Predictability of Output Growth and Inflation: A Multi-Horizon Survey Approach

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We develop an unobserved-components approach to study surveys of forecasts containing multiple forecast horizons. Under the assumption that forecasters optimally update their beliefs about past, current, and future state variables as new information arrives, we use our model to extract information on the degree of predictability of the state variable and the importance of measurement errors in the observables. Empirical estimates of the model are obtained using survey forecasts of annual GDP growth and inflation in the United States with forecast horizons ranging from 1 to 24 months, and the model is found to closely match the joint realization of forecast errors at different horizons. Our empirical results suggest that professional forecasters face severe measurement error problems for GDP growth in real time, while this is much less of a problem for inflation. Moreover, inflation exhibits greater persistence, and thus is predictable at longer horizons, than GDP growth and the persistent component of both variables is well approximated by a low-order autoregressive specification.

KEY WORDS: Fixed-event forecasts; Kalman filtering; Multiple forecast horizons; Survey data.

1. INTRODUCTION

Much can be learned by studying how forecasters update their beliefs about economic variables such as output growth or inflation through time. Specifically, data on forecasts recorded at multiple horizons reveal how persistent agents believe the underlying variables are, the value of new information arriving between updating points, and the importance of measurement errors surrounding economic variables.

The econometric analysis of multi-horizon survey data is, however, complicated by several factors. First, since forecasts are recorded at both long and short horizons, there is considerable overlap in the forecasts and forecast errors. Second, the fact that measurement errors in the underlying variables affect agents' forecasts introduces a signal extraction problem in agents' learning process, and causes further dependence in forecast errors measured at different horizons. For these reasons, only limited results are available using this type of "fixed event" data; see Nordhaus (1987), Swidler and Ketcher (1990), Davies and Lahiri (1995), Clements (1997), Isiklar, Lahiri, and Loungani (2006), and Lahiri and Sheng (2008).

This paper develops a new approach for extracting information on how rapidly agents learn about the state of the economy and characterizing their views about temporary and persistent components in the predicted variable. Specifically, we develop a framework for studying panels of forecasts containing numerous different forecast horizons ("large H") recorded for relatively few time periods ("small T"). The first contribution of this paper is to analytically reveal the rich information available by studying how forecasts of a variable measured at a low frequency (e.g., annual GDP growth) are updated at a higher frequency (monthly, in our case). We do so by modeling agents' learning problem—accounting for how they simultaneously backcast, nowcast and forecast past, current and future variables—in the context of a set of Kalman filter updating equations. We then seek to exploit this information using method-of-moments-based estimation techniques to match the properties of forecasts observed across different horizons with the moments implied by our model for agents' updating process. To conduct inference, we propose a method for simulating standard errors of the moments that are consistent with the underlying model. To our knowledge, this approach for modeling learning and conducting inference has not previously been considered in the literature. We also develop a maximum likelihood approach for estimating the parameters of the model.

The "large H" nature of our data enables us to answer a number of interesting questions that are intractable with forecasts of just one or two different horizons, such as the importance of measurement errors, the rate at which uncertainty about macroeconomic variables is resolved as the forecast horizon is reduced, and forecasters' beliefs about the current state of the economy (as measured by their "nowcasts" of GDP growth and inflation). It is of course no surprise that uncertainty about macroeconomic variables declines as the date the variable is revealed draws nearer; the novel aspect of this paper is to propose a model that theoretically explains both the level and the shape of this uncertainty as a function of the forecast horizon.

The second contribution of this paper is empirical: we use consensus forecasts of U.S. inflation and GDP growth over 1990–2004, and find many interesting results. Consistent with our model, we find that the rate of uncertainty resolution is faster at short and medium horizons than at long horizons, due in part to the presence of a persistent component in the predicted

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series, in part to forecasters' access to noisy data on current-period realizations. Measurement error appears to be important in forecasts of GDP growth but much less so for inflation, a finding that is consistent with other studies of measurement error in macroeconomic variables, but using different datasets; see, for example, Croushore and Stark (2001).

The plan of the paper is as follows. Section 2 presents our model for how forecasters update their predictions as the forecast horizon shrinks and discusses various estimation and inference methods. Section 3 presents empirical results using data from Consensus Economics over the period 1990–2004 and Section 4 concludes. Proofs and additional technical details are contained in appendices.

2. MULTI–HORIZON FORECAST ERRORS

We start by developing a model for how forecasters update their beliefs about macroeconomic variables such as output growth and inflation. Our analysis makes use of the rich information available in high frequency revisions of forecasts of a variable observed at a lower frequency, for example, monthly revisions to forecasts of annual inflation. Since we shall be concerned with flow variables that agents gradually learn about as new information arrives prior to and during the period of their measurement, the fact that part of the outcome may be known prior to the end of the measurement period (the "event date") introduces additional complications, and means that the timing of the forecasts has to be carefully considered.

Our analysis assumes that agents have a squared loss function over the forecast error, $e_{t|t-h} \equiv z_t - \hat{z}_{t|t-h}$, where z_t is the predicted variable, $\hat{z}_{t|t-h}$ is the forecast computed at time $t-h$, t is the event date and h is the forecast horizon. Other loss functions have been discussed by, for example, Patton and Timmermann (2007). One advantage of assuming squared loss is that it is easier to justify focusing on aggregate or consensus forecasts, as we shall be doing here, computed as an average of the individual forecasts. Under this loss function, the optimal h—period forecast is simply the conditional expectation of z_t given information at time $t-h$, $\mathcal{F}_{t-h}$:

$$\hat{z}_{t|t-h} = E[z_t|\mathcal{F}_{t-h}]. \tag{1}$$

To study agents' learning process we keep the event date, t, fixed and vary the forecast horizon, h.

2.1 A Benchmark Model

Since the predicted variable in our application is measured less frequently than the forecasts are revised, it is convenient to describe the target variable as a rolling sum of a higher-frequency variable. To this end, let y_t denote the single-period variable (e.g., monthly log-first differences of GDP or a log-price index tracking inflation), while the rolling sum of the 12 most recent single-period observations of y is denoted z_t:

$$z_t = \sum_{j=0}^{11} y_{t-j}. \tag{2}$$

Our model is based on a decomposition of y_t into a persistent (and thus predictable) first-order autoregressive component, x_t, and a temporary component, u_t:

$$\begin{aligned} y_t &= x_t + u_t, \\ x_t &= \phi x_{t-1} + \varepsilon_t, \qquad -1 < \phi < 1, \\ u_t &\sim \text{iid}(0, \sigma_u^2), \qquad \varepsilon_t \sim \text{iid}(0, \sigma_\varepsilon^2), \\ E[u_t \varepsilon_s] &= 0 \qquad \forall t, s. \end{aligned} \tag{3}$$

Here ϕ measures the persistence of x_t, while u_t and ε_t are innovations assumed to be both serially uncorrelated and mutually uncorrelated. Setting y_t to be a combination of an AR(1) process and an unpredictable process implies that y_t follows an ARMA(1, 1); see Granger and Newbold (1986) for example. Without loss of generality, we assume that the unconditional mean of x_t, and thus y_t and z_t, is zero.

Our use of a variable tracking monthly changes in GDP (y_t) is simply a modeling device: U.S. GDP figures are currently only available quarterly. Economic forecasters, however, can almost certainly be assumed to employ higher frequency data when constructing their monthly forecasts of GDP. Giannone, Reichlin, and Small (2008) and Aruoba, Diebold, and Scotti (2009), for example, propose methods to incorporate into macroeconomic forecasts news about the economy between formal announcement dates. When we take our model to data we focus on those aspects of the model that have empirical counterparts.

The assumption that the predicted variable contains a first-order autoregressive component, while clearly an approximation, is able to capture the presence of a persistent component in most macroeconomic data. For example, much of the dynamics in the common factors extracted from large cross-sections of macroeconomic variables by Stock and Watson (2002) is captured by low-order autoregressive terms. It is straightforward to allow more lags or other observed variables to enter in the forecasting model, although the latter approach is complicated by the existence of literally hundreds of economic state variables that could be adopted in such models, (Stock and Watson 2006), "real time" revisions to such data (Diebold and Rudebusch 1991) and uncertainty about which models agents actually use (Garratt et al. 2003).

We first present results under simple, but unrealistic, assumptions about the forecasters' information set in order to reveal some basic properties of the problem. We introduce more realistic assumptions in the next section. Under the assumption that both x_t and y_t are observed at time t, the simplicity of our benchmark model allows an analytic characterization of how the mean squared forecast error (MSE) evolves as a function of the forecast horizon (h):

Proposition 1. Suppose that y_t can be decomposed into a persistent component (x_t) and a temporary component (u_t) satisfying Equation (3) and forecasters minimize the squared loss given the information set $\mathcal{F}_t = \sigma([x_{t-j}, y_{t-j}], j = 0, 1, 2, \ldots)$. Then the mean squared forecast error as a function of the fore-

cast horizon is given by

$$E\left[e_{t|t-h}^2\right] = \begin{cases} 12\sigma_u^2 + \dfrac{1}{(1-\phi)^2} \\ \quad \times \left(12 - 2\dfrac{\phi(1-\phi^{12})}{1-\phi} + \dfrac{\phi^2(1-\phi^{24})}{1-\phi^2}\right)\sigma_\varepsilon^2 \\ \quad + \dfrac{\phi^2(1-\phi^{12})^2(1-\phi^{2h-24})}{(1-\phi)^3(1+\phi)}\sigma_\varepsilon^2, \\ \quad \text{for } h \geq 12 \\ h\sigma_u^2 + \dfrac{1}{(1-\phi)^2} \\ \quad \times \left(h - 2\dfrac{\phi(1-\phi^h)}{1-\phi} + \dfrac{\phi^2(1-\phi^{2h})}{1-\phi^2}\right)\sigma_\varepsilon^2, \\ \quad \text{for } h < 12. \end{cases}$$

Proposition 1 is proved in the Appendix A and is simple to interpret: The first term in the expression for the mean squared error captures the unpredictable component, u_t. The second term captures uncertainty about shocks to the remaining values of the persistent component, x_t, over the measurement period. The additional term in the expression for $h \geq 12$ comes from having to predict x_{t-11}, the initial value of the persistent component at the beginning of the measurement period.

To illustrate Proposition 1, Figure 1 plots the root mean squared error (RMSE) for $h = 1, 2, \ldots, 24$ using parameters similar to those we obtain in the empirical analysis for U.S. GDP growth. Holding the unconditional variance of annual GDP growth, σ_z^2, and the ratio of the transitory component variance to the persistent component variance, σ_u^2/σ_x^2, fixed we show the impact of varying the persistence parameter, ϕ. The figure shows the large impact that this parameter has on the shape of the RMSE function. For $h < 12$, the RMSE grows as a square root of the length of the forecast horizon if y has no persistent component ($\phi = 0$). Conversely, the presence of a persistent component gives rise to a more gradual decline in the forecast error variance as the horizon is reduced. Uncertainty is resolved more gradually, the higher the value of ϕ. Notice also how the change in RMSE gets smaller at the longest horizons, irrespective of the value of ϕ.

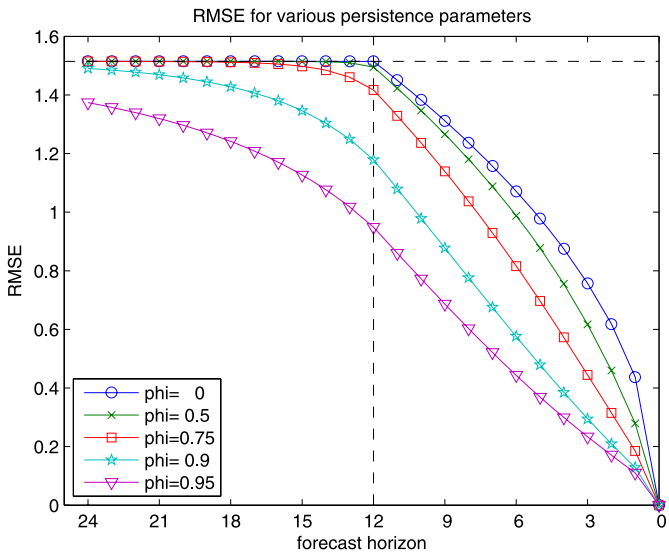

Figure 1. Root mean squared forecast errors (RMSE) as a function of the forecast horizon (h) for various degrees of persistence (ϕ) in the predictable component. The online version of this figure is in color.

2.2 Measurement Errors

Proposition 1 is helpful in establishing intuition for the drivers of how macroeconomic uncertainty gets resolved through time. However, it also has some significant shortcomings. Most obviously, it assumes that forecasters observe both the predicted variable, y, and its persistent component, x, without error, and so uncertainty vanishes completely as $h \to 0$. Macroeconomic variables are, however, to varying degrees, subject to measurement errors as reflected in data revisions and changes in benchmark weights. Such errors are less important for survey-based inflation measures such as the consumer price index (CPI). Revisions are, however, very common for measures of output, such as GDP; see, for example, Croushore and Stark (2001), Croushore (2006), and Corradi, Fernandez, and Swanson (2009).

Measurement errors make the forecasters' problem more difficult and introduce a signal extraction problem: the greater the measurement error, the noisier are past observations of y and hence the less precise the forecasters' readings of the state of the economy. They also mean that forecasters cannot simply "plug in" observed values of past y's during the measurement period ($h < 12$): these quantities must also be estimated.

To account for these effects, we cast our original model in state-space form with a state equation

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \phi \end{bmatrix}\begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} u_t \\ \varepsilon_t \end{bmatrix}, \quad \begin{bmatrix} u_t \\ \varepsilon_t \end{bmatrix} \sim \text{iid}\left(0, \begin{bmatrix} \sigma_u^2 & 0 \\ 0 & \sigma_\varepsilon^2 \end{bmatrix}\right). \tag{4}$$

Next assume that agents only observe y_t with error, and that x_t is unobserved. This setup is far more realistic for economic data which are often subject to measurement error and whose persistent components are not directly observable. If, for example, the measurement error is assumed iid then the measurement equation for this system becomes

$$\tilde{y}_t = y_t + \eta_t, \qquad \eta_t \sim \text{iid}(0, \sigma_\eta^2). \tag{5}$$

Faust, Rogers, and Wright (2005) find that revisions to U.S. GDP figures are essentially unpredictable, motivating the simple iid noise structure used above.

Despite its simplicity, this model does not yield a formula for the term structure of RMSE-values that is readily interpretable. The key difficulty that arises is best illustrated by considering "current-year" forecasts ($1 \leq h < 12$). When producing a current-year forecast at time $t - h$, economic agents must use past and current information to "backcast" realizations $y_{t-11}, \ldots, y_{t-h-1}$; they must also produce a "nowcast" for the current month y_{t-h}, and, finally, must predict future realizations, $y_{t-h+1}, \ldots, y_t$. When the persistent component, x_t, is not observable, the resulting forecast errors will generally be serially correlated even after conditioning on all information that is available to the agents. Handling this problem is difficult and requires expressing past, current, and future forecast errors in terms of the primitive shocks, u_t, ε_t, and η_t, which are serially uncorrelated. We show how to accomplish this for a more general model in the next section.

2.3 A More General Model

We now extend the state-space model introduced in the previous section in two directions. First, we allow the measurement error facing the forecasters to be either iid or follow an MA(1) process. These cases correspond to the measurement error occurring directly in the growth rate, or in the log-level of the series [which then becomes an MA(1) error term on the growth rate].

Second, the definition of the "annual" rate of change in the variable need not be the simple December-on-December change as assumed in Equation (2) above. Rather, it may be defined as the change in the "average" level of the series in one year relative to that in the previous year. This is the form of the annual variable used in our empirical analysis below. In Appendix B we show that various alternative definitions of the annual growth rate can be represented as simple weighted sums of the most recent 24 monthly growth rates:

$$z_t = \sum_{j=0}^{23} \omega_j y_{t-j}. \tag{6}$$

In the simple definition used in Equation (2) the weights equal one for each of the most recent 12 months and zero for the rest; in Appendix B we show that for other definitions the weights take an "inverted V" shape as a function of the lag.

Given the above considerations, and to more easily handle the backcasting, nowcasting, and forecasting aspects of the forecasters' problem, it is convenient to extend the state variable to include an additional error term and its lag (v_t and v_{t-1}), as well as 28 lags of the monthly growth rate, y_t. (Strictly, we only need 23 lags for the derivations in this section, but the additional 5 lags are required in Section 2.5 and create no additional complexity.) The state equation is then

$$\begin{aligned}
&\boldsymbol{\xi}_t = \mathbf{F}\boldsymbol{\xi}_{t-1} + \boldsymbol{\nu}_t, \quad \text{where} \\
&\boldsymbol{\xi}_t \equiv [\, x_t \quad v_t \quad v_{t-1} \quad y_t \quad \cdots \quad y_{t-28} \,]', \\
&\boldsymbol{\nu}_t \equiv [\, \varepsilon_t \quad v_t \quad 0 \quad \varepsilon_t + u_t \quad \mathbf{0}_{1\times 28} \,]', \\
&\mathbf{F} = \left[\begin{array}{ccc|c} \phi & 0 & 0 & \\ 0 & 0 & 0 & \\ 0 & 1 & 0 & \mathbf{0}_{4\times 29} \\ \phi & 0 & 0 & \\ \hline \multicolumn{3}{c|}{\mathbf{0}_{28\times 3}} & \mathbf{I}_{28} \quad \mathbf{0}_{28\times 1} \end{array}\right], \quad \text{and} \\
&\boldsymbol{\nu}_t \sim \text{iid}(\mathbf{0}, \mathbf{Q}), \quad \text{where} \\
&\mathbf{Q} = \left[\begin{array}{ccc|c} \sigma_\varepsilon^2 & 0 & 0 & \sigma_\varepsilon^2 & \\ 0 & \sigma_v^2 & 0 & 0 & \mathbf{0}_{4\times 28} \\ 0 & 0 & 0 & 0 & \\ \sigma_\varepsilon^2 & 0 & 0 & \sigma_\varepsilon^2 + \sigma_u^2 & \\ \hline \multicolumn{4}{c|}{\mathbf{0}_{28\times 4}} & \mathbf{0}_{28\times 28} \end{array}\right],
\end{aligned} \tag{7}$$

where $\mathbf{0}_{m\times n}$ is a $(m \times n)$ matrix of zeros, and $\mathbf{I}_k$ is a k-dimensional identity matrix.

While more complicated in appearance, the data-generating process for the key variables (x_t, y_t) is unchanged from the previous section. The measurement equation is generalized to allow for the possibility of an MA(1) measurement error:

$$\begin{aligned}
\tilde{\mathbf{y}}_t &= \mathbf{H}'\boldsymbol{\xi}_t + \mathbf{w}_t, \quad \text{where} \\
\mathbf{H} &\equiv [\, 0 \quad 1 \quad \lambda \quad 1 \quad \mathbf{0}_{1\times 28} \,]', \\
\mathbf{w}_t &\sim \text{iidN}(\mathbf{0}, \mathbf{R}), \quad \text{and} \\
E[\boldsymbol{\nu}_t \mathbf{w}_s'] &= 0 \qquad \forall s, t.
\end{aligned} \tag{8}$$

In our application the measurement variable is a scalar (and $\mathbf{w}_t = \eta_t$) but we will present our theoretical framework for the general case where $\tilde{\mathbf{y}}_t$ is a vector. Further, in our application the error term $\mathbf{w}_t$ is not strictly needed, as it is nested in the MA(1) error in Equation (8), but we include it for ease of comparison with the above model and with other state-space models. The simple iid noise structure is obtained by setting $\lambda = 0$. (In this case one should set either $\sigma_v^2 = 0$ or $\sigma_\eta^2 = 0$, as these parameters are not separately identified when $\lambda = 0$.) Alternatively, if the measurement errors faced by forecasters were iid in the levels of the series, this would suggest an MA(1) error structure in the growth rates, in which case we would set $\lambda = -1$ and $\sigma_\eta^2 = 0$.

The annual target variable, z_t, is defined as a weighted sum of the most recent 24 values of the monthly growth rates:

$$z_t = \sum_{j=0}^{23} \omega_j y_{t-j} = \boldsymbol{\gamma}_0' \boldsymbol{\xi}_t,$$

where

$$\begin{aligned}
\boldsymbol{\gamma}_0 &= [\, \mathbf{0}_{1\times 3} \quad \boldsymbol{\omega}' \quad \mathbf{0}_{1\times 5} \,]', \\
\boldsymbol{\omega} &= [\, \omega_0 \quad \omega_1 \quad \cdots \quad \omega_{23} \,]'.
\end{aligned}$$

Different choices for the weight vector $\boldsymbol{\omega}$, corresponding to different definitions of the annual variable, are discussed in Appendix B.

In generating their forecasts, we assume that our forecasters know the form and parameters of the data-generating process, presented in Equations (7) and (8), and we further assume that they use the Kalman filter to optimally predict (forecast, nowcast and backcast) the values of y_t needed for the forecast of the annual variable, z_t. Thus the learning problem faced by the forecasters in our model relates to the latent state of the economy (measured by x_t and y_t), but not to the parameters of the model. This simplification is necessitated by our short time series of data. We also assume that the forecaster has been using the Kalman filter long enough that all updating matrices are at their steady-state values. This is done simply to remove any "start of sample" effects that may or may not be present in the data. Let

$$\begin{aligned}
\tilde{\mathcal{F}}_t &= \sigma(\tilde{\mathbf{y}}_t, \tilde{\mathbf{y}}_{t-1}, \ldots, \tilde{\mathbf{y}}_1), \\
\hat{\boldsymbol{\xi}}_{t|t-1} &\equiv E[\boldsymbol{\xi}_t | \tilde{\mathcal{F}}_{t-1}] \equiv E_{t-1}[\boldsymbol{\xi}_t], \\
\hat{\mathbf{y}}_{t|t-1} &\equiv E[\tilde{\mathbf{y}}_t | \tilde{\mathcal{F}}_{t-1}] \equiv E_{t-1}[\tilde{\mathbf{y}}_t].
\end{aligned}$$

Following Hamilton (1994), define the following matrices:

$$\begin{aligned}
\mathbf{P}_{t+1|t} &\equiv E\big[\big(\boldsymbol{\xi}_{t+1} - \hat{\boldsymbol{\xi}}_{t+1|t}\big)\big(\boldsymbol{\xi}_{t+1} - \hat{\boldsymbol{\xi}}_{t+1|t}\big)'\big] \\
&= (\mathbf{F} - \mathbf{K}_t\mathbf{H}')\mathbf{P}_{t|t-1}(\mathbf{F}' - \mathbf{H}\mathbf{K}_t') + \mathbf{K}_t\mathbf{R}\mathbf{K}_t' + \mathbf{Q} \to \overline{\mathbf{P}}_1, \\
\mathbf{K}_t &\equiv \mathbf{F}\mathbf{P}_{t|t-1}\mathbf{H}(\mathbf{H}'\mathbf{P}_{t|t-1}\mathbf{H} + \mathbf{R})^{-1} \to \overline{\mathbf{K}}, \\
\mathbf{P}_{t|t} &\equiv E\big[\big(\boldsymbol{\xi}_t - \hat{\boldsymbol{\xi}}_{t|t}\big)\big(\boldsymbol{\xi}_t - \hat{\boldsymbol{\xi}}_{t|t}\big)'\big] \\
&= \mathbf{P}_{t|t-1} - \mathbf{P}_{t|t-1}\mathbf{H}\big(\mathbf{H}'\mathbf{P}_{t|t-1}\mathbf{H} + \mathbf{R}\big)^{-1}\mathbf{H}'\mathbf{P}_{t|t-1} \\
&\to \overline{\mathbf{P}}_1 - \overline{\mathbf{P}}_1\mathbf{H}(\mathbf{H}'\overline{\mathbf{P}}_1\mathbf{H} + \mathbf{R})^{-1}\mathbf{H}'\overline{\mathbf{P}}_1 \equiv \overline{\mathbf{P}}_0.
\end{aligned} \tag{9}$$

The convergence of $\mathbf{P}_{t|t-1}$, $\mathbf{P}_{t|t}$, and $\mathbf{K}_t$ to their steady-state values relies on $|\phi| < 1$, and we impose this in the estimation. To initialize these matrices we use their unconditional equivalents, $\mathbf{P}_{1|0} \equiv E[(\boldsymbol{\xi}_t - E[\boldsymbol{\xi}_t])(\boldsymbol{\xi}_t - E[\boldsymbol{\xi}_t])']$ and $\hat{\boldsymbol{\xi}}_{1|0} = E[\boldsymbol{\xi}_t]$. Estimates of the state variables are updated via

$$\hat{\boldsymbol{\xi}}_{t|t} = \hat{\boldsymbol{\xi}}_{t|t-1} + \mathbf{P}_{t|t-1}\mathbf{H}\big(\mathbf{H}'\mathbf{P}_{t|t-1}\mathbf{H} + \mathbf{R}\big)^{-1}\big(\tilde{\mathbf{y}}_t - \mathbf{H}'\hat{\boldsymbol{\xi}}_{t|t-1}\big), \quad (10)$$

while the multi-step prediction error uses

$$\begin{aligned} \hat{\boldsymbol{\xi}}_{t+h|t} &= \mathbf{F}^h \hat{\boldsymbol{\xi}}_{t|t}, \\ \mathbf{P}_{t+h|t} &\equiv E\big[\big(\boldsymbol{\xi}_{t+h} - \hat{\boldsymbol{\xi}}_{t+h|t}\big)\big(\boldsymbol{\xi}_{t+h} - \hat{\boldsymbol{\xi}}_{t+h|t}\big)'\big] \\ &= \mathbf{F}^h \mathbf{P}_{t|t}(\mathbf{F}')^h + \sum_{j=0}^{h-1} \mathbf{F}^j \mathbf{Q}(\mathbf{F}')^j \to \overline{\mathbf{P}}_h \quad \text{for } h \geq 1. \end{aligned} \quad (11)$$

The full set of MSE-values across different horizons can now be extracted from $\overline{\mathbf{P}}_h$:

$$\begin{aligned} \hat{z}_{t|t-h} &\equiv E[z_t|\tilde{\mathcal{F}}_{t-h}] = \boldsymbol{\gamma}_0' \hat{\boldsymbol{\xi}}_{t|t-h} \quad \text{yielding} \\ \text{MSE}_h &\equiv E[(z_t - \hat{z}_{t|t-h})^2] = \boldsymbol{\gamma}_0' \overline{\mathbf{P}}_h \boldsymbol{\gamma}_0, \quad \text{for } h \geq 0. \end{aligned} \quad (12)$$

Note that for $h < 12$ the optimal forecast $\hat{z}_{t|t-h}$ involves a combination of forecasts, $E[y_{t-h+j}|\tilde{\mathcal{F}}_{t-h}]$ for $j > 0$, nowcasts, $E[y_{t-h}|\tilde{\mathcal{F}}_{t-h}]$, and backcasts, $E[y_{t-h-k}|\tilde{\mathcal{F}}_{t-h}]$ for $k > 0$. Our use of an extended state equation means that these terms are all captured in the above expressions without having to handle them separately.

Figure 2 uses these equations to illustrate the impact of measurement error on the RMSE-values at different horizons. For this illustration, we set $\lambda = \sigma_v = 0$ and vary σ_η as a function of σ_u, so the measurement error variance is expressed in terms of the innovation variance for y. In the absence of measurement errors the RMSE will converge to zero as $h \to 0$, whereas in the presence of measurement error the RMSE will converge to some positive quantity. As the horizon, h, shrinks towards zero, the relative importance of measurement errors grows. Moreover, the RMSE function gets flatter as the size of the measurement error increases. Note, however, that measurement error plays no part for long-horizon forecasts, since its impact on overall uncertainty is small relative to other sources of uncertainty, and so Figure 2 resembles Figure 1 for long horizon forecasts. This also shows that the persistence (ϕ) and measurement error (σ_η^2) parameters are separately identified by jointly considering long and short horizon forecast errors, and illustrates the rich information contained in survey forecasts covering multiple forecast horizons.

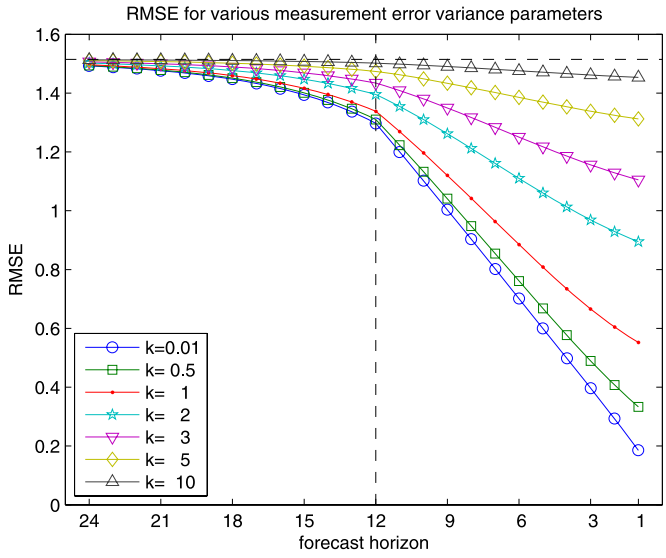

Figure 2. Root mean squared forecast errors (RMSE) as a function of the forecast horizon (h) for various levels of measurement error in the predicted variable (larger k implies greater meas. error). The online version of this figure is in color.

The analytical results in this section show that a simple model of the forecasting environment faced by macroeconomic forecasters in practice can accommodate a rich set of empirical phenomena: with just four free parameters a variety of RMSE patterns is obtained. Further, by studying such a model in detail we gain some quantitative insight into the key drivers of macroeconomic forecast errors. We next move on to matching the parameters of our model to data.

2.4 GMM Estimation

Our initial strategy for estimation is to choose the parameters that enable the model to match the observed forecast errors as closely as possible. To this end, we estimate the parameters using the Generalized Methods of Moments (GMM) based on the moment conditions obtained by matching the sample MSE, $T^{-1}\sum_{t=1}^{T} e_{t|t-h}^2$ at various forecast horizons to the population mean squared errors, $\text{MSE}_h(\boldsymbol{\theta})$, implied by our model. Our parameter estimates are obtained from

$$\hat{\boldsymbol{\theta}}_T \equiv \arg\min_{\boldsymbol{\theta}} \mathbf{g}_T(\boldsymbol{\theta})' \mathbf{W}_T \mathbf{g}_T(\boldsymbol{\theta}), \quad (13)$$

$$\mathbf{g}_T(\boldsymbol{\theta}) \equiv \frac{1}{T}\sum_{t=1}^{T} \begin{bmatrix} e_{t|t-1}^2 - \text{MSE}_1(\boldsymbol{\theta}) \\ e_{t|t-2}^2 - \text{MSE}_2(\boldsymbol{\theta}) \\ \vdots \\ e_{t|t-H}^2 - \text{MSE}_H(\boldsymbol{\theta}) \end{bmatrix}, \quad (14)$$

where $\boldsymbol{\theta} \equiv [\sigma_u^2, \sigma_\varepsilon^2, \phi, \sigma_v^2, \lambda]'$ and $\text{MSE}_h(\boldsymbol{\theta})$ is obtained using Proposition 1 or the updating equations leading to (12).

In situations with large H there are several over-identifying restrictions, and so the choice of weighting matrix, $\mathbf{W}_T$, in the GMM estimation is important. In our initial estimates we use the identity matrix as the weighting matrix so that all horizons get equal weight in the estimation procedure; this is not fully efficient, but is justified by our focus on modeling the entire term structure of forecast errors. For comparison, we also present efficient GMM estimates, using the inverse covariance matrix of the moment conditions as the weighting matrix. The covariance matrix of the sample moments is also used to compute standard errors and test the over-identifying conditions. In our application the sample is only $T = 14$ years long while we have $H = 24$ forecast horizons and so it is not feasible to estimate this matrix directly from the data since this would require controlling for the correlation between the sample moments induced by overlaps across the 24 horizons. Fortunately, given the simple structure of our model, for a given parameter value we can compute a model-implied covariance matrix of the sample moments. Under the assumption that the model is correctly

specified, this matrix captures the correlation between sample moments induced by overlaps and serial persistence.

To obtain $\overline{\mathbf{P}}_1$, $\overline{\mathbf{P}}_0$, and $\overline{\mathbf{K}}$ we simulate 100 nonoverlapping years of data and update $\mathbf{P}_{t|t-1}$, $\mathbf{P}_{t|t}$, and $\mathbf{K}_t$ following Hamilton (1994). We use these matrices at the end of the 100th year as estimates of $\overline{\mathbf{P}}_1$, $\overline{\mathbf{P}}_0$, and $\overline{\mathbf{K}}$. To obtain the covariance matrix of the moments, used to compute standard errors and the test of over-identifying restrictions, we use the model-implied covariance matrix of the moments, based on the parameter estimate from the first-stage GMM. This matrix is not available in closed-form and so we simulate 1000 nonoverlapping years of data to estimate it, imposing that the innovations to these processes ($\boldsymbol{\nu}_t$ and $\mathbf{w}_t$) are normally distributed, and using the expressions given above to obtain the Kalman filter forecasts.

We use only six forecast horizons ($h = 1, 3, 6, 12, 18, 24$) in the estimation, rather than the full set of 24, in response to studies of the finite-sample properties of GMM estimates (e.g., Tauchen 1986) which find that using many more moment conditions than required for identification leads to poor approximations from the asymptotic theory, particularly when the moments are highly correlated, as in our application. We have also estimated the models presented in this paper using the full set of 24 moment conditions and the results were qualitatively similar.

2.5 Maximum Likelihood Estimation

With the analytical formulas for the model-implied mean squared forecast errors given in the previous section, obtaining GMM estimates of the unknown model parameters is straightforward. Normality is sufficient but not necessary for the GMM estimates: under nonnormality, our approach is still applicable if we assume that forecasters construct their predictions as optimal linear projections rather than expectations. However, in this case the Kalman filter is no longer optimal, and another filter based on different distributional assumptions may perform better.

Under normality, however, GMM suffers from the usual drawback that it is less efficient than fully specified maximum likelihood (ML). In this section we describe the steps required to estimate the model by ML. This approach is complicated by the fact that forecasts of varying horizons appear in the survey across different months. We address this by extending the econometrician's measurement variable (i.e., the variable observable to us) to include a hypothetical full set of forecasts, with horizons from $h = 1$ to $h = 24$, at each month. This simplifies the algebra, but introduces the problem that not all of these variables are actually observable at each point in time. We use the approach presented in Aruoba, Diebold, and Scotti (2009) to address this problem.

2.5.1 The Econometrician's State and Measurement Variables. Recall from Section 2.3 that the forecasters' state and measurement variables are $\boldsymbol{\xi}_t$ and $\tilde{\mathbf{y}}_t$, respectively. In turn, the econometrician's state and measurement variables are

$$\begin{aligned}
\underset{(65\times 1)}{\boldsymbol{\Phi}_t} &\equiv [\underset{(32\times 1)}{\boldsymbol{\xi}_t'}, \underset{(32\times 1)}{\hat{\boldsymbol{\xi}}_{t|t}'}, \underset{(1\times 1)}{\tilde{\mathbf{y}}_t'}], \\
\underset{(30\times 1)}{\boldsymbol{\Gamma}_t} &\equiv [\hat{z}_{t+24,t}, \hat{z}_{t+23,t}, \ldots, \hat{z}_{t+1,t}, z_t, z_{t-1}, \ldots, z_{t-5}]'.
\end{aligned} \tag{15}$$

Hence $\tilde{\mathbf{y}}_t$, which is observed by forecasters but not by the econometrician, can be interpreted as a summary statistic for all the information forecasters observe that helps them predict the actual target variable. These variables give rise to a two-layered Kalman filter, as we will use the Kalman filter to handle the inference problem faced by the econometrician on top of the Kalman filter assumed to be employed by the forecasters in our sample.

We first show that we can write the econometrician's state variable as following a VAR(1). The forecaster's nowcast of the state variable is given by (see Hamilton 1994):

$$\begin{aligned}
\hat{\boldsymbol{\xi}}_{t|t} &= \mathbf{F}\hat{\boldsymbol{\xi}}_{t-1|t-1} + \overline{\mathbf{P}}_1\mathbf{H}(\mathbf{H}'\overline{\mathbf{P}}_1\mathbf{H} + \mathbf{R})^{-1}\left(\tilde{\mathbf{y}}_t - \mathbf{H}'\mathbf{F}\hat{\boldsymbol{\xi}}_{t-1|t-1}\right) \\
&= (\mathbf{I} - \bar{\mathbf{A}}_1\mathbf{H}')\mathbf{F}\hat{\boldsymbol{\xi}}_{t-1|t-1} + \bar{\mathbf{A}}_1\tilde{\mathbf{y}}_t, \quad \text{where} \\
\bar{\mathbf{A}}_1 &\equiv \overline{\mathbf{P}}_1\mathbf{H}(\mathbf{H}'\overline{\mathbf{P}}_1\mathbf{H} + \mathbf{R})^{-1}.
\end{aligned} \tag{16}$$

Next, we note that

$$\begin{aligned}
\tilde{\mathbf{y}}_t &= \mathbf{H}'\boldsymbol{\xi}_t + \boldsymbol{\eta}_t \\
&= \mathbf{H}'\mathbf{F}\boldsymbol{\xi}_{t-1} + \mathbf{H}'\boldsymbol{\nu}_t + \boldsymbol{\eta}_t, \quad \text{and so} \\
\hat{\boldsymbol{\xi}}_{t|t} &= (\mathbf{I} - \bar{\mathbf{A}}_1\mathbf{H}')\mathbf{F}\hat{\boldsymbol{\xi}}_{t-1|t-1} + \bar{\mathbf{A}}_1(\mathbf{H}'\mathbf{F}\boldsymbol{\xi}_{t-1} + \mathbf{H}'\boldsymbol{\nu}_t + \boldsymbol{\eta}_t) \\
&= (\mathbf{I} - \bar{\mathbf{A}}_1\mathbf{H}')\mathbf{F}\hat{\boldsymbol{\xi}}_{t-1|t-1} + \bar{\mathbf{A}}_1\mathbf{H}'\mathbf{F}\boldsymbol{\xi}_{t-1} + \bar{\mathbf{A}}_1\mathbf{H}'\boldsymbol{\nu}_t + \bar{\mathbf{A}}_1\boldsymbol{\eta}_t.
\end{aligned} \tag{17}$$

Pulling these together, we find that the econometrician's state variable follows a VAR(1):

$$\begin{aligned}
\boldsymbol{\Phi}_t &= \begin{bmatrix} \boldsymbol{\xi}_t \\ \hat{\boldsymbol{\xi}}_{t|t} \\ \tilde{\mathbf{Y}}_t \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{F} & \mathbf{0}_{32\times 32} & \mathbf{0}_{32\times 1} \\ \bar{\mathbf{A}}_1\mathbf{H}'\mathbf{F} & (\mathbf{I} - \bar{\mathbf{A}}_1\mathbf{H}')\mathbf{F} & \mathbf{0}_{32\times 1} \\ \mathbf{H}'\mathbf{F} & \mathbf{0}_{1\times 32} & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{\xi}_{t-1} \\ \hat{\boldsymbol{\xi}}_{t-1|t-1} \\ \tilde{\mathbf{Y}}_{t-1} \end{bmatrix} \\
&\quad + \begin{bmatrix} \boldsymbol{\nu}_t \\ \bar{\mathbf{A}}_1\mathbf{H}'\boldsymbol{\nu}_t + \bar{\mathbf{A}}_1\boldsymbol{\eta}_t \\ \mathbf{H}'\boldsymbol{\nu}_t + \boldsymbol{\eta}_t \end{bmatrix} \quad \text{or} \\
\boldsymbol{\Phi}_t &= \mathbf{F}^*\boldsymbol{\Phi}_{t-1} + \boldsymbol{\nu}_t^*, \quad \text{where} \\
V[\boldsymbol{\nu}_t^*] \equiv \mathbf{Q}^* &= \begin{bmatrix} \boldsymbol{\Sigma}_\nu & \boldsymbol{\Sigma}_\nu\mathbf{H}_1'\bar{\mathbf{A}} & \boldsymbol{\Sigma}_\nu\mathbf{H} \\ \bar{\mathbf{A}}_1'\mathbf{H}\boldsymbol{\Sigma}_\nu & \bar{\mathbf{A}}_1\mathbf{H}'\boldsymbol{\Sigma}_\nu\mathbf{H}\bar{\mathbf{A}}_1' + \bar{\mathbf{A}}_1\mathbf{R}\bar{\mathbf{A}}_1' & \bar{\mathbf{A}}_1\mathbf{H}'\boldsymbol{\Sigma}_\nu\mathbf{H} + \bar{\mathbf{A}}_1\mathbf{R} \\ \mathbf{H}'\boldsymbol{\Sigma}_\nu & \mathbf{H}'\boldsymbol{\Sigma}_\nu\mathbf{H}\bar{\mathbf{A}}_1' + \mathbf{R}'\bar{\mathbf{A}}_1' & \mathbf{H}'\boldsymbol{\Sigma}_\nu\mathbf{H} + \mathbf{R} \end{bmatrix}.
\end{aligned} \tag{18}$$

(Starred objects refer to the econometrician's inference problem.) Next we show that the econometrician's measurement

variables are linear functions of his state variables:

$$\boldsymbol{\Gamma}_t = \begin{bmatrix} \hat{z}_{t+24|t} \\ \vdots \\ \hat{z}_{t+1|t} \\ z_t \\ \vdots \\ z_{t-5} \end{bmatrix} = \left[\begin{array}{c|c|c} \mathbf{0}_{24\times 32} & \begin{matrix} \boldsymbol{\gamma}_0' F^{24} \\ \vdots \\ \boldsymbol{\gamma}_0' F \end{matrix} & \mathbf{0}_{24\times 1} \\ \hline \begin{matrix} \boldsymbol{\gamma}_0' \\ \boldsymbol{\gamma}_{-1}' \\ \vdots \\ \boldsymbol{\gamma}_{-5}' \end{matrix} & \mathbf{0}_{6\times 33} & \end{array}\right] \begin{bmatrix} \boldsymbol{\xi}_t \\ \hat{\boldsymbol{\xi}}_{t|t} \\ \tilde{\mathbf{Y}}_t \end{bmatrix} + \begin{bmatrix} w^*_{24|t} \\ \vdots \\ w^*_{1|t} \\ w^*_{0|t} \\ \vdots \\ w^*_{-5|t} \end{bmatrix},$$

$$\boldsymbol{\Gamma}_t = \mathbf{H}^{*\prime}\boldsymbol{\Phi}_t + \mathbf{w}^*_t, \quad \text{where} \tag{19}$$

$$\mathbf{w}^*_t \sim \text{iid}(\mathbf{0}, \mathbf{R}^*) \quad \text{and}$$

$$\boldsymbol{\gamma}_{-j} \equiv \begin{bmatrix} \mathbf{0}_{1\times(3+j)} & \boldsymbol{\omega}' & \mathbf{0}_{1\times(5-j)} \end{bmatrix}' \quad \text{for } j = 0, 1, \ldots, 5.$$

The vectors $\boldsymbol{\gamma}_{-j}$ generate the lagged annual variable z_{t-j} from the elements of the variable $\boldsymbol{\xi}_t$. The error term in the econometrician's measurement equation, $\mathbf{w}^*_t$, is included to accommodate the possibility of errors in the survey of forecasters' predictions. In the absence of such errors this variable can be set to zero (i.e., set $\mathbf{R}^* = 0$).

With the expressions in Equations (18) and (19) we have thus shown that the econometrician's problem fits into a standard state-space framework. We next discuss how we handle the fact that $\boldsymbol{\Gamma}_t$ is not completely observed by the econometrician.

2.5.2 Dealing With "Missing" Forecasts. Now we address the fact that in our dataset the econometrician does not get to observe a full set of 24 forecasts at each point in time. Rather, we only observe two forecasts, and possibly a realization of the annual target variable. For example, if date t is January, then the observed variable is $\ddot{\boldsymbol{\Gamma}}_t = [\hat{z}_{t+24|t}, \hat{z}_{t+12|t}]'$. If we were in April, then the measurement variable contains two forecasts and the value for the actual in the year ended in December, and so $\ddot{\boldsymbol{\Gamma}}_t = [\hat{z}_{t+21|t}, \hat{z}_{t+9|t}, z_{t-3}]'$.

To handle these "missing" forecasts, we follow the approach of Aruoba, Diebold, and Scotti (2009). Let $\mathbf{J}_t$ be a 30×1 vector of ones and zeros indicating which elements of $\boldsymbol{\Gamma}_t$ are observable at time t, and let $n_t = \boldsymbol{\iota}'\mathbf{J}_t$ be the number of ones in the vector $\mathbf{J}_t$. Define the "selection matrix" $\mathbf{S}_t$ as a $(n_t \times 30)$ matrix containing the rows of $\mathbf{I}_{30}$ that correspond to the elements of $\mathbf{J}_t$ that equal one. This allows us to write the $n_t \times 1$ subvector of $\boldsymbol{\Gamma}_t$, $\ddot{\boldsymbol{\Gamma}}_t$:

$$\ddot{\boldsymbol{\Gamma}}_t = \mathbf{S}_t\boldsymbol{\Gamma}_t. \tag{20}$$

As Aruoba, Diebold, and Scotti (2009) explain, the Kalman filter can be applied to problems with missing data by exploiting the above mapping from $\boldsymbol{\Gamma}_t$ to $\ddot{\boldsymbol{\Gamma}}_t$.

Let $\ddot{\mathcal{F}}_t = \sigma(\ddot{\boldsymbol{\Gamma}}_t, \ddot{\boldsymbol{\Gamma}}_{t-1}, \ldots, \ddot{\boldsymbol{\Gamma}}_1)$ denote the information set available to the econometrician at time t. In a minor abuse of notation we denote the econometrician's expectations with "hats," even though these expectations are based on a different (smaller) information set to that of the forecasters:

$$\begin{aligned} &\hat{\ddot{\boldsymbol{\Gamma}}}_{t|t-1} \equiv E[\ddot{\boldsymbol{\Gamma}}_t|\ddot{\mathcal{F}}_{t-1}], \qquad \hat{\boldsymbol{\Gamma}}_{t|t-1} \equiv E[\boldsymbol{\Gamma}_t|\ddot{\mathcal{F}}_{t-1}], \\ &\hat{\boldsymbol{\Phi}}_{t|t-1} \equiv E[\boldsymbol{\Phi}_t|\ddot{\mathcal{F}}_{t-1}]. \end{aligned} \tag{21}$$

To obtain the likelihood for this model, we need to obtain the residuals and their covariance matrix at each point in time. The residuals in the standard case with no "missing" forecasts are

$$\begin{aligned} \mathbf{e}_t &\equiv \boldsymbol{\Gamma}_t - \hat{\boldsymbol{\Gamma}}_{t|t-1} = (\mathbf{H}^{*\prime}\boldsymbol{\Phi}_t + \mathbf{w}^*_t) - \mathbf{H}^{*\prime}\hat{\boldsymbol{\Phi}}_{t|t-1} \\ &= \mathbf{H}^{*\prime}\big(\boldsymbol{\Phi}_t - \hat{\boldsymbol{\Phi}}_{t|t-1}\big) + \mathbf{w}^*_t \qquad \text{and} \\ V[\mathbf{e}_t] &= \mathbf{H}^{*\prime}\mathbf{P}^*_{t|t-1}\mathbf{H}^* + \mathbf{R}^* \equiv \mathbf{Q}_{t|t-1}, \quad \text{where} \\ \mathbf{P}^*_{t|t-1} &\equiv E\big[\big(\boldsymbol{\Phi}_t - \hat{\boldsymbol{\Phi}}_{t|t-1}\big)\big(\boldsymbol{\Phi}_t - \hat{\boldsymbol{\Phi}}_{t|t-1}\big)'\big]. \end{aligned} \tag{22}$$

The corresponding expressions for the residuals when we account for the fact that some of the forecasts are not observed at each point in time are

$$\begin{aligned} \ddot{\mathbf{e}}_t &\equiv \ddot{\boldsymbol{\Gamma}}_t - \hat{\ddot{\boldsymbol{\Gamma}}}_{t|t-1} = \mathbf{S}_t\big(\boldsymbol{\Gamma}_t - \hat{\boldsymbol{\Gamma}}_{t|t-1}\big) \qquad \text{and} \\ V[\ddot{\mathbf{e}}_t] &= \mathbf{S}_t\mathbf{Q}_{t|t-1}\mathbf{S}_t' \equiv \ddot{\mathbf{Q}}_{t|t-1}. \end{aligned} \tag{23}$$

Finally, we need expressions for obtaining the forecasts of the measurement and state variables, and for updating the variance matrices

$$\begin{aligned} \hat{\ddot{\boldsymbol{\Gamma}}}_{t|t-1} &\equiv \mathbf{S}_t\hat{\boldsymbol{\Gamma}}_{t|t-1} = \mathbf{S}_t\mathbf{H}^{*\prime}\hat{\boldsymbol{\Phi}}_{t|t-1} \qquad \text{and} \\ \hat{\boldsymbol{\Phi}}_{t|t-1} &= \mathbf{F}^*\hat{\boldsymbol{\Phi}}_{t-1|t-1}, \\ \hat{\boldsymbol{\Phi}}_{t|t} &= \hat{\boldsymbol{\Phi}}_{t|t-1} + \mathbf{P}^*_{t|t-1}\mathbf{H}^*\ddot{\mathbf{Q}}^{-1}_{t|t-1}\ddot{\mathbf{e}}_t, \\ \mathbf{P}^*_{t|t} &= \mathbf{P}^*_{t|t-1} - \mathbf{P}^*_{t|t-1}\mathbf{H}^*\mathbf{S}_t'\ddot{\mathbf{Q}}^{-1}_{t|t-1}\mathbf{S}_t\mathbf{H}^{*\prime}\mathbf{P}^*_{t|t-1}, \\ \mathbf{P}^*_{t+1|t} &= \mathbf{F}^*\mathbf{P}^*_{t|t}\mathbf{F}^* + \mathbf{Q}^*. \end{aligned} \tag{24}$$

With this in hand, we can now write down the log-likelihood for this problem, where at each point in time we "zero out" the impact of the unobserved measurement variables, and only compute the log-likelihood for those elements of $\boldsymbol{\Gamma}_t$ that we observe. Thus our log-likelihood is

$$\begin{aligned} \log L(\boldsymbol{\theta}) = -\frac{1}{2}\sum_{t=1}^{T}\big\{&n_t \log(2\pi) + \log\big|\ddot{\mathbf{Q}}_{t|t-1}(\boldsymbol{\theta})\big| \\ &+ \ddot{\mathbf{e}}_t'(\boldsymbol{\theta})\ddot{\mathbf{Q}}^{-1}_{t|t-1}(\boldsymbol{\theta})\ddot{\mathbf{e}}_t(\boldsymbol{\theta})\big\}, \end{aligned} \tag{25}$$

where we have made the dependence of $\ddot{\mathbf{Q}}_{t|t-1}$ and $\ddot{\mathbf{e}}_t$ on the unknown parameter vector explicit. As usual, we initialize the expectations and covariance matrices at their unconditional equivalents.

3. EMPIRICAL RESULTS

3.1 Data

Our data is taken from the Consensus Economics (CE) Inc. forecasts which comprise polls of private sector forecasters and are widely considered by organizations such as the IMF and the U.S. Federal Reserve. Each month participants are asked about their views of a range of variables for the major economies and the consensus (average) forecast is recorded. Our analysis

focuses on U.S. real GDP growth and Consumer Price Index (CPI) inflation for the current and subsequent year. This gives us 24 monthly next-year and current-year forecasts over the period January 1990 to December 2004 or a total of $24 \times 14 = 336$ monthly observations. Naturally our observations are not independent draws but are subject to a set of tight restrictions across horizons, as revealed by the analysis in the previous section.

The CE database tracks the views of professional forecasters. Economic interest in professional forecasts arises from the fact that these forecasts are used as inputs to the decisions of economic agents such as firms and consumers; see Granger and Machina (2006) for discussion, and that consensus forecasts from professional forecasters have been found to out-perform consensus forecasts from households in terms of forecast accuracy; see Ang, Bekaert, and Wei (2007). Further, the size and breadth of the professional forecasting industry makes this an interesting sector in its own right. For example, the CE survey we use draws on forecasts from over 70 unique institutions, including banks, nonfinancial corporations, and government agencies.

Our assumption that the forecasters efficiently update their forecasts on a monthly basis as new information becomes available is likely to be a better characterization of professional forecasters' behavior than households' behavior, as the latter have been found to update their forecasts rather less frequently (Carroll 2003). Our analysis also assumes that forecasters have a squared error loss function. Under this loss function, the objective of professional forecasters is to report an unbiased forecast for the eventual forecast user. Of course, if the objectives of the professional forecasters and the forecast users differ, then there could be principal-agent issues related to how professional forecasters generate their forecasts (see, e.g., Ottaviani and Sorensen 2006). This could also lead to distortions from using a consensus estimate rather than basing the analysis on individual forecasters' predictions or considering the dispersion in beliefs as done by Patton and Timmermann (2010). These issues are not addressed in our analysis.

As a prelude to our analysis of the RMSE function, we initially undertook a range of statistical tests that check for biases and serial correlation in the forecast errors. We tested for biases in the forecasts by testing whether the forecast errors were mean zero and by estimating "Mincer–Zarnowitz" (1969) (MZ) regressions and autocorrelation regressions

$$\begin{aligned} y_t &= \beta_0^h + \beta_1^h \hat{y}_{t|t-h} + \epsilon_{t|t-h}, \\ e_{t|t-h} &= \gamma_0^h + \gamma_1^h e_{t-12j|t-12j-h} + \upsilon_{t|t-h}. \end{aligned} \tag{26}$$

In the latter regression we set $j = 1$ for $h \leq 12$ and $j = 2$ for $h > 12$ to account for the fact that even perfectly optimal forecasts can generate forecast errors that are serially correlated at lags shorter than the forecast horizon. We test optimality by testing that $\beta_0^h = 0, \beta_1^h = 1$ in the MZ regression, and by testing that $\gamma_0^h = \gamma_1^h = 0$ in the forecast error regression, for $h = 1, \ldots, 24$. The results are presented in Table 1. For GDP growth, there was no evidence of significant forecast bias and only limited evidence against rationality in the MZ or forecast error regressions. For inflation, these tests revealed some evidence against forecast rationality at horizons beyond one year. The modeling framework described in the previous section assumes that forecasts at all horizons are rational, but does not require that we include the full set of horizons in the estimation, and so it is possible to drop the forecasts that fail one or more of these tests of rationality and estimate the model only using the remaining forecasts. In what follows we include all forecasts in the analysis for simplicity, and proceed to estimate the parameters of our model under the assumption that forecasters use information efficiently.

Table 1. Rationality tests for consensus forecasts of U.S. GDP growth and Inflation

	GDP growth			Inflation		
Horizon	Bias	MZ	Autocorr	Bias	MZ	Autocorr
1	0.85	0.95	0.23	0.76	0.60	0.30
2	0.78	0.96	0.26	0.84	0.79	0.64
3	0.47	0.72	0.13	0.25	0.34	0.34
4	0.50	0.28	0.42	0.13	0.30	0.28
5	0.42	0.23	0.42	0.11	0.26	0.27
6	0.67	0.05	0.59	0.14	0.27	0.25
7	0.80	0.30	0.70	0.10	0.11	0.28
8	0.83	0.45	0.75	0.42	0.06	0.82
9	0.34	0.48	0.48	0.92	0.18	0.97
10	0.15	0.30	0.04	0.62	0.11	0.74
11	0.06	0.17	0.00	0.58	0.16	0.35
12	0.08	0.21	0.08	0.95	0.04	0.44
13	0.13	0.32	0.02	0.51	0.00	0.68
14	0.16	0.34	0.00	0.33	0.00	0.23
15	0.25	0.50	0.04	0.16	0.00	0.03
16	0.49	0.79	0.09	0.09	0.00	0.00
17	0.61	0.87	0.08	0.13	0.04	0.00
18	0.67	0.87	0.24	0.09	0.03	0.00
19	0.68	0.63	0.28	0.06	0.04	0.00
20	0.69	0.32	0.27	0.04	0.01	0.00
21	0.66	0.28	0.22	0.05	0.01	0.00
22	0.68	0.25	0.20	0.06	0.03	0.00
23	0.69	0.20	0.25	0.04	0.01	0.00
24	0.62	0.84	0.21	0.04	0.02	0.00

NOTE: For horizons ranging from one to 24 months, this table presents p-values from three tests of forecast rationality for the consensus forecasts of GDP growth (columns 1–3) and inflation (columns 4–6). For each variable, the first column presents the results of a test for bias in the forecasts; the second column presents the p-values from a joint test that, for each horizon, h, $\beta_0^h = 0 \cap \beta_1^h = 1$ in the Mincer–Zarnowitz regression of the realized value of the target variable on the forecast: $y_t = \beta_0^h + \beta_1^h \hat{y}_{t|t-h} + \epsilon_{t|t-h}$; the third column in each panel presents the p-values from a test for zero mean and zero autocorrelation in the forecast errors, based on a regression of the forecast error on its corresponding monthly lag during the previous year (for horizons up to 12 months) or the year prior to that (for horizons greater than 12 months).

The CE survey defines the annual target variable as a rate of growth in an average of the level of the GDP or CPI series, rather than as a simple December-on-December change in the log-levels of these series. We discuss the exact form of the CE definition in Appendix B. In our analysis, we use the measure of the target variable published by Consensus Economics in the year after the measurement period, and in Appendix B we show that this variable can be represented as a weighted sum of (the unobserved) monthly changes in the log-level of these series.

3.2 Parameter Estimates and Tests

The simple benchmark model contains just three free parameters, namely the variance of the innovations in the temporary (σ_u^2) and persistent (σ_ε^2) components, and the persistence parameter, ϕ, for the predictable component. The expressions for the MSE as a function of h, stated in Proposition 1 for the

benchmark model and in Equation (12) for the model that allows for measurement error, enable us to use GMM to estimate the unknown parameters given a panel of forecast errors measured at various horizons. These parameters are not separately identifiable if forecasts for a single horizon are all that is available so access to multi-horizon forecasts is crucial to our analysis. Since the variance of the h-period forecast error grows linearly in σ_u^2 while σ_ε^2 and ϕ generally affect the MSE in a nonlinear fashion, these parameters can be identified from a sequence of MSE-values corresponding to different forecast horizons, h, provided at least three different horizons are available.

Figure 3 plots the RMSE-values for output growth and inflation at the 24 different horizons. (Note that we plot RMSE and not MSE, for ease of interpretation, and so these plots would not be linear in the case of $\phi = 0$.) In the case of output growth the RMSE shrinks from about 1.5% at the 24-month horizon to 1% at the 12-month horizon and 0.3% at the one-month horizon. For inflation it ranges from 0.7% at the two-year horizon to 0.5% at the 12-month horizon and less than 0.06% at the one-month horizon. Forecast precision improves systematically as the forecast horizon is reduced, as expected. Moreover, consistent with Proposition 1, the rate at which the RMSE declines is smaller in the next-year forecasts ($h \geq 12$) than in current-year forecasts ($h < 12$).

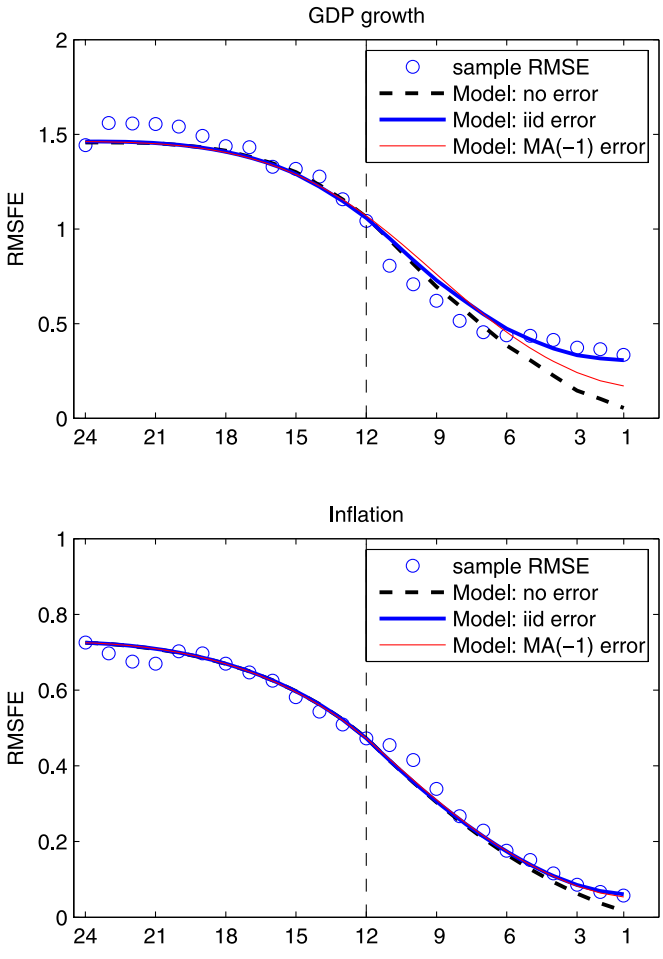

Figure 3. Root mean squared forecast errors (RMSE) for U.S. GDP growth and Inflation as a function of the forecast horizon, for three different models. The online version of this figure is in color.

The fitted values from the model without measurement error (where we impose $\sigma_v^2 = \lambda = 0$), also shown in Figure 3, clearly illustrate the limitation of this specification. This model assumes that forecasters get a very precise reading of the outcome towards the end of the current year and hence forces the fitted estimate of the RMSE to decline sharply at short forecast horizons. This property is clearly at odds with the GDP growth data and means that the benchmark model without measurement error does not succeed in simultaneously capturing the behavior of the RMSE at both the short and long horizons. For inflation forecasts the assumption of zero measurement error appears broadly consistent with the data. This is consistent with Croushore and Stark (2001) who report that revisions in reported GDP figures tend to be larger than those in reported inflation figures, and with Giannone, Reichlin, and Small (2008) who note that "nowcasting" GDP in real time is a difficult statistical task, whereas it is less so for inflation as reliable estimates of inflation are available at a monthly frequency.

Table 2 presents estimates of the unknown parameters obtained by GMM using the identity weight matrix, efficient GMM and maximum likelihood. First consider Panel A of Table 2 which presents parameter estimates and provides a formal test of the "no noise" model. Unsurprisingly, in view of Figure 3, this model is strongly rejected for GDP growth, and it is also rejected for inflation, indicating the need for a small but nonzero measurement error component. Notice that the estimate of ϕ is positive for GMM, but negative for efficient GMM. While this at first seems odd, it can be explained by considering the structure of the annual target variable and the weights used in efficient GMM. As shown in Appendix B, the annual target variable used by Consensus Economics can be expressed as a weighted sum of monthly changes, which induces a substantial amount of autocorrelation in the annual series, independent of the value of ϕ. For example, keeping the other parameters fixed at their values for GDP growth, the first-order autocorrelations in, the annual target variable when $\phi = -0.9$, 0, or 0.9 are 0.15, 0.22, and 0.55. Using the same parameters and the weighting scheme underlying the annual inflation variable we obtain first-order autocorrelations of 0.25, 0.25, and 0.57. Thus apparently large differences in this parameter do not translate to correspondingly large differences in the predicted behavior of the target variable.

Panel B of Table 2 presents parameter estimates for the model extended to allow for iid measurement errors which introduces an extra parameter, σ_v^2, reflecting the magnitude of measurement errors. This model passes the specification tests for both GDP growth and inflation and thus there is little statistical evidence against our simple specification, once measurement errors are considered. Of course, this does not mean that these simple specifications would be preferred with a longer time series of data, which might help identify richer dynamics in GDP growth or inflation. Panel B also reveals that the predictable component of inflation is more persistent than that in output growth, according to all three estimation methods.

Finally, we estimate a third model, which allows for an iid measurement error in the forecasters' observation of the log-level of these series, inducing an MA(1) error in their observation of its growth rate, with MA coefficient of -1. (With a longer sample of data it is possible to estimate the MA parameter freely, allowing the data to decide whether it equals 0,

Table 2. GMM parameter estimates of the consensus forecast model

	GDP growth			Inflation		
	GMM	GMM^{eff}	MLE	GMM	GMM^{eff}	MLE
			Panel A: No measurement error			
σ_u^2	0.000	0.000	0.186	0.037	0.000	0.017
	(–)	(–)	(0.081)	(0.035)	(–)	(0.106)
σ_ε^2	0.048	1.759	0.001	0.000	0.4259	0.000
	(0.043)	(1.249)	(0.000)	(0.001)	(0.132)	(0.001)
ϕ	0.586	−0.853	0.996	0.993	−0.999	0.999
	(0.249)	(0.185)	(0.002)	(0.275)	(0.383)	(0.001)
$\sigma_v^2 \times 10$	0.000	0.000	0.000	0.000	0.000	0.000
	(–)	(–)	(–)	(–)	(–)	(–)
λ	0.000	0.000	0.000	0.000	0.000	0.000
	(–)	(–)	(–)	(–)	(–)	(–)
J p-value	0.000	0.000	(–)	0.000	0.000	(–)
$Q_{\text{GMM}} \times 10^3$	36.743	16,504	7630.7	0.032	377.91	41.035
$\log L$	−2518.5	−1214.1	−211.3	−244.6	−2621.3	−64.95
			Panel B: iid measurement error			
σ_u^2	0.000	0.000	0.494	0.035	0.038	1.235
	(–)	(–)	(0.208)	(0.011)	(0.002)	(0.838)
σ_ε^2	0.033	0.029	0.002	0.000	0.000	0.005
	(0.041)	(0.010)	(0.000)	(0.001)	(0.000)	(0.001)
ϕ	0.663	0.671	0.997	0.974	0.999	0.999
	(0.292)	(0.077)	(0.002)	(0.223)	(0.105)	(0.001)
$\sigma_v^2 \times 10$	0.116	0.142	0.074	0.004	0.004	0.031
	(0.117)	(0.080)	(0.055)	(0.012)	(0.002)	(0.080)
λ	0.000	0.000	0.000	0.000	0.000	0.000
	(–)	(–)	(–)	(–)	(–)	(–)
J p-value	0.698	0.895	(–)	0.990	1.000	(–)
$Q_{\text{GMM}} \times 10^3$	13.409	55.396	155,952	0.001	0.015	1,327,033
$\log L$	−2417.6	−2416.5	−192.4	−684.26	−110.6	−60.304
			Panel C: MA(1) measurement error			
σ_u^2	0.000	0.000	0.499	0.026	0.023	0.238
	(–)	(–)	(0.208)	(0.011)	(0.002)	(0.838)
σ_ε^2	0.017	0.471	0.002	0.000	0.001	0.001
	(0.033)	(0.091)	(0.000)	(0.002)	(0.001)	(0.001)
ϕ	0.771	−0.976	0.996	0.928	0.915	0.999
	(0.320)	(0.061)	(0.002)	(0.262)	(0.107)	(0.001)
$\sigma_v^2 \times 10$	1.421	2.753	0.247	0.158	0.187	16.45
	(2.098)	(1.807)	(0.093)	(0.266)	(0.036)	(0.080)
λ	−1.00	−1.00	−1.00	−1.00	−1.00	−1.00
	(–)	(–)	(–)	(–)	(–)	(–)
J p-value	0.000	0.519	(–)	0.981	1.000	(–)
$Q_{\text{GMM}} \times 10^3$	29.766	2040.50	170,186	0.001	0.004	45,425
$\log L$	−2317.0	−1319.4	−200.4	−1464.8	−1608.4	−57.826

NOTE: The table reports estimates of the parameters of the Kalman filter model fitted to the Consensus Economics forecasts with standard errors in parentheses. Three estimation methods are considered: GMM using the identity weight matrix, efficient GMM (GMM^{eff}), and maximum likelihood (MLE). The p-values from the J-tests of over-identifying restrictions for the two GMM estimates are given in the third-last row. The final two rows presents the values of the GMM objective function, with identity weight matrix, and the log-likelihoods, at the estimated parameters. Panel A presents results when the model is estimated imposing that there is no measurement error (i.e., $\sigma_v^2 = \lambda = 0$), Panel B presents results when an iid measurement error (i.e., setting $\lambda = 0$) is considered, and Panel C presents results when a MA(1) measurement error is considered, imposing that $\lambda = -1$.

corresponding to iid noise, -1, corresponding to iid noise on the level, or some other value. In our short sample of data we found that this parameter was not well identified, and so we do not attempt to estimate it here.) The parameter estimates for this model are presented in Panel C of Table 2. For both GDP growth and inflation, we find quite similar results to the iid error case presented in Panel B: the MA(1) model is not rejected using the test of over-identifying restrictions, and the parameter estimates are similar. Note that the model for inflation is poorly estimated under ML. From both the value of the GMM objective function and the log-likelihood, we see that the iid measurement error is preferred for GDP growth, while the MA(1) error specification is preferred for inflation.

4. CONCLUSION

This paper considered survey forecasts of macroeconomic variables which hold the "event" date constant, while varying the length of the forecast horizon. We proposed a simple, parsimonious unobserved components model and developed tools for estimation and inference based on simulation methods that account for the forecasters' learning problem. Our methods can be used to estimate the size of measurement errors in the underlying variables and the degree of persistence in the data generating process.

Empirically, our analysis confirms several findings in the existing literature that were obtained using very different datasets: (1) Professional forecasters face severe measurement error problems for GDP growth in real time, while this is much less of an issue for inflation; (2) inflation exhibits greater persistence, and thus is predictable at longer horizons, than GDP growth; and (3) the persistent component of these variables is well approximated by a low-order autoregressive specification.

APPENDIX A

Proof of Proposition 1

Since $z_t = \sum_{j=0}^{11} y_{t-j}$ and $y_t = x_t + u_t$, where x_t is the persistent component, forecasting z_t given information h months prior to the end of the measurement period, $\mathcal{F}_{t-h} = \{x_{t-h}, y_{t-h}, x_{t-h-1}, y_{t-h-1}, \ldots\}$, requires accounting for the persistence in x. Note that

$$\begin{aligned} x_{t-h+1} &= \phi x_{t-h} + \varepsilon_{t-h+1}, \\ x_{t-h+2} &= \phi^2 x_{t-h} + \phi\varepsilon_{t-h+1} + \varepsilon_{t-h+2}, \\ &\vdots \\ x_t &= \phi^h x_{t-h} + \sum_{j=0}^{h-1} \phi^j \varepsilon_{t-j}. \end{aligned}$$

Adding up these terms we find that, for $h \geq 12$,

$$\begin{aligned} z_t &= \sum_{j=0}^{11} x_{t-j} + \sum_{j=0}^{11} u_{t-j} \\ &= \frac{\phi(1-\phi^{12})}{1-\phi} x_{t-12} + \frac{1}{1-\phi} \sum_{j=0}^{11} (1-\phi^{12-j}) \varepsilon_{t-12+1+j} \\ &\quad + \sum_{j=0}^{11} u_{t-j}. \end{aligned}$$

Thus the optimal forecast for $h \geq 12$ is

$$\hat{z}_{t|t-h} \equiv E[z_t|\mathcal{F}_{t-h}] = \sum_{j=0}^{11} E[y_{t-j}|\mathcal{F}_{t-h}] = \sum_{j=0}^{11} \phi^{h-j} x_{t-h},$$

so

$$\hat{z}_{t|t-h} = \frac{\phi^{h-11}(1-\phi^{12})}{1-\phi} x_{t-h} \quad \text{for } h \geq 12.$$

For the current-year forecasts ($h < 12$) the optimal forecast of z_t makes use of those realizations of y that have already been observed. Thus the optimal forecast is

$$\begin{aligned} \hat{z}_{t|t-h} &= \sum_{j=0}^{11} E[y_{t-j}|\mathcal{F}_{t-h}] = \sum_{j=h}^{11} y_{t-j} + \sum_{j=0}^{h-1} E[x_{t-j}|\mathcal{F}_{t-h}] \\ &= \sum_{j=h}^{11} y_{t-j} + \sum_{j=0}^{h-1} \phi^{h-j} x_{t-h}, \end{aligned}$$

so

$$\hat{z}_{t|t-h} = \sum_{j=h}^{11} y_{t-j} + \frac{\phi(1-\phi^h)}{1-\phi} x_{t-h} \quad \text{for } h < 12.$$

Using these expressions for the optimal forecasts, we can derive the forecast error, $e_{t|t-h} \equiv z_t - \hat{z}_{t|t-h}$, as a function of the forecast horizon. For $h \geq 12$,

$$\begin{aligned} e_{t|t-h} &= \sum_{j=0}^{11} u_{t-j} + \sum_{j=0}^{11} x_{t-j} - \frac{\phi^{h-11}(1-\phi^{12})}{1-\phi} x_{t-h} \\ &= \sum_{j=0}^{11} u_{t-j} + \sum_{j=0}^{11} \frac{1-\phi^{j+1}}{1-\phi} \varepsilon_{t-j} \\ &\quad + \sum_{j=12}^{h-1} \frac{\phi^{j-11}(1-\phi^{12})}{1-\phi} \varepsilon_{t-j}. \end{aligned}$$

In computing the variance of $e_{t|t-h}$ we exploit the fact that u and ε are independent of each other at all lags. For $h \geq 12$,

$$\begin{aligned} E\left[e_{t|t-h}^2\right] &= \sum_{j=0}^{11} E[u_{t-j}^2] + \sum_{j=0}^{11} \frac{(1-\phi^{j+1})^2}{(1-\phi)^2} E[\varepsilon_{t-j}^2] \\ &\quad + \sum_{j=12}^{h-1} \frac{\phi^{2j-22}(1-\phi^{12})^2}{(1-\phi)^2} E[\varepsilon_{t-j}^2] \\ &= 12\sigma_u^2 + \frac{\sigma_\varepsilon^2}{(1-\phi)^2} \left(12 - 2\frac{\phi(1-\phi^{12})}{1-\phi} \right. \\ &\quad \left. + \frac{\phi^2(1-\phi^{24})}{(1-\phi^2)} \right) + \frac{\phi^2(1-\phi^{12})^2(1-\phi^{2h-24})}{(1-\phi)^3(1+\phi)} \sigma_\varepsilon^2, \end{aligned}$$

as presented in the proposition. For $h < 12$ we have

$$\begin{aligned} e_{t|t-h} &= \sum_{j=0}^{11} y_{t-j} - \sum_{j=h}^{11} y_{t-j} - \frac{\phi(1-\phi^h)}{1-\phi} x_{t-h} \\ &= \sum_{j=0}^{h-1} u_{t-j} + \sum_{j=0}^{h-1} \frac{1-\phi^{j+1}}{1-\phi} \varepsilon_{t-j}, \end{aligned}$$

so

$$E\left[e_{t|t-h}^2\right] = \sum_{j=0}^{h-1} E[u_{t-j}^2] + \sum_{j=0}^{h-1} \frac{(1-\phi^{j+1})^2}{(1-\phi)^2} E[\varepsilon_{t-j}^2]$$
$$= h\sigma_u^2 + \frac{\sigma_\varepsilon^2}{(1-\phi)^2}\left(h - 2\frac{\phi(1-\phi^h)}{1-\phi} + \frac{\phi^2(1-\phi^{2h})}{1-\phi^2}\right).$$

APPENDIX B: REPRESENTATION OF THE ANNUAL TARGET VARIABLE

In this appendix we show that the yearly growth rates used by Consensus Economics (CE) as the target variables for their surveys can be represented as a weighted sum of monthly rates of growth. When the yearly variable is a simple December-on-December growth rate this representation is perfect; when the yearly variable is a growth rate involving averages of the level of the series within a calendar year (as in CE's definition) this representation is only an approximation, but one that is extremely accurate for realistic parameter values.

Let p_t be the level of a series (e.g., inflation or GDP growth) measured in month t. The CE yearly variables for inflation and GDP growth are defined as

$$z_t^{\text{INF}} \equiv \frac{\bar{p}_t}{\bar{p}_{t-12}} - 1 \qquad \text{and} \qquad z_t^{\text{GDP}} \equiv \frac{\tilde{p}_t}{\tilde{p}_{t-12}} - 1,$$

where

$$\bar{p}_t \equiv \frac{1}{12}\sum_{k=0}^{11} p_{t-k} \qquad \text{and} \qquad \tilde{p}_t \equiv \frac{1}{4}\sum_{k=0}^{3} p_{t-3k}.$$

We now show that we can represent z_t^{INF} and z_t^{GDP} as weighted sums of log-differences of p_t, $y_t \equiv \log(p_t/p_{t-1})$. First, take the inflation definition. We obtain an expression for $\bar{p}_t$ as a nonlinear function of the monthly growth rates:

$$p_{t-k} = p_{t-24}\exp\left\{\sum_{j=k}^{23} y_{t-j}\right\},$$

$$\bar{p}_t \equiv \frac{1}{12}\sum_{k=0}^{11} p_{t-k} = p_{t-24}\frac{1}{12}\sum_{k=0}^{11}\exp\left\{\sum_{j=k}^{23} y_{t-j}\right\}$$
$$= \exp\left\{\sum_{j=12}^{23} y_{t-j}\right\} p_{t-24}\frac{1}{12}\sum_{k=0}^{11}\exp\left\{\sum_{j=k}^{11} y_{t-j}\right\},$$

and

$$\bar{p}_{t-12} = p_{t-24}\frac{1}{12}\sum_{k=12}^{23}\exp\left\{\sum_{j=k}^{23} y_{t-j}\right\},$$

so

$$\frac{\bar{p}_t}{\bar{p}_{t-12}} = \frac{\exp\{\sum_{j=12}^{23} y_{t-j}\} p_{t-24}(1/12)\sum_{k=0}^{11}\exp\{\sum_{j=k}^{11} y_{t-j}\}}{p_{t-24}(1/12)\sum_{k=12}^{23}\exp\{\sum_{j=k}^{23} y_{t-j}\}}$$
$$= \frac{\exp\{\sum_{j=12}^{23} y_{t-j}\}\sum_{k=0}^{11}\exp\{\sum_{j=k}^{11} y_{t-j}\}}{\sum_{k=12}^{23}\exp\{\sum_{j=k}^{23} y_{t-j}\}}.$$

Note that we have not employed any approximations so far. Next we use the approximation $\exp\{a\} \approx 1 + a$ when $a \approx 0$. So

$$z_t^{\text{INF}} \equiv \frac{\bar{p}_t}{\bar{p}_{t-12}} - 1 \approx \log\left(\frac{\bar{p}_t}{\bar{p}_{t-12}}\right)$$
$$= \log\left(\frac{\exp\{\sum_{j=12}^{23} y_{t-j}\}\sum_{k=0}^{11}\exp\{\sum_{j=k}^{11} y_{t-j}\}}{\sum_{k=12}^{23}\exp\{\sum_{j=k}^{23} y_{t-j}\}}\right)$$
$$= \sum_{j=12}^{23} y_{t-j} + \log\left(\sum_{k=0}^{11}\exp\left\{\sum_{j=k}^{11} y_{t-j}\right\}\right) - \log\left(\sum_{k=12}^{23}\exp\left\{\sum_{j=k}^{23} y_{t-j}\right\}\right)$$
$$\approx \sum_{j=12}^{23} y_{t-j} + \log\left(\sum_{k=0}^{11}\left(1 + \sum_{j=k}^{11} y_{t-j}\right)\right) - \log\left(\sum_{k=12}^{23}\left(1 + \sum_{j=k}^{23} y_{t-j}\right)\right)$$
$$= \sum_{j=12}^{23} y_{t-j} + \log\left(12 + \sum_{k=0}^{11}\sum_{j=k}^{11} y_{t-j}\right) - \log\left(12 + \sum_{k=12}^{23}\sum_{j=k}^{23} y_{t-j}\right)$$
$$= \sum_{j=12}^{23} y_{t-j} + \log\left(12 + \sum_{k=0}^{11}(k+1)y_{t-k}\right) - \log\left(12 + \sum_{k=12}^{23}(k-11)y_{t-k}\right).$$

Then we apply this approximation again, to obtain $\log(12 + a) \approx \log(12) + a/12$ when $a \approx 0$. This allows

$$z_t^{\text{INF}} \approx \sum_{j=12}^{23} y_{t-j} + \log(12) + \frac{1}{12}\sum_{k=0}^{11}(k+1)y_{t-k} - \log(12) - \frac{1}{12}\sum_{k=12}^{23}(k-11)y_{t-k}$$
$$= \sum_{j=12}^{23} y_{t-j} + \frac{1}{12}\sum_{k=0}^{11}(k+1)y_{t-k} - \frac{1}{12}\sum_{k=12}^{23}(k-11)y_{t-k}$$
$$= \sum_{k=0}^{11}\frac{k+1}{12}y_{t-k} + \sum_{k=12}^{23}\frac{23-k}{12}y_{t-k}$$
$$\equiv \sum_{k=0}^{23}\omega_k^{\text{INF}} y_{t-k},$$

where

$$\omega_k^{\text{INF}} \equiv \begin{cases} 1 - |k-11|/12, & 0 \le k < 24 \\ 0, & k \ge 24. \end{cases}$$

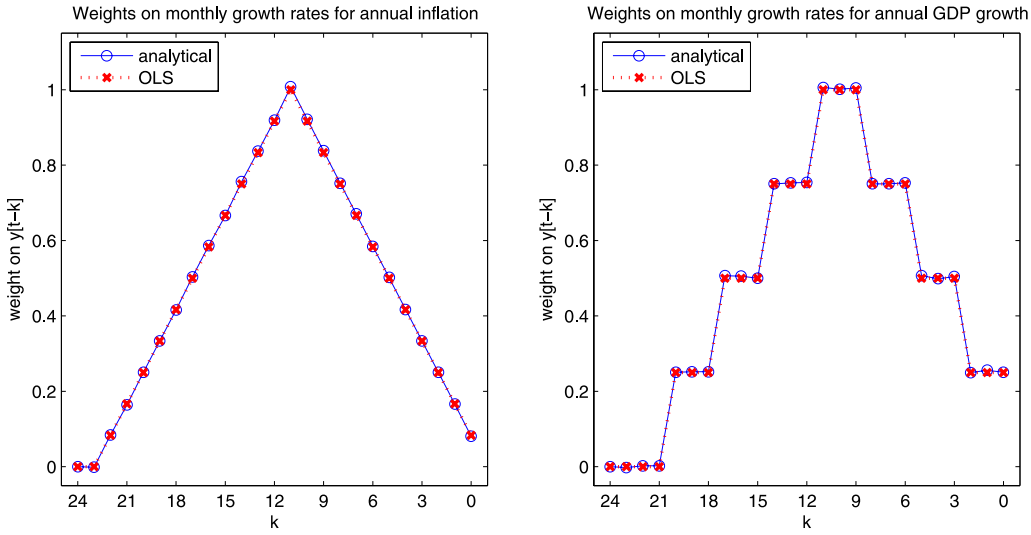

Figure B.1. Weights on monthly growth rates to obtain the annual target variables used by Consensus Economics via analytical approximation or by OLS estimation. The online version of this figure is in color.

A similar set of computations for the GDP target variable (details available upon request) yields the following weights:

$$z_t^{\text{GDP}} \equiv \frac{\tilde{p}_t}{\tilde{p}_{t-12}} - 1 \approx \sum_{k=0}^{23} \omega_k^{\text{GDP}} y_{t-k},$$

where

$$\omega_k^{\text{GDP}} = \begin{cases} 1 - |\lfloor k/3 \rfloor - 3|/4 & 0 \le k < 24 \\ 0, & k \ge 24, \end{cases}$$

and $\lfloor a \rfloor$ rounds a down to the nearest integer.

To confirm that the approximations used above are reasonable in practice, we ran the following simulation study. We generated 10,000 samples of 24 "months" of log-changes in the variable, y_t. We assume that the monthly growth rates are iid uniformly distributed in the range $[-5\%, +5\%]$. (Note that this degree of volatility is rather high, which works against the accuracy of our approximation.) We then computed the *exact* annual target variable (either inflation or GDP) using each of these 24 months of log-changes, and labeled this z_i^X, for $X \in \{\text{INF}, \text{GDP}\}$. Finally, we compute the R^2 of the analytical approximation for the true annual growth rates, across the 10,000 replications. These R^2-values were 0.9950 and 0.9949 respectively.

To compare the analytical weights with the optimal linear approximation to the annual variable using these monthly growth rates, we regress the exact annual target variable on the 24 monthly log-growth rates across the 10,000 replications:

$$z_i^{\text{INF}} = \sum_{j=0}^{23} \beta_j y_{24-j}^{(i)} + u_i, \qquad i = 1, 2, \ldots, 10{,}000,$$

$$z_i^{\text{GDP}} = \sum_{j=0}^{23} \gamma_j y_{24-j}^{(i)} + e_i.$$

The R^2 from these regressions were 0.9951 and 0.9949 respectively. The estimated coefficients, along with the analytical approximate weights derived above, are presented in Figure B.1. This figure shows that the analytical approximation is essentially identical to the optimal linear approximation. To see how volatility in this variable affects the approximation, we also considered monthly growth rates as distributed in the ranges $[-10\%, 10\%]$ and $[-2.5\%, 2.5\%]$. The R^2-values both slightly fell to 0.98 in the first case, and both rose to 0.999 in the second case.

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