to December 2014 sample.

β	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{P}}$	$\beta^{\mathcal{M}+}$	$\beta^{\mathcal{M}-}$	ME	BM	MOM	REV	IVOL	ILLIQ	R^2
Panel A: Weekly											
4.69											2.37
3.72											
	14.58	0.96	5.20	-13.80							5.07
	5.90	0.50	1.87	-3.71							
	10.85	-0.52	3.67	-13.58	-6.36	-1.94	0.08	-0.35	-1.06	-3.63	10.83
	5.92	-0.38	1.76	-5.03	-7.50	-2.30	2.54	-4.39	-1.40	-5.77	
Panel B: Monthly											
2.93											1.90
2.71											
	8.70	1.67	3.51	-3.36							4.45
	4.39	1.15	1.55	-1.30							
	4.42	-0.57	-0.65	-6.41	-4.89	-1.86	0.08	-0.25	0.98	-2.27	10.79
	3.55	-0.58	-0.44	-3.34	-7.06	-2.58	2.64	-4.15	1.56	-4.60	

4. Daily Data and Monthly Semibetas

The theory underlying the realized semibetas and the consistent estimation of the latent priced covariation components formally hinges on the use of ever finer sampled data, which motivates our analysis above based on high-frequency intraday data for the estimation of daily realized semibetas. Meanwhile, reliable high-frequency data is only available for a select set of stocks over a fairly recent sample period. In this section we extend our previous analysis, and investigate monthly semibetas constructed from daily data for a broader set of stocks over a longer sample period.

Specifically, we employ the CRSP daily database, expanding our sample period to

and $\beta^{\mathcal{M}-}$) semibetas generally increase with the horizon over which they are calculated; for additional details see the Supplemental Appendix.

Table 6: **Monthly Fama-Macbeth Regressions on Semibetas.** The table reports the estimated annualized risk premia and Newey-West robust t-statistics from overlapping monthly Fama-MacBeth cross-sectional predictive regressions. The monthly semibetas are calculated from daily data. All of the control variables are measured on the day prior to the monthly returns. The estimates are based on all of the common, non-penny, stocks in the CRSP data base from January 1963 to December 2017.

β	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{P}}$	$\beta^{\mathcal{M}^+}$	$\beta^{\mathcal{M}^-}$	ME	BM	MOM	REV	IVOL	ILLIQ	R^2
4.10											2.36
3.77											
	10.43	1.40	4.15	-6.42							5.22
	4.46	0.87	1.15	-2.03							
	8.66	-0.66	5.60	-14.09	-2.55	-0.47	0.06				10.70
	3.56	-0.43	1.42	-3.72	-4.93	-0.40	2.14				
	6.59	-1.90	6.33	-15.59	-2.08	-0.75	0.07	-0.12	-1.60	2.40	13.38
	2.85	-1.06	1.50	-3.82	-4.22	-0.66	2.61	-1.97	-1.48	2.44	

January 1963 until December 2017, and include all common publicly traded stocks.[15] Armed with this expanded data set, we then calculate monthly semibetas by replicating the sums over the intraday returns in equation (2) with the corresponding sums over the daily returns within the month. All in all, this provides us with 262,308 firm-month observations. We similarly calculate monthly up and downside betas, and monthly coskewness and cokurtosis measures by replacing the intraday sums in equations (11), (16) and (17), respectively, with the corresponding daily sums.

Putting these results further into perspective, it is well established that monthly returns are generally closer to being Normally distributed than daily returns (see, e.g., Campbell, Lo and MacKinlay, 1997; Engle, 2011). At longer monthly investment horizons, we are therefore less likely to find significant gains from the semibeta pricing framework, as under Normality it collapses to the traditional CAPM. Moreover, the use of daily returns in the estimation of monthly semibetas invariably blurs some of the asymmetric dependencies captured by the daily realized semibetas. Table 6, however, shows that the pricing relationships documented for the high-frequency-based daily semibetas are

[15] Specifically, we consider all stocks with CRSP codes 10 and 11. In line with previous work, we remove all "penny stocks," those with prices less than five dollars, to help alleviate biases arising from price discreteness; see, e.g., Harris (1994) and Amihud (2002).

also present in the monthly semibetas. In particular, in direct parallel to the second row of Table 2, only the risk premiums for $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}-}$ in the second row of Table 6 are significant. Further, mirroring the earlier high-frequency-based results, the explanatory power of the semibeta model is more than double that of the traditional CAPM reported in the top row, with an average cross-sectional R^2 of 5.22% compared with 2.36%.[16] The bottom two rows of Table 6 show that these results remain robust to the inclusion of the same set of controls used in Section 3. In short, our finding of differing risk prices for exposure to different semibetas is not specific to high-frequency data in a recent sample period; it holds true more generally for a much larger sample of stocks over a much longer sample period.

Table 7 additionally shows that the inclusion of the monthly up and downside betas and the monthly coskewness and cokurtosis measures do not affect this conclusion. In particular, consistent with Ang, Chen and Xing (2006a), the estimates in the second row imply that only downside beta risk is priced. Meanwhile, the inclusion of the semibetas in the cross-sectional regressions, reported in the third row of the table, renders the estimated risk premiums for both β^{+} and β^{-} insignificant.[17] In line with the earlier findings of Harvey and Siddique (2000) and others, the estimated risk premiums for the monthly CSK and CKT measures, reported in the fourth row of Table 7, are also both significant.[18] Importantly, however, the inclusion of the semibetas, as in the last row of Table 7, substantially increases the average monthly cross-sectional R^2 from 1.69% to 6.49%. The estimated risk premiums for $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}-}$ are also both strongly significant in the cross-sectional regressions that include CSK and CKT.

[16] Tests of the restriction that the risk premiums associated with the four semibetas are indeed the same, corresponding to the CAPM null hypothesis in equation (8), are rejected at the 5% level for 45% of the 659 months in the sample.

[17] The higher correlations between the monthly semibetas and the monthly up and downside betas result in less stable risk premium point estimates; summary statistics for the monthly betas and controls are provided in Appendix C. Nonetheless, the restrictions in equation (14), corresponding to symmetric pricing of semibetas, is rejected at the 5% level for 34% of the 659 monthly cross-sectional regressions, and the restrictions in equation (15), corresponding to only downside beta risk being priced, is rejected at the 5% level for 47% of the monthly regressions.

[18] The significance of the monthly CSK and CKT measures contrasts with the results in Table 4, and the lack of significance of the corresponding high-frequency-based daily measures.

Table 7: **Monthly Fama-Macbeth Regressions and Other Measures.** The table reports the estimated annualized risk premia and Newey-West robust t-statistics from overlapping monthly Fama-MacBeth cross-sectional predictive regressions. The monthly semibetas, up and downside betas, coskewness and cokurtosis measures are calculated from daily data. The estimates are based on all of the common, non-penny, stocks in the CRSP data base from January 1963 to December 2017.

$\beta^{\mathcal{N}}$	$\beta^{\mathcal{P}}$	$\beta^{\mathcal{M}^+}$	$\beta^{\mathcal{M}^-}$	β^+	β^-	CSK	CKT	R^2
10.43	1.40	4.15	-6.42					5.22
4.46	0.87	1.15	-2.03					
				1.06	3.16			3.42
				1.61	3.74			
12.37	2.90	2.41	-7.56	-6.64	-0.90			5.57
4.97	1.03	1.20	-2.48	-0.95	-0.28			
						5.00	1.98	1.69
						2.81	2.57	
18.11	-2.27	2.87	-12.09			12.10	-2.80	6.49
4.98	-1.04	0.81	-3.40			4.26	-3.43	

The next section demonstrates how these cross-sectional pricing relations may used in the formulation of superior investment strategies by betting on $\beta^{\mathcal{N}}$ and betting against $\beta^{\mathcal{M}^-}$.

5. Betting On, and Against, Semibetas

In this section we investigate trading strategies based on betas and semibetas. In the mean-semivariance framework, only $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ are priced, with the former carrying a positive risk premium and the latter a negative risk premium. Given this, we implement a semibeta strategy by considering the performance of an equal-weighted combination of "betting on $\beta^{\mathcal{N}}$" and "betting against $\beta^{\mathcal{M}^-}$" portfolios. We also examine the performance of each of these portfolios separately, as well as that of a long-short strategy based on the traditional market beta.[19]

[19] The Supplemental Appendix contains additional results for a portfolio which takes long positions in high $\beta^{\mathcal{N}}$ stocks and short positions in high $\beta^{\mathcal{M}^-}$ stocks, and a portfolio based on long positions in low $\beta^{\mathcal{M}^-}$ stocks and short positions in low $\beta^{\mathcal{N}}$ stocks. The performance of these two additional betting on and against semibeta portfolios are qualitatively similar to that of the semibeta portfolio presented here.

To avoid the critiques of Novy-Marx and Velikov (2018), we form the long-short portfolios using well-established methods. Firstly, we estimate betas and semibetas using standard, if modern, methods from high frequency econometrics, as described in Section 2 above. We then take a value-weighted long position in the top quintile and a value-weighted short position in the bottom quintile of stocks, rebalanced daily, to obtain zero-cost portfolios. We rely on continuously-compounded, as opposed to arithmetic, returns to facilitate the calculation of the cumulative portfolio returns over longer holding periods. We restrict the sample of stocks to the constituents of the S&P 500 index, thus explicitly excluding small, and potentially difficult to short micro-cap stocks. We use the popular four-factor model of Fama and French (1993) and Carhart (1997) (FFC4) and the five-factor model of Fama and French (2015) (FF5) to assess the risk-adjusted performance of the portfolios and estimate the corresponding alphas.

The top panel of Table 8, reports the average returns, standard deviations and annualized Sharpe ratios for the long-short portfolios. The average return on the semibeta portfolio is nearly double that of the beta portfolio, while the volatility is just over half that of the beta portfolio, combining to yield a Sharpe ratio of 1.05 compared with 0.30 for the traditional market beta portfolio.[20] The latter two columns show that both the long and the short leg of the semibeta portfolio contribute to this superior performance: the $\beta^{\mathcal{N}}$ portfolio generates much higher returns and comparable volatility to the standard beta portfolio, while the $\beta^{\mathcal{M}^-}$ portfolio generates somewhat higher returns with much lower volatility.

The lower panel of Table 8 reports the estimated FFC4 and FF5 alphas and factor loadings of the different portfolios. The traditional beta strategy generates an annualized alpha of 4.17% according to the FF5 factor model, with a t-statistic of just 1.77. The beta strategy generates no significant alpha according to the FFC4 factor model. In contrast, the semibeta strategy, and both of its underlying components, generate large

[20]Identical rankings of the four portfolios are obtained when using the sample mean of the squared demeaned negative daily returns in place of the daily sample variance in the calculation of downside Sharpe type ratios. Further details of these additional results are available in the Supplemental Appendix.

Table 8: **Betting On and Against Semibetas.** The top panel reports annualized descriptive statistics of the betting on and against (semi)beta strategies. All of the portfolios are self-financing based on value-weighted long-short positions rebalanced daily. The bottom panel reports the time-series regression estimates and Newey-West robust t-statistics for the FFC4 and FF5 factor models, along with the corresponding alphas in annualized percentage terms. The estimates are based on all of the S&P 500 constituent stocks and days in the 1993-2014 sample.

	β		Semi β		$\beta^{\mathcal{N}}$		$\beta^{\mathcal{M}^-}$	
Avg ret	4.98		9.76		10.98		7.66	
Std dev	16.57		9.30		16.89		8.00	
Sharpe	0.30		1.05		0.65		0.96	
α	2.19	4.17	8.35	9.68	8.05	10.62	7.76	7.88
	0.87	1.77	6.32	7.38	3.36	4.58	4.17	4.19
β_{MKT}	0.59	0.51	0.30	0.25	0.61	0.52	-0.02	-0.03
	67.61	55.78	64.70	47.90	73.15	56.89	-2.34	-3.44
β_{SMB}	0.30	0.12	0.30	0.21	0.40	0.22	0.20	0.20
	18.10	7.36	33.91	22.61	25.00	13.49	16.08	14.98
β_{HML}	-0.02	0.18	-0.01	0.11	-0.08	0.13	0.05	0.08
	-1.24	10.58	-1.61	11.10	-4.75	7.57	3.82	6.08
β_{MOM}	-0.24		-0.14		-0.22		-0.07	
	-19.53		-22.46		-19.01		-7.31	
β_{RMW}		-0.50		-0.28		-0.53		-0.04
		-22.15		-22.56		-23.70		-2.28
β_{CMA}		-0.35		-0.28		-0.44		-0.13
		-13.21		-19.09		-16.74		-5.95
R^2	58.15	60.26	55.92	59.55	60.59	64.38	6.72	7.42

and significant alphas, according to both the FFC4 and FF5 factor models. Annualized alphas range from 7.76% to 10.62%, with the corresponding t-statistics between 3.36 and 7.38.[21] These alphas will, of course, be reduced when accounting for transactions costs,

[21]The inclusion of a betting against beta (BAB) factor in the FF3 model results in an even larger alpha of 10.39% for the Semi β strategy, with a corresponding t-statistic of 8.44. To guard against potential biases in the unconditional alphas arising from temporal variation in conditional betas (see, e.g., Jagannathan and Wang (1996) and Lewellen and Nagel (2006)), we also calculate conditional alphas following the approach of Cederburgh and O'Doherty (2016) (cf. Section II.B). The same general conclusions remain true: the semibeta portfolios result in highly significant positive conditional alphas, while the conditional alphas for the standard beta portfolios are always insignificant. The magnitudes of the average conditional alphas for the Semi β and $\beta^{\mathcal{N}}$ portfolios are also very similar to the values reported in Table 8, while the average conditional alphas for the $\beta^{\mathcal{M}^-}$ portfolios are marginally lower. Further details of these additional results are available in the Supplemental Appendix.

and we analyze this in more detail below.

Looking at the estimated factor loadings, the conventional long-short β portfolio and the $\beta^{\mathcal{N}}$ portfolio exhibit fairly similar FFC4 and FF5 systematic risk exposures. Meanwhile, the estimated factor loadings for the $\beta^{\mathcal{M}^-}$ portfolio are markedly different. In contrast to the other portfolios, the $\beta^{\mathcal{M}^-}$ portfolio is close to market neutral. The FFC4 estimates further suggest that the portfolio contains a higher proportion of value stocks than the other portfolios, while the FF5 estimates point to decidedly lower exposures to the profitability and investment factors than any of the other portfolios. The combined semi β strategy naturally reflects these different risk profiles of the $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ portfolios.[22]

5.1. Betting On the Competition

To further buttress the superiority of the semibeta portfolio, Table 9 reports the results from analogously constructed up and downside beta, and coskewness and cokurtosis portfolios. Given the pertinent discussion in Ang, Chen and Xing (2006a) and Harvey and Siddique (2000), we consider value-weighted long-short positions based on the top and bottom quintiles betting on β^-, against β^+, against CSK, and on CKT. In parallel to the semibeta portfolios discussed above, we also consider equal-weighted combinations of the two respective pairs of measures, denoted "$\beta^- - \beta^+$" and "$CKT - CSK$" in the table.[23]

The top panel reveals that only the β^- and the combined up and downside beta portfolios have Sharpe ratios in excess of the conventional beta sorted portfolio, equal to

[22] To further explore these differences in risk profiles, we also calculated industry concentrations. The β and $\beta^{\mathcal{N}}$ portfolios again appear fairly similar along that dimension. Most noticeably, the $\beta^{\mathcal{M}^-}$ portfolio on average invest less in HiTech firms and more in Non-durables than the other two portfolios. Moreover, it is generally less concentrated with lower overall industry exposures. Further details are available in the Supplemental Appendix.

[23] Ang, Chen and Xing (2006a) note that β^+ tends to be positively correlated with β, leading to an ambiguous prediction for the sign of the relationship between β^+ and expected returns. To overcome this, they suggest sorting on the "relative" β^+, defined as $\beta - \beta^+$. We also implemented this approach and found that the resulting portfolio did indeed have a higher Sharpe ratio than the portfolio based solely on β^+. However, the FFC4 and FF5 alphas were small and statistically insignificant. We omit these results in the interests of space.

0.54 and 0.48, respectively, compared to 0.30 for the traditional beta portfolio. The β^+ and the CSK and CKT portfolios all have small, or even negative Sharpe ratios. Even the two highest Sharpe ratios, however, are substantially below those for the various semibeta-based strategies, presented in Table 8, which range from 0.65 to 1.05.

The lower panel of Table 9 further shows that the FFC4 and FF5 alphas for the CSK and CKT portfolios are all small and statistically insignificant. Only the β^- portfolio and the combined $\beta^- - \beta^+$ portfolio result in significant alphas, ranging from 2.56% to 6.80%, with t-statistics between 1.90 and 2.76. As one might expect, the estimated risk exposures for the β^- portfolio are fairly similar to the estimates for the semibeta portfolio reported in Table 8. However, in spite of these similarities in risk profiles, the annualized FFC4 and FF5 alphas for the combined semibeta portfolio are both larger and much more strongly significant than the alphas for the β^- portfolio, again highlighting the superior performance of the betting on and against semibeta strategy.

5.2. Longer Holding Periods

The daily rebalancing of the long-short (semi)beta strategies considered in Table 8 may be difficult to implement in practice. Instead, we now consider the performance of the same portfolio strategies based on less frequent weekly and monthly rebalancing, or equivalently longer weekly and monthly holding periods.

Table 10, in particular, shows that moving to weekly rebalancing badly affects the traditional beta strategy, with the the Sharpe ratio falling markedly from 0.30 to 0.08. Moreover, the FF5 alpha that was borderline significant with daily rebalancing becomes small and insignificant. By contrast, the semibeta strategy reported in the second set of columns continues to outperform. The semibeta Sharpe ratio does fall from 1.05 to 0.63, and the annualized alphas are also somewhat lower than the alphas obtained with more frequent daily rebalacing. However, both the FFC4 and FF5 alphas remain strongly significant, with t-statistics of 3.45 and 4.35, respectively.

Table 11 presents the corresponding results based on even less frequent monthly port-

Table 9: **Betting On the Competition.** The top panel reports annualized descriptive statistics of portfolios formed using up and downside betas, and coskewness and cokurtosis measures. All of the portfolios are self-financing based on value-weighted long-short positions rebalanced daily. The bottom panel reports the time-series regression estimates and Newey-West robust t-statistics for the FFC4 and FF5 factor models, along with the corresponding alphas in annualized percentage terms. The estimates are based on all of the S&P 500 constituent stocks and days in the 1993-2014 sample.

	β^- - β^+		β^-		β^+		CKT - CKS		CSK		CKT	
Avg ret	2.96		7.53		-3.61		-0.16		-1.21		0.54	
Std dev	5.45		15.56		14.67		6.30		7.70		9.55	
Sharpe	0.54		0.48		-0.25		-0.03		-0.16		0.06	
α	2.56	2.93	4.78	6.80	-1.67	-2.94	-0.60	-0.03	-1.44	-1.27	-0.11	0.86
	2.10	2.42	1.90	2.76	-0.69	-1.28	-0.49	-0.02	-0.86	-0.76	-0.06	0.47
β_{MKT}	0.04	0.03	0.55	0.47	-0.47	-0.41	0.13	0.11	0.02	0.01	0.24	0.21
	9.32	6.38	61.82	48.63	-54.89	-45.37	30.25	23.49	2.60	1.94	36.63	28.96
β_{SMB}	0.01	0.01	0.27	0.12	-0.25	-0.10	-0.02	-0.06	0.02	0.03	-0.06	-0.15
	1.01	1.43	16.31	7.10	-15.85	-6.02	-2.58	-7.10	1.87	2.11	-4.96	-11.23
β_{HML}	-0.02	-0.04	-0.04	0.13	-0.01	-0.20	-0.09	-0.06	-0.03	-0.06	-0.16	-0.06
	-2.53	-4.09	-2.07	7.05	-0.38	-11.98	-11.15	-6.98	-2.45	-4.85	-12.20	-4.72
β_{MOM}	0.04		-0.17		0.25		-0.01		0.04		-0.06	
	6.29		-14.09		21.19		-2.51		4.43		-7.07	
β_{RMW}		0.00		-0.43		0.42		-0.10		0.00		-0.21
		-0.30		-18.06		19.01		-9.15		0.26		-12.22
β_{CMA}		-0.01		-0.32		0.30		-0.04		0.03		-0.10
		-1.04		-11.57		11.42		-2.70		1.69		-5.05
R^2	2.67	1.93	52.93	55.64	50.17	50.32	17.43	18.91	0.89	0.58	27.91	30.36

Table 10: **Betting On and Against Semibetas, Weekly Rebalancing.** The top panel reports annualized descriptive statistics of the betting on and against (semi)beta strategies. All of the portfolios are self-financing based on value-weighted long-short positions rebalanced weekly. The bottom panel reports the time-series regression estimates and Newey-West robust t-statistics for the FFC4 and FF5 factor models, along with the corresponding alphas in annualized percentage terms. The estimates are based on all of the S&P 500 constituent stocks and days in the 1993-2014 sample.

	β		Semi β		$\beta^{\mathcal{N}}$		$\beta^{\mathcal{M}^-}$	
Avg ret	1.18		5.47		7.76		2.40	
Std dev	15.30		8.64		15.50		7.52	
Sharpe	0.08		0.63		0.50		0.32	
α	-1.11	0.71	4.37	5.66	5.54	7.71	2.40	2.84
	-0.46	0.31	3.45	4.35	2.52	3.46	1.46	1.72
β_{MKT}	0.52	0.44	0.25	0.20	0.51	0.43	-0.02	-0.04
	60.65	48.93	55.60	38.70	66.56	49.31	-3.40	-5.54
β_{SMB}	0.30	0.13	0.30	0.22	0.39	0.23	0.21	0.21
	18.43	7.83	35.58	23.57	26.64	14.41	19.12	17.76
β_{HML}	-0.08	0.08	-0.06	0.02	-0.12	0.05	0.00	-0.01
	-4.68	4.54	-7.02	2.26	-8.05	2.96	-0.10	-0.55
β_{MOM}	-0.20		-0.11		-0.20		-0.01	
	-17.14		-17.93		-19.12		-1.85	
β_{RMW}		-0.47		-0.26		-0.48		-0.05
		-21.38		-21.18		-22.57		-3.05
β_{CMA}		-0.26		-0.22		-0.35		-0.08
		-9.80		-14.89		-14.07		-4.48
R^2	51.93	54.10	47.31	51.03	53.19	56.27	7.21	8.44

folio rebalancing. The Sharpe ratio for the traditional beta strategy falls even further to -0.04, and the corresponding FFC4 and FF5 alphas are both negative, albeit not statistically significantly so. The semibeta portfolio, on the other hand, retains its appeal. The Sharpe ratio of 0.48 is obviously lower than the ratios obtained with daily and weekly rebalancing, and the annualized alphas are also both less than the daily and weekly alphas. Still, both of the alphas remain statistically significant, consistent with the analysis in Section 3.5, and the relationship between semibetas and expected returns holding true at daily, weekly, and monthly horizons.

Table 11: **Betting On and Against Semibetas, Monthly Rebalancing.** The top panel reports annualized descriptive statistics of the betting on and against (semi)beta strategies. All of the portfolios are self-financing based on value-weighted long-short positions rebalanced monthly. The bottom panel reports the time-series regression estimates and Newey-West robust t-statistics for the FFC4 and FF5 factor models, along with the corresponding alphas in annualized percentage terms. The estimates are based on all of the S&P 500 constituent stocks and days in the 1993-2014 sample.

	β		Semi β		$\beta^{\mathcal{N}}$		$\beta^{\mathcal{M}^-}$	
Avg ret	-0.64		4.04		3.10		4.33	
Std dev	14.43		8.39		14.72		7.18	
Sharpe	-0.04		0.48		0.21		0.60	
α	-2.48	-1.34	2.96	4.22	1.17	2.85	4.09	4.94
	-1.17	-0.64	2.30	3.24	0.55	1.34	2.83	3.42
β_{MKT}	0.46	0.41	0.23	0.19	0.46	0.39	0.00	-0.02
	62.38	50.99	51.68	36.54	62.55	47.54	0.90	-4.01
β_{SMB}	0.26	0.11	0.31	0.23	0.38	0.23	0.24	0.23
	18.67	7.27	36.10	24.59	26.93	14.85	25.18	22.62
β_{HML}	-0.08	0.05	-0.06	0.01	-0.08	0.08	-0.04	-0.06
	-5.79	3.22	-6.46	1.06	-5.48	5.15	-3.64	-5.76
β_{MOM}	-0.21		-0.10		-0.22		0.02	
	-20.65		-16.43		-21.44		2.22	
β_{RMW}		-0.42		-0.26		-0.45		-0.08
		-21.18		-21.10		-21.89		-5.86
β_{CMA}		-0.17		-0.19		-0.30		-0.08
		-7.03		-12.88		-12.57		-4.91
R^2	49.21	49.52	46.76	50.29	50.83	52.42	11.71	13.32

5.3. Transaction Costs

The results above pertaining to the profitability of the betting on and against semibeta strategy did not take into account the cost of actually implementing the portfolio positions. Such costs are clearly of practical importance. Hence, in this section we explicitly consider the impact of transaction costs.

To better replicate empirical practice we focus on the semibeta portfolio with monthly rebalancing. In parallel to existing work (e.g., Han, 2006; DeMiguel, Garlappi and Uppal, 2009; Liu, 2009), we assume that the transaction costs are proportional to the turnover of the portfolio, with the portfolio turnover computed simply as the sum of the turnover of the long and short legs of the portfolio. Consistent with the total roundtrip transaction cost estimates for large U.S. stocks reported in the literature (see, e.g., the estimates in Novy-Marx and Velikov, 2016), we consider costs of 0.5% and 1%, with 1% providing a conservative upper bound.

Rather than simply trading all the way to the positions that would be "optimal" in the absence of transaction costs, several practically-oriented procedures have been developed in the literature to help mitigate trading costs (e.g., Bertsimas and Lo, 1998; Engle and Ferstenberg, 2007; Obizhaeva and Wang, 2013). These procedures are typically geared towards the specific setting and strategy at hand and can be difficult to realistically implement. Instead, we follow the simple-to-implement idea of Garleanu and Pedersen (2013) of only *partially* adjusting the portfolio weights each period.

Specifically, let ω_t^F denote the vector of (fully-adjusted) semibeta portfolio weights in month t. The partially-adjusted portfolio weights for month t are then obtained as:[24]

$$\omega_t^P = \lambda \omega_{t-1}^P + (1-\lambda)\omega_t^F, \tag{18}$$

where the scalar parameter $0 < \lambda < 1$ governs the speed of adjustment. While such a partial-adjustment approach will help mitigate turnover, it will generally also dampen the signal. As such, the benefits will depend in a complicated way on the interaction between the particular strategy and the transaction costs that are incurred, and the best choice of λ must be determined on a case-by-case basis. We will not attempt to do so here. Instead, in line with similar uses of moving average filters in other situations, volatility estimation included, we simply set $\lambda = 0.95$ and initialize the weights by setting $\omega_1^P \equiv \omega_1^F$.[25]

Table 12 summarizes the performance of the resulting partially adjusted semibeta

[24] This same approach has also recently been implemented by Bollerslev, Hood, Huss and Pedersen (2018). Garleanu and Pedersen (2013) further suggest changing the "target portfolio" to one that is part-way between the currently fully-adjusted optimal portfolio and the best estimate of next period's optimal portfolio. We have not attempted to implement this additional refinement here.

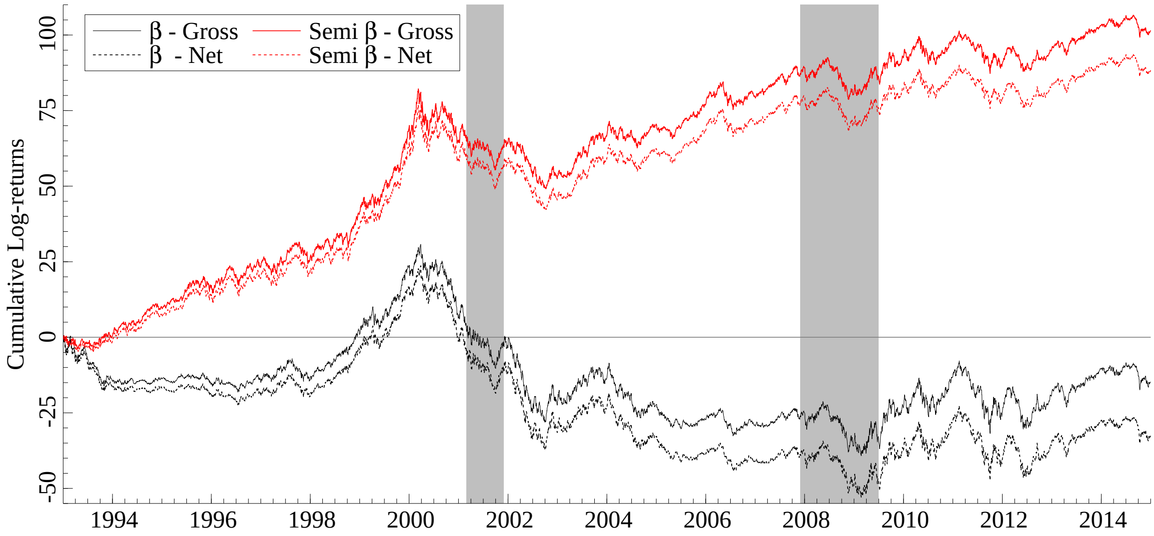
[25] The Supplemental Appendix contains additional results for alternative choices of λ, further highlighting the trade-off in signal retention and transaction cost reduction. It also contains additional results for alternative, more involved, procedures based on smoothing the semibeta estimates.

Table 12: **Betting On and Against Semibetas with Transaction Costs.** The top panel reports annualized descriptive statistics for the semibeta portfolios. The bottom panel reports the time-series regression estimates and Newey-West robust t-statistics for the FFC4 and FF5 factor models, along with the corresponding alphas in annualized percentage terms. All of the portfolios are self-financing based on value-weighted long-short positions determined by the combined semibeta strategy rebalanced monthly. T-cost refers to the roundtrip transaction costs. The left panel is identical to the second panel in Table 11 and fully adjusted portfolio weights. The right three panels report the results based on partially-adjusted portfolio weights, as discussed in the main text. The estimates are based on all of the S&P 500 constituent stocks and days in the 1993-2014 sample.

T-cost	0%		0%		0.5%		1.0%	
Adjustment	Full		Partial		Partial		Partial	
Avg ret	4.04		4.62		4.32		4.02	
Std dev	8.39		7.77		7.77		7.77	
Sharpe	0.48		0.59		0.56		0.52	
α	2.96	4.22	3.09	5.31	2.79	5.01	2.49	4.71
	2.30	3.24	3.05	5.33	2.76	5.03	2.46	4.72
β_{MKT}	0.23	0.19	0.25	0.17	0.25	0.17	0.25	0.17
	51.68	36.54	69.25	44.87	69.23	44.85	69.18	44.81
β_{SMB}	0.31	0.23	0.27	0.20	0.27	0.20	0.27	0.20
	36.10	24.59	40.61	27.95	40.58	27.93	40.54	27.89
β_{HML}	-0.06	0.01	-0.13	-0.11	-0.13	-0.11	-0.13	-0.11
	-6.46	1.06	-18.72	-14.51	-18.69	-14.49	-18.66	-14.46
β_{MOM}	-0.10		0.01		0.01		0.01	
	-16.43		2.59		2.59		2.59	
β_{RMW}		-0.26		-0.26		-0.26		-0.26
		-21.10		-27.01		-27.01		-27.00
β_{CMA}		-0.19		-0.20		-0.20		-0.20
		-12.88		-18.16		-18.15		-18.13
R^2	46.76	50.29	52.20	58.90	52.19	58.90	52.16	58.87

portfolios. For ease of comparison, the left-most panel presents the results using fully-adjusted portfolio weights with zero transaction costs, corresponding to the second panel in Table 11. The second panel in Table 12 presents the results for the partially-adjusted portfolio also in the absence of transaction costs. As the numbers show, partial adjustment of the weights slightly improves the performance, even before transaction costs: the average return is slightly higher, the volatility is slightly lower, leading to an increase in the Sharpe ratio from 0.48 to 0.59. Likewise, the FFC4 and FF5 alphas are both slightly

Figure 3: **Cumulative Returns for Beta and Semibeta Long-Short Portfolio Strategies**. The figure plots the cumulative percentage returns of long-short strategies based on beta and semibeta sorted value-weighted quintile portfolios. The shaded region represents NBER recession periods. The beta estimates and portfolio returns are based on all of the S&P 500 constituent stocks and days in the 1993-2014 sample.



higher for the portfolio based on partially-adjusted weights. Thus, the partial adjustment not only reduces turnover, it also appears to reduce the "noise" in the semibeta estimates, thereby strengthening the signal, in turn resulting in an overall slightly better performing semibeta portfolio.

The results in the third and fourth panels show the results for the same partially-adjusted semibeta portfolio subject to round-trip transaction costs of 0.5% and 1%, respectively.[26] The addition of transaction costs naturally lowers the average returns and Sharpe ratios. However, even with 1.0% roundtrip transaction costs, the Sharpe ratio of the partially-adjusted semibeta portfolio remains as high as 0.52. The FFC4 and FF5 alphas are also both economically large and statistically significant, with t-statistics of 2.46 and 4.72, respectively.

[26] The fully-adjusted semibeta portfolio performs very poorly in the presence of non-trivial transaction costs. With round-trip transaction costs of 1.0%, in particular, the Sharpe ratio equals -0.93, while the FFC4 and FF5 alphas equal -9.56% and -8.26%, respectively. Further details pertaining to this and other portfolios and transaction cost assumptions are presented in the Supplemental Appendix.

To visualize the timing of the returns, and more clearly contrast the performance of the semibeta strategy with the returns based on a traditional long-short beta strategy, Figure 3 plots the cumulative returns from both, where in both cases we rely on the partially-adjusted portfolio weights. The solid lines depict the cumulative returns ignoring transaction costs. The dashed lines show the returns that incorporate 1.0% roundtrip transaction costs. As the figure shows, the semibeta strategy performs well throughout most of the sample period, resulting in quite high cumulative returns at the end of the sample, even after incorporating 1% transaction costs. By contrast, and consistent with the idea of "betting against beta" advocated by Frazzini and Pedersen (2014), the traditional beta strategy performs poorly over much of the sample period, resulting in negative cumulative returns by the end of the sample, even without incorporating transaction costs.

6. Conclusion

We propose a new additive decomposition of the traditional market beta into four *semi*betas defined by the signed covariation between the market and individual asset returns: $\beta = \beta^{\mathcal{N}} + \beta^{\mathcal{P}} - \beta^{\mathcal{M}^+} - \beta^{\mathcal{M}^-}$. Consistent with the implications from a setup in which investors only care about downside risk, we find that only the two semibetas associated with negative market return variation are priced. At the same time, we strongly reject that the risk premiums for $\beta^{\mathcal{N}}$ and $-\beta^{\mathcal{M}^-}$ are the same, as is implied by a traditional downside beta model. We attribute this difference to arbitrage risk driving a wedge between the compensation for long versus short positions.

The results from a variety of specifications and empirical analyzes, involving different sampling frequencies, prediction horizons and a long list of additional controls, reveal that the annualized risk premium for $\beta^{\mathcal{N}}$ is around 23%, while the annualized risk premium for $\beta^{\mathcal{M}^-}$ is around -9%. By comparison the risk premium for the traditional market β is around 4%. We further establish that simple trading strategies that bet on $\beta^{\mathcal{N}}$ and against $\beta^{\mathcal{M}^-}$ leads to Sharpe ratios that are more than double that of the market.

Accounting for transaction costs, these same long-short semibeta strategies continue to produce economically large and strongly statistically significant risk adjusted alphas.

In conclusion: do not bet on or against beta, bet on *and* against the "right" semibetas.

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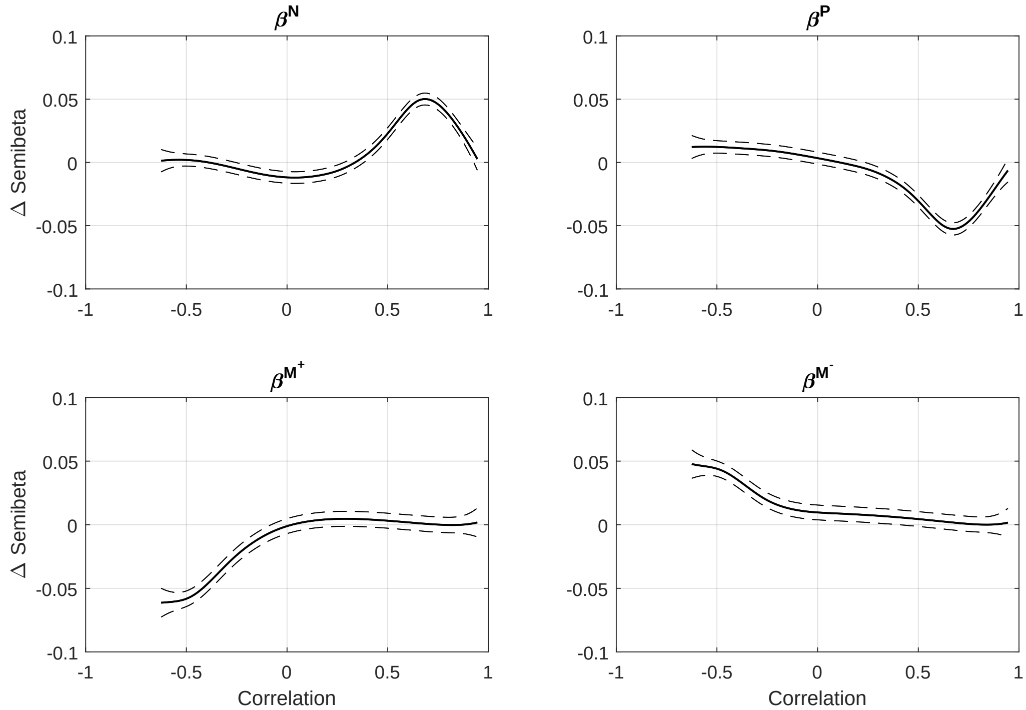
Appendix A. Asymmetric Dependencies

To help gauge whether the semibetas convey potentially useful additional information about asymmetric dependencies, it is instructive to compare the realized semibeta estimates to the limiting values that would obtain if the individual stock and aggregate market returns were jointly Normally distributed.

To that end, Figure A.1 reports the differences between the observed realized semibetas and the theoretical values that would hold under joint Normality, with positive values indicating greater correlation than under Normality. To facilitate the interpretation and more clearly highlight the differences, we report the results as a function of the daily realized correlations, averaged across all of the stocks and days in the sample.[27] The top panel reveals that realized negative semibetas are generally higher than they would be if returns were jointly Normally distributed, particularly for relatively highly correlated assets (e.g., for correlations between 0.4 and 0.9), where the confidence interval clearly excludes zero. Similarly, the realized positive semibetas are lower than would be expected under joint Normality. For the discordant semibetas, we find a similar story: β^{M-} is significantly larger (in magnitude) than would be expected under joint Normality, particularly for negatively correlated assets, while the opposite is true for β^{M+}. Taken together, these findings are consistent with the stylized fact that asset return dependence is stronger in downturns than upturns.

[27]More specifically, for each day and stock in the sample, we standardize all of the intraday returns to have unit daily variance. We then compute the daily realized covariance (correlation) and the four semibetas, averaging the estimates within correlation bins of width 0.01. Finally, we use a spline to smooth the differences from their implied Gaussian values.

Figure A.1: **Asymmetric Dependencies** The figure plots the deviations of the daily realized semibetas from their Gaussian limits as a function of the daily correlations between the individual stocks and the market, along with pointwise 99% confidence intervals. The estimates are averaged across all of the S&P 500 constituent stocks and days in the January 1993 to December 2014 sample.



Appendix B. Additional Control Variables

- Size (ME). Following Fama and French (1993), a firm's size is measured by its market value of equity: the product of closing price and the number of shares outstanding. Market equity is updated daily, we use its natural logarithm to reduce skewness.

- Book-to-Market (BM). Following Fama and French (1992), Book-to-Market is computed in June of year t, as the ratio of book value of common equity in fiscal year $t-1$ to the market value of equity in December of year $t-1$. Book value of equity is defined as book value of stockholder' equity (SEQ), plus balance-sheet deferred taxes (TXDB) and investment tax credit (ITCB, if available), minus book value of preferred stock (PSTK).

- Momentum (MOM). Following Jegadeesh and Titman (1993), momentum is the compound gross return from day $t-252$ through day $t-21$; i.e. skipping the short-term reversal month. The measure is computed only if a minimum of 100 days is available.

- Reversal (REV). As in Jegadeesh (1990) and Lehmann (1990), the short-term reversal is the return on days $t-20$ to $t-1$.

- Idiosyncratic Volatility (IVOL). Following Ang, Hodrick, Xing and Zhang (2006b), this is calculated as the daily updated standard deviation of the day $t-20$ to $t-1$ residuals from the daily return regression:

$$r_{t,i} - r_t^f = \alpha_i + \beta_i(f_t - r_{t,i}^f) + \gamma_i SMB_{t,i} + \phi_i HML_{t,i} + \epsilon_{t,i},$$

  where $r_{t,i}$ and f_t denote the daily stock and market return, r_t^f denotes the risk free rate, and $SMB_{t,i}$ and $HML_{t,i}$ denote the daily size and value factors for stock i.

- Illiquidity (ILLIQ). Following Amihud (2002), illiquidity for stock i is defined as:

$$ILLIQ_{t,i} = \frac{1}{20}\sum_{j=1}^{20}\left(\frac{|r_{t-j,i}|}{volume_{t-j,i} \times price_{t-j,i}}\right)$$

  We take the natural logarithm to reduce the skewness and the impact of outliers.

- Turnover (TO). Following Kumar (2009), we calculate turnover as the previous day's volume divided by shares outstanding.

Appendix C. Summary Statistics

Table C.1: **Descriptive Statistics TAQ Sample.** Panel A reports the time series averages of the cross-sectional means, medians and standard deviations. Panel B reports the time series averages of the cross-sectional correlations. The daily realized semibetas, up and donwside betas, coskewness and cokurtosis measures are all constructed from fifteen minutes intraday returns. The sample consists of all S&P 500 constituent stocks from January 1993 to December 2014.

Panel A: Cross-Sectional Summary Statistics

	β	$\beta^{\mathcal{P}}$	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{M}^+}$	$\beta^{\mathcal{M}^-}$	β^+	β^-	CSK	CKT	ME	BM	MOM	REV	IVOL	ILLIQ
Mean	0.92	0.72	0.68	0.27	0.25	0.92	0.90	0.00	1.40	15.51	0.50	17.58	1.46	1.63	-16.94
Median	0.83	0.61	0.57	0.16	0.15	0.81	0.80	0.00	1.48	15.51	0.42	11.79	1.09	1.38	-17.06
StDev	1.06	0.49	0.47	0.36	0.34	1.32	1.40	0.40	1.28	1.34	1.09	43.05	9.90	1.01	1.61

Panel B: Cross-Sectional Correlations

	β	$\beta^{\mathcal{P}}$	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{M}^+}$	$\beta^{\mathcal{M}^-}$	β^+	β^-	CSK	CKT	ME	BM	MOM	REV	IVOL	ILLIQ
β	1.00	0.67	0.66	-0.33	-0.33	0.78	0.77	0.00	0.65	0.05	-0.02	0.04	-0.01	0.13	-0.06
$\beta^{\mathcal{P}}$		1.00	0.44	0.06	0.18	0.82	0.29	0.30	0.32	-0.12	-0.03	0.02	0.04	0.34	0.10
$\beta^{\mathcal{N}}$			1.00	0.19	0.06	0.28	0.82	-0.30	0.32	-0.13	-0.03	0.02	-0.08	0.36	0.10
$\beta^{\mathcal{M}^+}$				1.00	0.38	-0.49	-0.05	-0.16	-0.41	-0.29	-0.01	-0.03	-0.08	0.36	0.26
$\beta^{\mathcal{M}^-}$					1.00	-0.05	-0.49	0.16	-0.41	-0.27	-0.01	-0.03	0.04	0.34	0.26
β^+						1.00	0.29	0.33	0.49	0.05	-0.02	0.03	0.07	0.11	-0.05
β^-							1.00	-0.34	0.49	0.03	-0.02	0.03	-0.09	0.12	-0.05
CSK								1.00	-0.01	0.00	0.00	-0.01	0.02	0.00	0.00
CKT									1.00	0.22	-0.03	0.03	0.00	-0.12	-0.20
ME										1.00	-0.09	0.07	0.03	-0.35	-0.88
BM											1.00	-0.03	0.00	-0.05	0.09
MOM												1.00	0.03	0.00	-0.11
REV													1.00	0.06	-0.03
IVOL														1.00	0.27
ILLIQ															1.00

Table C.2: **Descriptive Statistics CRSP Sample.** Panel A reports the time series averages of the cross-sectional means, medians and standard deviations. Panel B reports the time series averages of the cross-sectional correlations. The daily realized semibetas, up and donwside betas, coskewness and cokurtosis measures are all constructed from fifteen minutes intraday returns. The sample consists of all of the common, non-penny, stocks in the CRSP database from January 1963 to December 2017.

Panel A: Cross-Sectional Summary Statistics

	β	$\beta^{\mathcal{P}}$	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{M}^+}$	$\beta^{\mathcal{M}^-}$	β^+	β^-	CSK	CKT	ME	BM	MOM	REV	IVOL	ILLIQ
Mean	0.98	0.75	0.60	0.21	0.16	1.00	0.96	-0.03	1.28	14.23	0.73	15.00	1.46	1.51	-3.26
Median	0.91	0.67	0.54	0.15	0.10	0.90	0.90	-0.03	1.34	14.15	0.66	11.11	0.99	1.32	-3.27
StDev	0.75	0.45	0.35	0.20	0.17	0.96	1.06	0.29	0.82	1.32	0.49	31.97	8.61	0.81	0.79

Panel B: Cross-Sectional Correlations

	β	$\beta^{\mathcal{P}}$	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{M}^+}$	$\beta^{\mathcal{M}^-}$	β^+	β^-	CSK	CKT	ME	BM	MOM	REV	IVOL	ILLIQ
β	1.00	0.79	0.72	-0.30	-0.29	0.83	0.75	0.04	0.65	0.01	-0.08	0.06	-0.01	0.27	0.27
$\beta^{\mathcal{P}}$		1.00	0.43	-0.08	0.06	0.92	0.34	0.30	0.39	-0.08	-0.08	0.05	-0.04	0.48	0.39
$\beta^{\mathcal{N}}$			1.00	0.09	-0.09	0.34	0.90	-0.28	0.41	-0.11	-0.08	0.05	0.02	0.44	0.40
$\beta^{\mathcal{M}^+}$				1.00	0.23	-0.46	-0.02	-0.13	-0.44	-0.26	-0.01	-0.01	-0.01	0.48	0.35
$\beta^{\mathcal{M}^-}$					1.00	-0.04	-0.50	0.12	-0.44	-0.22	0.01	-0.01	-0.03	0.46	0.30
β^+						1.00	0.31	0.31	0.52	0.03	-0.06	0.05	-0.04	0.24	0.21
β^-							1.00	-0.30	0.54	0.00	-0.07	0.05	0.03	0.18	0.22
CSK								1.00	0.03	0.01	-0.01	-0.02	-0.03	-0.01	0.00
CKT									1.00	0.23	-0.06	0.04	0.01	-0.22	-0.12
ME										1.00	-0.22	0.07	0.02	-0.36	-0.59
BM											1.00	0.00	0.01	-0.04	0.14
MOM												1.00	-0.01	0.00	-0.18
REV													1.00	-0.03	-0.09
IVOL														1.00	0.64
ILLIQ															1.00