DAILY HOUSE PRICE INDICES: CONSTRUCTION, MODELING, AND LONGER-RUN PREDICTIONS

TIM BOLLERSLEV, a,b,c* ANDREW J. PATTON^a AND WENJING WANG^d

^a Department of Economics, Duke University, Durham, NC, USA
 ^b NBER, Cambridge, MA, USA
 ^c CREATES, Aaarhus, Denmark
 ^d Quantitative Research Group, Moody's Analytics, Inc., San Francisco, CA, USA

SUMMARY

We construct *daily* house price indices for 10 major US metropolitan areas. Our calculations are based on a comprehensive database of several million residential property transactions and a standard repeat-sales method that closely mimics the methodology of the popular monthly Case–Shiller house price indices. Our new daily house price indices exhibit dynamic features similar to those of other daily asset prices, with mild autocorrelation and strong conditional heteroskedasticity of the corresponding daily returns. A relatively simple multivariate time series model for the daily house price index returns, explicitly allowing for commonalities across cities and GARCH effects, produces forecasts of longer-run monthly house price changes that are superior to various alternative forecast procedures based on lower-frequency data. Copyright © 2015 John Wiley & Sons, Ltd.

Received 15 April 2014; Revised 19 March 2015



Supporting information may be found in the online version of this article.

There are many ways to measure changes in house prices, but the Standard & Poor's/ Case–Shiller index has become many economists' favored benchmark in recent years.

Wall Street Journal, 25 September 2012

1. INTRODUCTION

For many US households their primary residence represents their single largest financial asset holding: the Federal Reserve estimated the total value of the US residential real estate market at \$16 trillion at the end of 2011, compared with \$18 trillion for the US stock market (as estimated by the Center for Research in Security Prices). Consequently, changes in housing valuations importantly affect households' saving and spending decisions, and in turn the overall growth of the economy (see also the discussion in Holly *et al.*, 2010). A number of studies (e.g. Case *et al.*, 2011) have also argued that the wealth effect of the housing market for aggregate consumption is significantly larger than that of the stock market. The recent economic crisis, which arguably originated with the precipitous drop in housing prices beginning in 2006, directly underscores this point.

Set against this background, we (i) construct new *daily* house price indices for 10 major US metropolitan areas based on a comprehensive database of publicly recorded residential property

^{*} Correspondence to: Tim Bollerslev, Department of Economics and Fuqua School, Duke University, Durham, NC 27708, USA. E-mail: boller@duke.edu

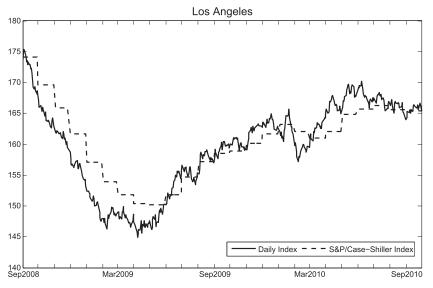


Figure 1. Daily and monthly house price indices for Los Angeles

transactions;¹ (ii) show that the dynamic dependencies in the new daily housing price series closely mimic those of other asset prices, and that these dynamic dependencies along with the crosscity correlations are well described by a standard multivariate generalized autoregressive conditional heteroskedasticity (GARCH) type model; and (iii) demonstrate that this relatively simple daily model allows for the construction of improved longer-run monthly and quarterly housing price forecasts compared with forecasts based on existing monthly and/or quarterly indices.

Our new daily house price indices are based on the same 'repeat-sales' methodology underlying the popular S&P/Case–Shiller monthly indices (Shiller, 1991) and the Federal Housing Finance Agency's quarterly indices. Measuring the prices at a daily frequency helps alleviate potential 'aggregation biases' that may plague the traditional coarser monthly and quarterly indices if the true prices change at a higher frequency. More timely house prices are also of obvious interest to policymakers, central bankers, developers and lenders alike, by affording more accurate and timely information about the housing market and the diffusion of housing prices across space and time (see, for example, the analysis in Brady, 2011).² Even though actual housing decisions are made relatively infrequently, potential buyers and sellers may also still benefit from more timely price indicators.

The need for higher-frequency daily indexing is perhaps most acute in periods when prices change rapidly, with high volatility, as observed during the recent financial crisis and its aftermath. To illustrate, Figure 1 shows our new daily house price index along with the oft-cited monthly S&P/Case–Shiller index for Los Angeles from September 2008 to September 2010. The precipitous drop in the daily index over the first 6 months clearly leads the monthly index. Importantly, the daily index also shows the uptick in housing valuations that occurred around April 2009 some time in advance of the monthly index. Similarly, the more modest rebound that occurred in early 2010 is also first clearly manifest in the daily index.

¹ To the best of our knowledge, this represents the first set of house price indices at the daily frequency analyzed in the academic literature. Daily residential house price indices constructed on the basis of a patent-pending proprietary algorithm are available commercially from Radar Logic Inc.

² Along these lines, the analysis in Anundsen (2014) also suggests that real-time econometric modeling could have helped in earlier detection of the fundamental imbalances underlying the recent housing market collapse.

Systematically analyzing the features of the dynamics of the new daily house price indices for all of the 10 metropolitan areas in our sample, we find that, in parallel to the daily returns on most other broadly defined asset classes, they exhibit only mild predictability in the mean, but strong evidence of volatility clustering. We show that the volatility clustering within and across the different house price indices can be satisfactorily described by a multivariate GARCH model. The correlation between the daily returns on the city indices is much lower than the correlation observed for the existing monthly return indices. However, as we aggregate the daily returns to monthly and quarterly frequencies, we find that the correlations increase to levels consistent with those observed for existing lower-frequency indices. Furthermore, we show that the new daily indices result in improved house price index forecasts, not solely by more quickly identify turning points as suggested by Figure 1 for Los Angeles, but also more generally for longer weekly and monthly forecast horizons and other sample periods, thus directly underscoring the informational advantages of the new daily indices *vis-á-vis* the existing published monthly indices.

The rest of the paper is organized as follows. The next section provides a review of house price index construction and formally describes the S&P/Case–Shiller methodology. Section 3 describes the data and the construction of our new daily prices series. Section 4 briefly summarizes the dynamic and cross-sectional dependencies in the daily series, and presents our simple multivariate GARCH model designed to account for these dependencies. Section 5 demonstrates how the new daily series and our modeling thereof may be used in more accurately forecasting the corresponding longer-run returns. Section 6 concludes. Additional results are provided as supporting information in an online supplementary Appendix.

2. HOUSE PRICE INDEX METHODOLOGIES

The construction of house price indices is plagued by two major difficulties. Firstly, houses are heterogeneous assets; each house is a unique asset, in terms of its location, characteristics, maintenance status, etc., all of which affect its price. House price indices aim to measure the price movements of a hypothetical house of average quality, with the assumption that average quality remains the same across time. In reality, average quality has been increasing over time, because newly built houses tend to be of higher quality and more in line with current households' requirements than older houses. Detailed house qualities are not always available or not directly observable, so when measuring house prices at an aggregate level it is difficult to take the changing average qualities of houses into consideration. The second major difficulty is sale infrequency. For example, the average time interval between two successive transactions of the same property is about 6 years in Los Angeles, based on our dataset described in Section 3 below. Related to that, the houses sold at each point in time may not be a representative sample of the overall housing stock.

Three main methodologies have been used to overcome the above-mentioned difficulties in the construction of reliable house price indices (see, for example, the surveys by Cho, 1996; Rappaport, 2007; Ghysels *et al.*, 2013). The simplest approach relies on the median value of all transaction prices in a given period. The National Association of Realtors employs this methodology and publishes median prices of existing home sales monthly for both the national and four Census regions. The median price index has the obvious advantage of calculation simplicity, but it does not control for heterogeneity of the houses actually sold.

A second, more complicated, approach uses a hedonic technique, to price the 'average quality' house by explicitly pricing its specific attributes. The US Census Bureau constructs its Constant Quality (Laspeyres) Price Index of New One-Family Houses Sold using a hedonic method. Although this method does control for the heterogeneity of houses sold, it also requires more advanced estimation procedures and much richer data than are typically available (see, for example, the recent study by Baltagi *et al.*, 2014), who rely on a sophisticated unbalanced spatial panel model.

A third approach relies on repeat sales. This is the method used by both Standard & Poor's and the Office of Federal Housing Finance Agency (FHFA). The repeat-sales model was originally introduced by Bailey *et al.* (1963) and subsequently modified by Case and Shiller (1989). The specific model currently used to construct the S&P/Case–Shiller indices was proposed by Shiller (1991) (see Clapp and Giaccotto, 1992; Meese and Wallace, 1997; for a comparison of the repeat-sales method with other approaches).³

As the name suggests, the repeat-sales method estimates price changes by looking at repeated transactions of the same house. This provides some control for the heterogeneity in the characteristics of houses, while only requiring data on transaction prices and dates. The basic models, however, are subject to some strong assumptions (see, for example, the discussion in Cho, 1996; Rappaport, 2007). Firstly, it is assumed that the quality of a given house remains unchanged over time. In practice, of course, the quality of most houses changes through aging, maintenance or reconstruction. This in turn causes a so-called 'renovation bias'. Secondly, repeat-sales indices exploit information only from houses that have been sold at least twice during the sampling period. This subset of all houses may not be representative of the entire housing stock, possibly resulting in a 'sample selection bias'. Finally, as noted above, all of the index construction methods are susceptible to 'aggregation bias' if the true average house price fluctuates within the estimation window.⁴

Our new daily home price indices are designed to mimic the popular S&P/Case–Shiller house price indices for the 'typical' prices of single-family residential real estate. They are based on a repeat-sales method and the transaction dates and prices for all houses that sold at least twice during the sample period. If a given house sold more than twice, then only the non-overlapping sale pairs are used. For example, a house that sold three times generates sale pairs from the first and second transaction, and the second and third transaction; the pair formed by the first and third transaction is not included.

More precisely, for a house j that sold at times s and t at prices $H_{j,s}$ and $H_{j,t}$, the standard repeat-sales model postulates that

$$\beta_t H_{j,t} = \beta_s H_{j,s} + \sqrt{2}\sigma_w w_{j,t} + \sqrt{(t-s)}\sigma_v v_{j,t}, \quad 0 \le s < t \le T$$

$$\tag{1}$$

where the house price index at any given time τ , computed across all houses *j* that sold between time 0 and *T*, is defined by the inverse of β_{τ} . The last two terms on the right-hand side account for 'errors', relative to the prices predicted by the aggregate index, in the sale pairs, with $\sqrt{2}\sigma_w w_{j,t}$ representing the 'mispricing error', and $\sqrt{(t-s)}\sigma_v v_{j,t}$ representing the 'interval error'. Mispricing errors are included to allow for imperfect information between buyers and sellers, potentially causing the actual sale price of a house to differ from its 'true' value. The interval error represents a possible drift over time in the value of a given house away from the overall market trend, and is therefore scaled by the (square root of the) length of the time interval between the two transactions. The error terms $w_{j,t}$ and $v_{j,t}$ are assumed independent and identically standard normal distributed.

The model in equation (1) lends itself to estimation by a multi-stage generalized least square type procedure (for additional details, see Case and Shiller, 1987), and each pair of sales of a given house $(H_{j,s}, H_{j,t})$ represents a data point to be used in estimation. We adopt a modified version of this method to construct our daily indices, described in detail in Section 3.1 below. In the standard estimation procedure, a 'base' period must be chosen, to initialize the index, and the S&P/Case–Shiller

³ Meese and Wallace (1997), in particular, point out that repeat-sales models can be viewed as special cases of hedonic models, assuming that the attributes, and the shadow prices of these attributes, do not change between sales. Thus, if the additional house characteristic data were widely available, it would clearly be preferable to use a hedonic pricing model.

⁴ Calhoun *et al.* (1995) compare repeat-sales indices over annual, semi-annual, quarterly as well as monthly intervals, and conclude that aggregation bias arises for all intervals greater than 1 month. By analogy, if the true housing values fluctuate *within* months, the standard monthly indices are likely to be biased. We formally test this conjecture below.

indices use January 2000. All index values prior to the base period are estimated simultaneously. After the base period, the index values are estimated using a chain-weighting procedure that conditions on previous values. This chain-weighting procedure is used to prevent revisions of previously published index values. Finally, the S&P/Case–Shiller indices are smoothed by repeating a given transaction in three successive months, so that the index for a given month is based on sale pairs for that month and the preceding two months.

3. DAILY HOUSE PRICE INDICES

The transaction data used in our daily index estimation is obtained from DataQuick, a property information company. The database contains information about more than 100 million property transactions in the USA from the late 1990s to 2012. We focus our analysis on the 10 largest Metropolitan Statistical Areas (MSAs), as measured in the year 2000. Further details pertaining to the data and the data-cleaning procedures are provided in the supplementary Appendix.

3.1. Estimation

The repeat-sales index estimation based on equation (1) is not computationally feasible at the daily frequency, as it involves the simultaneous estimation of several thousand parameters: the daily time spans for the 10 MSAs range from 2837 for Washington, DC, to 4470 days for New York. To overcome this difficulty, we use an expanding-window estimation procedure: we begin by estimating daily index values for the final month in an initial start-up period, imposing the constraint that all of the earlier months in the period have only a single *monthly* index value. Restricting the daily values to be the same within each month for all but the last month drastically reduces the dimensionality of the estimation problem. We then expand the estimation period by 1 month, and obtain daily index values for the new 'last' month. We continue this expanding estimation procedure through to the end of our sample period. This results in an index that is 'revision proof', in that earlier values of the index do not change when later data becomes available. Finally, similar to the S&P/Case–Shiller methodology, we normalize all of the individual indices to 100 based on their average values in the year 2000.

One benefit of the estimation procedure we adopt is that it is possible to formally test whether the 'raw' daily price series actually exhibit significant intra-monthly variation. In particular, following the approach used by Calhoun *et al.* (1995) to test for 'aggregation biases', we test the null hypothesis that the estimates of $\beta_{i,\tau}$ for MSA *i* are the same for all days τ within a given calendar month against the alternative that these estimates differ within the month. These tests strongly reject the null for all months and all 10 metropolitan areas; further details concerning the actual *F*-tests are available on request. We show below that this statistically significant intra-monthly variation also translates into economically meaningful variation and corresponding gains in forecast accuracy compared to the forecasts based on coarser monthly index values only.

3.2. Noise Filtering

Owing to the relatively few transactions that are available on a given day, the raw daily house price indices are naturally subject to measurement errors—an issue that does not arise so prominently for monthly indices.⁵ To help alleviate this problem, it is useful to further clean the data and extract more accurate estimates of the true latent daily price series. Motivated by the use of similar techniques for

⁵ The average number of transactions per day ranges from a low of 49 for Las Vegas to a high of 180 for Los Angeles. Measurement errors are much less of an issue for monthly indices, as they are based on approximately 20 times as many observations; i.e. around 1000–3500 observations per month.

extracting the 'true' latent price process from high-frequency data contaminated by market microstructure noise (e.g. Owens and Steigerwald, 2006; Corsi *et al.*, 2014), we rely on a standard Kalman filter-based approach to do so.

Specifically, let $P_{i,t}$ denote the true latent index for MSA *i* at time *t*. We assume that the 'raw' price indices for each of the MSAs constructed in the previous section, say $P_{i,t}^* = 1/\beta_{i,t}$ where *i* refers to the specific MSA, are related to the true latent price indices by

$$\log P_{i,t}^* = \log P_{i,t} + \eta_{i,t} \tag{2}$$

and the $\eta_{i,t}$ measurement errors are assumed to be serially uncorrelated. For simplicity of the filter, we further assume that the true index follows a random walk with drift:

$$r_{i,t} \equiv \Delta \log P_{i,t} = \mu_i + u_{i,t} \tag{3}$$

where $\eta_{i,t}$ and $u_{i,t}$ are mutually uncorrelated. It follows readily by substitution that

$$r_{i,t}^* \equiv \Delta \log P_{i,t}^* = r_{i,t} + \eta_{i,t} - \eta_{i,t-1}$$
(4)

Combining equations (3) and (4), this in turn implies an MA(1) error structure for the 'raw' returns, with the value of the MA coefficient determined by the variances of $\eta_{i,t}$ and $u_{i,t}$, σ_{η}^2 and σ_{u}^2 . This simple MA(1) structure is consistent with the sample autocorrelations for the raw return series reported in Figure A.1 in the supplementary Appendix.

Interpreting equations (3) and (4) as a simple state-space system, μ , σ_{η}^2 and σ_u^2 may easily be estimated by standard (quasi-)maximum likelihood methods. This also allows for the easy filtration of the 'true' daily returns, $r_{i,t}$, by a standard Kalman filter (see, for example, Hamilton, 1994). The Kalman filter implicitly assumes that $\eta_{i,t}$ and $u_{i,t}$ are i.i.d. normal. If the assumption of normality is violated, the filtered estimates are interpretable as best linear approximations. The Kalman filter parameter estimates reported in the supplementary Appendix imply that the noise-to-signal (σ_{η}/σ_u) ratios for the daily index returns range from a low of 6.48 (Los Angeles) to a high of 15.18 (Boston), underscoring the importance of filtering out the noise.

The filtered estimates of the latent 'true' daily price series for Los Angeles are depicted in Figure 2 (similar plots for all 10 cities are available in Figure A.2 in the supplementary Appendix). For comparison, we also include the raw daily prices and the monthly S&P/Case–Shiller index. Looking first at the top panel for the year 2000, the figure clearly illustrates how the filtered daily index mitigates the noise in the raw price series. At the same time, the filtered prices also point to discernible within-month variation compared to the step-wise constant monthly S&P/Case–Shiller index.

The bottom panel of Figure 2 reveals a similar story for the full 1995–2012 sample period. The visual differences between the daily series and the monthly S&P/Case–Shiller index are obviously less glaring on this scale. Nonetheless, the considerable (excessive) variation in the raw daily prices coming from the noise is still evident. We will consequently refer to and treat the filtered series as *the* daily house price indices in the analysis below.⁶

The online supplementary Appendix provides further frequency-based comparisons of the daily indices with the traditional monthly S&P/Case–Shiller indices, and the potential loss of information in going from a daily to a monthly observation frequency. In sum, because of the 3-month smoothing window used in the construction of the monthly S&P/Case–Shiller indexes, they essentially 'kill' all of the within-quarter variation in the new daily indices, while delaying all of the longer-run information

⁶ The 'smoothed' daily prices constructed from the full sample look almost indistinguishable from the filtered series shown in the figures. We purposely rely on filtered rather than smoothed estimates to facilitate the construction of meaningful forecasts.

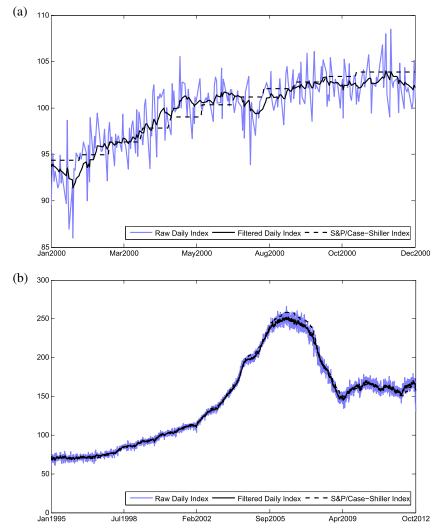


Figure 2. Raw and filtered daily house price indices for Los Angeles: (a) 3 January 2000 to 29 December 2000; (b) 3 January 1995 to 23 October 2012

by more than a month. We turn next to a detailed analysis of the time series properties of the new daily indices.

4. TIME SERIES MODELING OF DAILY HOUSING RETURNS

To facilitate the formulation of a multivariate model for all of the 10 city indices, we restrict our attention to the common sample period from June 2001 to September 2012. Excluding weekends and federal holidays, this yields 2843 daily observations.

4.1. Summary Statistics

Summary statistics for each of the 10 daily series are reported in Table I. Panel A gives the sample means and standard deviations for each of the index levels. Standard unit root tests clearly suggest that

				Tab	le I. Daily su	Table I. Daily summary statistics	cs			
	Los Angeles	Boston	Chicago	Denver	Miami	Las Vegas	San Diego	San Francisco	New York	Washington, DC
<i>Panel A: Level</i> Mean SD	evel 177.764 41.121	145.561 13.381	128.901 21.631	118.049 4.605	162.896 48.351	136.511 48.568	164.473 34.058	137.702 27.169	159.450 25.877	170.039 34.830
Panel B: Returns Mean SD AR(1) – – LB(10) (0	eturns 0.015 0.347 -0.059 67.877	0.008 0.351 0.047 21.935	-0.002 0.599 0.008 24.362	0.003 0.303 -0.018 16.838	0.006 0.428 -0.034 17.742	-0.006 0.370 0.061 59.549	0.010 0.387 -0.005 15.065	0.005 0.509 -0.113 269.509	0.011 0.291 0.049 13.335	0.015 0.502 -0.018 24.977
Panel C: S Mean SD AR(1) LB(10)	Panel C: Squared returns Acan 0.121 Acan 0.121 Acan 0.121 Acan 0.121 Acan 0.200 Acan 0.213 Acan 0.121 Acan 0.121 Acan 0.121 Acan 0.133 Acan 182.307	0.123 0.260 0.102 109.914	0.358 1.269 0.075 102.316	$\begin{array}{c} 0.092\\ 0.242\\ 0.021\\ 33.414 \end{array}$	0.183 0.336 0.107 445.189	0.137 0.369 0.071 85.348	0.150 0.270 0.037 50.715	0.259 0.616 0.042 179.632	0.085 0.170 0.042 53.109	0.252 0.607 0.132 106.434
<i>Note</i> : The ta first-order au	<i>Note:</i> The table reports summary statistics 1 first-order autocorrelation coefficient. LB(10	fficient. LB(10	for each of the) refers to the Lj	10 MSAs for th ung-Box portm	he June 2001 t tanteau test for	to September 20 up to 10th-order	12 sample perio r serial correlatio	or each of the 10 MSAs for the June 2001 to September 2012 sample period: a total of 2843 daily observations. AR(1) refers to the Ljung–Box portmanteau test for up to 10th-order serial correlation. The 95% critical value for this test is 18.31	aily observations value for this test	for each of the 10 MSAs for the June 2001 to September 2012 sample period: a total of 2843 daily observations. AR(1) denotes the) refers to the Ljung-Box portmanteau test for up to 10th-order serial correlation. The 95% critical value for this test is 18.31.

the price series are non-stationary, and as such the sample moments in panel A need to be interpreted with care; further details concerning the unit root tests are available upon request. In the following, we therefore focus on the easier-to-interpret daily return series.

The daily sample mean returns reported in panel B are generally positive, ranging from a low of -0.006 (Las Vegas) to a high of 0.015 (Los Angeles and Washington, DC). The standard deviation of the most volatile daily returns, 0.599 (Chicago), is double that of the least volatile returns, 0.291 (New York). The first-order autocorrelations are fairly close to zero for all of the cities, but the Ljung–Box χ^2_{10} tests for up to 10th-order serial correlation indicate significant longer-run dynamic dependencies in many of the series.

The corresponding results for the squared daily returns reported in panel C indicate very strong dynamic dependencies. This is also evident from the plot of the 10 daily return series in Figure 3, which show a clear tendency for large (small) returns in an absolute sense to be followed by other large (small) returns. This directly mirrors the ubiquitous volatility clustering widely documented in the literature for other daily speculative returns. Further, consistent with the evidence for other financial asset classes, there is also a commonality in the volatility patterns across most of the series. In particular, the magnitude of the daily price changes for each of the 10 cities were generally fairly low from 2004 to 2007 compared to their long-run average values. Correspondingly, and directly in line with the dynamic dependencies observed for other asset prices, there was a sizable increase in the magnitude of the typical daily house price change for the majority of the cities concurrent with the onset of the 2008–2010 financial crisis, most noticeably so for Miami, Las Vegas and San Francisco.

4.2. Modeling Conditional Mean Dependencies

The summary statistics discussed above point to the existence of some, albeit relatively mild, dynamic dependencies in the daily conditional means for most of the cities. Some of these dependencies may naturally arise from a common underlying dynamic factor that influences housing valuations nationally. In order to accommodate both city-specific and national effects within a relatively simple linear structure, we postulate the following model for the conditional means of the daily returns:

$$E_{t-1}(r_{i,t}) = c_i + \rho_{i1}r_{i,t-1} + \rho_{i5}r_{i,t-5} + \rho_{im}r_{i,t-1}^m + b_{ic}r_{c,t-1}^m$$
(5)

where $r_{i,t}^m$ refers to the (overlapping) 'monthly' returns defined by the summation of the corresponding daily returns:

$$r_{i,t}^{m} = \sum_{j=0}^{19} r_{i,t-j} \tag{6}$$

and the composite (national) return $r_{c,t}$ is defined as a weighted average of the individual city returns:

$$r_{c,t} = \sum_{i=1}^{10} w_i r_{i,t} \tag{7}$$

with the weights identical to those used in the construction of the composite 10 city monthly S&P/Case–Shiller index, which are 0.212, 0.074, 0.089, 0.037, 0.050, 0.015, 0.055, 0.118, 0.272 and 0.078. The own fifth lag of the returns is included to account for any weekly calendar effects. The inclusion of the own monthly returns and the composite monthly returns provides a parsimonious way of accounting for longer-run city-specific and common national dynamic dependencies. This particular formulation is partly motivated by the heterogeneous autoregressive (HAR) model proposed

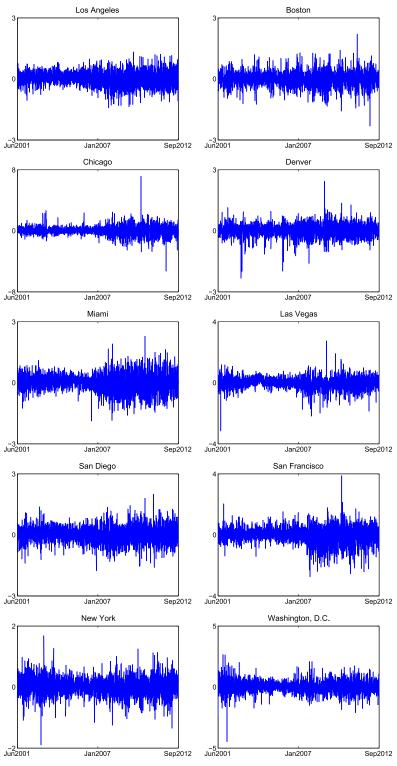


Figure 3. Daily housing returns

by Corsi (2009) for modeling so-called realized volatilities, and we will refer to it as an HAR-X model for short. This is not the absolutely *best* time series model for each of the 10 individual daily MSA indices. The model does, however, provide a relatively simple and easy-to-implement common parametric specification that fits all of the 10 cities reasonably well.⁷

We estimate this model for the conditional mean simultaneously with the model for the conditional variance described in the next section via quasi-maximum likelihood. The estimation results in Table II reveal that the ρ_1 and ρ_5 coefficients associated with the own lagged returns are mostly, though not uniformly, insignificant when judged by the robust standard errors reported in parentheses. Meanwhile, the b_c coefficients associated with the composite monthly return are significant for nine out of the 10 cities. Still, the one-day return predictability implied by the model is fairly modest, with the average daily R^2 across the 10 cities equal to 0.024, ranging from a low of 0.007 (Denver) to a high of 0.049 (San Francisco). This mirrors the low R^2 s generally obtained from time series modeling of other daily financial returns (e.g. Tsay, 2010).

The adequacy of the common specification for the conditional mean in equation (5) is broadly supported by the tests for up to 10th-order serial correlation in the residuals $\varepsilon_{i,t} \equiv r_{i,t} - E_{t-1}(r_{i,t})$ from the model reported in panel C of Table II. Only two of the tests are significant at the 5% level (San Francisco and Washington, DC) when judged by the standard χ^2_{10} distribution. At the same time, the tests for serial correlation in the squared residuals $\varepsilon^2_{i,t}$ from the model, given in the bottom two rows of panel C, clearly indicate strong nonlinear dependencies in the form of volatility clustering.

4.3. Modeling Conditional Variance and Covariance Dependencies

Numerous parametric specifications have been proposed in the literature to describe volatility clustering in asset returns. Again, in an effort to keep our modeling procedures simple and easy to implement, we rely on the popular GARCH(1,1) model (Bollerslev, 1986) for describing the dynamic dependencies in the conditional variances for all of the 10 cities:

$$Var_{t-1}(r_{i,t}) \equiv h_{i,t} = \omega_i + \kappa_i \varepsilon_{i,t-1}^2 + \lambda_i h_{i,t-1}$$

$$\tag{8}$$

The results from estimating this model jointly with the the conditional mean model described in the previous section are reported in panel B of Table II together with robust standard errors following Bollerslev and Wooldridge (1992) in parentheses.

The estimated GARCH parameters are all highly statistically significant and fairly similar across cities. Consistent with the results obtained for other daily financial return series, the estimates for the sum $\kappa + \lambda$ are all very close to unity (and just above for Chicago, at 1.002) indicative of a highly persistent, but eventually mean-reverting, time-varying volatility process. The high persistence might also in part reflect breaks in the overall levels of the volatilities, most notably around 2007 for several of the cities. As such, it is possible that even better-fitting in-sample models could be obtained by explicitly allowing for structural breaks. At the same time, with the time of the breaks unknown a priori, these models will not necessarily result in better out-of-sample forecasts (see, for example, the discussion in Pesaran and Timmermann, 2007; Anderson and Tian, 2014).

Wald tests for up to 10th-order serial correlation in the resulting standardized residuals, $\varepsilon_{i,t}/h_{i,t}^{1/2}$, reported in panel C, suggest that little predictability remains, with only two of the cities (Las Vegas and San Francisco) rejecting the null of no autocorrelation at the 5% level, and none at the 1% level.

⁷ Importantly, for the proper modeling of longer-run dynamic dependencies and forecast horizons beyond those analyzed here, the model does not incorporate any cointegrating relationships among the MSA indices. More sophisticated structural panel data models involving longer time spans of data explicitly allowing for cointegration between housing prices and real income have been estimated by Holly *et al.* (2010), among others.

	/ HAR-X-GA
:	Daily
	Table II.

Note: Panels A and B report quasi maximum likelihood estimates (QMLE) of HAR-X-GARCH models with robust standard errors in parentheses. Panel C reports Wald test statistics Washington, DC $\begin{array}{c} 0.084 \\ (0.010) \end{array}$ $\begin{array}{c} 0.067 \\ (0.043) \end{array}$ $0.032 \\ (0.006)$ 0.965 (0.007) 18.906 (0.042) 36.247 (0.000) 12.855 (0.232) 11.998 (0.285) 0.032(0.020) 1.245(0.884) 0.012(0.024) -0.035(0.007)0.027 0.998 New York 8.650 (0.566) $\begin{array}{c} 0.004 \\ (0.017) \end{array}$ -0.029 (0.006) $\begin{array}{c} 0.054 \\ (0.006) \end{array}$ 0.041 (0.023) $0.026 \\ (0.005)$ 0.969 (0.006) 7.133 (0.713) 36.577 (0.000) 6.386 (0.782) 0.908 (0.538) 0.040(0.020) 0.0330.995 San Francisco 40.312 (0.000) 0.016 (0.006) 0.983 (0.07) 25.641 (0.004) 72.156 (0.000) 18.762 (0.043) -0.222 (0.223) -0.094(0.018)0.160 (0.022) -0.014(0.006)0.056 (0.00) 0.0490.023(0.026) 0.999 or up to 10th-order serial correlation in the (squared) residuals and standardized residuals, with corresponding *p*-values in parentheses. $= c_i + \rho_{i,1}r_{i,t-1} + \rho_{i,s}r_{i,t-s} + \rho_{i,m}r_{i,t-1}^m + b_{i,c}r_{c,t-1}^m + \varepsilon_{i,t}$ $\varepsilon_{i,t}|\Omega_{t-1} \sim N(0,h_{i,t})$ $h_{i,t} = \omega_i + \kappa_i \varepsilon_{i,t-1}^2 + \lambda_i h_{i,t-1}$ San Diego 41.513 (0.000) 16.156 (0.095) $\begin{array}{c} 0.014 \\ (0.007) \end{array}$ (0.660)-0.037(0.020)-0.013 (0.006) (0.010)0.024(0.028) $\begin{array}{c} 0.985 \\ (0.008) \end{array}$ 8.953 (0.537) -0.036 (0.022) 0.030 0.240 (3.221) 0.060 0.999 7.677 Las Vegas 4.946 (0.895) $\begin{array}{c} 0.004 \\ (0.016) \end{array}$ 0.006(0.039) 0.017 (0.004) 0.035 (0.007) $\begin{array}{c} 0.014 \\ (0.013) \end{array}$ 0.017(0.006) $0.982 \\ (0.006)$ 18.086 (0.054) 46.849 (0.000) 20.148 (0.028) -0.111(0.368) 0.0440.999 Miami 9.426 (0.492) -0.006 (0.032) (0.008) $0.016 \\ (0.014)$ $\begin{array}{c} 0.013 \\ (0.003) \end{array}$ 0.986 (0.002) 8.952 (0.537) 50.471 (0.000) 6.576 (0.765) 1.152(0.942) -0.008 (0.006) -0.034(0.020)0.060 0.027 0.999 Denver 22.875 (0.011) 25.424 (0.005) $\begin{array}{c} 0.010 \\ (0.101) \end{array}$ $\begin{array}{c} 0.943 \\ (0.017) \end{array}$ $11.144 \\ (0.346)$ 14.344(0.158) -0.011(0.008) $0.034 \\ (0.012)$ -0.015 (0.052) (0.018)0.215 (0.103) 0.977-0.074 (5.338) 0.020 0.007 Chicago 15.071 (0.130) 24.616 (0.006) -0.006 (0.010) 0.056(0.009) 0.946(0.009) 15.178 (0.126) 56.910 (0.000) 0.094 (0.163) 0.005(0.011) -0.023 (0.007) (0.008)0.009 0.075 (0.058) 0.049 1.002 ri,t Boston 0.056(0.010) 62.011 (0.000) 11.878 (0.293) 24.289 (0.007) -0.014(0.005) $0.230 \\ (0.074)$ 0.926 (0.012) 10.934(0.363)-0.302 (0.769) 0.030(0.022) (0.017)(0.007)0.018 0.982 0.00 0.039 Panel C: Serial correlation tests 51,1 (0.091) Los Angeles 92.430 (0.000) 12.511 (0.252) $1.710 \\ (0.678)$ -0.080 (0.020) $0.054 \\ (0.020)$ -0.014 (0.007) 0.059 (600.0) 0.039 $0.013 \\ (0.015)$ $\begin{array}{c} 0.020 \\ (0.008) \end{array}$ 0.979 (0000) 0.99911.003 (0.357) Panel B: Variance Panel A: Mean c ($\times 10^{-2}$) $\omega (\times 10^{-2})$ $\varepsilon_{i,t} h_{i,t}^{-1/2}$ $arepsilon_{i,t}^2 h_{i,t}^{-1}$ $\kappa + \lambda$ ρ_m \mathbb{R}^2 $\varepsilon_{i,t}$ $\varepsilon_{i,t}^2$ δ 5 b_c 2 \sim

The tests for serial correlation in the squared standardized residuals, $\varepsilon_{i,t}^2/h_{i,t}$, reject the null for four cities, perhaps indicative of some remaining predictability in volatility not captured by this relatively simple model. However, for the majority of cities the specification in equation (8) appears to provide a satisfactory fit. The dramatic reduction in the values of the test statistics for the squared residuals compared to the values reported in the second row of panel C is particularly noteworthy.

The univariate HAR-X-GARCH models defined by equations (5) and (8) indirectly incorporate commonalities in the cross-city returns through the composite monthly returns $r_{c,t}$ included in the conditional means. The univariate models do not, however, explain the aforementioned commonalities in the volatilities observed across cities and the corresponding dynamic dependencies in the conditional covariances of the returns.

The constant conditional correlation (CCC) model proposed by Bollerslev (1990) provides a particularly convenient framework for jointly modeling the 10 daily return series by postulating that the temporal variation in the conditional covariances are proportional to the products of the conditional standard deviations. Specifically, let $\mathbf{r}_t \equiv [r_{1,t}, \ldots, r_{10,t}]'$ and $D_t \equiv \text{diag} \{h_{1t}^{1/2}, \ldots, h_{10,t}^{1/2}\}$ denote the 10 × 1 vector of daily returns and 10 × 10 diagonal matrix with the GARCH conditional standard deviations along the diagonal, respectively. The GARCH-CCC model for the conditional covariance matrix of the returns may then be succinctly expressed as

$$\operatorname{Var}_{t-1}(\mathbf{r}_t) = D_t R D_t \tag{9}$$

where R is a 10 × 10 matrix with ones along the diagonal and the conditional correlations in the off-diagonal elements. Importantly, the R matrix may be efficiently estimated by the sample correlations for the 10 × 1 vector of standardized HAR-X-GARCH residuals; i.e. the estimates of D_t^{-1} [$\mathbf{r}_t - E_{t-1}(\mathbf{r}_t)$]. The resulting estimates are reported in Table A.5 in the supplementary Appendix.⁸

4.4. Temporal Aggregation and Housing Return Correlations

The estimated conditional correlations from the HAR-X-GARCH-CCC model for the daily index returns reported in the supplementary Appendix average only 0.022. By contrast, the unconditional correlations for the monthly S&P/Case–Shiller index returns calculated over the same time period average 0.708, and range from 0.382 (Denver–Las Vegas) to 0.926 (Los Angeles–San Diego). The discrepancy between the two sets of numbers may appear to call into question the integrity of our new daily indices and/or the time-series models for describing the dynamic dependencies therein; however, conditional daily correlations and the unconditional monthly correlations are not directly comparable. In an effort to more directly compare the longer-run dependencies inherent in our new daily indices with the traditional monthly S&P/Case–Shiller indices, we aggregate our daily return indices to a monthly level by summing the daily returns are reported in the lower triangle of panel B in Table III. These numbers are obviously much closer, but generally still below the 0.708 average unconditional correlation for the published monthly S&P/Case–Shiller indices.

However, as previously noted, the monthly S&P/Case–Shiller indices are artificially 'smoothed', by repeating each sale pair in the 2 months following the actual sale. As such, a more meaningful comparison of the longer-run correlations inherent in our new daily indices with the correlations in the S&P/Case–Shiller indices is afforded by the unconditional quarterly (60 days) correlations reported

⁸ We also estimated the dynamic conditional correlation (DCC) model of Engle (2002), resulting in only a very slight increase in the maximized value of the (quasi-)log-likelihood function.

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Panel A: Daily (lower triangle) and weekly (upper triangle)	er triangle) and w	eekly (upper 1	triangle)							
Los Angeles	Ι	$\underset{0.065}{\textbf{0.117}}$	$\underset{0.124}{\textbf{0.061}}$	0.066 0.073	$\underset{0.219}{\textbf{0.197}}$	$\underset{0.250}{\textbf{0.172}}$	$\underset{0.240}{\textbf{0.198}}$	$\begin{array}{c} 0.280 \\ 0.309 \\ \end{array}$	$\underset{0.145}{\textbf{0.164}}$	$\underset{0.204}{\textbf{0.156}}$
Boston	$\underset{0.026}{\textbf{0.017}}$		$0.033 \\ 0.068$	0.068 0.128	$\underset{0.130}{\textbf{0.130}}$	$\underset{0.121}{\textbf{0.121}}$	$\underset{0.063}{\textbf{0.143}}$	$\underset{0.054}{\textbf{0.118}}$	$\underset{0.128}{\textbf{0.128}}$	$\underset{0.129}{\textbf{0.129}}$
Chicago	$\begin{array}{c} 0.002 \\ 0.019 \end{array}$	-0.007 -0.001		$\underset{0.108}{\textbf{0.108}}$	$\underset{0.149}{\textbf{0.077}}$	$\underset{0.064}{\textbf{0.058}}$	$\underset{0.042}{\textbf{0.049}}$	$\underset{0.148}{\textbf{0.084}}$	$\underset{0.115}{\textbf{0.102}}$	0.068 0.089
Denver	$0.001 \\ -0.003$	0.023 0.031	-0.002 -0.003	I	$\underset{\scriptstyle 0.100}{\textbf{0.100}}$	$0.092 \\ 0.110$	$\underset{0.090}{\textbf{0.100}}$	0.060 0.106	$\underset{0.006}{\textbf{0.053}}$	$\begin{array}{c} \textbf{0.084} \\ \textbf{0.090} \end{array}$
Miami	$\underset{0.069}{\textbf{0.069}}$	$0.047_{0.043}$	$\underset{0.046}{\textbf{0.024}}$	$0.044 \\ 0.047$		$\underset{0.239}{\textbf{0.173}}$	$\underset{0.214}{\textbf{0.178}}$	$\underset{0.176}{\textbf{0.165}}$	$\underset{0.169}{\textbf{0.187}}$	$\underset{0.183}{\textbf{0.150}}$
Las Vegas	0.060 0.077	$\underset{0.049}{\textbf{0.051}}$	$\underset{0.032}{\textbf{0.015}}$	$\underset{0.027}{\textbf{0.038}}$	$\underset{0.054}{\textbf{0.053}}$		$\underset{0.209}{\textbf{0.165}}$	$\underset{0.162}{\textbf{0.147}}$	$\underset{0.060}{\textbf{0.123}}$	$\underset{0.173}{\textbf{0.142}}$
San Diego	$0.077 \\ 0.072$	0.059 0.053	-0.006 $_{0.022}$	$\underset{0.042}{\textbf{0.045}}$	0.056 0.060	0.058 0.065		0.171 0.263	$\underset{0.169}{\textbf{0.148}}$	$\underset{0.127}{\textbf{0.137}}$
San Francisco	$\underset{0.235}{\textbf{0.183}}$	$0.037 \\ 0.038$	0.037 0.065	0.006 -0.003	0.057	$\begin{array}{c} 0.052 \\ 0.068 \end{array}$	0.069 0.066	Ι	$\underset{0.137}{\textbf{0.138}}$	$\underset{0.151}{\textbf{0.136}}$
New York	$\underset{0.041}{\textbf{0.032}}$	$\begin{array}{c} 0.0011 \\ 0.000 \end{array}$	0.047 0.061	-0.009 -0.002	$\underset{0.063}{\textbf{0.063}}$	$0.010 \\ -0.002$	0.027 0.029	$\underset{0.031}{\textbf{0.024}}$		$\underset{0.088}{\textbf{0.149}}$
Washington, DC	$\underset{0.045}{\textbf{0.045}}$	$\underset{0.034}{\textbf{0.038}}$	$\underset{0.024}{\textbf{0.017}}$	$\underset{0.041}{\textbf{0.032}}$	$\underset{0.038}{\textbf{0.041}}$	$\underset{0.034}{\textbf{0.049}}$	$\underset{0.027}{\textbf{0.033}}$	$\underset{0.038}{\textbf{0.038}}$	$\underset{0.043}{\textbf{0.044}}$	Ι
Panel B: Monthly (lower triangle) and quarterly (upper triangle	ower triangle) and	quarterly (up	oper triangle)							
Los Angeles	I	$\underset{0.621}{\textbf{0.634}}$	0.530 0.602	$\underset{0.506}{\textbf{0.463}}$	$0.730 \\ 0.852$	$\underset{0.837}{\textbf{0.600}}$	$\underset{0.906}{\textbf{0.731}}$	$\underset{0.834}{\textbf{0.724}}$	$\underset{0.747}{\textbf{0.759}}$	$0.733_{0.856}$
Boston	$\underset{0.348}{\textbf{0.382}}$		$\underset{0.655}{\textbf{0.451}}$	$\underset{0.559}{\textbf{0.400}}$	$\underset{0.507}{\textbf{0.616}}$	$0.533 \\ 0.522$	0.624 0.673	$\underset{0.623}{\textbf{0.594}}$	$\underset{0.735}{\textbf{0.643}}$	$\underset{0.688}{\textbf{0.688}}$
Chicago	$\underset{0.344}{\textbf{0.266}}$	$0.207 \\ 0.320$	I	0.323 0.502	$\underset{0.612}{\textbf{0.519}}$	$\underset{0.510}{\textbf{0.417}}$	$\underset{0.567}{\textbf{0.513}}$	$\begin{array}{c} 0.500 \\ 0.667 \\ 0.667 \end{array}$	$0.572 \\ 0.767$	$0.532 \\ 0.675$
Denver	$0.251 \\ 0.355$	$\underset{0.254}{\textbf{0.254}}$	$\begin{array}{c} 0.138 \\ 0.293 \end{array}$	I	$\underset{0.370}{\textbf{0.457}}$	$0.391 \\ 0.317$	0.454 0.557	$\underset{0.625}{\textbf{0.416}}$	$\underset{0.411}{\textbf{0.458}}$	$\underset{0.513}{\textbf{0.456}}$
Miami	0.493	$\underset{0.277}{\textbf{0.384}}$	$0.274 \\ 0.355$	$\begin{array}{c} 0.271 \\ 0.239 \end{array}$		$\underset{0.797}{\textbf{0.591}}$	$\underset{0.769}{\textbf{0.769}}$	$\underset{0.754}{\textbf{0.669}}$	$0.734 \\ 0.761$	0.697 0.801
Las Vegas	0.395 0.633	0.328 0.322	$\begin{array}{c} 0.210 \\ 0.233 \end{array}$	$0.229 \\ 0.201$	$\underset{0.547}{\textbf{0.404}}$	I	$0.589 \\ 0.782$	0.558 0.657	$0.599 \\ 0.659$	0.582 0.708
San Diego	0.497 0.626	$\underset{0.307}{\textbf{0.388}}$	0.260 $_{0.276}$	$\begin{array}{c} 0.266 \\ 0.351 \end{array}$	$\underset{0.570}{\textbf{0.468}}$	$\underset{0.497}{\textbf{0.400}}$	Ι	$\underset{0.822}{\textbf{0.678}}$	$0.731 \\ 0.711 \\ 0.711$	$\underset{0.824}{\textbf{0.694}}$
San Francisco	$\begin{array}{c} 0.511 \\ 0.623 \end{array}$	$\underset{0.288}{\textbf{0.288}}$	$\underset{0.404}{\textbf{0.253}}$	$\underset{0.427}{\textbf{0.216}}$	$\underset{0.527}{\textbf{0.424}}$	$\underset{0.417}{\textbf{0.343}}$	$\underset{0.600}{\textbf{0.435}}$		$0.700 \\ 0.663$	$0.677_{0.791}$
New York	$\underset{0.478}{\textbf{0.505}}$	$\underset{0.415}{\textbf{0.384}}$	$\underset{0.427}{\textbf{0.318}}$	$\underset{0.149}{\textbf{0.247}}$	$\underset{0.496}{\textbf{0.496}}$	$\underset{0.354}{\textbf{0.383}}$	$\underset{0.430}{\textbf{0.480}}$	$\underset{0.394}{\textbf{0.431}}$	Ι	$\underset{0.761}{\textbf{0.738}}$
Washington, DC	$\begin{array}{c} 0.469 \\ 0.603 \end{array}$	0.366 0.375	$0.277_{0.385}$	$0.253 \\ 0.309$	$0.444_{0.515}$	$\underset{0.444}{\textbf{0.368}}$	$\underset{0.551}{\textbf{0.433}}$	$\underset{0.486}{\textbf{0.414}}$	$\underset{0.437}{\textbf{0.437}}$	[

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in the upper triangle of panel B in Table III. There, we find an average correlation of 0.668, and a range of 0.317 (Denver–Las Vegas) to 0.906 (Los Angeles–San Diego), which are quite close to the corresponding numbers for the published S&P/Case–Shiller index returns.

These comparisons, of course, say nothing about the validity of the HAR-X-GARCH-CCC model for the daily returns, and the low daily *conditional* correlations estimated by that model. As a further model specification check, we therefore also consider the model-implied longer-run correlations, and study how these compare with the sample correlations for the actual longer-run aggregate returns.

The top number in each element of panels A and B of Table III gives the median model-implied unconditional correlations for the daily, weekly, monthly and quarterly return horizons, based on 500 simulated sample paths. The bottom number in each element is the corresponding sample correlations for the actual longer-run aggregated returns. Although the daily unconditional correlations in panel A are all close to zero, the unconditional correlations implied by the model gradually increase with the return horizon, and almost all of the quarterly correlations are in excess of one-half. Importantly, the longer-run model-implied correlations are all in line with their unconditional sample analogues.

To further illuminate this feature, Figure 4 presents the median model-implied and sample correlations for return horizons ranging from 1 day to a quarter, along with the corresponding simulated 95% confidence intervals implied by the model for the Los Angeles–New York city pair. The model provides a very good fit across all horizons, with the actual correlations well within the confidence bands. The corresponding plots for all of the 45 city pairs, presented in Figure A.3 in the supplementary Appendix, tell a similar story.

Taken as whole, these results clearly support the idea that the longer-run cross-city dependencies inherent in our new finer-sample daily house price series are consistent with those in the published coarser monthly S&P/Case–Shiller indices. The results also confirm that the joint dynamic dependencies in the daily returns are well described by the relatively simple HAR-X-GARCH-CCC model, in turn suggesting that this model could possibly be used in the construction of improved house price index forecasts over longer horizons.

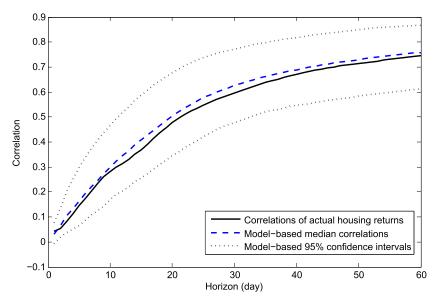


Figure 4. Unconditional return correlations for Los Angeles and New York

5. FORECASTING HOUSING INDEX RETURNS

One of the major potential benefits from higher-frequency data is the possibility of constructing more accurate forecasts by using models that more quickly incorporate new information. The plot for Los Angeles discussed in the Introduction alludes to this idea. In order to more rigorously ascertain the potential improvements afforded by the daily house price series and our modeling thereof, we consider a comparison of the forecasts from the daily HAR-X-GARCH-CCC model with different benchmark alternatives.

Specifically, consider the problem of forecasting the 20-day ('monthly') return on the house price index for MSA i:

$$r_{i,t}^{(m)} \equiv \sum_{j=0}^{19} r_{i,t-j} \tag{10}$$

for forecast horizons ranging from h = 20 days ahead to h = 1 day ahead.⁹ When h = 20 this corresponds to a simple one-step-ahead forecast for 1-month returns, but for h < 20 an optimal forecast will contain a mixture of observed data and a forecast for the return over the remaining part of the month. We will use the period June 2001 to June 2009 as our in-sample period, and the period July 2009 to September 2012 as our out-of-sample period, with all of the model parameters estimated once over the fixed in-sample period.¹⁰

Our simplest benchmark forecast is based purely on end-of-month data, and is therefore *not* updated as the horizon shrinks. We consider a simple AR(1) for these monthly returns:

$$r_{i,t}^{(m)} = \phi_0 + \phi_1 r_{i,t-20}^{(m)} + e_{i,t}$$
(11)

As the forecast is not updated through the month, the forecast made at time t - h is simply the AR(1) forecast made at time t - 20:

$$\hat{r}_{i,t-h}^{\text{Mthly}} = \hat{\phi}_0 + \hat{\phi}_1 r_{i,t-20}^{(m)} \tag{12}$$

Our second benchmark forecast is again purely based on monthly data, but now we allow the forecaster to update the forecast at time t - h, which may be in the middle of a month. We model the incorporation of observed data by allowing the forecaster to take a linear combination of the monthly return observed on day t - h and the 1-month-ahead forecast made on that day:

$$\hat{r}_{i,t-h}^{\text{Interp}} = \left(1 - \frac{h}{20}\right) r_{i,t-h}^{(m)} + \frac{h}{20} \left(\hat{\phi}_0 + \hat{\phi}_1 r_{i,t-h}^{(m)}\right)$$
(13)

Our third forecast fully exploits the daily return information, by using the actual returns from time t - 19 to t - h as the first component of the forecast, as these are part of the information set at time t - h, and then using a 'direct projection' method to obtain a forecast for the remaining *h*-day return based on the 1-month return available at time t - h. Specifically:

$$\hat{r}_{i,t-h}^{\text{Direct}} = \sum_{j=h}^{19} r_{i,t-j} + \hat{\beta}_0^{(h)} + \hat{\beta}_1^{(h)} r_{i,t-h}^{(m)}$$
(14)

⁹ In the forecast literature, this is commonly referred to as a 'fixed event' forecast design; see Nordhaus (1987) for an early analysis of such problems.

¹⁰ In a preliminary version of the paper we used an earlier vintage of the DataQuick database that ended in June 2009, which is how we chose this particular sample-split point. That preliminary version of the paper did not consider any out-of-sample comparisons, and so the results presented here are close to 'true', rather than 'pseudo', out-of-sample.

where $\beta_0^{(h)}$ and $\beta_1^{(h)}$ are estimated from the projection:

$$\sum_{j=0}^{h-1} r_{i,t-j} = \beta_0^{(h)} + \beta_1^{(h)} r_{i,t-h}^{(m)} + u_{i,t}$$
(15)

Finally, we consider a forecast based on the HAR-X-GARCH-CCC model presented in the previous section. Like the third forecast, this forecast uses the actual returns from time t - 19 to t - h as the first component, and then iterates the expression for the conditional daily mean in equation (5) forward to get forecasts for the remaining h days:

$$\hat{r}_{i,t-h}^{\text{HAR}} = \sum_{j=h}^{19} r_{i,t-j} + \sum_{j=0}^{h-1} \hat{E}_{t-h} \left[r_{i,t-j} \right]$$
(16)

Given the construction of the target variable, we expect the latter three forecasts ('Interp', 'Direct', 'HAR') all to beat the 'Mthly' forecast for all horizons less than 20 days. If intra-monthly returns have dynamics that differ from those of monthly returns, then we expect the latter two forecasts to beat the 'Interp' forecast. Finally, if the HAR-X-GARCH-CCC model presented in the previous section provides a better description of the true dynamics than a simple direct projection, then we would expect the fourth forecast to beat the third.

Figure 5 shows the resulting root mean squared errors (RMSEs) for the four forecasts as a function of the forecast horizon, when evaluated over the July 2009 to September 2012 out-of-sample period. The first striking, though not surprising, feature is that exploiting higher-frequency (intra-monthly) data leads to smaller forecast errors than a forecast based purely on monthly data. All three of the forecasts that use intra-monthly information outperform the model based solely on end-of-month data. The only exception to this is for Las Vegas at the h = 20 horizon, where the HAR model slightly underperforms the monthly model.

Another striking feature of Figure 5 is that the more accurate modeling of the daily dynamic dependencies afforded by the HAR-X-GARCH-CCC model results in lower RMSEs across *all* forecast horizons for eight of the 10 cities. For San Francisco and Las Vegas the direct projection forecasts perform essentially as well as the HAR forecasts, and for Denver and Los Angeles the improvement of the HAR forecast is small (but positive for all horizons). For some of the cities (Boston, Miami and Washington, DC, in particular) the improvements are especially dramatic over longer horizons.

The visual impression from Figure 5 is formally underscored by Diebold–Mariano tests, reported in Table IV. Not surprisingly, the HAR forecasts significantly outperform the monthly forecasts for horizons of 1, 5 and 10 days, for all 10 cities and the composite index. At the 1-month horizon, a tougher comparison for the model, the HAR forecasts are significantly better than the monthly model forecasts for four out of 10 cities, as well as the composite index, and are never significantly beaten by the monthly model forecasts. Almost identical conclusions are drawn when comparing the HAR forecasts to the 'interpolation' forecasts, supporting the conclusion that the availability of daily data clearly holds the promise of more accurate forecasts, particularly over shorter horizons, but also even at the monthly level.

The bottom row of each panel in Table IV compares the HAR forecasts with those from a simple direct projection model. Such forecasts have often been found to perform well in comparison with 'iterated' forecasts from more complicated dynamic models. By contrast, the Diebold–Mariano tests reported here suggest that the more complicated HAR forecasts generally perform better than the direct projection forecasts. For no city–horizon pair does the direct projection forecast lead to significantly lower out-of-sample forecast RMSE than the HAR forecasts, while for many city–horizon pairs the

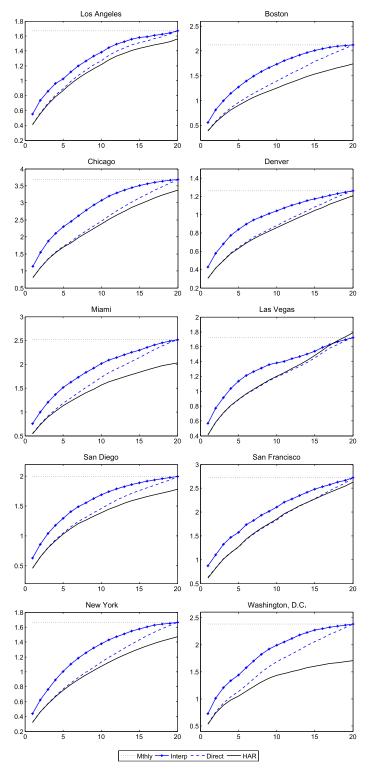


Figure 5. Forecast RMSEs as a function of forecast horizon (1–20 days)

			1				and ty. Drouth internation torcast comparison was				
	Composite	Composite Los Angeles	Boston	Chicago	Denver	Miami	Las Vegas	San Diego	San Francisco	New York	Washington, DC
Panel A: One day ahead $(h = 1)$ Mthly vs. HAR9.240Interp vs. HAR8.707Direct vs. HAR1.599	v ahead (h = 1 9.240 8.707 1.599) 8.337 10.171 1.381	7.378 7.623 2.943	10.060 6.242 -0.176	9.845 9.249 1.224	8.680 11.415 2.785	9.981 8.569 0.126	9.929 10.786 -0.276	8.067 7.865 3.139	8.981 8.609 -0.012	9.142 10.293 2.173
Panel B: One week ahead $(h = 5)$ Mthly vs. HAR4.956Interp vs. HAR4.071Direct vs. HAR4.495	ek ahead (h = 4.956 4.071 4.495	5) 4.458 2.964 1.200	3.876 4.856 3.580	5.412 5.466 1.514	5.126 6.724 1.141	5.087 5.882 2.669	6.682 4.501 -0.298	6.581 4.761 0.768	4.258 5.349 -0.373	5.268 4.588 0.562	4.981 5.304 3.212
Panel C: Two weeks ahead $(h = 10)$ Mthly vs. HAR4.544Interp vs. HAR4.372Direct vs. HAR5.668	eks ahead (h = 4.544 4.372 5.668	10) 2.751 1.478 0.828	3.799 3.617 3.567	6.647 4.586 2.640	4.343 4.042 0.763	4.078 3.333 2.585	5.204 2.489 -0.214	5.847 3.598 1.342	3.453 2.954 -0.381	5.261 2.973 0.964	4.392 3.798 3.563
Panel D: One month ahead $(h = 20)$ Mthly vs. HAR — Interp vs. HAR 6.762 Direct vs. HAR —	mth ahead (h = 	= 20) 	3.553	4.117	0.830	2.211	-0.511		0.941		4.268
<i>Note:</i> The table reports the Diebold–Mariano test statistics for equal predictive accuracy against the alternative that the HAR forecast outporties and Direct. The test statistics are asymptotically standard Normal under the null of equal predictive accuracy. The tests are based on the September 2012. The Mthly, Interp and Direct models are all identical when $h = 20$, so only one set of test statistics are reported in Panel D	ports the Diebo The test statistic The Mthly, Inter	ld–Mariano test s are asymptotic p and Direct mo	statistics fo ally standar dels are all i	r equal predi d Normal unc dentical whe	ctive accura der the null $c_{\rm n}$ n $h = 20$, s	cy against th of equal prec o only one s	ie alternative t lictive accurac et of test statis	hat the HAR for y. The tests are ttics are reporte	orecast outperform based on the out-o d in Panel D.	is the other thr of-sample perio	<i>Note</i> : The table reports the Diebold–Mariano test statistics for equal predictive accuracy against the alternative that the HAR forecast outperforms the other three forecasts: Mthly, Interp and Direct. The test statistics are asymptotically standard Normal under the null of equal predictive accuracy. The tests are based on the out-of-sample period from July 2009 to September 2012. The Mthly, Interp and Direct models are all identical when $h = 20$, so only one set of test statistics are reported in Panel D.

Table IV. Diebold-Mariano forecast comparison tests

 $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

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J. Appl. Econ. **31**: 1005–1025 (2016) DOI: 10.1002/jae reverse is true. In particular, for Boston, Miami and Washington, DC, the HAR forecasts significantly beat the direct projection forecasts across all four horizons, and for the composite index this is true for all but the shortest horizon.

6. CONCLUSION

We present a set of new *daily* house price indices for 10 major US Metropolitan Statistical Areas spanning the period from June 2001 to September 2012. The indices are based on the repeat-sales method of Shiller (1991), and use a comprehensive database of several million publicly recorded residential property transactions. We demonstrate that the dynamic dependencies in the new daily housing price series closely mimic those of other financial asset prices, and that the dynamics, along with the cross-city correlations, are well described by a standard multivariate GARCH-type model. We find that this simple daily model allows for the construction of improved daily, weekly and monthly housing price index forecasts compared to the forecasts based solely on monthly price indices.

The new 'high frequency' house price indices developed here open the possibility for many other applications. Most directly, by providing more timely estimates of movements in the housing market, the daily series should be of immediate interest to policymakers and central banks. Combining the daily house price series with other daily estimates of economic activity may also afford better and more up-to-date insights into changes in the macro economy more broadly. We leave further work along these liens for future research.

ACKNOWLEDGEMENTS

We would like to thank the Editor (M. Hashem Pesaran) and two anonymous referees for their helpful comments. We would also like to thank Pat Bayer, Jia Li, George Tauchen and seminar participants at the 2013 NBER-NSF Time Series Conference in Washington, DC, the 2013 North American Summer Meeting of the Econometric Society in Los Angeles and the Duke Financial Econometrics Lunch Group for their many helpful comments and suggestions. Bollerslev also gratefully acknowledges the support from an NSF grant to the NBER and CREATES funded by the Danish National Research Foundation (DNRF78).

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