Modeling and Forecasting (Un)Reliable Realized Covariances for More Reliable Financial Decisions

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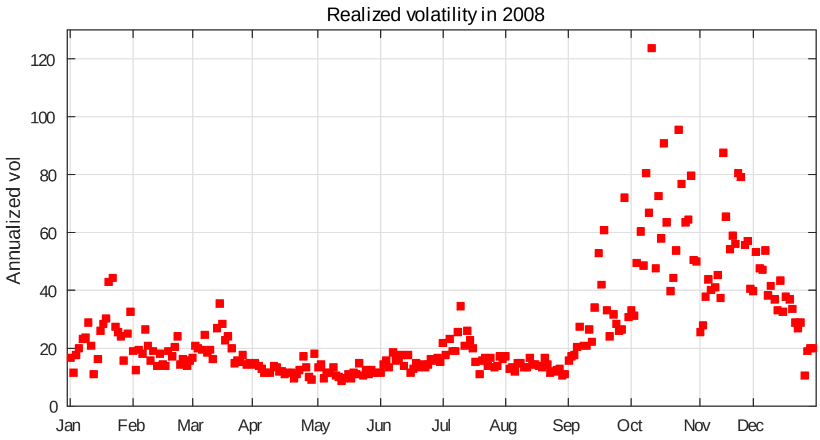
Motivation

- Models for risk and covariation are critical inputs in portfolio decisions and risk management.
  - Trillions of dollars invested and traded on the basis of such models
  - Thousands of research papers developing such models
- So demand is high, but supply is high too. What could possibly be left to discuss?
- We propose exploiting another source of information to improve models, both univariate and multivariate, based on high frequency realized measures

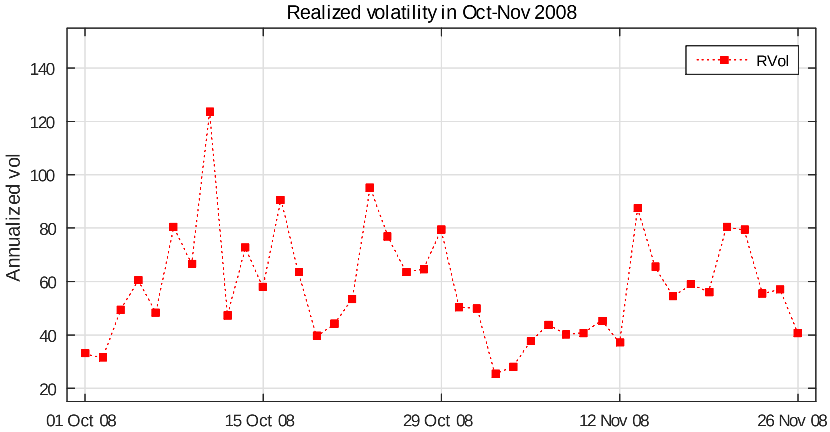
S&P 500 realized volatility in 2008

Annualized std dev ranges from 8.6% to 124%



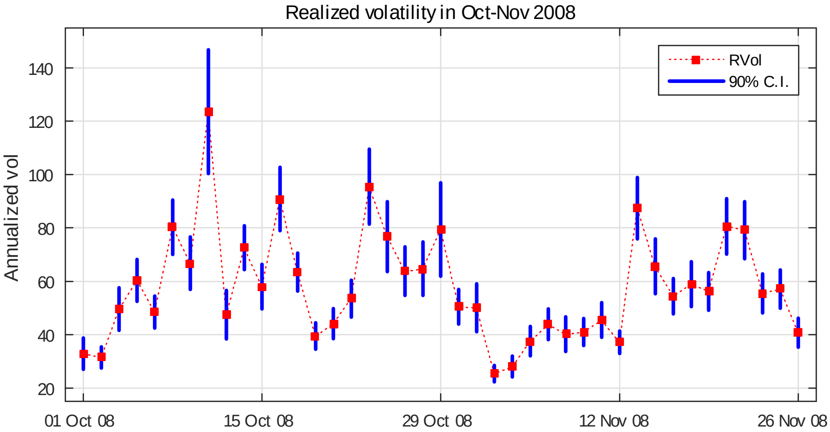
S&P 500 realized volatility in late 2008

Annualized std dev ranges from 25.4% to 124% (peaks on Oct 10)



S&P 500 realized volatility in late 2008, with 90% conf int

Volatility varies, and so does our *ability to estimate* volatility



Measurement errors in financial data I

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  - **Accounting data**: Beaver, Kettler and Scholes (1970, *TAR*), Easton and Monahan (2005, *TAR*), and others.
  - **Hedge fund data**: smoothing (Getmansky, Lo and Makaraov 2004 *JFE*), strategic reporting (Patton Ramadorai and Streatfield 2015 *JF*)
  - **Volatility measures**: Andersen and Bollerslev (1998 *IER*), Barndorff-Nielsen and Shephard (2002, *JRSS-B*)

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- In almost all cases, there is not much we can do:
  - Acknowledge their presence, attempt to infer resulting direction of bias
  - Use an instrumental variable, if one can be found and defended
  - Aggregate (eg, across firms or time) in attempt to average away errors

Measurement errors in financial data II

- Recent work in high frequency econometrics allows us to **directly estimate** degree of measurement error in volatility measures

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  - In some applications, can be used to bias-correct estimators (eg, Andersen, et al. 2005 *Ecta*)

Measurement errors in financial data II

- Recent work in high frequency econometrics allows us to **directly estimate** degree of measurement error in volatility measures
  - In some applications, can be used to bias-correct estimators (eg, Andersen, et al. 2005 *Ecta*)
  - ★ We use it to improve on existing methods for forecasting covariance matrices, and thus decisions based on these covariance matrices

Summary of main results

1. Models that use this information almost always outperform corresponding models that ignore it.
   - Measures of statistical accuracy (in-sample and out-of-sample) are significantly improved using this information
2. Portfolio decisions are significantly improved when using this information
   - Min variance portfolios that use this info have lower turnover and variance
   - "Management fees" for switching to our models range from 50 bps to 9%
3. Our methods substantially outperform existing shrinkage methods that do not exploit information about measurement errors.
   - Ledoit & Wolf (2003, 2004), Jagannathan & Ma (2003), DeMiguel et al. (2009)
4. Lower turnover, from less noisy forecasts, makes daily re-allocation profitable
   - Transaction costs eliminate gains from standard models, but not ours.

Outline

1. Motivation
2. Using measurement error information in predictive models
3. Empirical analysis of U.S. equity returns
   - In-sample and out-of-sample forecasting analysis
   - Portfolio decisions: minimum variance and tracking portfolio construction
   - Different re-balancing frequencies
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Realized measures and measurement errors I

- Consider simple realized variance:

$$RV_t = \sum_{s=1}^{m} r_{st}^2$$

where t denotes a day, s denotes an intra-daily period (eg, a 5-minute period) and m is the number of intra-daily periods (eg, 79).

- Under some assumptions, Barndorff-Nielsen and Shephard (2002, *JRSS-B*) show that

$$\sqrt{m}\,(RV_t - IV_t) \xrightarrow{\mathcal{L}_s} MN\,(0, 2IQ_t) \quad \text{as } m \to \infty$$

where IV is the "integrated variance" and IQ is the integrated quarticity:

$$IV_t = \int_{t-1}^{t} \sigma^2\,(s)\,ds \quad \text{and} \quad IQ_t = \int_{t-1}^{t} \sigma^4\,(s)\,ds$$

Realized measures and measurement errors II

- Critically, BNS (2002) also provide a way to estimate the asymptotic variance, "realized quarticity":

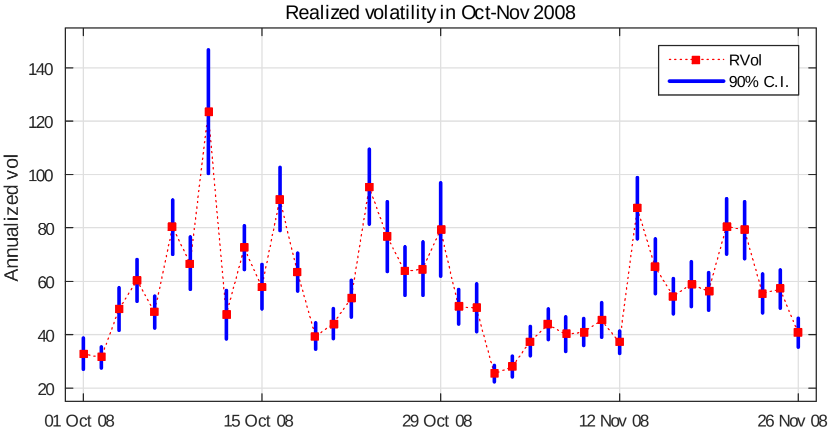
$$RQ_t = \frac{m}{3} \sum_{s=1}^{m} r_{st}^4 \xrightarrow{p} IQ_t \quad \text{as } m \to \infty$$

- With this theory in hand, we can:

1. Estimate volatility (model free) for a given firm on a single day, AND
2. Estimate the *accuracy* of our volatility estimate on each day.

S&P 500 realized volatility in late 2008, with 90% conf int

Volatility varies, and so does our *ability to estimate* volatility



Exploiting the errors

- Consider the simple HAR model of Corsi (2009, *JFEC*):

$$RV_t = \beta_0 + \beta_d RV_{t-1} + \beta_w \overline{RV}_{t-5|t-2} + \beta_m \overline{RV}_{t-22|t-6} + \varepsilon_t$$

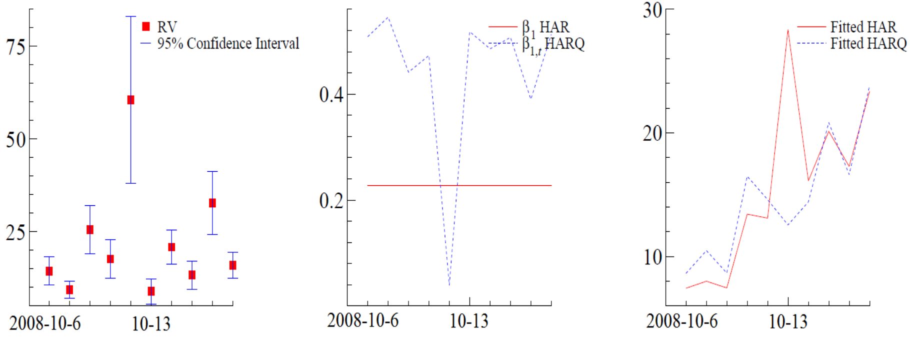
- We know the RHS variables are measured with error, *and* that the measurement error is time-varying.
- Bollerslev, Patton and Quaedvlieg (2015, *JoE*) attempt to capture these features by extending the HAR model to include a "Q" term:

$$\begin{aligned} \beta_{d,t} &= \bar{\beta}_d + \beta_q \widetilde{RQ}_t^{1/2} \\ \widetilde{RQ}_t^{1/2} &\equiv RQ_t^{1/2} - \overline{RQ^{1/2}} \end{aligned}$$

- We expect β_q to be **negative**:
  - When measurement error (captured by RQ) is high, RV gets a lower weight in the forecast ⇒ shrinkage towards the mean
  - When RQ is low, RV gets greater weight ⇒ more accurate information about current level of volatility

Measurement error and weight on RV

HARQ reacts less to RV when measured with greater error



The multivariate problem and portfolio decisions

- We consider a vector of asset returns $\mathbf{r}_t = [r_{1,t}, ..., r_{N,t}]'$, with $(N \times N)$ integrated covariance matrix Σ_t:

$$\Sigma_t = \int_{t-1}^{t} \Sigma(s)\, ds$$

- We consider estimating this matrix using the multivariate "realized kernel" (RK) of Barndorff-Nielsen, Hansen, Lunde and Shephard (2011, *JoE*).

- That paper provides asymptotic distribution theory for this estimator:

$$m^{1/5}\left(\text{vech}RK_t - \text{vech}\Sigma_t\right) \xrightarrow{\mathcal{L}_s} MN\left(\,0\,,\;3.77 \times IQ_t\right)$$

and the (large) matrix IQ_t can be estimated using the methods of Barndorff-Nielsen and Shephard (2004, *Ecta*).

Summarizing multivariate measurement error

- For a collection of N asset returns, the BNS asymptotic covariance will be $\frac{1}{2}N(N+1) \times \frac{1}{2}N(N+1)$, and will have

$$\dim\left\{ \ \text{vech}\left(IQ_t\right) \ \right\} = \frac{1}{8}\left(N^4 + 2N^3 + 3N^2 + 2N\right)$$

unique elements. Eg, for $N = 10$ it is 1540.

- We consider only using the (square-root) *diagonal* elements of the IQ matrix (denoted $\boldsymbol{\pi}_t$)
- This captures (time-varying) measurement in each element of the realized kernel, but does not attempt to exploit estimates of the *covariances* between the measurement errors.

EWMA(Q)

- Firstly, consider the simple exponentially-weighted moving average filter. This can be represented as:

$$\begin{aligned} \mathbf{v}_t &= (1-\alpha)\,\mathbf{v}_{t-1} + \alpha\mathbf{s}_{t-1} \\ \text{where } \mathbf{s}_t &= \text{vech}\,(RK_t) \end{aligned}$$

where α determines the exponential decay rate. Rather than fix this parameter (eg, at 0.03) we estimate it using QML.

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- We extend this model to the following "EWMAQ" model:

$$\begin{aligned} \mathbf{v}_t &= (1-\alpha_{t-1}) \circ \mathbf{v}_{t-1} + \boldsymbol{\alpha}_{t-1} \circ \mathbf{s}_{t-1} \\ \boldsymbol{\alpha}_{t-1} &= \alpha + \alpha_Q \widetilde{\boldsymbol{\pi}}_{t-1} \end{aligned}$$

- We expect $\theta_Q < 0$.

HAR(Q)

- Next we consider extending to exploit measurement error information is the multivariate HAR model due to Chiriac and Voev (2010, *JAE*). Consider the "scalar" version of this model:

$$\begin{aligned} \mathbf{s}_t &= \theta_0 + \theta_d \mathbf{s}_{t-1} + \theta_w \bar{\mathbf{s}}_{t-5|t-2} + \theta_m \bar{\mathbf{s}}_{t-22|t-6} + \varepsilon_t \\ \text{where } \mathbf{s}_t &= \text{vech}\left(RK_t\right) \\ \bar{\mathbf{s}}_{t-h|t-i} &= \frac{1}{h-i+1} \sum_{j=i}^{h} \mathbf{s}_{t-j} \end{aligned}$$

where all coefficients are scalars.

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  where all coefficients are scalars.
- We extend this model to the following "HARQ" model:

$$\begin{aligned} \mathbf{s}_t &= \theta_0 + \boldsymbol{\theta}_{d,t-1} \circ \mathbf{s}_{t-1} + \theta_w \bar{\mathbf{s}}_{t-5|t-2} + \theta_m \bar{\mathbf{s}}_{t-22|t-6} + \varepsilon_t \\ \boldsymbol{\theta}_{d,t-1} &= \bar{\theta}_d + \theta_Q \tilde{\boldsymbol{\pi}}_{t-1} \end{aligned}$$

- We again expect $\theta_Q < 0$.

HAR-DRD(Q)

- Next decompose the covariance matrix into a diagonal matrix of standard deviations and the correlation matrix:

$$S_t = D_t R_t D_t$$

- Oh and Patton (2015, *JoE*) suggested modelling each of the individual variances using a HAR model, and then modelling the correlation matrix using the scalar multivariate HAR of Chiriac and Voev (2010).
  - This allows more flexibility than the scalar MV HAR, but is easy to ensure positive definiteness.
- We incorporate measurement error information into the univarate volatility models (which comprise the D_t matrix)
  - This model has $5N + 4$ parameters compared with 5 for the HARQ model
- We do not attempt to use the information in the correlation model.
  - As correlations are bounded we anticipate less gains there.

HEAVY(Q)

- Finally, Noureldin, Shephard and Sheppard (2012, *JAE*) proposed the multivariate HEAVY model:

$$V_t = \mathbb{E}\left[\mathbf{r}_t \mathbf{r}_t' | \mathcal{F}_{t-1}\right]$$

  is the conditional covariance matrix of the vector of returns. Let $\mathbf{v}_t$ =vech(V_t) . Then set:

$$\mathbf{v}_t = (I - b - a\kappa)\,\lambda_V + b\mathbf{v}_{t-1} + a\mathbf{s}_{t-1}$$

- We extend this to include a measure of estimation error:

$$\begin{aligned}\mathbf{v}_t &= (I - b - \mathbf{a}_{t-1}\kappa)\,\lambda_V + b\mathbf{v}_{t-1} + \mathbf{a}_{t-1} \circ \mathbf{s}_{t-1} \\ \mathbf{a}_{t-1} &= a + a_Q \tilde{\boldsymbol{\pi}}_{t-1}\end{aligned}$$

- We follow Engle, Pakel, Shephard and Sheppard (2014) and estimate this model by composite likelihood.

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Data

- We use data on the SPY ETF and 10 Dow Jones stocks
  - AmEx, Boeing, Chevron, DuPont, GE, IBM, JP Morgan, Coca Cola, MSFT, Exxon
- Sample period: April 1997 to Dec 2013, $T = 5267$.

	Realized Variance		*Realized Quarticity*	
	Mean	**St Dev**	**Mean**	**St Dev**
SPY	0.954	1.165	3.882	13.933
AmEx	3.906	55.941	3.694	12.797
Boeing	2.782	16.425	1.714	5.192
Chevron	1.975	13.185	0.671	1.775
⋮	⋮	⋮	⋮	⋮

- Important for this paper: **quarticity is far from constant** through time.

In-sample estimation results: EWMA

Robust standard errors in parentheses

	RiskMetrics	$\widehat{\text{RiskMetrics}}$	EWMA	EWMAQ
α	0.06	0.013	0.079	0.102
	(–)	(0.002)	(0.003)	(0.004)
α_Q				-0.004
				(0.001)
QLIKE	16.395	15.662	15.408	15.393
Frobenius	14.295	13.892	12.324	12.255

- Using high frequency data improves the EWMA model, and using info on measurement error improves it even further:
- The "Q" variable is strongly significant
- Weight on daily information goes up (on average)

In-sample estimation results: MV HAR

Robust standard errors in parentheses

	HAR	HARQ
θ_d	0.247	0.541
	(0.040)	(0.040)
θ_w	0.410	0.333
	(0.038)	(0.038)
θ_m	0.244	0.113
	(0.038)	(0.038)
θ_Q		-0.043
		(0.018)

- The "Q" variable is strongly significant
- Weight on daily information goes up (on average)

In-sample estimation results: HEAVY

Robust standard errors in parentheses

	HEAVY	HEAVYQ
a	0.106	0.148
	(0.009)	(0.009)
b	0.876	0.825
	(0.004)	(0.004)
a_Q		-0.026
		(0.012)

- The "Q" variable is strongly significant
- Weight on daily information goes up (on average)

Out-of-sample forecast comparisons

- We next consider out-of-sample comparisons of these forecasting models
- Forecasts are based on a rolling window of 1000 days, updated each day
- Compare "std" and "Q" models using Diebold-Mariano (1995, *JBES*)
- Find the *set* of best models using the "Model Confidence Set" (Hansen, et al. 2011, *Ecta*)

Out-of-sample forecast comparisons

Avg Frobenius and QLIKE distance, Diebold-Mariano tests (*)

	Frobenius				*QLIKE*			
	HAR	DRD	EW	HVY	HAR	DRD	EW	HVY
Std	12.31	12.13	12.38	12.47	14.38	14.14	14.11	14.05
Q	12.11	11.98	12.18	12.16	14.16	13.90	14.09	14.00
diff	0.20*	0.16*	0.20*	0.31*	0.22*	0.24*	0.01*	0.05*

- "Q" beats "standard" version for all four model comparisons: average distance is significantly higher (at 0.05 level)

Out-of-sample forecast comparisons

Avg Frobenius and QLIKE distance, Diebold-Mariano tests (*) and MCS results (bold)

	Frobenius				*QLIKE*			
	HAR	DRD	EW	HVY	HAR	DRD	EW	HVY
Std	12.31	**12.13**	12.38	12.47	14.38	14.14	14.11	**14.05**
Q	**12.11**	**11.98**	**12.18**	**12.16**	14.16	**13.90**	**14.09**	**14.00**
diff	0.20*	0.16*	0.20*	0.31*	0.22*	0.24*	0.01*	0.05*

- "Q" beats "standard" version for all four model comparisons: average distance is significantly higher (at 0.05 level)
- "Q" models are almost always in the "Model Confidence Set;" std models are almost never.

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Improved portfolio decisions: min variance portfolios I

- Consider the global minimum variance portfolio problem:

$$\mathbf{w}_t^* = \arg\min_{w} \mathbf{w}_t' H_{t|t-1} \mathbf{w}_t \quad \text{s.t.} \quad \mathbf{w}_t' \boldsymbol{\iota} = 1$$
$$= \frac{H_{t|t-1}^{-1} \boldsymbol{\iota}}{\boldsymbol{\iota}' H_{t|t-1}^{-1} \boldsymbol{\iota}}$$

- We will then compare covariance matrix forecasts by the out-of-sample variance of the estimated minimum variance portfolios they generate.
- We will also compare their **turnover**:

$$\text{Turn}_t = \sum_{i=1}^{N} \left| w_{t+1}^{*(i)} - w_t^{*(i)} \frac{1 + r_t^{(i)}}{1 + \mathbf{w}_t^{*\prime} \mathbf{r}_t} \right|$$

Improved portfolio decisions: min variance portfolios II

- We consider **transaction costs** proportional ($\mathbf{c}$) to the portfolio turnover, with $\mathbf{c} \in \{0, 1, 2\}\,\%$, (Fleming et al. 2003, *JFE*; Brown & Smith 2011, *MS*).

- The after-costs returns are then:

$$r_{pt} = \mathbf{w}_t^{*\prime}\mathbf{r}_t - \mathbf{c} \times \mathrm{Turn}_t$$

- Finally, we compute the realized utility for quadratic utility investor:

$$\mathcal{U}\left(r_{pt};\gamma\right) = \left(1 + r_{pt}\right) - \frac{1}{2}\frac{\gamma}{1+\gamma}\left(1 + r_{pt}\right)^2$$

- And use this to compute a "management fee," Δ, that makes the investor indifferent between models a and b:

$$\frac{1}{T}\sum_{t=1}^{T}\mathcal{U}\left(r_{pt}^{(a)};\gamma\right) = \frac{1}{T}\sum_{t=1}^{T}\mathcal{U}\left(r_{pt}^{(b)} - \Delta;\gamma\right)$$

Global min variance portfolio results: Vol, turn, and S.R.

Std dev and turnover are both lower in all cases for "Q" models

	HAR		DRD		EWMA		HEAVY	
	std	Q	std	Q	std	Q	std	Q
Mean	3.08	3.16	3.61	4.37	3.46	3.87	3.85	4.12
S.D.	14.92	14.78	15.02	**14.57**	14.97	**14.64**	14.92	**14.61**
Turn	0.52	0.39	0.39	0.34	0.14	0.10	0.17	0.12
$\text{Sharpe}^{c=0\%}$	0.206	0.214	0.241	0.300	0.231	0.264	0.258	0.282
$\text{Sharpe}^{c=1\%}$	0.118	0.148	0.175	0.241	0.208	0.248	0.229	0.261
$\text{Sharpe}^{c=2\%}$	0.030	0.082	0.109	0.183	0.185	0.231	0.200	0.240

- Q models generate lower OOS variances for estimated min. var. portfolio
- Turnover is also reduced (less "noisy" forecasts)

Global min variance portfolio results: Management fees

"Management fees" for moving from nonQ to Q models are up to 168 bps

		HAR	DRD	EWMA	HEAVY
$c = 0\%$	Δ_1	9.7	82.6*	46.2	32.2
	Δ_{10}	27.3	142.6*	90.9*	73.3*
$c = 1\%$	Δ_1	44.3*	95.6*	55.9*	44.9*
	Δ_{10}	61.9*	155.6*	100.7*	86.0*
$c = 2\%$	Δ_1	78.8*	108.6*	65.7*	57.6*
	Δ_{10}	96.4*	168.7*	110.4*	98.7*

Tracking error minimization results: Volatility and turnover

Std dev and turnover are both lower in all cases for "Q" models

- Next we consider tracking portfolio construction
  - We find the weights on the 10 stocks that track the SPY
  - Then measure the OOS variance of the tracking error

	HAR		DRD		EWMA		HEAVY	
	std	Q	std	Q	std	Q	std	Q
S.D.	6.618	**6.489**	6.610	**6.521**	6.636	**6.513**	6.637	**6.506**
Turn	0.173	0.102	0.134	0.114	0.063	0.045	0.080	0.057

Tracking error minimization results: Management fees

"Management fees" for moving from nonQ to Q models are up to 75 bps

		HAR	DRD	EWMA	HEAVY
$c = 0\%$	Δ_1	29.1	27.8	33.6	32.5
	Δ_{10}	39.1	35.0*	43.4	42.8
$c = 1\%$	Δ_1	46.9*	42.8*	38.2	38.5
	Δ_{10}	56.9*	50.0*	47.9*	48.8
$c = 2\%$	Δ_1	64.6*	47.8*	42.7*	44.5*
	Δ_{10}	74.6*	55.0*	52.5*	54.8*

Comparison with existing shrinkage methods I

- Next, we compare our approach with leading methods from the literature.
- We take the "DRD" model forecasts as the baseline and consider various ways of improving/shrinking those forecasts:

1. **No shrinkage** at all
2. **Dynamic shrinkage** using "Q" information: the DRDQ model
3. **1/N** (DeMiguel, Garlappi and Uppal 2009): Just use an equal-weighted portfolio
4. **Jagannathan and Ma**: Impose short-sales constraint when constructing optimal portfolio weights

Comparison with existing shrinkage methods II

Then we consider shrinkage methods based on:

$$H_{t|t-1} = \alpha_{t-1} F_{t-1} + (1 - \alpha_{t-1}) S_{t-1}$$

where S_t is the realized kernel, F_t is the shrinkage target and α_t controls the degree of shrinkage.

5. **Single factor** (Ledoit and Wolf 2003): keep variances unshrunk, but shrink correlations towards an estimate based on a one-factor model.
6. **Equicorrelation** (Voev 2008): keep variances unshrunk, but shrink correlations towards an estimate based on an equicorrelation structure.
7. **Identity** (Ledoit and Wolf 2004): Shrink entire covariance matrix towards the identity matrix.
   - In methods 5–7, we use the "optimal" shrinkage factor (α_t) from LW (03).

Global min variance portfolio results: Vol and S.R.

Std dev is lowest, and Sharpe ratio is highest, for DRDQ

	DRDQ	DRD	J-Ma	1/N	*Shrinkage* Factor	*Shrinkage* Equicorr	*Shrinkage* Identity
Mean	4.371	3.612	3.922	1.044	3.700	3.723	2.753
S.D.	**14.57**	15.02	15.30	18.58	14.99	14.98	15.07
Turn	0.339	0.391	0.322	0.009	0.369	0.361	0.303
$\text{Sharpe}^{c=0\%}$	0.300	0.241	0.256	0.056	0.247	0.249	0.183
$\text{Sharpe}^{c=1\%}$	0.241	0.175	0.203	0.055	0.185	0.188	0.132
$\text{Sharpe}^{c=2\%}$	0.183	0.109	0.150	-0.054	0.123	0.127	0.081

Global min variance portfolio results: Management fees

"Mgmt fees" for moving from given method to DRDQ model: 40 to 900 bps

All relative to DRDQ					*Shrinkage*		
		DRD	J-Ma	1/N	Factor	Equicorr	Identity
$c = 0\%$	Δ_1	82.6*	51.7*	399.2*	73.3*	70.9*	169.3*
	Δ_{10}	146.0*	150.0*	997.1*	129.0*	125.8*	237.2*
$c = 1\%$	Δ_1	95.6*	47.4*	316.2*	80.9*	76.5*	160.3*
	Δ_{10}	155.6*	145.7*	914.1*	136.7*	131.4*	228.1*
$c = 2\%$	Δ_1	108.6*	43.1*	233.1*	88.6*	82.1*	151.3*
	Δ_{10}	168.7*	141.5*	831.1*	144.4*	137.0*	219.1*

Outline

1. Motivation
2. Using measurement error information in predictive models
3. Empirical analysis of U.S. equity returns
   - In-sample and out-of-sample forecasting analysis
   - Portfolio decisions: minimum variance and tracking portfolio construction
   - **Different re-balancing frequencies**
4. Summary

Global min variance portfolio: daily, weekly, monthly

"Q" information also improves models at weekly and monthly frequencies

	Daily		*Weekly*		*Monthly*	
	std	Q	std	Q	std	Q
Mean	3.61	4.37	3.12	3.26	3.24	3.36
S.D.	15.02	14.57	15.37	14.73	15.59	15.54
Turn	0.39	0.34	0.11	0.12	0.03	0.03
$\text{Sharpe}^{c=0\%}$	0.241	0.300	0.203	0.221	0.205	0.217
$\text{Sharpe}^{c=1\%}$	0.175	0.241	0.185	0.201	0.200	0.211
$\text{Sharpe}^{c=2\%}$	0.109	0.183	0.166	0.181	0.195	0.206

- Q models generate lower OOS variances for estimated min var portfolio
- Turnover is also reduced (less "noisy" forecasts)

Global minimum variance portfolio: daily, weekly, monthly

"Management fees" for moving from nonQ to Q models are signif even for monthly models

		Daily	*Weekly*	*Monthly*
$c = 0\%$	Δ_1	82.6*	23.1	16.9
	Δ_{10}	142.6*	108.9*	61.6*
$c = 1\%$	Δ_1	95.6*	22.0	16.8
	Δ_{10}	155.6*	107.8*	61.5*
$c = 2\%$	Δ_1	108.6*	20.9	16.8
	Δ_{10}	168.7*	106.6*	61.5*

Gains from daily re-balancing?

- When incorporating transaction costs, one might expect that the gains from re-balancing more frequently are eroded.
- We now compare **monthly** DRDQ with **daily** DRD and DRDQ models
  - And **weekly** DRDQ models with **daily** DRD and DRDQ models
- Our hope is that the daily DRDQ model has reduced the "spurious turnover" sufficiently that it is competitive with lower-frequency re-balancing in the presence of realistic transaction costs.

Global min variance portfolio results: Management fees

"Management fees" for switching from lower frequency to daily model

		Weekly (Q)		Monthly (Q)	
	Daily	std	Q	std	Q
$c = 0\%$	Δ_1	31.2		92.4*	
	Δ_{10}	-6.6		152.7*	
$c = 1\%$	Δ_1	-38.1		1.7	
	Δ_{10}	-75.9*		61.9*	
$c = 2\%$	Δ_1	-107.3*		-88.9*	
	Δ_{10}	-145.2*		-28.8	

Global min variance portfolio results: Management fees

"Management fees" for switching from lower frequency to daily model

		Weekly (Q)		Monthly (Q)	
	Daily	std	Q	std	Q
$c = 0\%$	Δ_1	31.2	113.7*	92.4*	174.9*
	Δ_{10}	-6.6	136.0*	152.7*	295.4*
$c = 1\%$	Δ_1	-38.1	57.5*	1.7	97.3*
	Δ_{10}	-75.9*	79.7*	61.9*	217.7*
$c = 2\%$	Δ_1	-107.3*	1.3	-88.9*	61.1*
	Δ_{10}	-145.2*	23.5	-28.8	140.0*

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4. **Summary**

Summary

- We propose using information about *time-varying* measurement error in high frequency volatility measures to improve forecasts.
- The degree of measurement error also needs to be estimated, and so whether it actually helps is an empirical question.
- We find that incorporating information about measurement error leads to statistically significant and economically meaningful gains:
    1. Models that use this information almost always outperform corresponding models that ignore it.
    2. Minimum variance and tracking portfolios that use this information have lower turnover and lower variance
- Our methods substantially outperform existing shrinkage methods that do not exploit information about measurement errors.

Appendix

Out-of-sample comparisons: low/high meas. error days

Avg QLIKE loss. Bold=in 90% MCS. *=Q beats nonQ

	Full sample	Lower 95% $\|\|\pi_t\|\|$	Upper 5% $\|\|\pi_t\|\|$
HAR	14.382	13.275	32.651
HARQ	14.159*	13.206*	**32.190***
HAR-DRD	14.140	**13.022**	32.603
HAR-DRDQ	**13.896***	**12.990***	**31.050***
EWMA	14.105	13.109	32.686
EWMAQ	**14.091***	13.122	32.643*
HEAVY	**14.051**	13.041	32.263
HEAVYQ	**14.004***	13.050	32.258

Global minimum variance portfolio results - no short sales

Std dev and turnover are both lower in all cases for “Q” models

	HAR		DRD		EWMA		HEAVY	
	std	Q	std	Q	std	Q	std	Q
Mean	3.75	3.93	3.92	4.63	3.93	4.36	4.23	4.67
S.D.	15.37	14.98	15.30	14.86	15.26	**14.82**	15.25	**14.78**
Turn	0.39	0.28	0.32	0.28	0.10	0.07	0.13	0.09
$\text{Sharpe}^{c=0\%}$	0.244	0.262	0.256	0.312	0.257	0.294	0.277	0.316
$\text{Sharpe}^{c=1\%}$	0.180	0.214	0.203	0.264	0.241	0.282	0.256	0.300
$\text{Sharpe}^{c=2\%}$	0.116	0.167	0.150	0.217	0.224	0.270	0.234	0.284

- Q models generate lower OOS variances for estimated min. var. portfolio
- Turnover is also reduced (less “noisy” forecasts)

Global minimum variance portfolio results - no short sales

"Management fees" for moving from nonQ to Q models are up to 168 bps

		HAR	DRD	EWMA	HEAVY
$c = 0\%$	Δ_1	23.1	77.8*	62.3	64.7
	Δ_{10}	76.1	137.6*	114.7*	120.0*
$c = 1\%$	Δ_1	49.7	88.4	57.0	60.6
	Δ_{10}	102.8*	148.3*	116.4*	126.6*
$c = 2\%$	Δ_1	76.4*	99.0*	64.3*	70.3*
	Δ_{10}	129.4*	158.9*	123.9*	133.2*

Global minimum variance portfolio results

Gains from applying shrinkage to our DRDQ model?

	DRDQ	J-Ma	*Shrinkage on DRDQ*		
			Factor	Equicorr	Identity
Mean	4.371	4.633	4.546	4.605	4.088
S.D.	14.57	14.86	14.53	**14.52**	14.60
Turn	0.339	0.280	0.322	0.316	**0.260**
$\text{Sharpe}^{c=0\%}$	0.300	0.312	0.313	**0.317**	0.280
$\text{Sharpe}^{c=1\%}$	0.241	**0.264**	0.257	0.262	0.235
$\text{Sharpe}^{c=2\%}$	0.183	**0.217**	0.201	0.207	0.190

Global minimum variance portfolio results

"Management fees" for applying shrinkage to the DRDQ model?

All relative to DRDQ			*Shrinkage on DRDQ*		
		J-Ma	Factor	Equicorr	Identity
$c = 0\%$	Δ_1	-17.8	-18.0	-24.1	28.9
	Δ_{10}	-22.4	-22.7	-30.2*	33.4*
$c = 1\%$	Δ_1	-32.4*	-22.2	-29.8*	9.0
	Δ_{10}	-36.7*	-26.9	-35.9*	13.5
$c = 2\%$	Δ_1	-51.2*	-26.4	-35.5*	-10.9
	Δ_{10}	-47.4*	-31.0	-41.6*	-6.4