

Dynamic Copula Models and High Frequency Data

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26 August 2014

S.A.1 Co-jumps and realized correlation

This appendix describes how we obtain an estimate of the continuous and jump decomposition of realized correlation. This decomposition is based on Bollerslev, Li and Todorov (2012). First, define Time-of-Day (TOD) volatility as the ratio of the diffusive variation over each five-minute interval relative to the average for the whole day.

$$TOD_i = \frac{\frac{1}{T} \sum_{t=1}^T r_{i,t}^2 \mathbf{1} \{ |r_{i,t}| \leq \tau \sqrt{BV_t \wedge RV_t} \times m^{-0.49} \}}{\frac{1}{mT} \sum_{t=1}^T \sum_{j=1}^m r_{j,t}^2 \mathbf{1} \{ |r_{j,t}| \leq \tau \sqrt{BV_{s/n} \wedge RV_{s/n}} \times m^{-0.49} \}}, \quad i = 1, 2, \dots, m \quad (2)$$

where m is the number of intra-daily observations, τ is a tuning parameter (set to 2.5 as in Bollerslev et al. (2012)), BV_t is Barndorff-Nielsen and Shephard's (2006) bipower variation estimator, and \wedge refers to the minimum between two quantities. The the threshold used to identify jump returns is:

$$\alpha_i = \tau \sqrt{(BV_{i/n} \wedge RV_{i/n}) TOD_{i-[i/n]n}} \text{ where } i = 1, \dots, mT \quad (3)$$

Bollerslev et al. (2012) identify the time intervals that include a realized jump as those for which $|r_{i,t}| \geq \alpha_i m^{-0.49}$. If a given time interval is identified to contain a jump for both stock A and stock B , and the jumps are in the same direction, then the interval is considered to contain a “co-jump.” Then define the “non-jump” product of returns as:

$$\widehat{r_{t,j}^A r_{t,j}^B} = \begin{cases} 0, & \text{if "co-jump" detected} \\ r_{t,j}^A r_{t,j}^B, & \text{otherwise} \end{cases} \quad (4)$$

The realized covariance that excludes intervals with co-jumps is given by:

$$\widehat{RCov}_t^{AB} = \sum_{j=1}^m \widehat{r_{t,j}^A r_{t,j}^B}. \quad (5)$$

The continuous part of realized correlation is defined as

$$RCorr_t^{(CTS)} = \frac{\widehat{RCov}_t^{AB}}{\sqrt{RV_t^A RV_t^B}}$$

and the contribution of the co-jumps to total realized correlation is given by:

$$RCorr_t^{(JMP)} = RCorr_t - RCorr_t^{(CTS)} \quad (6)$$

References

- [1] Bandorff-Nielsen, O.E., and N. Shephard, 2006, "Econometrics of Testing for Jumps in Financial Economics Using Bipower Variation", *Journal of Financial Econometrics*, 4, 1–30.
- [2] Bollerslev T., S.Z. Li and V. Todorov, 2012, "Jump Tails, Extreme Dependencies and the Distribution of Stock Returns", *Journal of Econometrics*, forthcoming.

S.A.2 Additional tables and figures

Table A.1. GAS Specification

	Microsoft and Apple		Exxon and Chevron		Celgene and J&J				
	Normal	R. Gumbel Student's t	Normal	R. Gumbel Student's t	Normal	R. Gumbel Student's t			
ω	0.241 (0.137)	-0.187 (0.084)	0.225 (0.092)	0.182 (0.370)	0.041 (0.055)	0.143 (0.291)	0.002 (0.007)	-0.099 (0.028)	0.002 (0.008)
α	0.089 (0.027)	0.148 (0.063)	0.120 (0.027)	0.098 (0.044)	0.132 (0.047)	0.120 (0.053)	0.023 (0.013)	0.201 (0.044)	0.024 (0.015)
β	0.752 (0.142)	0.789 (0.106)	0.783 (0.109)	0.921 (0.158)	0.888 (0.140)	0.938 (0.124)	0.996 (0.012)	0.946 (0.016)	0.996 (0.014)
ν^{-1}			0.116 (0.018)			0.137 (0.023)			0.042 (0.019)
log L	312.620	319.720	341.350	1474.20	1463.70	1526.00	114.650	103.100	117.340

Notes: This table presents the estimated parameters from the GAS model presented in equation (3) of the main paper, for the copula of three pairs of assets.

Table A.2: GRAS with realized correlation (1 min)

	<i>Microsoft and Apple</i>		<i>Exxon and Chevron</i>		<i>Celgene and J&J</i>				
	Normal	R. Gumbel	Student's t	Normal	R. Gumbel	Student's t	Normal	R. Gumbel	Student's t
ω	0.162 (2.149)	-0.392 (0.188)	0.143 (0.093)	0.712 (2.595)	-0.049 (0.063)	0.649 (0.292)	0.006 (0.219)	-0.025 (0.568)	0.006 (0.043)
α	0.067 (2.479)	0.100 (0.063)	0.111 (0.038)	0.148 (2.017)	0.165 (0.045)	0.175 (0.042)	0.029 (0.062)	0.057 (0.084)	0.031 (0.026)
β	0.666 (0.299)	0.758 (0.128)	0.707 (0.141)	0.578 (0.285)	0.618 (0.192)	0.599 (0.166)	0.967 (0.168)	0.989 (0.251)	0.971 (0.148)
ν^{-1}			0.113 (0.020)			0.131 (0.021)			0.040 (0.023)
γ_{RC}	0.529 (2.973)	0.566** (0.275)	0.525** (0.245)	0.548 (2.281)	0.407** (0.204)	0.606*** (0.242)	0.090 (0.463)	0.048 (0.974)	0.077 (0.358)
log L	323.50	330.80	352.48	1502.80	1486.10	1548.30	118.53	114.32	120.86
$p\text{-val}_{GAS}$	0.53	0.04	0.04	0.54	0.06	0.02	0.96	0.83	0.85

Notes: This table presents the estimated parameters from the GRAS model presented in equation (4) of the main paper, for the copula of three pairs of assets. The coefficient on realized correlation (based on 1-minute sampling) is denoted γ_{RC} . Estimates of γ_{RC} that are significant at the 1%, 5%, 10% level are denoted with 3, 2 and 1 asterisks respectively. The bottom row of this table presents the p -value from a test that the realized measure is equal to zero, and thus that the GRAS model simplifies to a GAS model.

Table A.3: GRAS with realized correlation (10 min)

	<i>Microsoft and Apple</i>		<i>Exxon and Chevron</i>		<i>Celgene and J&J</i>				
	Normal	R. Gumbel	Student's t	Normal	R. Gumbel	Student's t	Normal	R. Gumbel	Student's t
ω	0.019 (0.123)	-0.411 (0.182)	0.074 (0.098)	0.384 (0.197)	-0.177 (0.066)	0.305 (0.167)	0.002 (0.013)	-0.151 (0.253)	0.002 (0.014)
α	0.025 (0.033)	0.080 (0.063)	0.079 (0.040)	0.129 (0.032)	0.137 (0.041)	0.145 (0.040)	0.024 (0.019)	-0.016 (0.069)	0.024 (0.019)
β	0.863 (0.173)	0.777 (0.129)	0.784 (0.115)	0.646 (0.128)	0.703 (0.113)	0.679 (0.118)	0.954 (0.042)	0.927 (0.101)	0.959 (0.040)
ν^{-1}			0.111 (0.021)			0.136 (0.021)			0.038 (0.023)
γ_{RC}	0.286** (0.156)	0.527*** (0.195)	0.376*** (0.113)	0.674*** (0.204)	0.454*** (0.146)	0.687*** (0.208)	0.123* (0.095)	0.226 (0.390)	0.111* (0.086)
log L	325.01	330.58	350.71	1504.00	1487.90	1547.30	121.01	115.19	122.98
$p\text{-val}_{GAS}$	0.09	0.03	0.01	0.00	0.00	0.00	0.10	0.90	0.10

Notes: This table presents the estimated parameters from the GRAS model presented in equation (4) of the main paper, for the copula of three pairs of assets. The coefficient on realized correlation (based on 10-minute sampling) is denoted γ_{RC} . Estimates of γ_{RC} that are significant at the 1%, 5%, 10% level are denoted with 3, 2 and 1 asterisks respectively. The bottom row of this table presents the p -value from a test that the realized measure is equal to zero, and thus that the GRAS model simplifies to a GAS model.

Table A.4: GRAS with realized correlation (15 min)

	<i>Microsoft and Apple</i>		<i>Exxon and Chevron</i>		<i>Celgene and J&J</i>				
	Normal	R. Gumbel	Student's t	Normal	R. Gumbel	Student's t	Normal	R. Gumbel	Student's t
ω	0.009 (0.142)	-0.341 (0.166)	0.129 (0.110)	0.437 (0.243)	-0.209 (0.096)	0.325 (0.195)	0.002 (0.020)	-0.099 (0.085)	0.002 (0.019)
α	0.014 (0.034)	0.091 (0.061)	0.095 (0.041)	0.135 (0.035)	0.147 (0.047)	0.154 (0.044)	0.021 (0.024)	0.065 (0.042)	0.022 (0.025)
β	0.926 (0.171)	0.795 (0.129)	0.753 (0.117)	0.619 (0.165)	0.648 (0.168)	0.672 (0.153)	0.939 (0.063)	0.963 (0.036)	0.945 (0.056)
ν^{-1}			0.107 (0.020)			0.136 (0.024)			0.037 (0.024)
γ_{RC}	0.157* (0.111)	0.393*** (0.171)	0.324*** (0.100)	0.669*** (0.234)	0.523*** (0.206)	0.670*** (0.243)	0.158* (0.111)	0.173* (0.131)	0.144* (0.109)
log L	322.33	327.38	347.77	1499.30	1485.90	1545.50	122.49	116.30	124.38
$p\text{-val}_{GAS}$	0.08	0.03	0.00	0.00	0.00	0.01	0.07	0.09	0.09

Notes: This table presents the estimated parameters from the GRAS model presented in equation (4) of the main paper, for the copula of three pairs of assets. The coefficient on realized correlation (based on 15-minute sampling) is denoted γ_{RC} . Estimates of γ_{RC} that are significant at the 1%, 5%, 10% level are denoted with 3, 2 and 1 asterisks respectively. The bottom row of this table presents the p -value from a test that the realized measure is equal to zero, and thus that the GRAS model simplifies to a GAS model.

Table A.5: GRAS with transformed realized correlation

	<i>Microsoft and Apple</i>		<i>Exxon and Chevron</i>		<i>Celgene and J&J</i>				
	Normal	R. Gumbel	Student's t	Normal	R. Gumbel	Student's t	Normal	R. Gumbel	Student's t
ω	0.116 (0.157)	-0.453 (0.258)	0.121 (0.116)	0.608 (0.270)	-0.087 (0.060)	0.632 (0.230)	0.014 (0.049)	-0.028 (0.581)	0.012 (0.031)
α	0.050 (0.035)	0.084 (0.064)	0.088 (0.038)	0.138 (0.036)	0.153 (0.046)	0.172 (0.043)	0.026 (0.033)	0.055 (0.101)	0.027 (0.032)
β	0.679 (0.262)	0.733 (0.193)	0.692 (0.184)	0.600 (0.177)	0.624 (0.178)	0.549 (0.171)	0.847 (0.142)	0.989 (0.240)	0.873 (0.114)
ν^{-1}			0.112 (0.023)			0.134 (0.030)			0.036 (0.023)
γ_{RC}	0.232** (0.129)	0.251*** (0.118)	0.235*** (0.094)	0.205*** (0.082)	0.150*** (0.059)	0.268*** (0.084)	0.183* (0.113)	0.024 (0.403)	0.153* (0.116)
log L	326.80	331.55	353.03	1505.30	1489.20	1550.00	120.28	114.71	122.04
$p\text{-val}_{GAS}$	0.07	0.04	0.02	0.02	0.02	0.00	0.05	0.82	0.09

Notes: This table presents the estimated parameters from the GRAS model presented in equation (4) of the main paper, for the copula of three pairs of assets. The coefficient on realized correlation, transformed using the arc tan function, is denoted γ_{RC} . Estimates of γ_{RC} that are significant at the 1%, 5%, 10% level are denoted with 3, 2 and 1 asterisks respectively. The bottom row of this table presents the p -value from a test that the realized measure is equal to zero, and thus that the GRAS model simplifies to a GAS model.

Table A.6: GRAS with daily, weekly, and monthly realized correlations

	<i>Microsoft and Apple</i>		<i>Exxon and Chevron</i>		<i>Celgene and J&J</i>				
	Normal	R. Gumbel Student's t	Normal	R. Gumbel Student's t	Normal	R. Gumbel Student's t			
ω	0.084 (0.113)	-0.375 (0.148)	0.133 (0.094)	0.535 (0.262)	-0.218 (0.108)	0.496 (0.240)	0.002 (0.066)	-0.437 (0.711)	0.001 (0.068)
α	0.044 (0.033)	0.073 (0.057)	0.081 (0.037)	0.151 (0.034)	0.158 (0.047)	0.195 (0.039)	0.025 (0.036)	0.059 (0.102)	0.027 (0.037)
β	0.740 (0.167)	0.764 (0.122)	0.716 (0.113)	0.569 (0.181)	0.565 (0.211)	0.554 (0.179)	0.886 (0.313)	0.843 (0.302)	0.918 (0.311)
ν^{-1}			0.109 (0.020)			0.125 (0.021)			0.035 (0.021)
γ_{RC}	0.620*** (0.252)	0.743*** (0.248)	0.666*** (0.188)	0.735*** (0.278)	0.530*** (0.201)	0.744*** (0.267)	0.454*** (0.217)	1.198** (0.667)	0.350* (0.232)
γ_W	-0.242*** (0.157)	-0.171 (0.202)	-0.215* (0.140)	-0.540*** (0.351)	-0.245 (0.264)	-0.376 (0.315)	-0.312** (0.757)	-1.109 (0.871)	-0.213 (0.714)
γ_M	0.122* (0.143)	-0.043 (0.193)	0.052 (0.141)	0.566* (0.380)	0.353 (0.388)	0.522* (0.356)	0.077 (0.736)	0.465 (1.023)	0.011 (0.713)
log L	330.37	334.53	355.53	1509.90	1490.90	1552.20	121.71	116.48	123.23
$p\text{-val}_{GRAS}$	0.04	0.16	0.04	0.02	0.20	0.03	0.56	0.22	0.78
$p\text{-val}_{GRAS}$	0.10	0.41	0.16	0.10	0.25	0.15	0.58	0.30	0.77

Notes: This table presents the estimated parameters from the GRAS model presented in equation (11) of the main paper, but using realized correlations over a week or a month in place of 5-day and 22-day averages of daily realized correlations.

Table A.7: GRAS with realized correlation, comparison of all pairs

	<i>Apple</i>	<i>Celgene</i>	<i>Chevron</i>	<i>J&J</i>	<i>Exxon</i>	<i>Microsoft</i>
<i>Apple</i>	-	0.16	0.50	0.69	0.08**	0.00***
<i>Celgene</i>	0.10*	-	0.01***	0.02**	0.09**	0.01***
<i>Chevron</i>	0.84	0.05**	-	0.04**	0.00***	0.28
<i>J&J</i>	0.88	0.00***	0.08**	-	0.05**	0.87
<i>Exxon</i>	0.04**	0.00***	0.00***	0.09**	-	0.10*
<i>Microsoft</i>	0.00***	0.02**	0.10*	0.80	0.03**	-

Notes: This table presents the p -value from a test that the realized measure is equal to zero, and thus that the GRAS model simplifies to a GAS model, where the coefficients are significant at the 1%, 5%, 10% level are denoted with 3, 2 and 1 asterisks respectively. The values under the diagonal are the p -values corresponding to the Rotated Gumbel and the upper values correspond to the Student's t specification.

Table A.8: Marginal distribution (Realized GARCH Margins)

	<i>Microsoft</i>	<i>Apple</i>	<i>Exxon</i>	<i>Chevron</i>	<i>Celgene</i>	<i>J&J</i>
<i>Panel A: Conditional variance</i>						
Constant	0.057 (0.001)	0.361 (0.008)	0.163 (0.006)	0.134 (0.004)	0.230 (0.014)	0.082 (0.002)
GARCH	0.600 (0.006)	0.592 (0.004)	0.587 (0.018)	0.640 (0.006)	0.781 (0.005)	0.629 (0.005)
RM	0.444*** (0.007)	0.320*** (0.002)	0.329*** (0.012)	0.301*** (0.004)	0.158*** (0.001)	0.309*** (0.005)
<i>Panel B: Realized Equation</i>						
Constant	0.081 (0.003)	-1.929 (0.317)	-0.688 (0.054)	-0.690 (0.058)	-3.404 (1.314)	-0.142 (0.050)
Variance	0.727 (0.0003)	1.105 (0.010)	1.244 (0.016)	1.197 (0.015)	1.383 (0.028)	1.159 (0.041)
Leverage 1	-0.128 (0.005)	-0.600 (0.047)	-0.314 (0.012)	-0.263 (0.012)	-2.452 (1.323)	-0.157 (0.004)
Leverage 2	0.064 (0.001)	0.920 (0.065)	0.333 (0.007)	0.288 (0.008)	1.923 (0.860)	0.261 (0.007)
log L	12,113	15,217	11,866	12,132	18,679	9,535
<i>Panel C: Skew t density</i>						
DoF	5.483	7.282	12.568	19.559	4.969	6.133
Skew	-0.002	0.038	-0.119	-0.105	0.012	-0.015
<i>Panel D: GoFtests</i>						
<i>KS p-value</i>	0.370	0.363	0.348	0.308	0.685	0.904
<i>CvM p-value</i>	0.337	0.101	0.166	0.245	0.576	0.969

Notes: The first panel presents parameter estimates for the Realized GARCH model of the conditional variance for Hansen, Huang and Shek (2011). The second panel presents parameter estimates for the Realized GARCH model (realized measure equation) of the conditional variance. The third panel presents parameter estimates for Hansenís (1994) skew t density for the standardized residuals. The bottom panel presents simulation-based p-values from tests of the goodness-of-fit of the density specification..

Table A.9: GRAS with realized correlation (and Realized GARCH Margins)

	<i>Microsoft and Apple</i>		<i>Exxon and Chevron</i>		<i>Celgene and J&J</i>				
	Normal	R. Gumbel	Student's t	Normal	R. Gumbel	Student's t	Normal	R. Gumbel	Student's t
ω	0.063 (0.110)	-0.618 (0.333)	0.060 (0.096)	0.560 (0.222)	-0.202 (0.091)	0.501 (0.206)	0.017 (0.040)	-0.096 (0.557)	0.018 (0.041)
α	0.033 (0.037)	0.064 (0.064)	0.067 (0.034)	0.146 (0.034)	0.142 (0.040)	0.151 (0.038)	0.035 (0.029)	0.107 (0.065)	0.037 (0.033)
β	0.659 (0.209)	0.686 (0.200)	0.673 (0.171)	0.561 (0.159)	0.580 (0.176)	0.568 (0.038)	0.817 (0.122)	0.959 (0.243)	0.818 (0.130)
ν^{-1}			0.102 (0.018)			0.103 (0.023)			0.052 (0.033)
γ_{RC}	0.718** (0.333)	0.885** (0.423)	0.733*** (0.288)	0.751*** (0.274)	0.600*** (0.220)	0.826*** (0.291)	0.478** (0.244)	0.175* (0.127)	0.492** (0.281)
log L	337.59	343.56	360.44	1,515.3	1,491.2	1,542.3	125.03	121.94	128.53
$p\text{-val}_{GAS}$	0.020	0.050	0.010	0.020	0.030	0.010	0.090	0.090	0.080

Notes: This table presents the estimated parameters from the GRAS model presented in equation (4) of the main paper, for the copula of three pairs of assets. The coefficient on realized correlation is denoted γ_{RC} . Estimates of γ_{RC} that are significant at the 1%, 5%, 10% level are denoted with 3, 2 and 1 asterisks respectively. The bottom row of this table presents the p -value from a test that the realized measure is equal to zero, and thus that the GRAS model simplifies to a GAS model.

Table A.10: Out-of-sample comparison of density forecasts
Censored likelihood approach

		Lower tail region probability				
		0.01	0.05	0.10	0.25	1.00
<i>Microsoft and Apple</i>						
GRAS vs. Const	Normal	1.847**	2.901***	3.785***	3.557***	1.536*
	R. Gumbel	1.955**	2.488***	3.227***	3.472***	2.000***
	Student's t	2.313***	2.897***	3.833***	3.637***	2.014***
GRAS vs. GAS	Normal	1.311*	1.941**	3.084***	3.268***	1.192
	R. Gumbel	0.734	1.030	1.871**	2.269***	0.997
	Student's t	1.494*	1.606*	2.566***	2.470***	0.932
<i>Exxon and Chevron</i>						
GRAS vs. Const	Normal	4.048***	9.190***	11.232***	9.646***	2.770***
	R. Gumbel	1.021	4.133***	3.709***	3.611***	3.530***
	Student's t	4.627***	9.454***	10.596***	10.090***	3.236***
GRAS vs. GAS	Normal	3.906***	8.362***	9.406***	8.597***	1.081
	R. Gumbel	0.043	2.567***	1.514***	1.026	1.475*
	Student's t	3.798***	7.601***	7.767***	7.930***	1.618*
<i>Celgene and J&J</i>						
GRAS vs. Const	Normal	1.811**	4.823***	7.211***	12.679***	3.100***
	R. Gumbel	2.398***	5.539***	7.307***	10.031***	1.429*
	Student's t	1.966***	5.085***	7.515***	13.008***	2.586***
GRAS vs. GAS	Normal	1.940**	4.776***	6.850***	11.108***	0.623
	R. Gumbel	2.845***	5.648***	7.576***	9.877***	-0.375
	Student's t	2.091***	4.972***	7.096***	11.433***	0.138

Notes: This table presents t -statistics from pair-wise comparisons of the out-of-sample likelihoods of competing density forecasts. We consider five regions of support over which to compare the competing density forecasts: the joint lower 0.01, 0.05, 0.10 and 0.25 tails, as well as the entire support. For a given copula specification (Normal, rotated Gumbel and Student's t) we compare specifications of the dynamics: Constant versus GRAS and GAS versus GRAS. Test statistics that are significant at the 1%, 5%, 10% level (one-sided) are denoted with 3, 2 and 1 asterisks respectively.

Table A.11: Summary Statistics, Forecast portfolio choice

	Microsoft and Apple		Exxon and Chevron		Celgene and J&J	
	<i>in-sample</i>					
Mean	-0.054	0.117	0.022	0.019	0.115	0.017
Std dev	2.358	3.241	1.578	1.472	4.329	1.550
Skewness	-0.232	-0.079	0.115	-0.127	-0.180	-1.088
Kurtosis	10.307	5.573	6.917	4.779	6.939	17.827
Correl (lin/rank)	0.422/0.482		0.776/0.761		0.083/0.142	
	<i>out-sample</i>					
Mean	0.005	0.122	0.022	0.039	0.049	0.002
Std dev	2.015	2.578	1.928	2.073	2.508	1.127
Skewness	0.226	-0.269	0.215	0.264	0.102	0.241
Kurtosis	12.424	7.671	17.549	15.210	9.120	14.588
Correl (lin/rank)	0.463/0.468		0.907/0.851		0.392/0.343	

Table A.12: Management fees and realized portfolio return performance (constrained)

		Relative risk aversion				
		1	3	7	10	20
<i>Microsoft and Apple</i>						
GRAS vs. Const	Normal	-18.960***	-1.349	-0.620	-0.428	-0.217
	R. Gumbel	2.659	1.145	0.457	0.295	0.122
	Student's t	4.493	4.532	1.582	1.106	0.538
GRAS vs. GAS	Normal	-18.514***	-4.479**	-1.775*	-1.240*	-0.610*
	R. Gumbel	2.459	1.166	0.164	0.043	0.000
	Student's t	-1.820	-3.845*	-2.079**	-1.487***	-0.757***
<i>Exxon and Chevron</i>						
GRAS vs. Const	Normal	0.499	-0.677	-0.038	0.039	0.028
	R. Gumbel	-1.724	3.800	1.451	1.004	0.475
	Student's t	0.897	2.403	0.877	0.642	0.314
GRAS vs. GAS	Normal	-5.265	-2.103	-0.693	-0.446	-0.269
	R. Gumbel	-4.483	-2.027	-0.414	-0.309	-0.146
	Student's t	0.368	5.531*	2.612*	1.973*	0.926*
<i>Celgene and J&J</i>						
GRAS vs. Const	Normal	4.487	2.059	0.636	0.370	0.193
	R. Gumbel	9.253	-2.153	-0.864	-0.618	-0.353
	Student's t	1.787	9.148***	3.673***	2.589***	1.255***
GRAS vs. GAS	Normal	4.072	1.773	0.602	0.363	0.199
	R. Gumbel	14.677	0.184	0.086	0.084	0.027
	Student's t	-0.882	3.745*	1.521*	1.090*	0.538*

Notes: This table presents the “management fee,” in basis points per year, that an investor with risk aversion given in the column titles would be willing to pay to switch from the Constant or GAS model to the GRAS model. A block bootstrap is used to ascertain the significance of these fees, and estimates that are significant at the 1%, 5%, 10% level (one-sided) are denoted with 3, 2 a

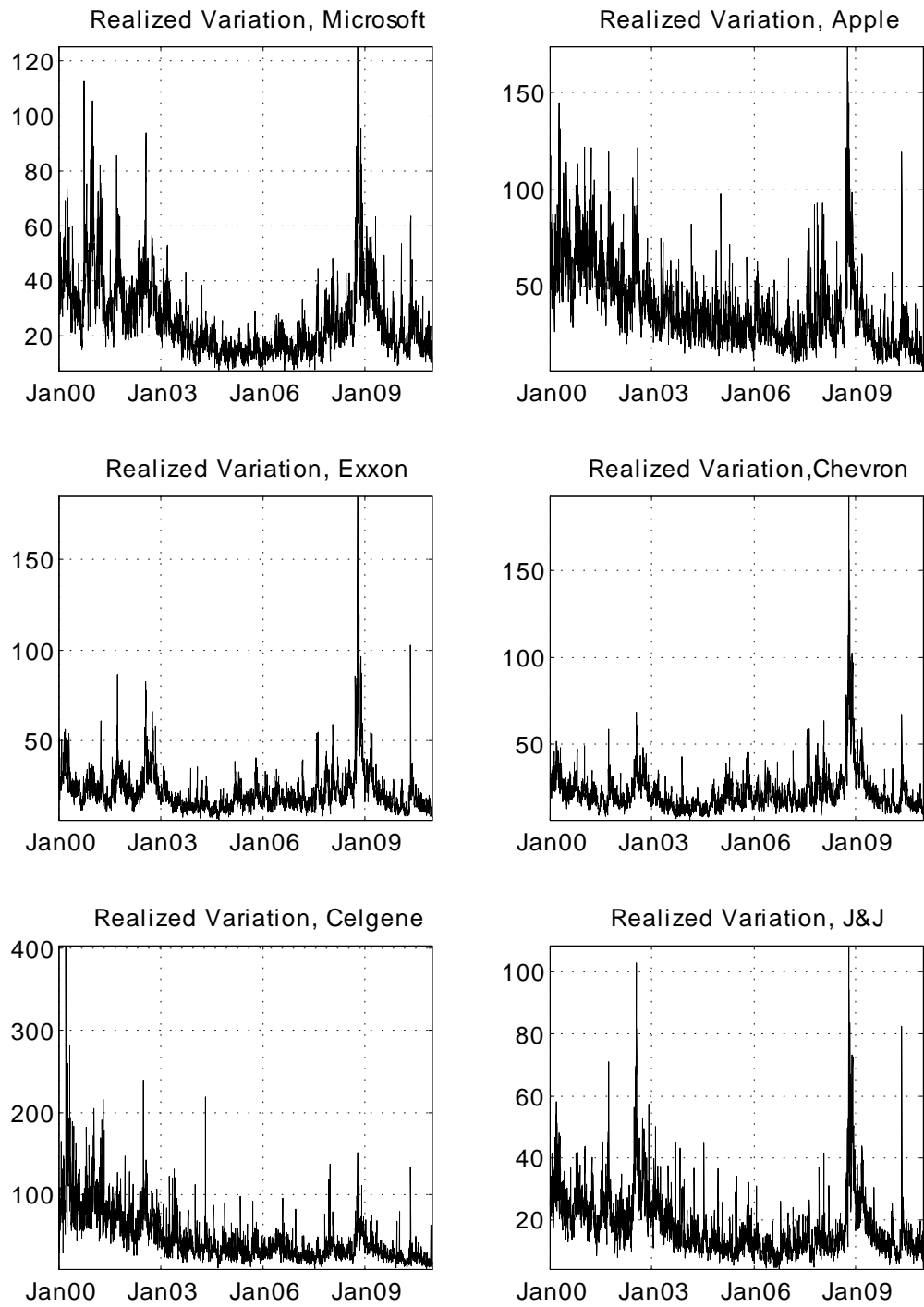


Figure 1: This Figure shows the time series of the annualized 5-minute realized volatility (standard deviation) for six different stocks. 15

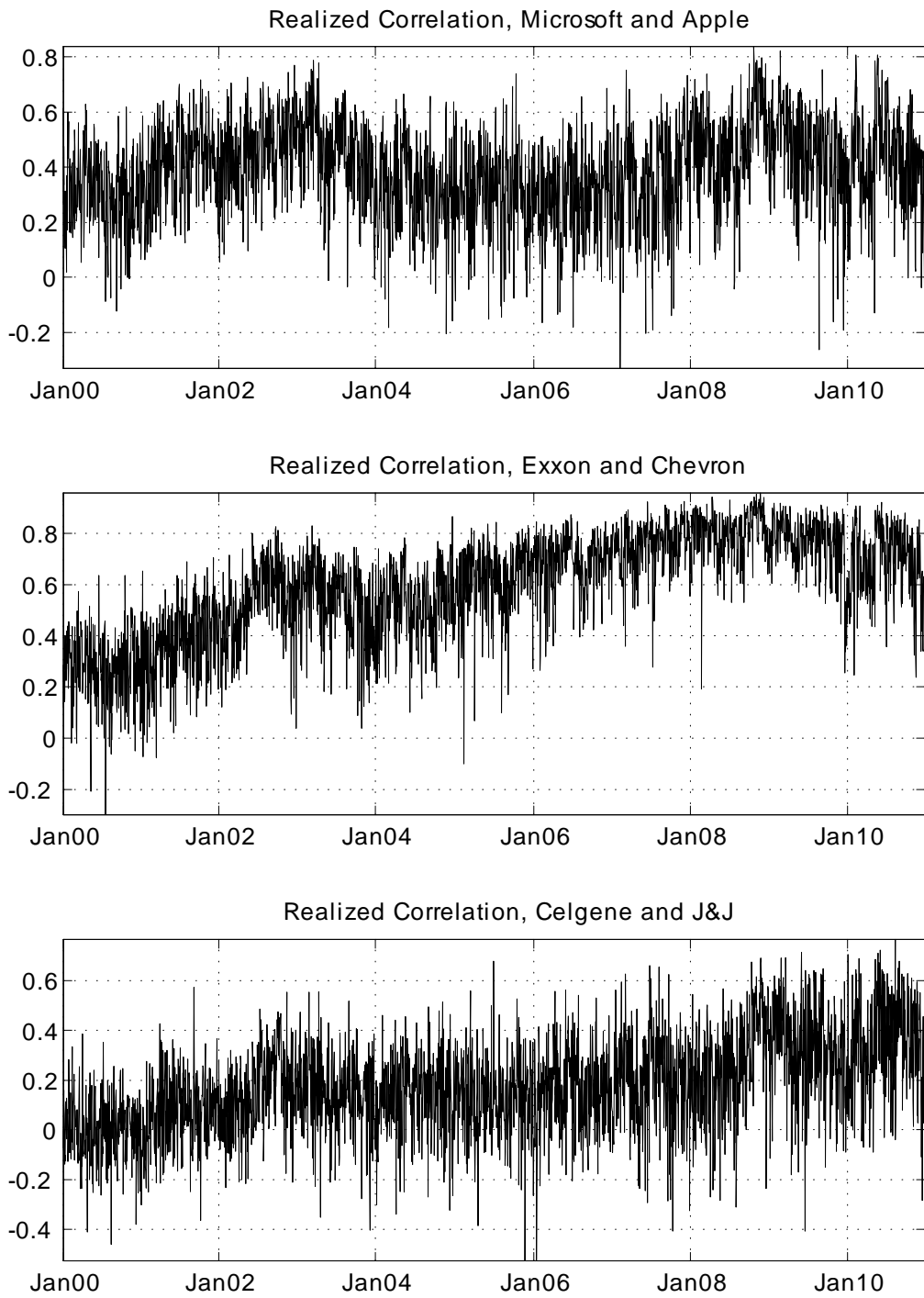


Figure 2: This Figure shows the time series of the 5-minute realized correlation for three different pairs of stocks.

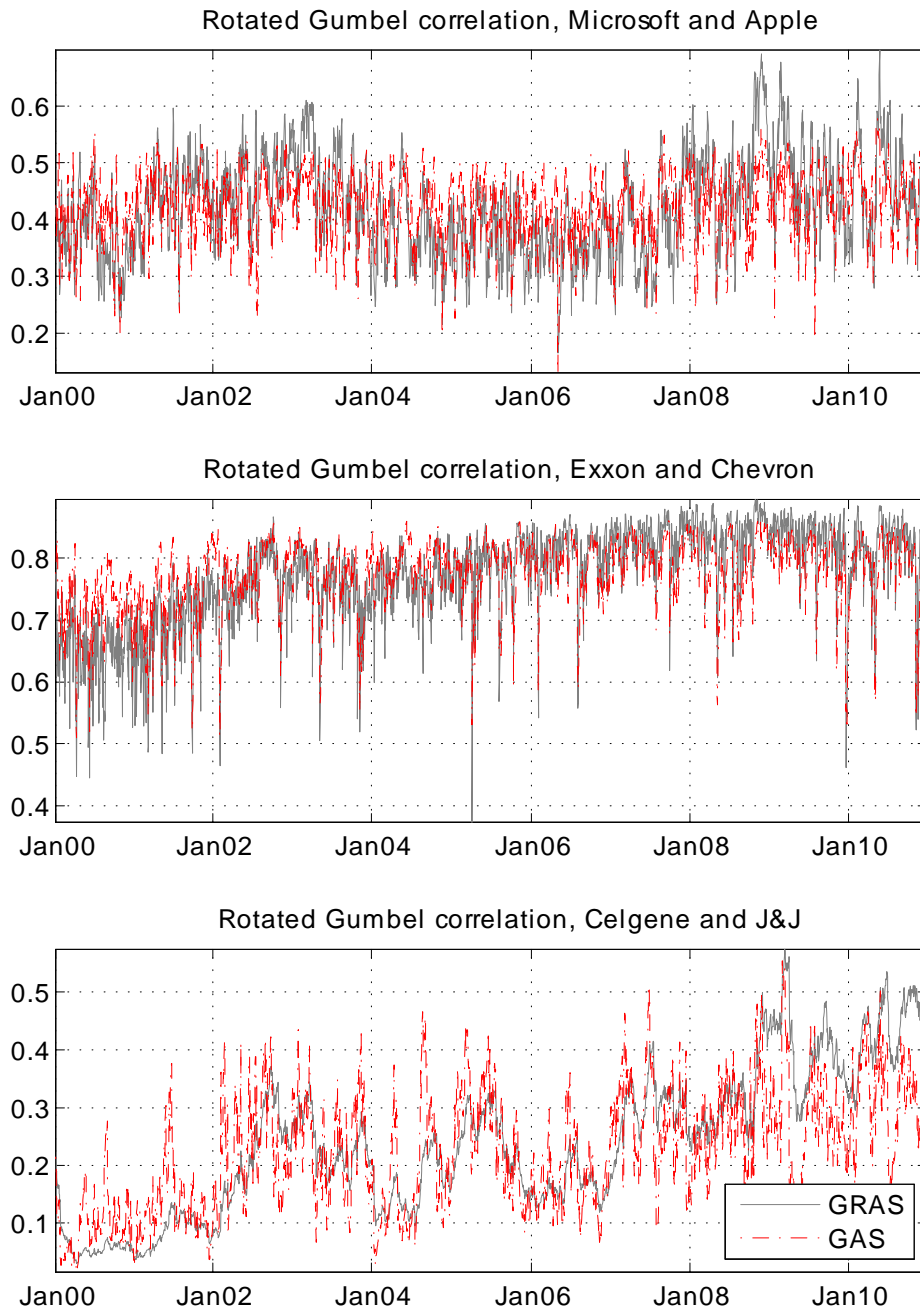


Figure 3: This Figure shows the estimated correlation parameter of the Rotated Gumbel GRAS model (solid line) against the estimated correlation parameter of the Rotated Gumbel GAS model (dotted line) for three different pairs of stocks.

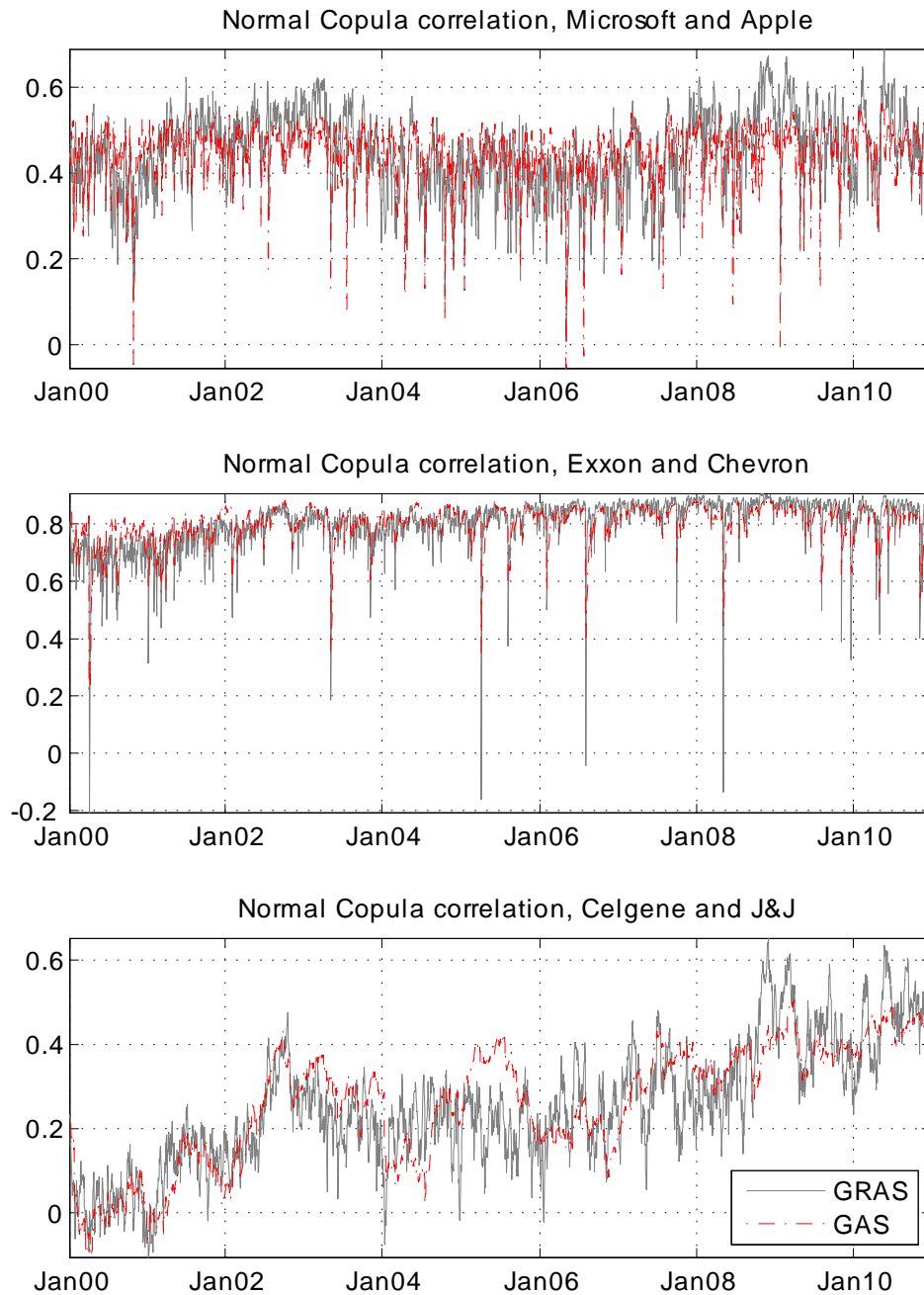


Figure 4: This Figure shows the estimated correlation parameter of the Normal GRAS model (solid line) against the estimated correlation parameter of the Normal GAS model (dotted line) for three different pairs of stocks.

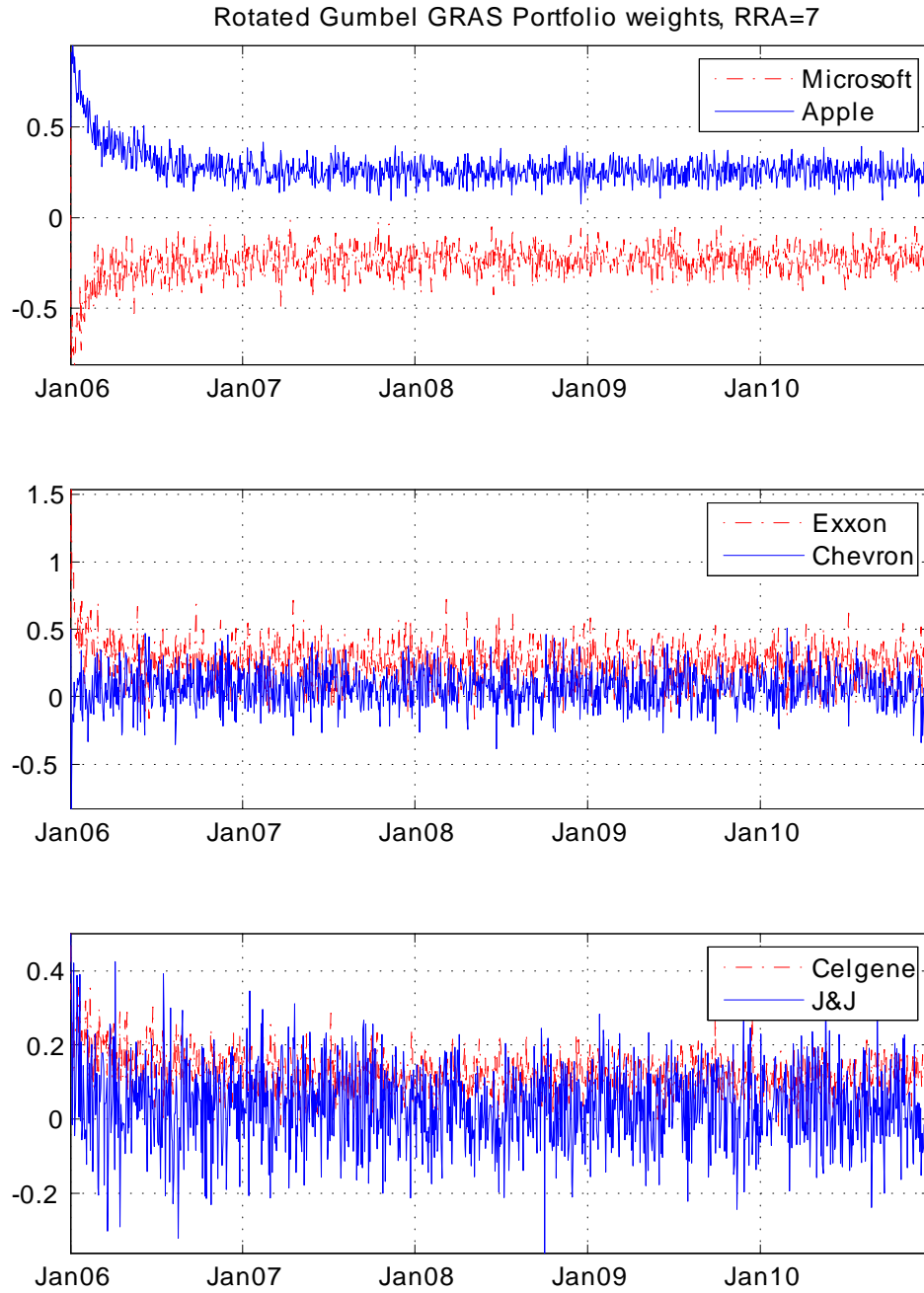


Figure 5: This Figure shows the portfolio weights of the portfolio problem for three pairs of assets for a level of risk aversion of 7 under the Rotated Gumbel GRAS model.

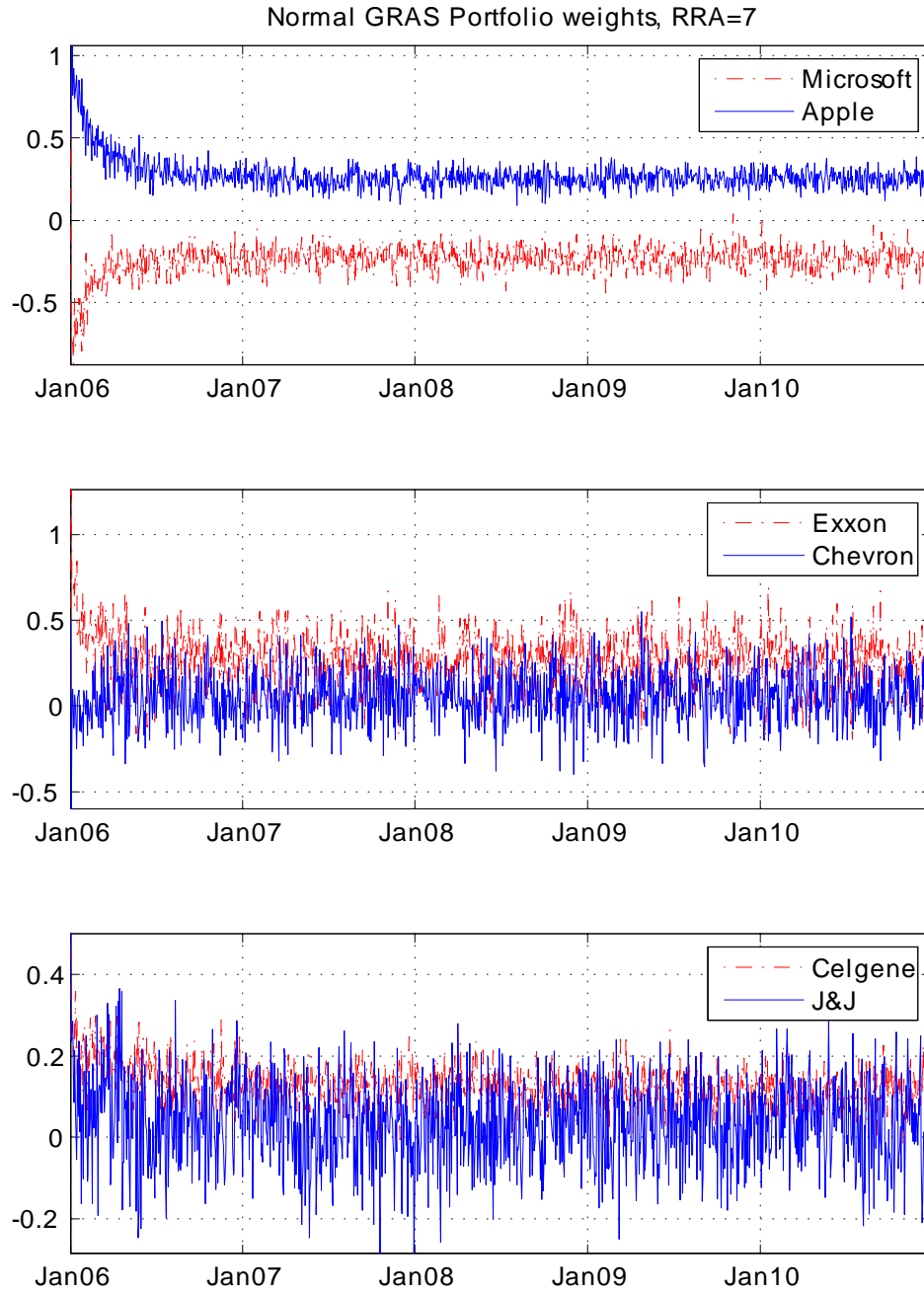


Figure 6: This Figure shows the portfolio weights of the portfolio problem for three pairs of assets for a level of risk aversion of 7 under the Normal GRAS model.

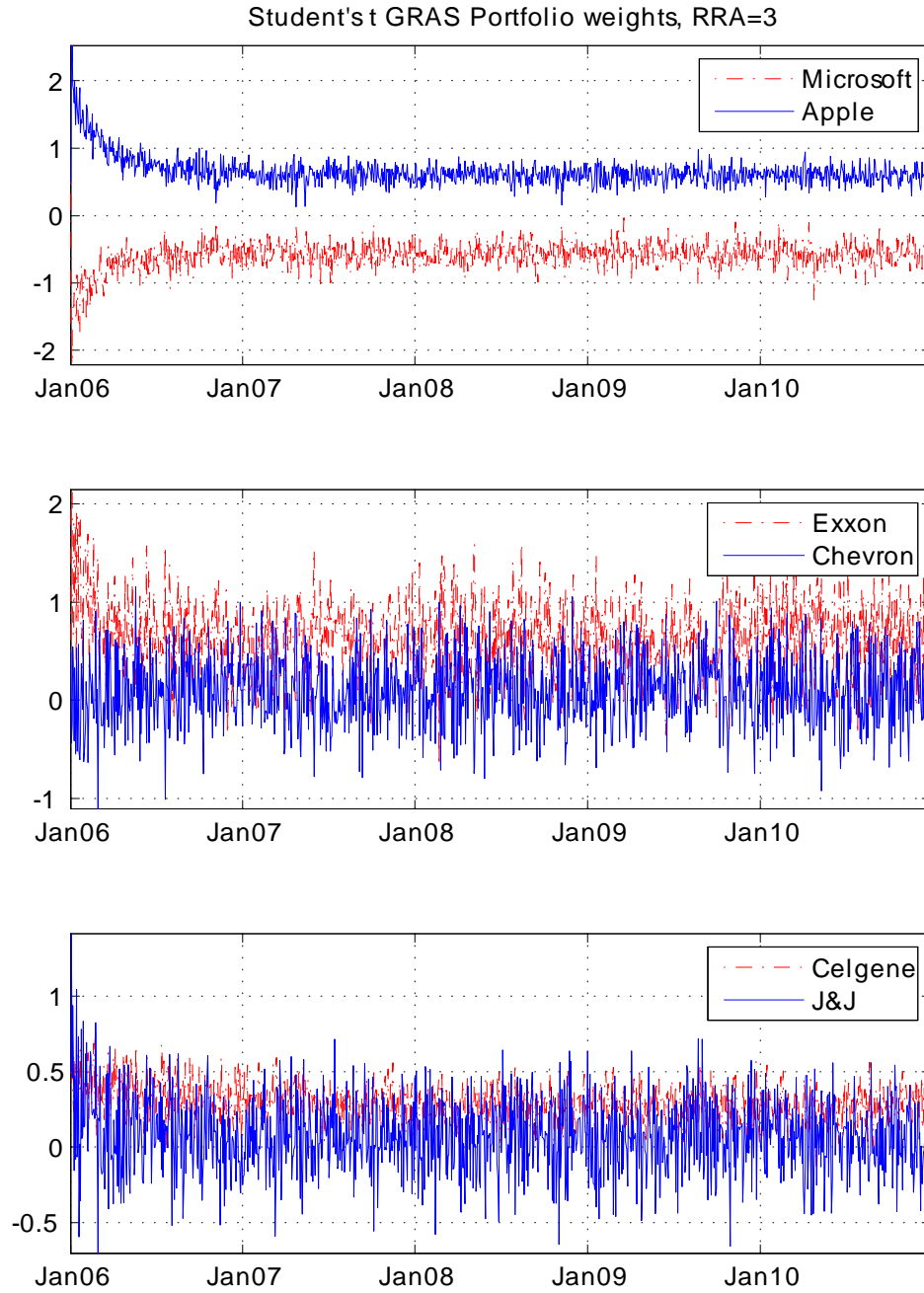


Figure 7: This Figure shows the portfolio weights of the portfolio problem for three pairs of assets for a level of risk aversion of 3 under the Student's t GRAS model.

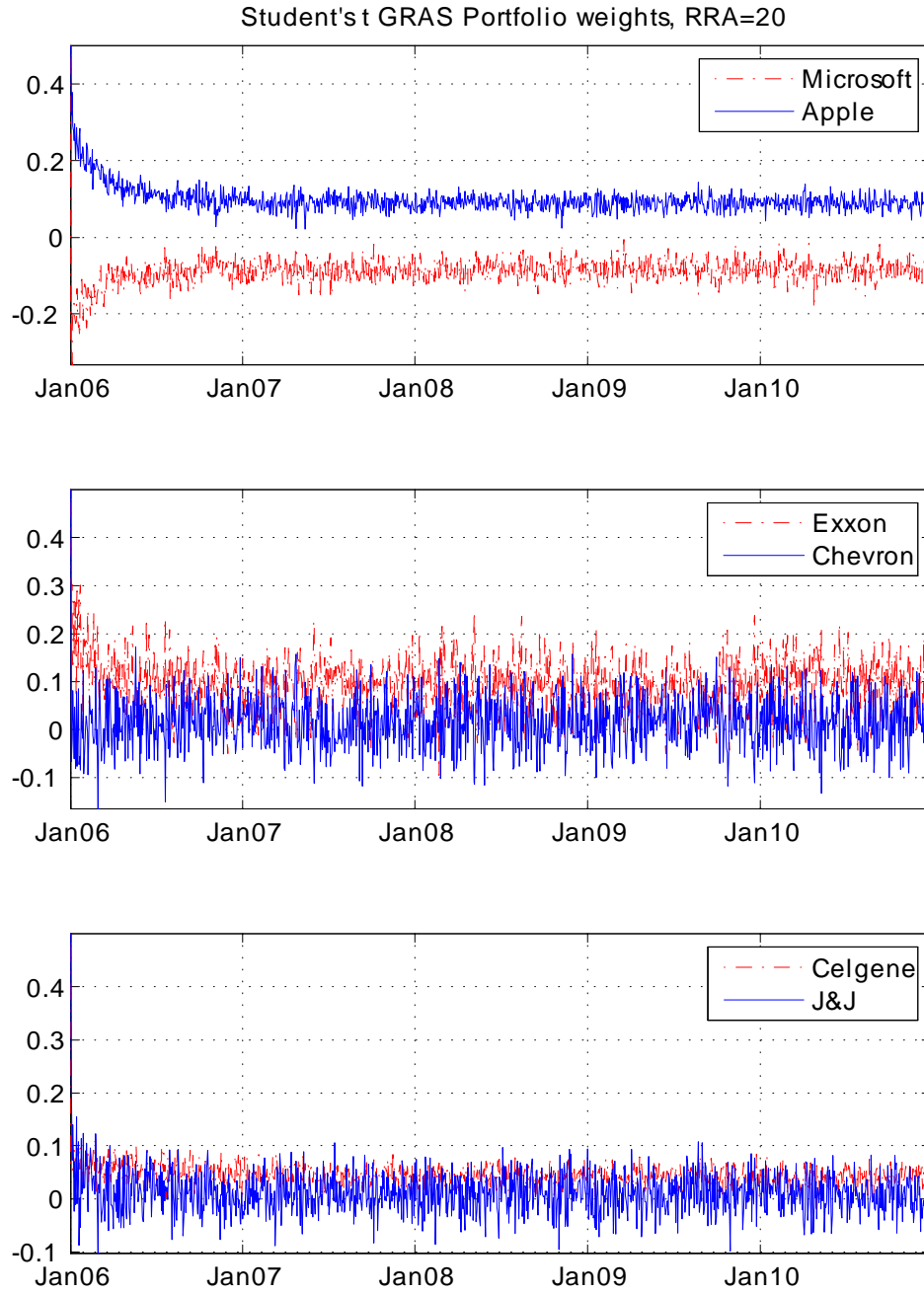


Figure 8: This Figure shows the portfolio weights of the portfolio problem for three pairs of assets for a level of risk aversion of 20 under the Student's t GRAS model.