On the Out-of-Sample Importance of Skewness and Asymmetric Dependence for Asset Allocation

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Outline of talk

- Motivation
- Definition of asymmetric dependence
- Set-up of the problem
  - Data
  - Investor's utility function and optimisation problem
  - Density models: mean, variance, skewness and copula
  - Investment strategies
  - Portfolio performance measures
- Results
  - Unconstrained versus short sales constrained
  - Economic significance
  - Statistical significance

Motivation: stock returns are non-normal

- The distribution of stock returns are widely reported as be being **skewed**, see Kraus and Litzenberger (1976), Harvey and Siddique (1999,2000), *inter alia.*

- Recent studies report that stock returns are more higly correlated in bear markets than bull markets – a form of **asymmetric dependence**, see Erb *et al.* (1994), Longin and Solnik (2001), Ang and Chen (2002).

Describing asymmetric dependence

- There are a number of ways of trying to measure and present asymmetric dependence

- One simple way is to look at *exceedence correlations* , see Longin and Solnik (2001) and Ang and Chen (2002):

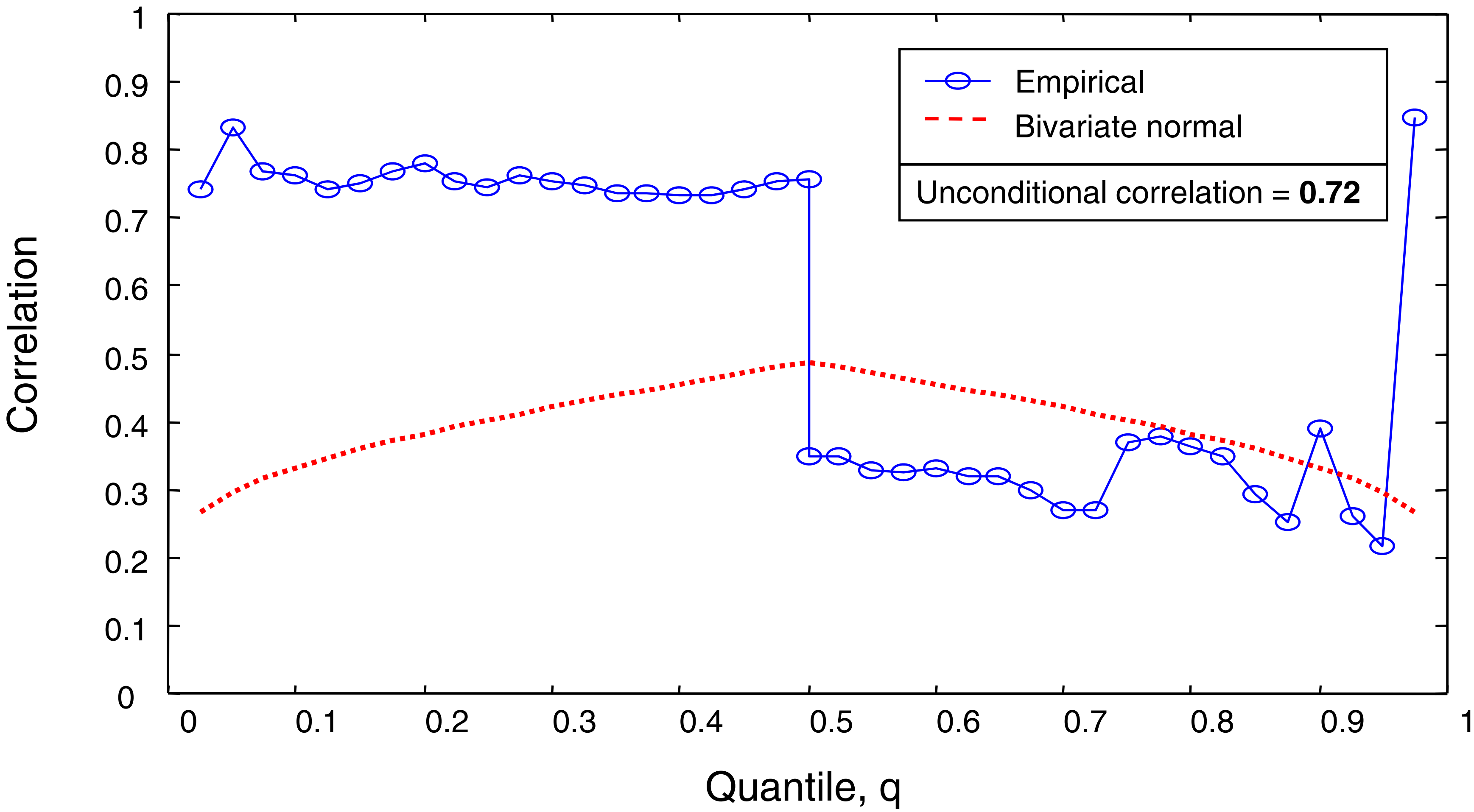
Correl [X , Y | Quantile(X) < q , Quantile(Y) < q], for $q \leq 0.5$

Correl [X , Y | Quantile(X) > q , Quantile(Y) > q], for $q \geq 0.5$

- [*I don't use this measure in the modelling stage, but it is useful for preliminary analysis of the data.*]

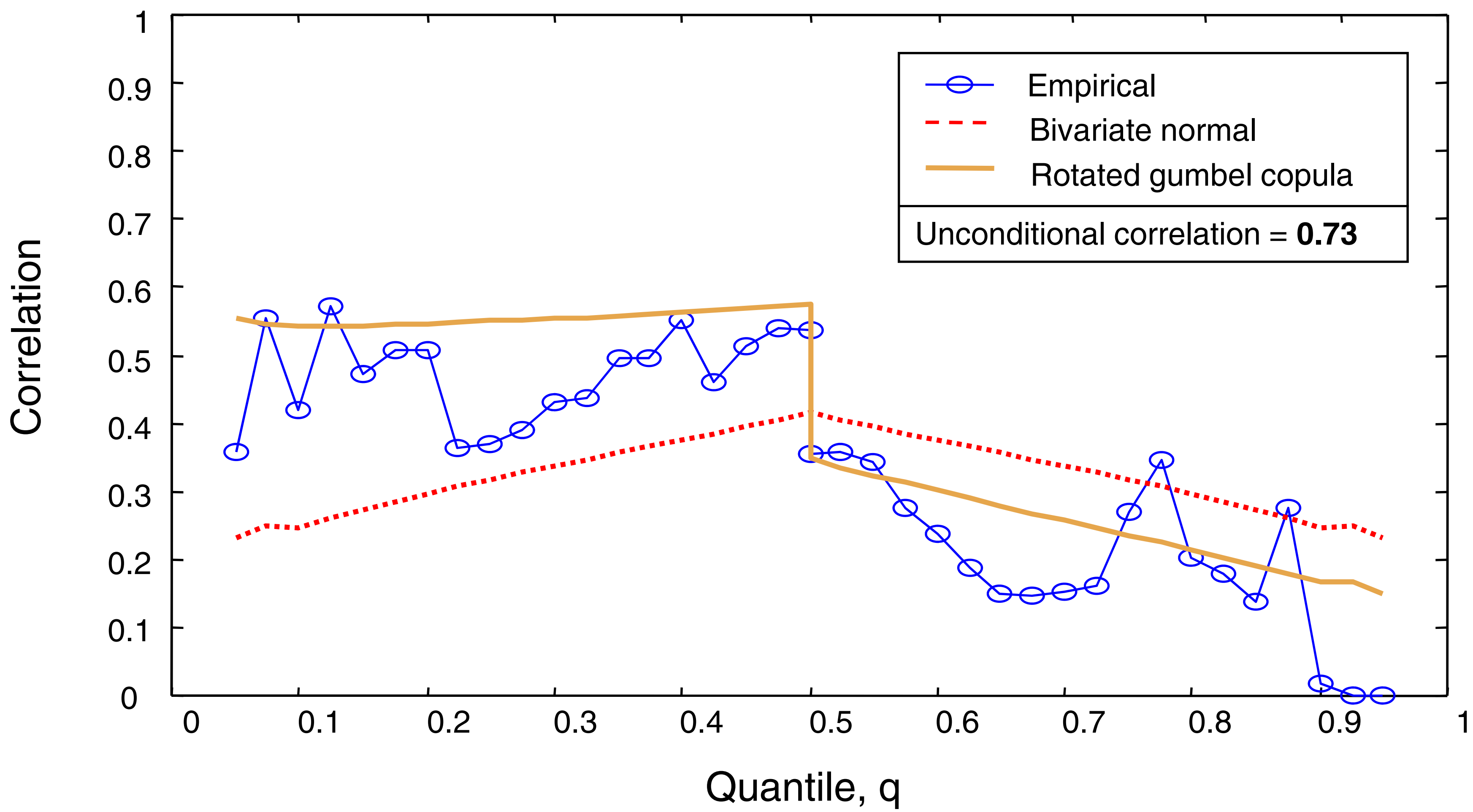
Asymmetric dependence

Exceedence correlations between raw excess returns



Asymmetric dependence

Exceedence correlations between transformed residuals



Goal of this research

- The presence of skewness and/or asymmetric dependence violates the assumption that stock returns are normally distributed

- I attempt to determine the economic and statistical significance of these non-normalities for a particular pair of indices, in the context of out-of-sample asset allocation

- I find substantial economic significance, and moderate statistical significance

Investor's optimisation problem

- The investor's optimisation problem is:

$$\omega_t^* = \arg\max_{\omega} \hat{E}_{t-1}[U(\omega_x X_t + \omega_y Y_t \})]$$

$$\equiv \arg\max_{\omega} \iint U(\omega_x x + \omega_y y).\hat{h}_t(x, y).dx.dy$$

$$= \arg\max_{\omega} \iint U(\omega_x x + \omega_y y).\hat{f}_t(x).\hat{g}_t(y).\hat{c}_t(\hat{F}_t(x), \hat{G}_t(y))dx.dy$$

where U is a CRRA utility function with RRA of 1, 3, 7, 10 and 20.

Data and Estimation

- Monthly data from Jan 1954 to Dec 1999 on a U.S. risk-free asset, a small cap and a big cap stock index.
  - In-sample period: Jan 1954 – Dec 1989, 420 obs
  - Out-of-sample period: Jan 1990 – Dec 1999, 120 obs
- Model selection is done only once, using the in-sample data.
- Parameters of the model are estimated recursively throughout the out-of-sample period.

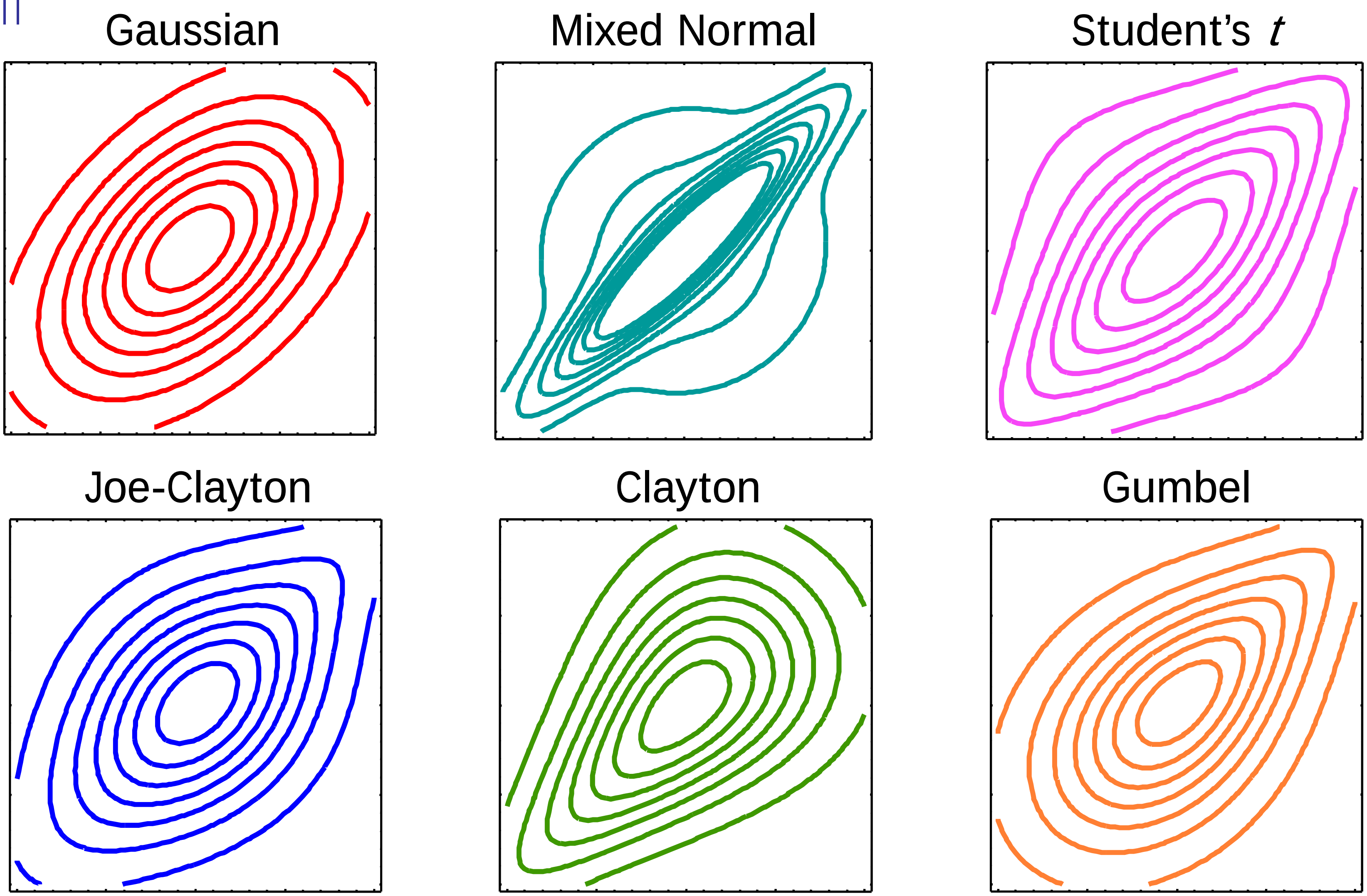
Copulas and Sklar's theorem

- Sklar (1959) showed that we may decompose the distribution of (X,Y) into three parts:

$$H(x, y) \Leftrightarrow C(F(x), G(y)) \quad \forall x, y$$

- Joint dist'n of X and Y → $H(x, y)$
- *Copula* of X and Y → C
- Marginal dist'n of X → $F(x)$
- Marginal dist'n of Y → $G(y)$

All of these distributions have N(0,1) marginal distributions and ρ=0.50



The density models

- I compare the performance of three density models.
  - All have AR models for the mean, and TARCH models for the variance
  - All use DIV, RF and SPR as explanatory variables

1. The first assumes a bivariate normal density
2. The second allows for time-varying skewness, via Hansen's (1994) skewed *t*, but imposes a normal copula
3. The third allows for time-varying skewness and chooses the optimal copula model from a set of 9 possible copulas (selects the 'rotated Gumbel' copula)

The asset allocation decision rules

1. 100% weight in small caps
2. 100% weight in big caps
3. 50% weight in each stock index
4. Optimise using unconditional distribution
5. Optimise using a bivariate normal
6. Optimise using a skewed *t* – Normal copula
7. Optimise using a skewed *t* –flexible copula

Portfolio performance measures

- I use four measures of portfolio performance:

1-3. Mean to risk ratios:

  - Mean / standard deviation (Sharpe ratio)
  - Mean / 5% Value-at-Risk
  - Mean / 5% Expected Shortfall

4. Management fee

  - A more interpretable value than average realised utility
  - This is a fee, expressed in basis points per year, that a particular investor would be willing to pay to switch from a 50:50 portfolio to another portfolio.

Short sales constraints

- Short sales constraints have two interpretations in this context:

1. Economically they reflect the constraints that many market participants face, and so possibly make the study more realistic

1. Econometrically they can be interpreted as an 'insanity filter', preventing the hypothetical investor from taking extreme positions in the market.

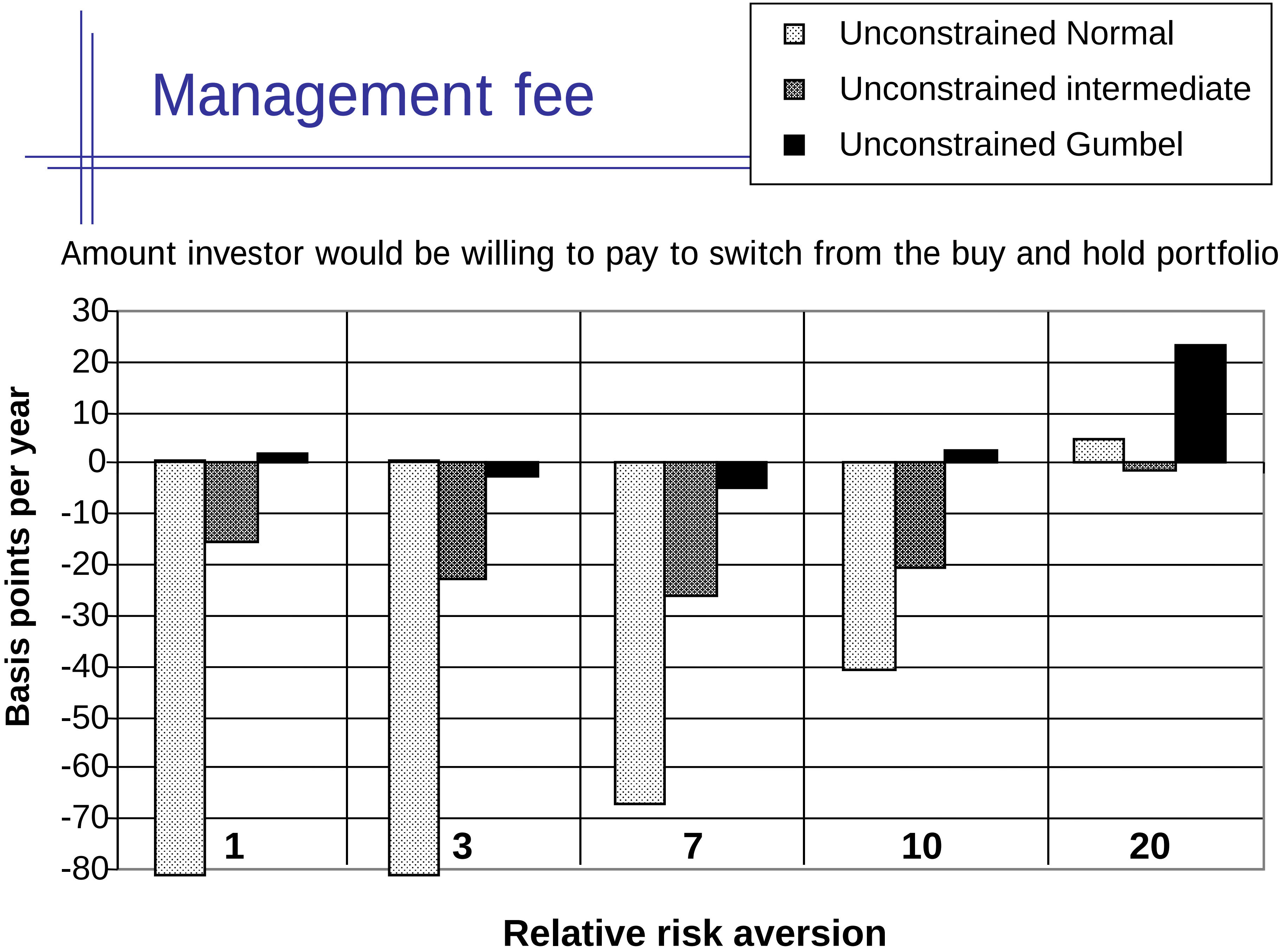
   $\rightarrow$ Stock and Watson (1999), for example, find that such filters improve forecast accuracy from non-linear models.

Economic significance

- Gumbel model out-performs the normal model 16 out of 20 comparisons
  - Overall average out-performance is 16.7%
  - Average out-performance in management fee is 41 (1) basis points for unconstrained (constrained) investors.
- Gumbel model out-performs the ‘intermediate’ model in all 20 comparisons
  - Overall average out-performance is 52.3%
  - Average out-performance in management fee is 21 (1.5) basis points for unconstrained (constrained) investors

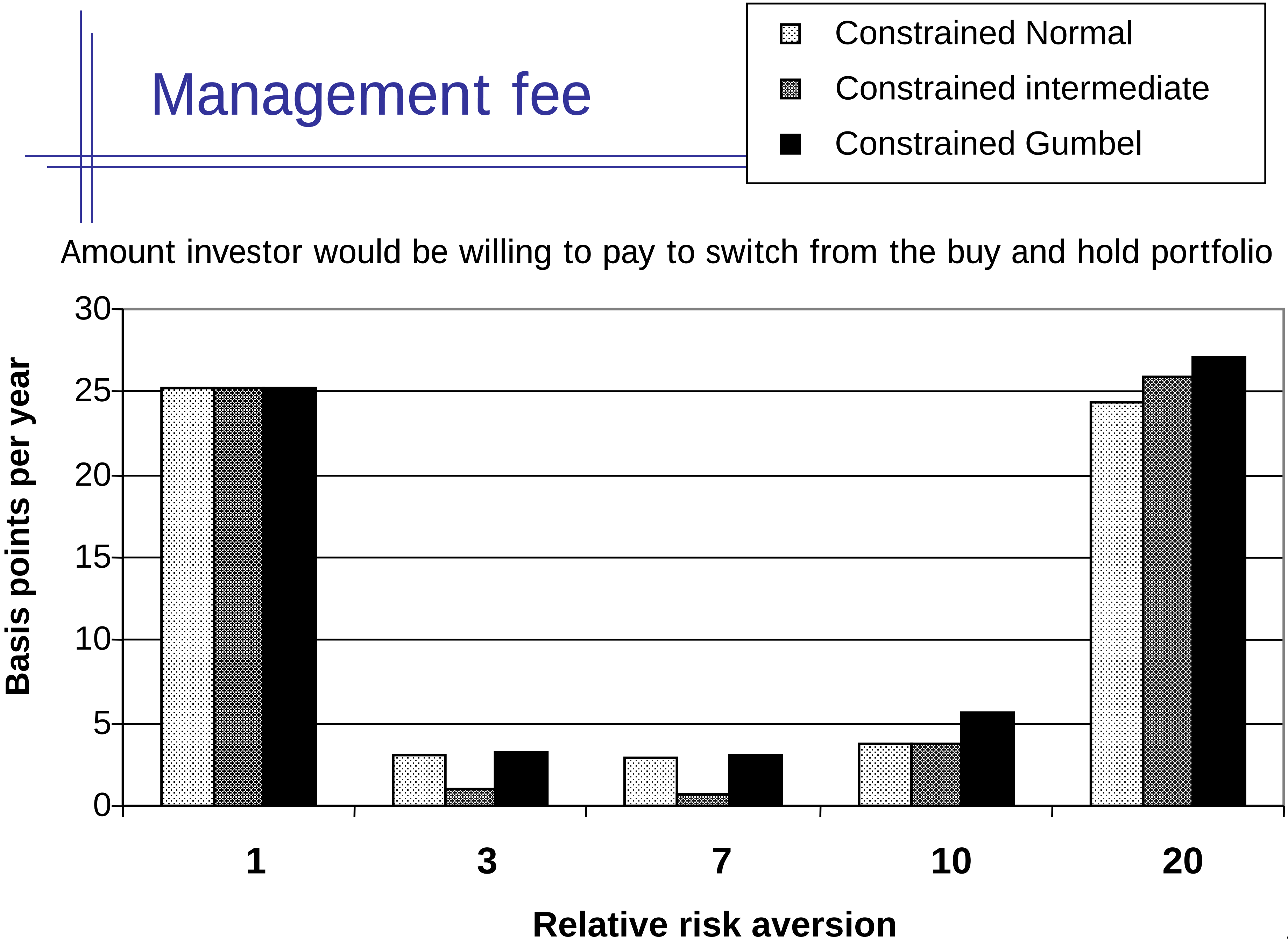
Management fee

Amount investor would be willing to pay to switch from the buy and hold portfolio



Management fee

Amount investor would be willing to pay to switch from the buy and hold portfolio



Pair-wise comparison bootstrap tests

- Focussing on results using realised utility:

Unconstrained investors:

- Gumbel model significantly outperformed *both* the Normal and intermediate models for all levels of risk aversion

- Normal and intermediate models were not distinguishable

Pair-wise comparison bootstrap tests

Short sales constrained investors:

- Gumbel out-performed Normal model for high risk aversion (RRA=10 and 20) while Normal out-performed Gumbel for RRA=1
- Gumbel outperformed the intermediate model for all levels of risk aversion
- Normal and intermediate models were again indistinguishable

Bootstrap reality check results

- Reject benchmark portfolio if ‘consistent’ p-value is less than 0.10

Benchmark portfolio: Normal

	Unconstrained			Short sales constrained		
RRA	Lower	Consistent	Upper	Lower	Consistent	Upper
1	N/A	N/A	N/A	0.316	0.316	0.896
3	N/A	N/A	N/A	0.586	0.667	0.792
7	0.042	**0.042**	0.042	0.746	0.792	0.842
10	0.034	**0.034**	0.034	0.373	0.384	0.593
20	0.117	0.185	0.309	0.082	**0.082**	0.535

Bootstrap reality check results

- Reject benchmark portfolio if ‘consistent’ p-value is less than 0.10

Benchmark portfolio: Intermediate

	Unconstrained			**Short sales constrained**		
RRA	Lower	Consistent	Upper	Lower	Consistent	Upper
1	0.126	0.126	0.126	0.556	0.556	0.932
3	0.066	**0.066**	0.317	0.319	0.368	0.470
7	0.067	**0.067**	0.305	0.349	0.394	0.493
10	0.023	**0.023**	0.224	0.380	0.511	0.579
20	0.238	0.380	0.380	0.151	0.161	0.611

Summary of Results

- Capturing skewness and asymmetric dependence leads to better portfolio performance:
  - Noteworthy, as in many cases simpler models do best in out-of-sample comparisons
  - For these assets, it seems that asymmetric dependence is more important than skewness
  - Statistical significance of improvement is moderate
- Short sales constraints improve portfolio decisions made using out-of-sample density forecasts
- Economic significance is greatest for unconstrained investors, eg: hedge funds.

Future work

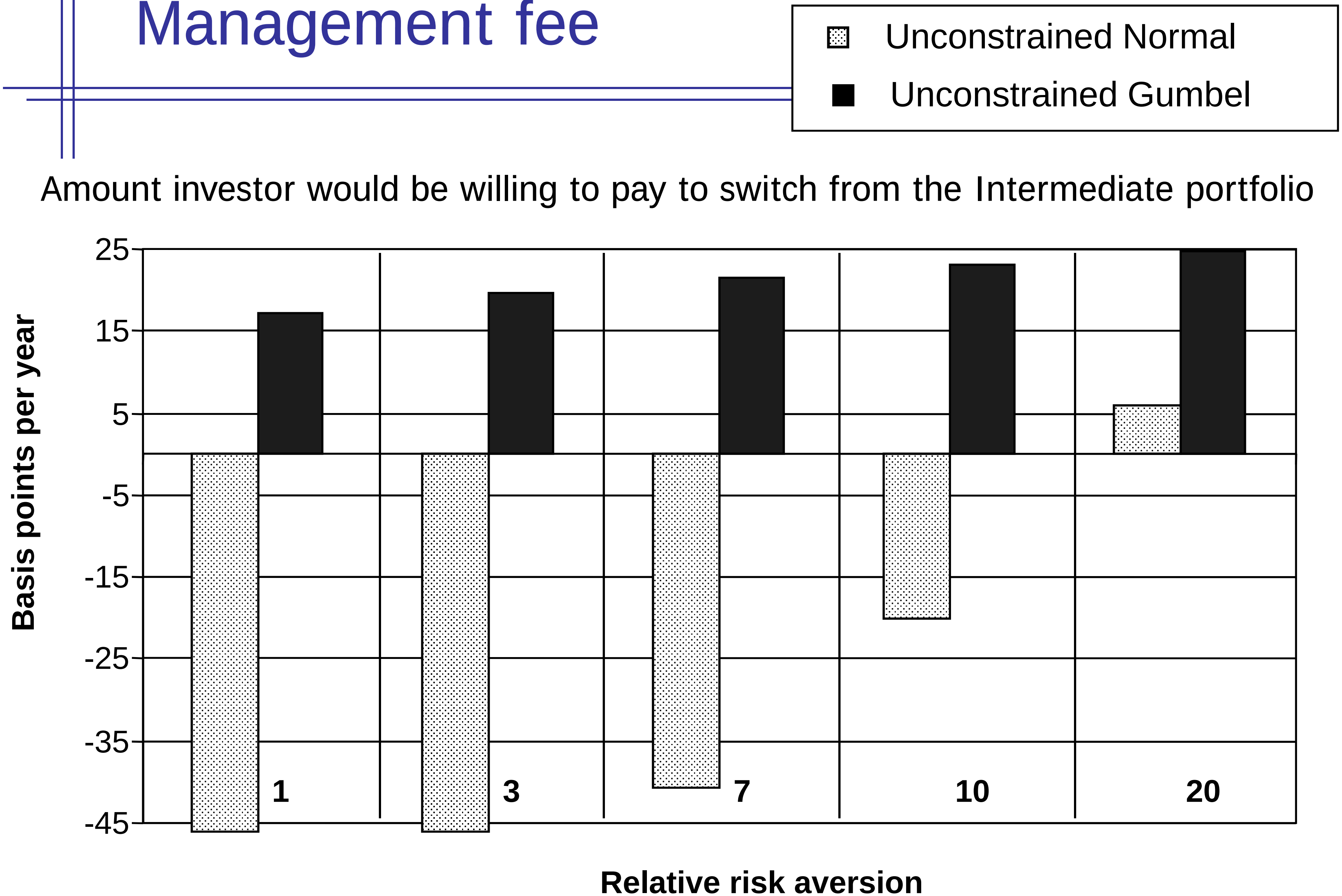
1. Impact of parameter estimation uncertainty on all of these results

2. Compare flexible parametric methods, like mine or those of Ang and Bekaert (2001), with nonparametric methods like those of Brandt (1999) and Aït-Sahalia and Brandt (2001)?

3. Extensions to higher dimensions: are the improvements even greater, or does estimation error dominate?

Management fee

Amount investor would be willing to pay to switch from the Intermediate portfolio



Management fee

Amount investor would be willing to pay to switch from the Intermediate portfolio