Monotonicity in Asset Returns: New Tests with Applications to the Term Structure, the CAPM and Portfolio Sorts

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Motivation

- Many finance theories predict a monotonic relationship between expected returns and other variables:
  - The liquidity preference hypothesis predicts higher average returns for longer-dated bonds [Richardson, Richardson and Smith, 1992]
  - The CAPM predicts higher average returns for higher beta stocks
  - Theories of momentum predict higher average performance for high past performance [Johnson, 2002]
- The full set of implications of such monotonic patterns is generally *not* explored in empirical analysis.
- Conventionally, a test is conducted by forming portfolios of stocks ranked by a particular characteristic, and then testing that the top-minus-bottom average return differential is significant and of the predicted sign.

Portfolio sorts in the literature

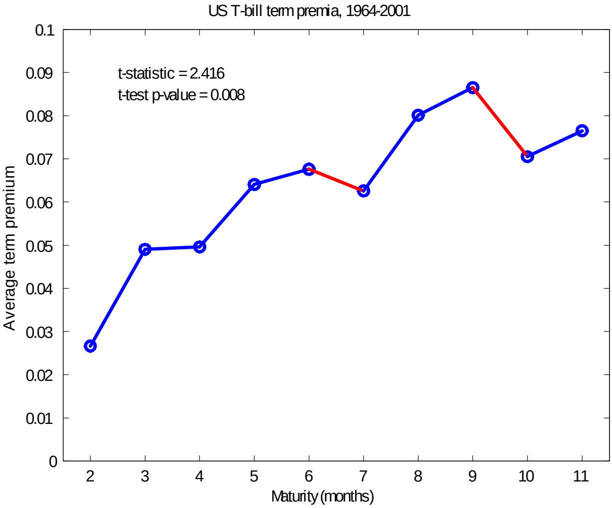
- **One-way sorts**:
  - book-to-market: Basu (1977, 1983), Fama and French (1992, 2006)
  - firm size: Banz (1981), Reinganum (1981), Berk (1995)
  - financial constraints: Lamont, Polk and Saa-Requejo (2001)
  - liquidity: Pastor and Stambaugh (2003)
  - default risk: Vassalou and Xing (2004)
  - volatility: Ang, Hodrick, Xing and Zhang (2006)
  - 'downside' risk: Ang, Chen and Xing (2006)
  - momentum, performance persistence: Jegadeesh and Titman (1993), Carhart (1997)
- **Double sorts**: momentum and size (Rouwenhorst (1998)), financial constraints and R&D expenditures (Li (2007))
- **Triple sorts**: Daniel, Grinblatt, Titman and Wermers (1997) and Vassalou and Xing (2004)

Monotonicity at the WFA 2009

1. Easley and O'Hara (2008): In the presence of ambiguity, the bid-ask spread is monotonically increasing in the degree of ambiguity
2. Kelly and Ljungqvist (2009): Average returns are monotonically increasing (less negative) in the number of analysts that continue to cover a stock after another analyst ceases coverage
3. Li and Palomino (2008): Expected returns are monotonically decreasing in the degree of price rigidity in the firm's industry
4. Wu, Huang, Liu and Rhee (2009): Expected returns are monotonically increasing in their "extreme downside risk" measure
5. Choi, Getmansky, Henderson and Tookes (2009): Security issuance is monotonically increasing in capital supply (for convertible bonds)

Is this relationship significantly positive?

Average T-bill term premia, 1964-2001



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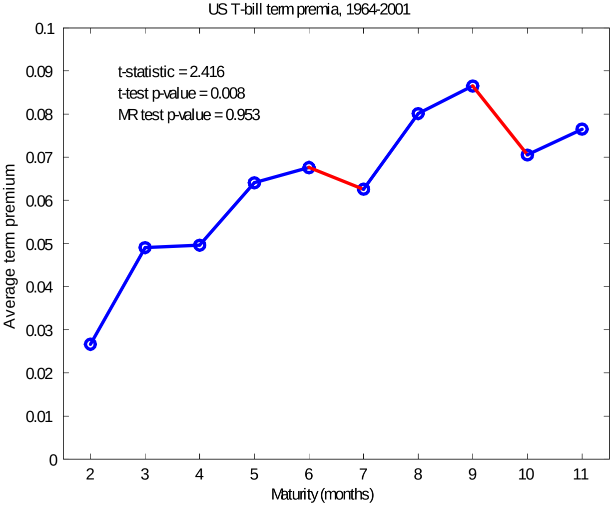

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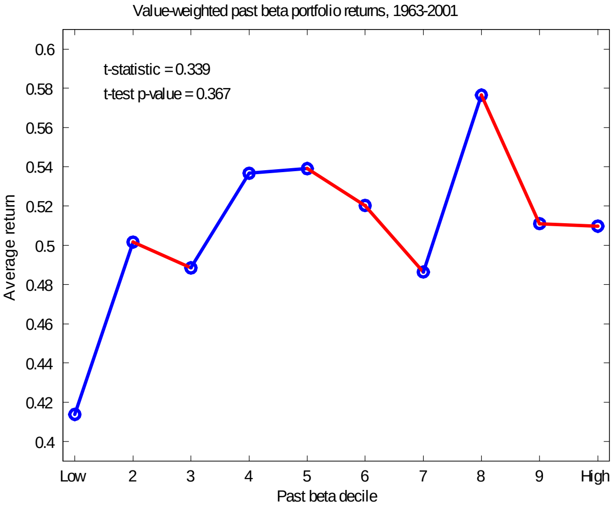
Expected Returns on CAPM beta decile portfolios, 1963-2001



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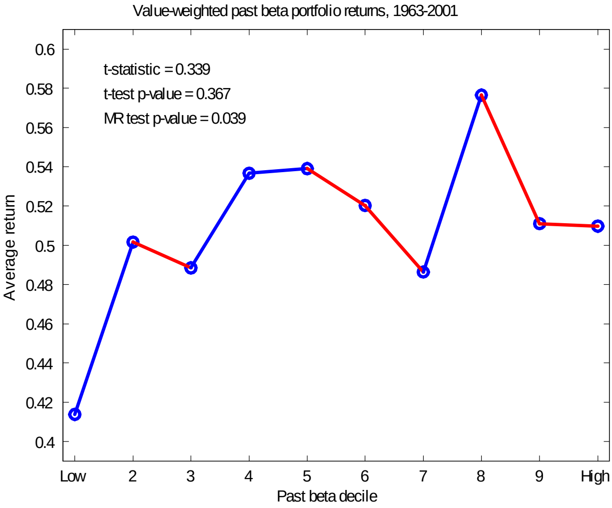

Contributions of this paper

- This paper proposes a test of the *monotonic relationship* between expected returns on assets (e.g., portfolios) sorted on some variable.
- Such a test is more directly related to the predictions of economic theories ($\partial\mu/\partial Z > 0$)
- Our "MR" tests are nonparametric, powerful, and easy to implement via the bootstrap.

Contributions of this paper, cont'd

- Our MR test generalises to cover several interesting cases:

1. Sorts based on **multiple variables:** two-way sorts, three-way sorts, etc.

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1. Sorts based on **multiple variables:** two-way sorts, three-way sorts, etc.
2. Monotonic relationships in **other parameters** of interest: risk-adjusted returns (alphas), or factor loadings (betas) etc.
3. **Piece-wise** monotonic relationships: a U-shaped or inverse-U shaped relationship, etc.

Outline of the talk

1. Motivation of tests of monotonicity
2. **Theory for the test for a monotonic relationship**
   1. Null and alternative hypotheses
   2. Two-way and D-way sorts
   3. Conducting the test via the bootstrap
3. Empirical findings
   1. Portfolio sorts on CAPM beta
   2. Monotonicity of the term premium
   3. Two-way sorts
4. Summary and conclusions

Portfolio sorts and trading strategies

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  - short the bottom ranked asset and invest in the top ranked asset, reaping the difference in expected returns
- If interest is limited to establishing such a trading strategy *and* it is possible to short the bottom-ranked stocks then the standard approach may suffice.
- If interest is focussed on testing the predictions of a theory that ranks stocks based on variables proxying for risk (or liquidity, or similar) then the *complete* cross-sectional pattern in expected returns should be used.

Testing for a monotonic relationship in expected returns

- Let μ_i, $i = 1, 2, ..., N$, be the expected return on the i^{th} asset obtained from a ranking on some characteristic

- Economic theory often suggests that an increasing $(\mu_{i-1} < \mu_i)$ or decreasing $(\mu_{i-1} > \mu_i)$ pattern in expected returns.

- We take as our null hypothesis the *absence* of any relationship or a relationship of the wrong sign, and seek to reject this in favour of the relationship predicted by the theory:

$$\begin{aligned} H_0 &: \mu_1 \geq \mu_2 \geq ... \geq \mu_N \\ H_1 &: \mu_1 < \mu_2 < ... < \mu_N \end{aligned}$$

- This is parallel to standard practice: the theory is only endorsed if the data provides statistically significant evidence against the null in favour of the predicted relationship.

Testing for a monotonic relationship in expected returns

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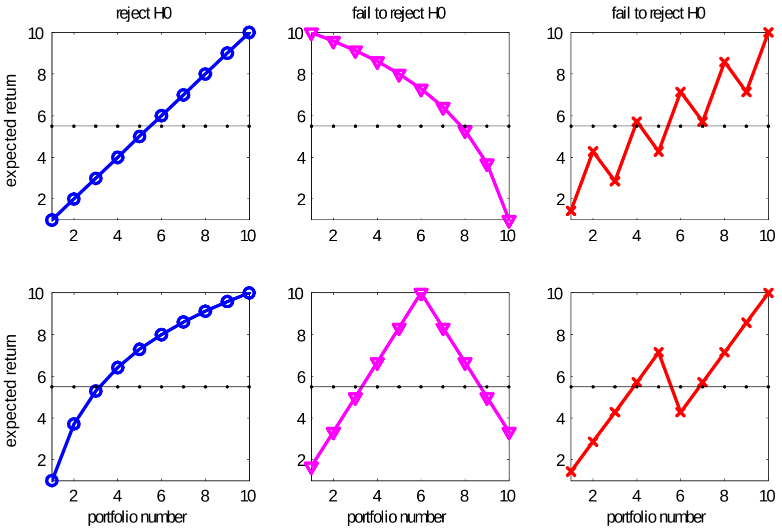
\end{aligned}
$$

- Our alternative is a multivariate one-sided hypothesis: there are many possible violations of H_0 that are not consistent with H_1
- Our test will only look for deviations of H_0 "in the direction" of H_1
  - We do not look for evidence against H_0 in the direction of a non-monotonic relationship, nor do we look for evidence of a monotonic relationship in the 'wrong' direction.
  - Thus a rejection of the null is evidence of a relationship consistent with the theory

Three types of patterns in expected returns



Wolak's test for a monotonic relationship

- An alternative approach to test for (the absence of) a monotonic relationship was provided by Wolak (1989) and implemented by Richardson, Richardson and Smith (1992).
- In that test the null and alternative hypotheses are:

$$
\begin{aligned}
H_0 &: \mu_1 \leq \mu_2 \leq ... \leq \mu_N \\
H_1 &: \mu_i > \mu_j \text{ for some } i < j
\end{aligned}
$$

- Here the weakly monotonic relationship is entertained under the null
  - Limited power (due to short samples or noisy data) may mean that a failure to reject the null of a monotonic relationship does not add much confidence to the conjectured relationship
  - Further, the null also includes the case of no relationship $(\mu_i = \mu_j)$
- We present the results of both tests for comparison

Implementing the MR test

- Let

$$\begin{aligned} \hat{\Delta}_i &= \hat{\mu}_i - \hat{\mu}_{i-1}, \;\; i = 2, .., N \\ \text{where} \;\; \hat{\mu}_i &\equiv \frac{1}{T} \sum_{t=1}^{T} r_{it} \end{aligned}$$

- Then the null and the alternative can be rewritten as

$$\begin{aligned} H_0 &: \;\; \Delta_i = 0, \;\; i = 2, ..., N \\ H_1 &: \;\; \min_{i=2,..,N} \Delta_i > 0. \end{aligned}$$

- To see this, note that if the *smallest* value of $\Delta_i = \mu_i - \mu_{i-1} > 0$, then we must have $\mu_i > \mu_{i-1}$ for *all* portfolios $i = 2, ..., N$. This motivates our choice of test statistic:

$$J_T = \min_{i=2,..,N} \hat{\Delta}_i \quad \text{or} \quad J_T = \min_{i=2,..,N} \hat{\Delta}_i / \hat{\sigma}_{\Delta i}$$

Two-way sorts and D-way sorts

- For an $N \times K$ table, the number of non-redundant inequalities implied by the alternative hypothesis is $2KN - N - K$, or $2N(N-1)$ if $K = N$
    - For a 5×5 table, 40 inequalities are implied
    - For a 10×10 table 180 inequalities are implied
- For a D-dimensional table with N elements in each dimension the number of inequalities is $DN^{D-1}(N-1)$
    - For $5 \times 5 \times 5$ table, 300 inequalities are implied
    - For $3 \times 3 \times 3 \times 3$ table, 216 inequalities are implied
- This shows how complicated and how rich the full set of relations implied by theory can be when applied to D-way portfolio sorts.

Conducting the test for a monotonic relationship

- Under standard conditions we know that

$$\sqrt{T}\left([\hat{\mu}_1, \ldots, \hat{\mu}_N]' - [\mu_1, \ldots, \mu_N]'\right) \to^d N(\mathbf{0}, \Omega)$$

- This is not so useful in our case as:

1. Requires estimating Ω, which is large if the number of individual portfolios is even moderately-sized.

2. We are interested in the distribution of

$$\min_{i=2,\ldots,N} \left(\hat{\mu}_i - \hat{\mu}_{i-1}\right)$$

   which is a non-standard test statistic, and requires simulation from the asymptotic distribution.

A bootstrap test for a monotonic relationship

- We instead draw on the theory in White (2000, Econometrica), developed for controlling for 'data snooping', who justifies the use of the bootstrap to obtain critical values

- We use the vector 'stationary bootstrap' of Politis and Romano (1994) to generate new samples of returns from the true sample.
  - This preserves any cross-sectional correlation
  - Accounts for autocorrelation and heteroskedasticity
  - Accounts for non-normality of returns

- This approach easily handles many inequality tests and thus two-way or D-way sorts are manageable.

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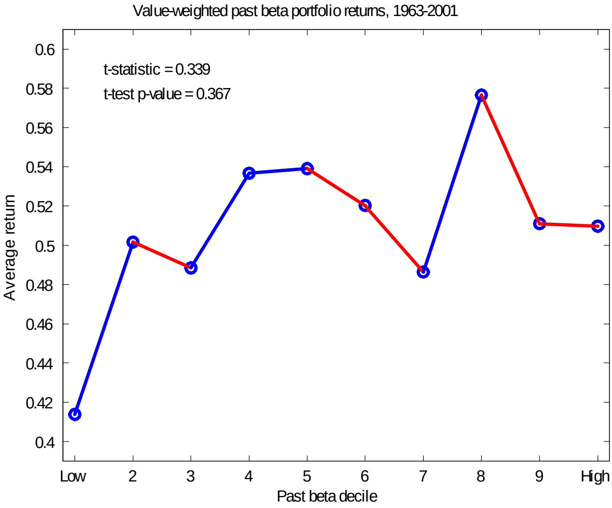
Portfolio sorts on CAPM beta

- We now present results of tests for a relationship between ex-ante estimates of CAPM beta and subsequent returns, using the same data as Ang, Chen and Xing (2006)
  - Each month, stocks are sorted into deciles using estimates of beta based on the past year of daily returns, and value-weighted portfolios are formed
  - If the CAPM holds, we would expect a monotonically increasing pattern in average returns
- We also study whether the post-ranked betas of these portfolios are monotonically increasing: failure of this property would suggest that past betas have little predictive content for future betas, perhaps due to instability

Ex-ante CAPM beta and expected returns

Value-weighted portfolios, 1963-2001



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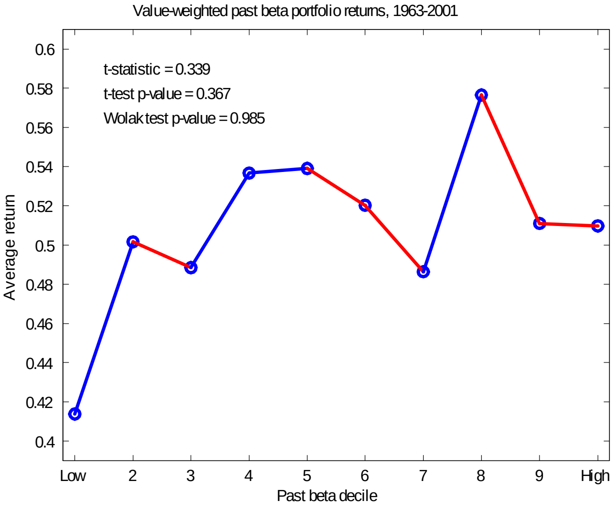

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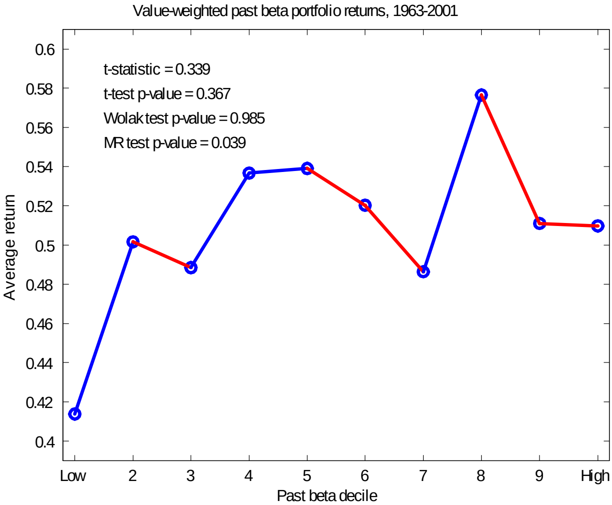

Ex-ante CAPM beta and ex-post betas

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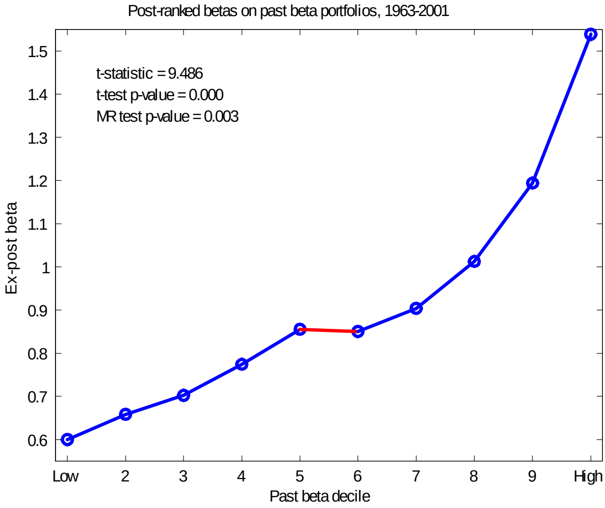
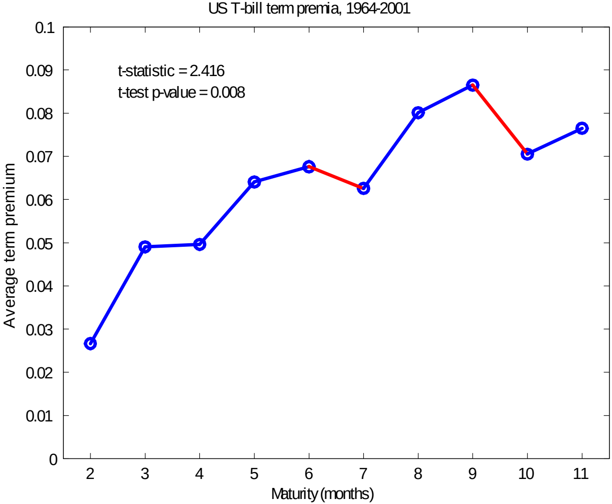


Testing Monotonicity of the Term Premium

- Fama (1984), McCulloch (1987) and Richardson, Richardson and Smith (1992) studied the implication of the liquidity preference hypothesis that term premia on Treasury securities should be increasing in time to maturity.
    - Fama (1984) used Bonferroni bounds to test for evidence *against* monotonicity, and found such evidence for the 9-month vs. 10-month bills
    - Richardson, Richardson and Smith (1992) studied a longer time series of data (1962-1990) using the more powerful Wolak (1989) test, and also strongly rejected monotonicity, over the full sample.
    - RRS also found that this rejection was due to the 1964-1972 sub-period, after which monotonicity could not be rejected.
- We re-visit this question using our MR test, using data from 1964-2001, and maturities from 2 to 11 months.

Term Premia and Time to Maturity

US T-bills, 1964-2001



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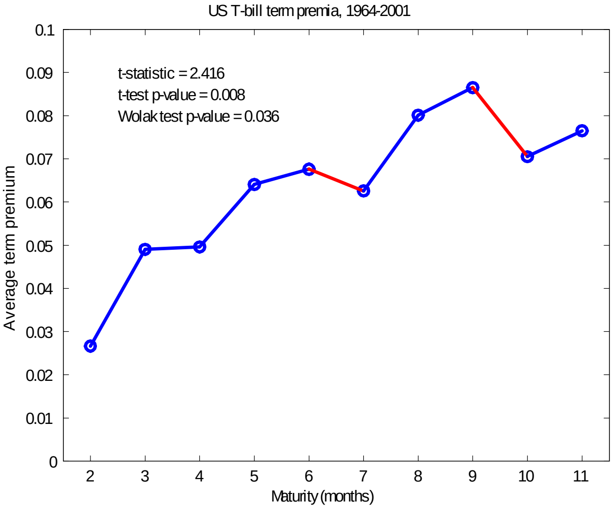

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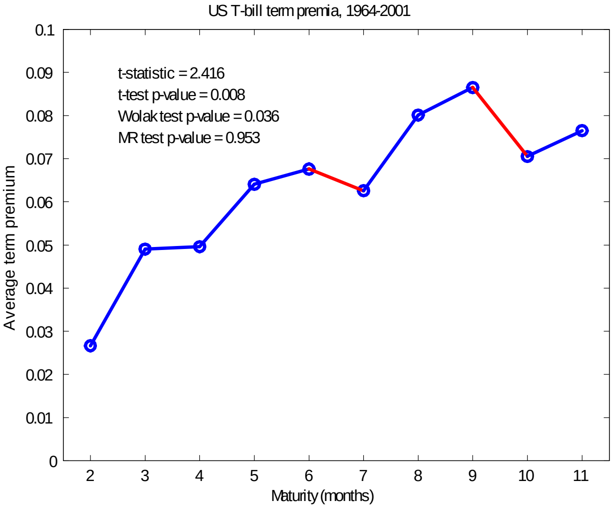

Term Premia and Time to Maturity

US T-bills, by sub-sample, from Table 2 Panel B

Tests of monotonicity of term premia

	top minus	t-test		MR	Wolak	Bonf.
Sample	bottom	t-stat	p-val	p-val	p-val	p-val
1964–2001	**0.050**	**2.416**	**0.008**	**0.953**	**0.036**	**0.020**
1964–1972	0.026	0.908	0.182	0.983	0.007	0.004
1973–2001	0.057	2.246	0.012	0.633	0.340	0.704

Two-way portfolio sorts

- We next examine some two-way portfolio sorts, using data from Ken French's web site.
- We look at 5×5 portfolios sorted on size and four other factors: book-to-market, momentum, short-term reversal and long-term reversal.
    - These sorts are "independent" double sorts
    - Our tests apply equally well to "independent" or "conditional" double sorts.

Two-way portfolio sorts

Size and Book-to-Market, 1963-2006, from Table 4 Panel A

	Growth	2	3	4	Value	MR pval	Joint pval
Market equity			Book-to-market ratio				
Small	0.71	1.30	1.34	1.55	1.66	**0.02**	
2	0.88	1.14	1.41	1.46	1.52	**0.00**	
3	0.89	1.21	1.21	1.33	1.51	0.06	**0.00**
4	1.00	0.99	1.22	1.33	1.37	**0.04**	
Big	0.88	0.97	0.98	1.07	1.07	**0.02**	
MR pval	0.69	0.40	0.41	0.07	**0.03**		
Joint MR pval			0.34				0.08

Two-way portfolio sorts

Size and Momentum, 1963-2006, from Table 4 Panel B

	Losers	2	3	4	Winners	MR pval	Joint pval
Market equity			Momentum				
Small	0.36	1.15	1.42	1.56	1.97	**0.00**	
2	0.42	1.03	1.26	1.50	1.78	**0.00**	
3	0.60	0.98	1.12	1.23	1.73	**0.00**	0.15
4	0.60	0.99	1.03	1.24	1.58	**0.01**	
Big	0.65	0.88	0.77	0.98	1.27	0.55	
MR pval	0.89	0.14	**0.00**	0.12	**0.02**		
Joint MR pval			0.71				0.55

Summary and conclusions

- Theoretical research in financial economics often generates a prediction of a monotonic relationship between an asset's expected return and some characteristic of the asset

- This paper presents a new, nonparametric, direct test of whether such a prediction is borne out in the data.

- We see two principal uses for the new "MR" test:

  1. As a descriptive statistic for monotonicity in expected returns or other functions of returns (eg, slope coefficients)
  2. As a formal test of a theoretical model that predicts a monotonic relationship in the data

- Matlab code to replicate all results in this paper is available at:

  `www.econ.ox.ac.uk/members/andrew.patton/code.html`