

Table 3: Ratio of estimator MSE to 1SMLE MSE, rank correlation = 0.25

	<i>MSMLE</i>	<i>EMSMLE</i>	<i>MSMLE</i>	<i>EMSMLE</i>
	$n_x = 1500$		$n_x = 3000$	
	$n_y/n_x = 0.25$			
μ_x	0.2323	0.2298	0.2585	0.2576
ϕ_x	0.2911	0.2860	0.2662	0.2616
ω_x	0.0349	0.0717	0.1096	0.2588
α_x	0.2015	0.3760	0.2684	0.7540
β_x	0.0597	0.1347	0.1512	0.4569
μ_y	0.9798	6.2254	0.9883	5.4261
ϕ_y	1.0446	7.7988	1.0623	5.7681
ω_y	1.0389	97.336	1.2272	55.915
α_y	1.2395	375.19	1.3074	16.065
β_y	1.0338	73.445	1.3243	33.769
κ_c	0.2422	4.1904	0.9662	2.2087
	$n_y/n_x = 0.50$			
μ_x	0.5039	0.5009	0.5128	0.5063
ϕ_x	0.5451	0.5360	0.6033	0.5928
ω_x	0.2026	0.4042	0.5273	1.2707
α_x	0.5719	1.0552	0.5414	1.5318
β_x	0.2979	0.6524	0.5552	1.6871
μ_y	0.9577	5.1416	0.9232	4.8861
ϕ_y	1.2059	6.4554	1.1373	6.0349
ω_y	2.1917	128.04	1.3163	11.047
α_y	1.2634	8.4137	1.2498	6.4429
β_y	1.9359	84.523	1.3261	5.9812
κ_c	0.9345	6.0718	1.0063	1.5156
	$n_y/n_x = 0.75$			
μ_x	0.7272	0.7241	0.7807	0.7748
ϕ_x	0.8142	0.8054	0.8513	0.8371
ω_x	0.6917	1.3817	0.7481	1.7849
α_x	0.8088	1.4706	0.7973	2.0635
β_x	0.7203	1.5914	0.7692	2.2745
μ_y	0.9637	2.1639	0.9857	2.2372
ϕ_y	1.1122	2.6064	1.0789	2.4776
ω_y	1.0220	3.8734	1.0792	3.2528
α_y	1.1820	3.6073	1.1425	3.9945
β_y	1.0647	2.9962	1.0902	3.1587
κ_c	0.9244	1.0207	0.9784	1.1176

Notes: This table presents the ratio of the mean-squared error of the multi-stage maximum likelihood estimator (MSMLE) and the one-step efficient maximum likelihood estimator (EMSMLE) of a given parameter to the one-stage maximum likelihood estimator (1SMLE) of that parameter. A value less than (greater than) one indicates that the estimator has lower (higher) MSE than the 1SMLE. $\mu_x, \phi_x, \omega_x, \alpha_x$ and β_x correspond to the mean, AR parameter, GARCH constant, GARCH innovation and GARCH smoothing parameters for the first margin. These are similarly defined for the second margin. κ_c indicates the parameter of the copula. n_x is the number of observations on the first margin, and n_y/n_x is the ratio of the number of observations on the second margin to those on the first. We set $n_c = n_y$. All simulations were done with 1000 replications.

Table 4: Ratio of estimator MSE to 1SMLE MSE, rank correlation = 0.50

	<i>MSMLE</i>	<i>EMSMLE</i>	<i>MSMLE</i>	<i>EMSMLE</i>
	$n_x = 1500$		$n_x = 3000$	
	$n_y/n_x = 0.25$			
μ_x	0.2512	0.2491	0.2715	0.2698
ϕ_x	0.3756	0.3714	0.3758	0.3709
ω_x	0.0902	0.1879	0.1241	0.2984
α_x	0.3129	0.6131	0.3703	0.9971
β_x	0.1258	0.2990	0.1916	0.5832
μ_y	1.0657	32.818	1.2447	6.6575
ϕ_y	1.5353	37.174	1.4986	8.3499
ω_x	2.8973	436.21	2.4352	1320.3
α_y	1.9235	18360	1.5584	250.74
β_y	2.1528	452.09	1.9792	718.27
κ_c	1.0268	13.713	0.8707	3.9978
	$n_y/n_x = 0.50$			
μ_x	0.5464	0.5425	0.5581	0.5544
ϕ_x	0.8418	0.8265	0.7321	0.7214
ω_x	0.6582	1.3151	0.6206	1.5039
α_x	0.7865	1.4819	0.7677	2.0066
β_x	0.7003	1.5804	0.6144	1.8866
μ_y	0.9710	5.2243	1.1331	5.6302
ϕ_y	1.6103	8.9915	1.4310	7.4175
ω_y	2.8625	477.96	1.7947	11.119
α_y	1.7234	37.273	1.4951	7.9972
β_y	2.6130	220.12	1.6615	6.5082
κ_c	0.8991	2.9769	1.0339	2.2217
	$n_y/n_x = 0.75$			
μ_x	0.8058	0.7968	0.8094	0.8033
ϕ_x	1.1858	1.1676	1.2523	1.2300
ω_x	3.1082	5.6013	1.4269	3.3519
α_x	1.1944	2.2870	1.2159	3.2775
β_x	1.9165	4.0172	1.3165	3.9560
μ_y	1.0865	2.5000	1.1060	2.5808
ϕ_y	1.4481	3.4861	1.6464	4.0011
ω_y	1.7967	5.8550	1.7373	5.2610
α_y	1.5795	4.3542	1.6066	5.5399
β_y	1.7066	4.3850	1.6842	4.8457
κ_c	0.9467	1.2843	0.9714	1.3357

Notes: This table presents the ratio of the mean-squared error of the multi-stage maximum likelihood estimator (MSMLE) and the one-step efficient maximum likelihood estimator (EMSMLE) of a given parameter to the one-stage maximum likelihood estimator (1SMLE) of that parameter. A value less than (greater than) one indicates that the estimator has lower (higher) MSE than the 1SMLE. $\mu_x, \phi_x, \omega_x, \alpha_x$ and β_x correspond to the mean, AR parameter, GARCH constant, GARCH innovation and GARCH smoothing parameters for the first margin. These are similarly defined for the second margin. κ_c indicates the parameter of the copula. n_x is the number of observations on the first margin, and n_y/n_x is the ratio of the number of observations on the second margin to those on the first. We set $n_c = n_y$. All simulations were done with 1000 replications.

Table 5: Ratio of estimator MSE to 1SMLE MSE, rank correlation = 0.75

	MSMLE	EMSMLE	MSMLE	EMSMLE
	$n_x = 1500$		$n_x = 3000$	
	$n_y/n_x = 0.25$			
μ_x	0.2921	0.2896	0.2811	0.2790
ϕ_x	0.6762	0.6645	0.5910	0.5826
ω_x	0.4246	0.8552	0.6470	1.5659
α_x	0.5765	1.0606	0.7094	1.9049
β_x	0.4383	0.9849	0.6592	2.0008
μ_y	1.1884	9.1711	1.1063	5.7635
ϕ_y	2.8080	33.257	2.7113	14.901
ω_y	4.1007	949.22	7.9919	238.66
α_y	3.1254	782.74	2.9529	25.426
β_y	4.1306	502.88	5.5663	104.08
κ_c	1.4216	42.352	0.9913	9.4008
	$n_y/n_x = 0.50$			
μ_x	0.5581	0.5525	0.5841	0.5809
ϕ_x	1.1393	1.1236	1.2162	1.1996
ω_x	0.2172	0.4453	1.4974	3.5455
α_x	1.3921	2.5570	1.4794	3.8855
β_x	0.5663	1.2811	1.5576	4.5994
μ_y	1.1415	5.8964	1.1449	5.7048
ϕ_y	2.5993	14.392	2.4983	13.127
ω_y	7.0964	66.693	4.0292	25.208
α_y	3.0955	25.174	2.9409	17.395
β_y	5.1363	30.271	3.5603	14.101
κ_c	1.0805	4.8292	1.0139	3.2043
	$n_y/n_x = 0.75$			
μ_x	0.8779	0.8747	0.8327	0.8249
ϕ_x	1.9364	1.9115	1.9923	1.9494
ω_x	3.2606	6.6357	2.6665	6.3185
α_x	2.2108	4.3404	2.3814	6.1699
β_x	2.7484	6.3401	2.6128	7.6227
μ_y	1.1610	2.7589	1.1882	2.6908
ϕ_y	2.6674	6.3029	2.4473	5.7116
ω_y	5.4271	18.380	3.4318	11.270
α_y	3.4128	9.9011	3.2746	10.479
β_y	4.7810	12.786	3.4261	10.177
κ_c	0.9381	1.6233	1.0123	1.6235

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