

# The Dynamics of Retail Oligopoly

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## Abstract

This paper examines competition between supermarket chains using a dynamic model of strategic investment. Employing a unique eight year panel dataset of store level observations that includes every supermarket operating in the United States, we propose and estimate a fully dynamic model of chain level competition. Using a structural model of dynamic oligopoly where firms compete each period in a static stage game, we estimate the dynamic parameters of the model using the methods proposed in Bajari, Benkard, and Levin (2002). The estimation takes place in two stage. In the first stage, the static parameters governing the outcome of product market competition are estimated using a differentiated products discrete choice demand system. We then employ a second, two-step procedure in which policy functions are first estimated from each firm's observed actions and outcomes are then matched to a (Markov perfect) equilibrium condition using forward simulation. The parameters of the structural model will then be used to evaluate the competitive impact from the introduction of superstores using the stochastic algorithm developed in Pakes & McGuire (2001).

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# 1 Introduction

Retail firms account for a surprising fraction of economic activity. In particular, retailers employ over 20% of the private sector workforce and produce nearly 13% of US GDP. Furthermore, mass merchandisers like Wal-Mart and Target have led the way in developing and diffusing innovative information technologies, often forcing upstream producers to lower prices and make complementary cost reducing investments. The rise of the “big box” format and a continued emphasis on one stop shopping has both increased the variety of products and lowered their costs. At the same time, many retail industries have become highly concentrated. Most “category killers” now compete locally with only one or two rivals. In some categories, like office supplies, there are only two or three chains nationwide. Viewed more broadly, these industries exhibit a highly skewed size distribution: a few giant chains compete with a large number of small local players. While the explosion in variety and reduction in price is unambiguously beneficial to consumers, the increase in concentration may be cause for concern. Fear of increased in concentration and impact on small businesses has triggered many municipalities to pass zoning laws restricting entry of ‘big-box’ stores to their markets. The goal of this paper is to develop a model of retail chain competition in which the impact of restrictions on competition between retail chains on competition, prices and consumer and total welfare can be evaluated.

The theoretical framework proposed in this paper is based on the Markov perfect equilibrium (MPE) framework of Ericson & Pakes (1995), in which firms make competitive investments that increase the quality of their products. In the context of retail competition, in which firms operate a chain of individual stores, quality is a function of the total number of stores operated by each firm, the individual characteristics of their stores, and their overall format (conventional supermarket or Supercenter). Clearly, allowing firms to adjust all of these features independently would yield an impossibly complex state space. Instead, our strategy is to focus on a single dimension of quality (store density) and allow firms to differ by format (supermarket or Supercenter). Product market competition is modeled using a discrete choice model of demand, in which firms may also differ by a measure of “perceived” quality that does not vary over time. We assume that the economically relevant features of the industry can be encoded into a state vector that includes each

firm's store density, its overall format, and its perceived level of quality. Firms receive state dependent payoffs in the product market and influence the evolution of the state vector through their entry, exit, and investment decisions. In particular, firms adjust their chain size each period by either opening new stores or closing existing ones. Each of these actions are subject to idiosyncratic shocks, which are treated as private information. Equilibrium occurs when firms choose strategies that maximize their expected discounted profits, given the expected strategies of their rivals.

Our empirical strategy is to estimate this model of competition using a unique panel that follows the entire supermarket industry over eight consecutive periods (years). Our estimator is based on the two-step procedure proposed by Bajari, Benkard & Levin (2002) and implemented in a similar context to ours by Ryan (2004). In the first step, we recover the firm's policy functions governing entry, exit, and investment. These functions characterize firms beliefs regarding the evolution of their state variables and the actions of their competitors. We also estimate the per-period payoff that each firm receives as a function of the current state. In the second step, we use the structure of the MPE to recover the parameters that make those beliefs optimal. Following Bajari et al. (2002), this is accomplished via forward simulation. Using the observed policy functions, we then simulate various future plays of the game, comparing the optimal strategies to various alternatives and identifying the parameters that rationalize the observed policies. Since we have recovered the structural parameters of the underlying model, we will then be able to perform policy experiments using a variant of the Pakes-McGuire (1994) algorithm. In particular, we aim to evaluate the impact of zoning laws that prevent the growth of Supercenters on investment, market structure, and consumer welfare.

This paper builds on both the sizable literature on estimating static entry games as well as more recent work on dynamic games. Until recently, the empirical entry literature has mainly employed static models of competition. As a consequence, the early papers were somewhat limited in scope, focusing primarily on characterizing the number of firms that could fit into markets of various size. In a series of seminal papers, Bresnahan & Reiss examined the relative importance of strategic and technological factors in determining market structure (Bresnahan & Reiss (1987, 1990, 1991)). By comparing the threshold market size at which only a single firm could survive to that which could sustain a second entrant, the

authors were able to distinguish empirically between the impact of sunk costs and the role of price competition. Berry (1992) extended this analysis to include both heterogeneity across firms and the impact of firm characteristics. More recently, Mazzeo (2002) and Seim (2004) have extended the static approach to incorporate various aspects of product differentiation, documenting the empirical relevance of both location and quality. In all of these studies, firms are assumed to provide only a single product. Moreover, a static setting clearly limits our ability to evaluate either merger policy or changes in the environment, as these are fundamentally dynamic questions. The emphasis on static (really two-period) frameworks was a direct result of the complexity associated with estimating a fully dynamic model of competition. Until recently, the burden was virtually insurmountable, as estimation required solving explicitly for an MPE via a nested fixed-point procedure that placed strong restrictions on the size of the state space. This computational burden placed severe restrictions on the ability to model complex interactions. However, the application of two-step estimation techniques has eased the burden substantially (Aguirregabiria & Mira (2002), Bajari et al. (2002), Pakes, Ostrovsky & Berry (2002), and Pesendorfer & Schmidt-Dengler (2002)), opening the door to much more realistic competitive frameworks.<sup>1</sup> Our goal is to use these methods to estimate a fully dynamic model of entry in which firms are able to constantly adjust their level of quality. Our paper is closest to the work of Ryan (2004), who estimates a dynamic model of entry and investment in the cement industry. Using a panel of firms in geographically distinct markets, he is able to recover the full cost structure of the industry and evaluate the welfare impact of a change in environmental policy.

The paper is organized as follows. Section 2 describes the construction of the dataset. Section 3 describes the theoretical framework. The empirical framework is described in Section 4. The results of the first and second steps of the estimation are presented in Section 5, while the results of the policy experiments (TBD) will be contained in Section 6. Section 7 concludes.

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<sup>1</sup>See Benkard (2004) for an early application of these methods to learning and strategic pricing in the commercial aircraft industry.

## 2 Data

The data for the supermarket industry are drawn from yearly snapshots of the Trade Dimension's Retail Tenant Database spanning the years 1997 to 2004, while market specific population growth rates are drawn from the U.S. Census. Trade Dimensions collects store level data from every supermarket operating in the U.S. for use in their *Marketing Guidebook* and *Market Scope* publications, as well as selected issues of *Progressive Grocer* magazine. The data are also sold to marketing firms and food manufacturers for marketing purposes. The (establishment level) definition of a supermarket used by Trade Dimensions is the government and industry standard: a store selling a full line of food products and generating at least \$2 million in yearly revenues. Foodstores with less than \$2 million in revenues are classified as small convenience stores and are not included in the dataset. Firms in this segment operate very small stores and compete with only the smallest grocery stores.

Information on average weekly volume, store size, number of checkouts, number of employees (full time equivalents), and the overall format of the store (e.g. Supercenter or conventional supermarket) is gathered through quarterly surveys sent to store managers. These surveys are then compared with similar surveys given to the principal food broker assigned to each store and are then further verified through repeated phone calls. Each store is assigned a unique identifier that remains with the store regardless of ownership, which we used to construct the overall panel. In addition, each store has a unique firm identifier, which we used to identify the ultimate owner. The availability of reliable firm identifiers is critical in the supermarket industry since parent firms will often operate stores under several "flag names," especially when the stores have been acquired through a merger. Initially, to avoid problems of false exits and entries, we will treat stores acquired in a merger as having always belonged to their final owner. Also, if a firm is taken private or bought out by a public holding company, we do not treat it as an entry or exit.

Previous empirical studies of the supermarket industry supports the division of retail food market into two distinct submarkets: supermarkets and grocery stores (Ellickson (2002b), Smith (2004)). Supermarkets compete in a tight regional oligopolies that do not compete significantly with the much smaller and fragmented grocery segment. Furthermore, the number of firms in these oligopolies do not increase with market size, yielding

an equilibrium number of entrants that is stationary with respect to population growth. Our retail database includes both types of firms. Since we are primarily concerned with competition between retail oligopolists and require a market structure that is stationary, we will focus on only the “top” firms in each market. Specifically, we include in our panel only those firms that served at least 5% the market (MSA) in which they operated in any period. Because these top firms do not compete with the firms in the fringe,<sup>2</sup> this should not introduce any selection problems.

The discrete choice model we use to characterize product market competition requires us to specify and collect data on the sales of the outside good. Obvious consumer alternatives to supermarkets include grocery stores, convenience stores, liquor stores, restaurants, and cafeterias. Therefore, we assume that total sales of the outside good are equal to the combined sales of all food and beverage stores (NAICS 445 - of which supermarkets are a subset) and all foodservice and drinking establishments (NAICS 772) less the sales accounted for by supermarkets alone. Data on total sales is taken from the 1997 Census of Retail Trade. To construct the share of the outside good, we use the Census dataset to construct an MSA specific multiplier characterizing the ratio of total sales in both categories (445 and 772) to total sales in supermarkets alone (NAICS 44511). We then use this multiplier to impute the total sales in both categories for each MSA in our dataset, using the observed revenue of the supermarkets as our baseline measure of sales. We are therefore assuming that the ratio is constant over time.

Estimating this demand system also requires data on firm level prices, which we acquired from the American Chamber of Commerce Researchers Association (ACCRA). The ACCRA collects data from over 250 U.S. towns and cities on prices of various retail products (26 of which are grocery items) for use in the construction of their *Cost of Living Index*. The ACCRA sends representatives to several supermarkets in each geographic market with the goal of collecting a representative sample of prices at the major chains. They are given a specific list of products for which to collect individual prices (e.g. 50 oz. Cascade dishwashing powder). We purchased their disaggregated dataset, meaning that we observe the store name and individual prices for each product. We then used these individual prices to construct a price index (using the same weights employed by ACCRA) for each store in

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<sup>2</sup>Ellickson (2002b) and Smith (2004) both present empirical results that support this claim.

their dataset (using the total basket price yields similar results). Since we are modeling competition at the firm level, we then aggregated the indices up to the level of the firm (in each market) and matched these price indices to the corresponding firms in our panel, yielding a total of 679 MSA/firm level observations on price. Since ACCRA only began recording the names of the individual stores in 2004, we have prices for only a single period. Summary statistics are provided in Table 1.

Table 1: Summary Statistics

	Format	
	Supercenter	Supermarket
Store Size	60.2 (9.63)	37.6 (14.84)
Checkouts	29.3 (6.38)	10.6 (4.10)
Stores per Market	3.3 (3.88)	9.70 (16.4)
Market Share	15.8 (11.4)	17.0 (13.0)
Basket Price	46.2 (3.57)	54.5 (5.94)
Firms per MSA	.70 (.64)	4.38 (1.42)

Store size is in 1000s of square feet.

### 3 Model

Our model of competition between retail chains is based on the Ericson & Pakes (1995) dynamic oligopoly framework. The game is in discrete time with an infinite horizon. We observe  $M$  distinct geographic markets ( $m = 1, \dots, M$ ), taken here to be the 331 U.S. Metropolitan Statistical Areas (MSAs). For each market/period combination, we observe a set of incumbent firms who are currently active in the market. Firms differ by format (type), either conventional supermarket or Supercenter, and we require that all outlets operated by a particular firm be of a single format. We further assume the existence of two potential entrants per period, one of each type, who choose whether or not to enter the market in that period. If they choose not to enter, they are replaced by new potential entrants in the subsequent period.

In each period, each potential entrant privately observes an idiosyncratic shock determining their sunk cost of entry. Based on this private draw, these firms decide whether to enter as a firm of the corresponding type. At the same time, incumbent firms of both types choose whether or not to exit. Next, both the incumbent firms who have chosen not to exit and any new entrants choose their optimal levels of investment. In particular, new entrants choose how many stores to build and incumbents choose whether to close existing stores or open new ones. Incumbent firms then compete in the product market, receiving a second private shock to the payoffs from this static phase of competition. After collecting their payoffs from the product market, the incumbent firms who chose to exit receive a scrap payment and the new entrants pay the sunk cost of entry. Finally, the investments of the entrants and continuing incumbents mature and the state vector updates accordingly.

In period  $t$ , each market can therefore be described by a set of  $N$  triples that define the current state vector  $s_t \in S$ . The components of the state vector are the firm’s type (assigned upon entry), its perceived quality (which is fixed over time), and its “store density” (the number of stores it operates per capita).<sup>3</sup> Given the state ( $s_t$ ) at time  $t$ , firms make entry, exit, price, and investment decisions, denoted collectively by the set  $A$ . Following the standard practice in the dynamic oligopoly literature, we focus only on pure strategy Markov Perfect Equilibrium (MPE) and assume that the equilibrium observed in the data is unique.

Given the Markov profile (strategy vector)  $\sigma$  mapping states into actions ( $\sigma : S \rightarrow A$ ), the value function of firm  $j$  can be written in recursive form as:

$$V_j(s|\sigma(s)) = \pi_j(\sigma(s)) + \beta_r \int V_j(s'|\sigma(s, \nu)) dP(s'|\sigma(s), s) \quad (1)$$

where the distribution  $P(s'|\sigma(s), s)$  characterizes the transition between states and  $\beta_r$  is a known discount factor. For a strategy profile  $\sigma$  to be an equilibrium, we then require that there be no firm  $j$  and alternative (Markovian) strategy  $\sigma'$  such that firm  $j$  prefers

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<sup>3</sup>The class of dynamic models we use for both estimation and simulation require state variables that remain stationary. Although population is free to grow without bound and the number of stores operated by the most successful chains rarely decreases, the number of stores per capita is relatively stable. Furthermore, due to the importance of endogenous fixed investments, the number of firms is also quite stable, both over time and across markets. In particular, the dynamics of retail chain growth can be modelled using a pure vertical differentiation model like Pakes & McGuire (1994) where the improvement of the outside good corresponds directly to increases in population. If a firm does not invest to counteract population growth, its “quality” (i.e. store density) will deteriorate relative to its rivals and it will eventually be forced to exit.

the alternative strategy  $\sigma'$  to  $\sigma$  given that all of its rivals use profile  $\sigma$ . Specifically,  $\sigma$  is an MPE if

$$V_j(s|\sigma_j, \sigma_{-j}) \geq V_j(s|\sigma'_j, \sigma_{-j}) \quad (2)$$

for all  $j, s$ , and  $\sigma'_j$ . It is this set of inequalities that forms the basis for the second step of the estimation procedure. Intuitively, the first step of the estimation procedure involves recovering (as flexibly as possible) estimates of the policy functions  $\sigma(s)$ , which can then be used to construct  $P(s'|\sigma(s), s)$ . We also recover the static portion of payoffs as a function of  $s$ . With these estimates in hand, we then use the equilibrium condition (2) to recover the remaining (dynamic) parameters, by simulating many future paths of play and finding the parameters that make the observed policies optimal.

## 4 Estimation Strategy

In this section, we describe our estimation strategy. The parameter estimates that we recover here are then used as inputs to the Pakes/McGuire algorithm in order to simulate the welfare effects of eliminating the superstore format. The model is estimated using the two-step procedure proposed by Bajari et al. (2002). In the first step, we estimate the policy functions that govern the transition between the states of the market, as well as the per-period profit function for the firms who are active in the market. In the second step, the rest of the parameters are estimated by simulating the behavior of firms in the market given the first step estimates and an equilibrium condition. The parameters recovered in the second step include the fixed and marginal costs of investment, the scrap value of exiting the market, and the distribution of sunk entry costs. The following subsection describe both steps in detail.

### 4.1 Per-Period Profits

Before we can recover the dynamic parameters, we must first determine the profit function that characterizes per period profits, as well as the policy functions that govern each firm's optimal strategy profiles. The current subsection describes the estimation of the demand system, which allows us to recover an estimate of each firm's profit from the static stage game as well as a measure of their perceived quality. The following subsection describes

how we estimate policy functions, using observed actions.

Estimating the dynamic parameters of the investment game requires specifying the payoffs that firms receive each period as a function of the full vector of state variables. We parameterize this relationship by estimating a static model of competition that fully characterizes the per-period profits from the static stage game. Intuitively, we estimate a discrete choice model of grocery demand (and supply) that yields an estimate of the profit that was actually earned by each firm as well as its perceived quality. We then estimate the correspondence between the state vector and these equilibrium payoffs using a semi non-parametric regression. This is then used to predict the payoffs each firm can expect to earn given the current state.

Competition in each period is modeled as a static stage game, using a standard differentiated products model in which consumers have unit demand for a shopping trip. Suppressing the market subscript for brevity, consumer  $i$ 's conditional indirect utility from shopping at firm  $j$  in period  $t$  is given by

$$u_{ijt} = x_{jt}\beta - \alpha p_{jt} + \xi_{jt} + \epsilon_{ijt} \quad (3)$$

where  $x_{jt}$  is a 2-dimensional vector of observed firm characteristics,  $p_{jt}$  is the price charged by firm  $j$  in period  $t$ ,  $\xi_{jt}$  is the mean of the unobserved characteristics of firm  $j$  in period  $t$  (its “perceived quality”), and  $\epsilon_{ijt}$  is an i.i.d. “logit” error (i.e. Type I Extreme Value with unit scale). The characteristics we observe for each firm include the number of stores they operate per capita (i.e. their store density  $d_{jt}$ ) and the firm’s format ( $type_j$ ), which is either conventional supermarket or Supercenter.<sup>4</sup> We also recover the part of  $\xi_{jt}$  that does not vary over time and use it as a measure of quality. We treat the remaining (time varying) component as an i.i.d. shock.

Following Berry (1994), we represent the mean utility of firm  $j$  in period  $t$  as

$$\delta_{jt} = x_{jt}\beta - \alpha p_{jt} + \xi_{jt} \quad (4)$$

and estimate the parameters  $\beta$  and  $\alpha$  using the standard “Berry logit”. In particular, given the assumption that the error term  $\epsilon_{ijt}$  is extreme value, the market share of firm  $j$  in period

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<sup>4</sup>Recall that we have assumed that all stores operated by a single firm are a single format.

$t$  is given by the well known logit formula

$$\mathcal{S}_{jt} = \frac{e^{x_{jt}\beta - \alpha p_{jt} + \xi_{jt}}}{1 + \sum_{k=1}^J e^{x_{kt}\beta - \alpha p_{kt} + \xi_{kt}}} \quad (5)$$

Normalizing the utility of the outside good to zero and constructing the ratio of shares yields the following estimation equation for the parameters of interest:

$$\ln\left(\frac{\mathcal{S}_{jt}}{\mathcal{S}_{0t}}\right) = x_{jt}\beta - \alpha p_{jt} + \xi_{jt} \quad (6)$$

In order to proceed to estimation, we need to construct the share of the outside good, which is taken here to be all food and beverage stores and foodservice and drinking establishments (NAICS 445 and 772) that are not supermarkets. Shares are then constructed as revenue shares in each market (MSA) in each period.

Equation (6) can be estimate by two-stage least squares (2SLS), provided we can identify valid instruments for prices. We look to the cost side in constructing these instruments, using the total square footage of all stores operated by the firm (in all markets), the total number of stores, and the total number of employees. All three measures are proxies for scale which, given the prevalence of quantity discounts, is likely to have a large influence on the cost of goods sold. Since we only observe prices in a single period (but observe everything else in the model - including the instruments - in all periods), we estimate the first stage regression using only the period and firms for which we have prices. We then use the first stage estimates to construct predicted prices for all firms in all periods, and estimate (6) using a 2SLS random effects estimator.

For the supply side of our static product market, we assume that firms compete Nash in prices. Since we have aggregated up to the level of the firm in each MSA, each supermarket can be treated as a single product firm whose profits in the static product market are given by

$$\tilde{\pi}_j(p, x, \xi) = (p_j - mc_j(q_j)) M \mathcal{S}_j(p, x, \xi) - C_j$$

where we have suppressed the time subscript for brevity.  $\mathcal{S}_j(p, x, \xi)$  is then the share of firm  $j$ ,  $M$  is the size of the market (population), and  $C_j$  is the fixed cost of production. Assuming the existence of a pure strategy equilibrium, the price vector  $p$  must satisfy the standard first order conditions of profit maximization:

$$\mathcal{S}_j(p, x, \xi) + (p_j - mc_j(q_j)) \frac{\partial \mathcal{S}_j(p, x, \xi)}{\partial p_j} = 0$$

or more compactly

$$p_j - mc_j = -\mathcal{S}_j / \left[ \frac{\partial \mathcal{S}_j}{\partial p_j} \right]$$

Since  $\frac{\partial \mathcal{S}_j}{\partial p_j} = \alpha_j \mathcal{S}_j (1 - \mathcal{S}_j)$  in this simple logit model, we can then express the gross profit margin of firm  $j$  as

$$\frac{p_j - mc_j}{p_j} = \frac{1}{\alpha_j p_j (1 - \mathcal{S}_j)}$$

allowing us to recover estimates of profit for every firm in the dataset simply by multiplying total revenue by the corresponding margin. Since we have now recovered the unobserved portion of our state vector ( $\xi_j$ ) and a measure of realized profits ( $\tilde{\pi}_j$ ), we are ready to estimate the reduced form profit function that links the state vector ( $s$ ) to per period payoffs.

Instead of explicitly solving for the equilibrium in the static stage game (which is not, in general, unique), we simply aim to characterize the mapping from state variables to payoffs as flexibly as possible. Since the number of firms changes from period to period, we clearly cannot condition on the entire state vector. Instead, we assume that the profit function can be approximated by

$$\ln \left( \frac{\tilde{\pi}_j}{pop} \right) = g_{\pi_1}(\xi_j, \bar{\xi}_{-j}, N^{SM}, N^{SC}) + g_{\pi_2}(d_j, \bar{d}_{-j}) + \varepsilon_\pi \quad (7)$$

where  $N^{SM}$  denotes the number of active firms operating conventional supermarkets,  $N^{SC}$  the number of active firms operating Supercenters,  $\xi_j$  is the perceived quality of firm  $j$ ,  $\bar{\xi}_{-j}$  is the average quality of its rivals,  $d_j$  is the store density of firm  $j$ , and  $\bar{d}_{-j}$  is the average store density of its rivals. For tractability, we assume that  $g_{\pi_1}(\cdot, \cdot)$  is simply a linear function of its arguments but allow  $g_{\pi_2}(\cdot, \cdot)$  to be an unknown, continuous function. We assume that the function  $g_{\pi_2}(\cdot, \cdot)$  is monotonically decreasing in  $\bar{d}_{-j}$  and monotonically increasing in  $d_j$ .<sup>5</sup> We then estimate the profit function (7) using a B-spline expansion for the function  $g_{\pi_2}$ . This method is a natural choice for a semi non-parametric relationship like (7) since it accommodates restrictions on the non-parametric part of the equation in a straightforward manner (see ? for more details). Specifically, the function  $g_{\pi_2}$  is approximated using a cubic spline in which the support of  $(d_j, \bar{d}_{-j})$  is divided into a  $4 \times 4$  equally spaced grid.

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<sup>5</sup>Since this property holds for the unrestricted estimator over most of its support as well as most reasonable economic settings, we have chosen to impose this restriction directly.

## 4.2 Entry, Exit, and Investment Costs

The previous subsection described how we connect each firm's state vector to its per period profit function. Of course, firms can influence the evolution of the state vector through their entry, exit, and investment decisions. A firm's full per period payoff function depends on the costs of each of these actions, which in turn depend on whether a firm is an entrant, a continuing incumbent, or an incumbent who has chosen to exit.

Since new entrants compete in the product market only in the period following their entry decision, their initial payoff is simply a function of the fixed cost of entry ( $ENTRY_j$ ) and their initial investment ( $I^e$ ) in store density:

$$\pi_j(s) = -ENTRY_j - C^e(I^e)$$

In our empirical analysis, we assume that  $C^e(\cdot)$  takes a simple quadratic form.

Incumbent firms differ from entrants in that they 1) obtain a payoff from the product market in the current period and 2) may choose to exit the market. Although incumbent firms that remain active choose how much to invest or de-invest in obtaining next period's state, these decisions do not impact *product market* payoffs in the current period. Therefore, their payoff function for continuing incumbents is given by:

$$\pi_j(s) = \tilde{\pi}_j(s) - C^{in}(I)$$

We also assume a quadratic form for  $C^{in}(I)$ , although it is allowed to vary according to whether the investment is positive (the firm opens additional stores) or negative (the firm closes existing stores).

Finally, incumbents that choose to exit have a payoff function given by

$$\pi_j(s) = \tilde{\pi}_j(s) + EXIT$$

where  $EXIT$  is the scrap value associated with closing down all remaining stores and exiting the market for good.

### 4.3 Policy Functions

Firms choose entry, exit, and investment policies that maximize the expected discounted stream of future profits. The probability of entry and exit can be written as

$$\begin{aligned} P(\text{entry}|s) &= \int g_{ent}(s_j, ENTRY_j) dF(ENTRY_j) \\ P(\text{exit}|s) &= g_{ex}(s_j) \end{aligned}$$

where the exit rule depends only on the current state, but the entry rule is also a function of the firms private draw from the distribution of sunk entry costs. Since exit and entry strategies typically take the form of simple cutoff rules in many dynamic oligopoly models, we assume that both conditional probabilities can be approximated using probit models.

Firms also follow optimal policy functions regarding investment. Since firms make entry decision by type (we assume that there exists one potential entrant of each type) and are assigned a value of perceived quality ( $\xi$ ) randomly, their investment policy functions govern only the transitions of store density ( $d_j$ ). In particular, firms choose the optimal number of stores to operate in the subsequent period as a function of the current state  $s$

$$d'_j = g_{inv}(s)$$

which we approximate with a linear regression.

As in the case of the profit functions, we cannot condition on the full set of state variables for any of these policy functions. Therefore, we assume that each relationship can be approximated using a small subset of the information in  $s$ .

### 4.4 Cost Parameters

The estimation of the demand system, profit correspondence, and policy functions constitute the first step of our estimation strategy. These estimates describe the law of motion of the state vector and the level of profit associated with each state. In the second stage of our two-step procedure, we recover estimates of the remaining parameters of the payoff and cost functions by finding the set of parameters that make the firm's observed policy functions optimal. Following Bajari et al. (2002), we specify the functional form for costs such that profit function is linear in the remaining parameters. This assumption plays a key

role in reducing the computational burden of the simulation estimator used to recover the parameters that make the observed policy functions optimal. We now describe the structure of this profit function.

Note that we have already recovered the first component of the profit function, namely the per-period profit  $\tilde{\pi}$ , in the prior stage. The net profit ( $\pi$ ) is simply the per-period profit less the cost associated with adjusting one's state. Parameterizing this function requires some additional notation. Let  $ACTIVE_t$  equal 1 if the firm is active at period  $t$ ,  $\Delta d_t^+ = (d_{t+1} - d_t)^+$ , and  $\Delta d_t^- = (d_{t+1} - d_t)^-$ , where  $(x)^+ = \max\{0, x\}$  and  $(x)^- = \max\{0, -x\}$ . Therefore,  $\Delta d_t^+$  is simply the net increase in  $d$  and  $\Delta d_t^-$  the net decrease. The cost function is then given by

$$\begin{aligned}
C_t = & (ENTRY + \sigma_{ent} \cdot \varepsilon_t^{ent}) \cdot 1_{[ACTIVE_t=1, ACTIVE_{t-1}=0]} \\
& + \phi_0 \cdot 1_{[\Delta d_t^+ > 0]} + \phi_1 \Delta d_t^+ \\
& + \gamma_0 \cdot 1_{[\Delta d_t^- > 0]} + \gamma_1 \Delta d_t^- \\
& + (EXIT) \cdot 1_{[ACTIVE_t=0, ACTIVE_{t-1}=1]}
\end{aligned} \tag{8}$$

where  $1_{[\cdot]}$  is an indicator function used to distinguish positive and negative investment. The overall profit is given by  $\pi_t = \tilde{\pi}_t - C_t$  and the present value of profit by  $\Pi = \sum_{t=0}^{\infty} \beta^t \pi_t$ .

Using the estimates from the first step, which fully characterize what each firm chooses to do in all situations, we can then simulate the evolution of the market under various initial conditions. Using many such simulations, we find the parameters that make the observed policy functions optimal by minimizing the profitable deviations from these observed strategies. In practice, we first pick an initial state, which is simply a vector of length  $n + 1$  containing the quality of the  $n$  firms in the market and the change in population from the previous period. We then draw a set of shocks and determine the actions of the firms who are in the market using their observed exit and investment policies. We also add to the market any firms who have chosen to enter, based upon their entry policy. This produces a vector of store densities (and active firms) for the next period, whose length depends on the number of firms who have chosen to remain active, exit, or enter. Using this iterative procedure, we then simulate the actions of the firms 100 periods into the future, producing a stream of profits  $\{\pi_t\}_{t=0}^{100}$ . The length of this simulation is chosen to ensure that the remaining period payoffs have a negligible present value.

In order to identify the parameters in (8), we simulate the behavior of the firms under two alternative scenarios. Under the first scenario, we simulate the future outcomes assuming that all firms use the optimal strategies recovered in the first stage, which we denote by  $\sigma$ . Under the second scenario, we assume that a single firm deviates from the optimal strategy, while its competitors continue to follow the optimal strategies. We denote the (single firm) alternative strategy by  $\sigma'$ . If  $\sigma$  is indeed the optimal strategy, then choosing an alternative strategy  $\sigma'$  rather than  $\sigma$  while the competitors continue to follow  $\sigma$  should yield a lower present value of profits. This condition should hold in any market, regardless of initial conditions. Specifically, we should find that  $\pi_j(\sigma, \sigma_{-j}, s_0; \theta) \geq \pi_j(\sigma', \sigma_{-j}, s_0; \theta)$ , where  $s_0$  is the initial state of the market and  $\theta$  is the vector of parameters of the cost function in (8). Since this restriction should hold in all cases, we can estimate  $\theta$  using a minimum-distance criteria based on this inequality. The goal is to find a parameter  $\theta$  such that the squared differences between  $\pi_i(\sigma, \sigma_{-j}, s_0; \theta)$  and  $\pi_i(\sigma', \sigma_{-j}, s_0; \theta)$  are minimized for the cases where the inequality  $\pi_j(\sigma, \sigma_{-j}, s_0; \theta) \geq \pi_j(\sigma', \sigma_{-j}, s_0; \theta)$  is violated. This parameter is estimated using the following integral

$$\int 1_{[\pi_j(\sigma, \sigma_{-j}, s_0; \theta) - \pi_j(\sigma', \sigma_{-j}, s_0; \theta) < 0]} [\pi_j(\sigma, \sigma_{-j}, s_0; \theta) - \pi_j(\sigma', \sigma_{-j}, s_0; \theta)]^2 dG(\sigma', s_0)$$

where  $G(\sigma', s_0)$  is an arbitrary distribution on the possible perturbations on the strategy  $\sigma$  and starting state  $s_0$ . In practice, we evaluate this integral by perturbing the vector  $\sigma$  (to generate alternative strategies), choosing the starting point  $s_0$  randomly from starting points observed in our sample. As a result, we evaluate the above integral using all the markets that appear in our data.

## 5 Empirical results

In this section, we report the empirical results from the first and second stages of estimation. Recall that the first stage yields estimates of the transition probabilities governing the movement between states, as well as the function that determines the mapping between states and per-period profits. It is important to emphasize that the goal of the first stage is simply to characterize what firms actually do. The primary structural parameters are estimated in the second stage by finding the parameters that make those actions optimal.

Table 2: Demand Parameters

	Constant	Stores/Pop	SuperC	Price
	.438 (.110)	5.14 (.062)	.182 (.037)	-1.82 (.057)
$R^2$	0.43			
First Stage $F$ -statistic	34.7			
Number of Observations	13417			
Number of Firms	2197			
Estimated Gross Margin	.348 (.062)			

Robust t-statistics in parentheses.

## 5.1 Empirical Results from Step 1

We start by discussing the results of the demand estimation. Recall that the ultimate goal here is to recover a mapping from state vectors to per-period profits. This procedure involves a number of steps. First, we estimate the discrete choice demand system, using a “Berry inversion” to construct a standard random effects estimator. This procedure yields estimates of the margins charged by each of the firms in the market. Using these margins, we are then able to construct predicted profits using the observed revenues of each firm. The mapping between states and profits is then estimated using the semi non-parametric regression described above.

The results of the demand estimation are presented in Table 2. The coefficients all have the expected signs and are significant at all levels. Not surprisingly, store density has a strong and positive impact on demand, as does the dummy variable indicating that the firm operates Supercenters. The coefficient on price is negative and large enough in magnitude that the every firm is pricing on the elastic portion of the demand curve, as the theory requires. The average predicted gross margin is 35%, which is reasonably close to industry estimates (typically 26%). Using these margins, we then construct estimates of each firm’s per-period profit and connect them to the state variables using a semi non-parametric regression estimated separately by type of firm. The results of this exercise are presented in Figure 2, in which we see that profit is indeed increasing in own store density and decreasing in the density of ones rivals. Since the results from this estimator cannot be presented in tabular form, we also include the results of a simple linear regression of per capita profit

on the number of firms of each type ( $N^{SC}$  and  $N^{SM}$ ), a firm’s own store density ( $d_j$ ), the average density of its competitors ( $\bar{d}_{-j}$ ), and corresponding measures of perceived quality ( $\xi_j$  and  $\bar{\xi}_{-j}$ ). These results, which are provided simply for illustration (since the second stage simulations use the semi non-parametric estimator only) are presented in Table 3. Again, the coefficients all have the expected signs and are significant at all levels. Specifically, profits are strongly increasing in own store density and own quality and decreasing in the average density and quality of a firm’s rivals. The number of competing firms has a negative impact which is stronger for rivals of the same type.

Table 3: Profit Functions

Dependent Variable: $\ln(\pi/pop)$		
	Supermarkets	Supercenters
Own Store Density ( $d_j$ )	4.88 (.141)	6.75 (.672)
Rival’s Store Density ( $\bar{d}_{-j}$ )	-1.69 (.247)	-.974 (.514)
Supercenters ( $N^{SC}$ )	-.048 (.032)	-.454 (.102)
Supermarkets ( $N^{SM}$ )	-.090 (.014)	-.084 (.026)
Own Quality ( $\xi_j$ )	.873 (.038)	.557 (.070)
Rival’s Quality ( $\bar{\xi}_{-j}$ )	-.639 (.045)	-.294 (.120)
Constant	2.43 (.124)	3.76 (.301)
$R^2$	.84	.77
Number of Observations	532	147

Robust t-statistics in parentheses.

The second set of first stage results are aimed at characterizing the policy functions of incumbent firms and potential entrants. These policy functions are estimated as four separate regressions, broken out by firm type. The first policy function is the one governing exit and is estimated using a probit on the same covariates used in the profit regressions plus the rate of population growth. The empirical results from the probits for conventional supermarkets and Supercenters are presented in the first columns of Tables 5 and 6 respectively. Conventional supermarkets are less likely to exit a market if they operate more stores, if they are higher quality, and if the market is growing. They are more likely to exit if their

rivals operate more stores, they have more rivals of either type, and if their rivals are higher quality. Supercenters are less likely to exit a market if they operate more stores. All other marginal effects are positive, although own quality is insignificantly different from zero. The positive (though insignificant) impact of population growth may reflect a preference for less urban markets.

Turning next to the entry decision, we report coefficients from a probit using the same covariates as exit (excluding the own values). The results are presented in the second columns of Tables 5 and 6. For the conventional supermarkets, we find that entry is less likely in markets with a greater number of stores, more Supercenter firms, and markets that are growing. There is more entry in markets with more supermarket firms and a higher level of rival quality. The strongest effect is the positive impact of existing supermarket firms, which probably reflects unobserved features of demand. Among Supercenters, all marginal effects (including population growth) are negative. Again, this may reflect a tendency among these firms to target less urban markets.

Finally, we consider the initial level of store density chosen by new entrants and the subsequent level of store density chosen by incumbents. These regressions are estimated using OLS. The results are presented in the third and fourth columns of Tables 5 and 6. For new entrants to the conventional supermarket group, we find that firms choose higher levels of investment when rivals operate more stores, when there is more competition from either type of firm, and when the market is growing. They invest less if their rivals are higher quality. Incumbent supermarket firms investment decisions are primarily driven by their existing store density. However, they also invest more when they are higher quality or when the market is growing. Supermarket firms invest less if rivals operate more stores, are higher quality, or exist in greater numbers. Entrants to the Supercenter segment invest more when facing a greater number of supermarket firms and higher quality rivals. They invest less when their rivals have more stores, when there are more Supercenter firms in the market, and when the market is growing. Finally, incumbent Supercenter firms invest more when they have a greater store density and when markets are growing. They invest less when they face more rivals of either type, a greater average store density, and high quality rivals. Somewhat strangely, they invest less if they are higher quality, but this effect is not statistically significant.

## 5.2 Empirical Results from Step 2

In this part of the estimation we recover the cost parameters in (8). This is done in two steps: we first recover the parameters associated with the cost of investing and de-investing, and then we recover the distribution of sunk costs of entry using an additional set of simulations. Table 4 contains the first set of parameter estimates.

Table 4: Investmet costs and exit value

	Units	Supermarkets	Supercenters
Exit Value (EXIT)	$10^6\$$	0.432 (1.75)	0.924 (0.212)
FC of Positive Investment ( $\phi_0$ )	$10^6\$$	-0.026 (0.026)	-0.0754 (0.0188)
MC of Positive Investment ( $\phi_1$ )	$10^6\$/10^4sqft$	-0.168 (0.079)	-0.549 (0.089)
FC of Negative Investment ( $\gamma_0$ )	$10^6\$$	-0.039 (0.040)	-0.058 (0.042)
MC of Negative Investment ( $\gamma_1$ )	$10^6\$/10^4sqft$	0.038 (0.086)	0.163 (0.0846)

The distribution of sunk cost is estimated using an additional procedure.

## 6 Policy Experiments

This section describes work in progress. For now, we simply provide a road map intended to clarify the purpose of the estimators described above and their use in conducting the policy experiments that constitute the central empirical contribution of this paper.

The parameters and distributions estimated in the previous section will be used here as inputs for a policy experiment. In particular, we will use the algorithm described in Pakes & McGuire (1994) (PM) to study two different scenarios. In the first, we allow both regular supermarkets and super-centers to coexist in the market, with no restrictions imposed on either. In the second, we restrict the model to allow only regular stores to exist in the market. We then use the PM-algorithm to simulate the behavior of the firms under each scenario. This will allow us to compare both producer and consumer surplus, as well as the size distribution of firms and quality provided in equilibrium.

The original PM-algorithm must be extended along several dimensions to incorporate the additional features specific to our setting. Most of these extensions have already been

addressed in the existing literature. First, we must accommodate two types of competitors: super-centers and regular supermarkets. The two types will be able to pursue different strategies and react differently to similar situations. Second, since we do not observe the actual level of investment, we must assume that the *cost* of investment is the random variable, rather than whether or not an investment is successful (as is the case in the original algorithm). Third, we will allow firms to make both positive and negative investments, reflecting the fact that some firms may choose to downsize their operations and sell off some of their existing stores. Fourth, we will allow firms to choose their initial level of quality, rather than having to enter at a predetermined quality level. Finally, we will allow multiple entries in each period, to accommodate the behavior observed in the data. The first two tasks described above are the most challenging, requiring a switch to the stochastic version of the PM-algorithm described in Pakes & McGuire (2001). Since these modifications to the PM-algorithm are in various stages of completion, a more comprehensive discussion will be added in a future version of the paper.

## 7 Conclusion

TBD

Table 5: Policy Parameters for Supermarkets

	Exit Probit	Entrants Investment	Incumbents Investment
Dependent Variable	$P(\text{exit}   X)$	$d'_{jm}$	$d'_{jm}$
Own Store Density ( $d_j$ )	-2.22 (.204)		-.136 (.097)
Rival Store Density ( $\bar{d}_{-j}$ )	-.354 (.432)	-2.89 (.711)	-.136 (.218)
Supercenters ( $N^{SC}$ )	.033 (.040)	-.172 (.059)	.006 (.023)
Supermarkets ( $N^{SM}$ )	.053 (.019)	.087 (.028)	-.014 (.010)
Own Quality ( $\xi_j$ )	-.300 (.040)		.094 (.022)
Rival's Quality ( $\bar{\xi}_{-j}$ )	.097 (.068)	-.154 (.100)	-.179 (.037)
Population Growth	.091 (.094)	-.047 (.138)	.126 (.054)
ln(Population)	-.193 (.027)	-.315 (.040)	.137 (.014)
Constant	-.869 (.226)		
Pseudo $R^2$	.065	.048	.010
Log Likelihood	-2143.9	-1584.4	-10534.2
Observations	10269	2371	9347

Robust t-statistics in parentheses.

Table 6: Policy Parameters for Super Centers

	Exit Probit	Entrants Investment	Incumbents Investment
Dependent Variable	$P(exit   X)$	$d'_{jm}$	$d'_{jm}$
Own Store Density ( $d_j$ )	-8.62 (5.32)		-.045 (.866)
Rival Store Density ( $\bar{d}_{-j}$ )	1.87 (2.19)	-2.17 (.899)	.839 (.604)
Supercenters ( $N^{SC}$ )	.723 (.211)	-.807 (.096)	-.439 (.093)
Supermarkets ( $N^{SM}$ )	.080 (.103)	-.123 (.041)	.037 (.033)
Own Quality ( $\xi_j$ )	-.199 (.317)		-.053 (.100)
Rival's Quality ( $\bar{\xi}_{-j}$ )	.564 (.426)	-.531 (.142)	-.598 (.124)
Population Growth	.477 (.419)	-.388 (.210)	.384 (.159)
ln(Population)	-.471 (.212)	-.117 (.054)	.695 (.052)
Constant	-2.12 (1.55)		
Pseudo $R^2$	.193	.080	.140
Log Likelihood	-86.3	-560.6	-1159.6
Observations	1553	2319	1410

Robust t-statistics in parentheses.

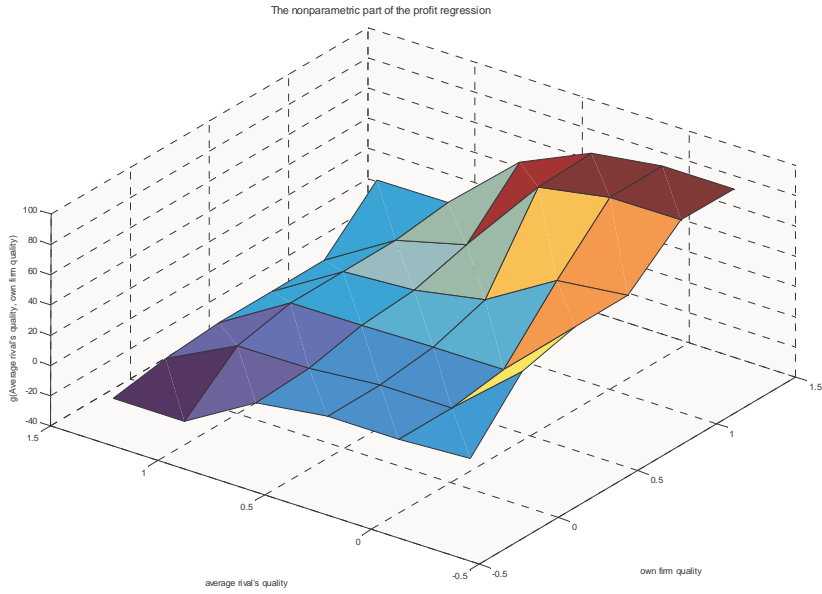


Figure 2: Estimated profit function

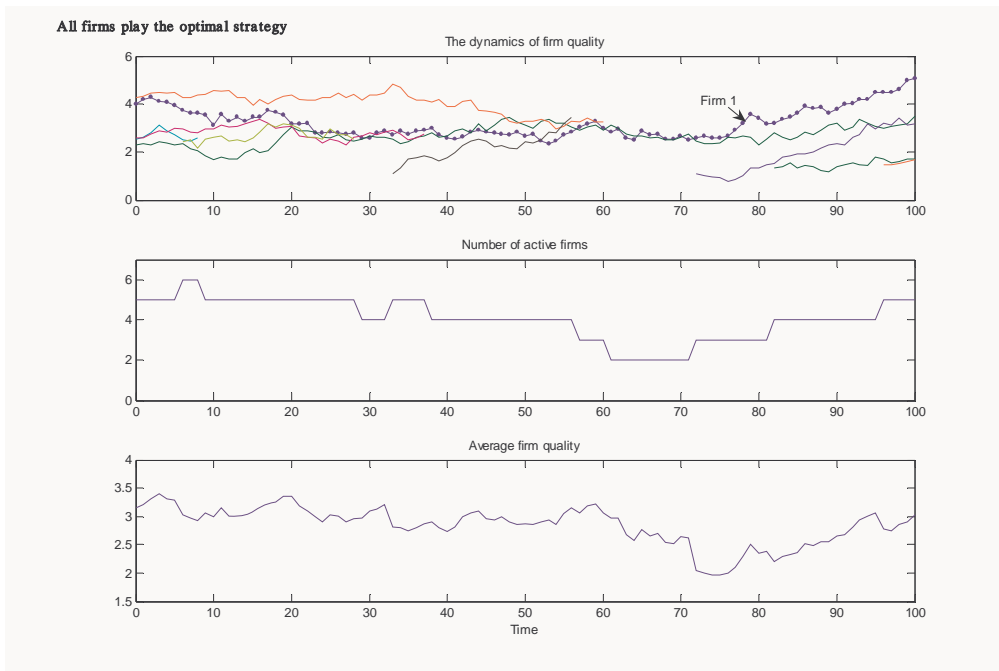


Figure 3: Simulation where all firms follow  $\sigma$

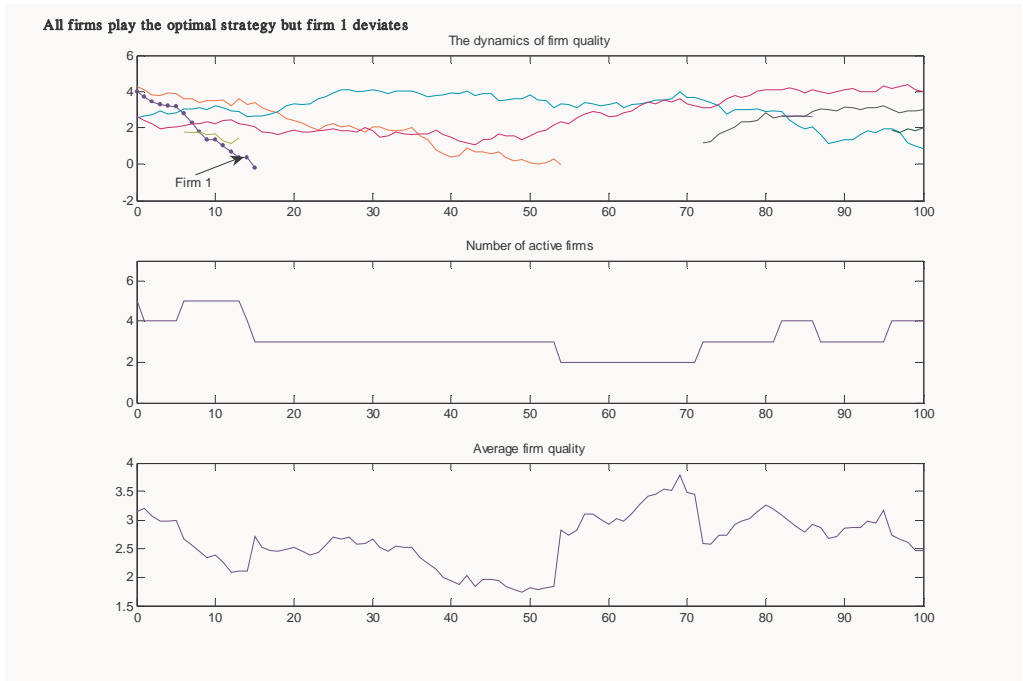


Figure 4: Simulation where firm 1 deviates

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