Glossary to ARCH (GARCH)*

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Abstract:
This paper provides an encyclopedic type reference guide to the long list of ARCH acronyms that have been used in the literature. In addition to the acronyms association with specific parametric models, I have also included descriptions of abbreviations association with more general procedures and ideas that figure especially prominently in the ARCH literature. With a few exceptions, I have restricted the list of acronyms to those which have appeared in already published studies.

Keywords:
ARCH, GARCH, ARCH-M, EGARCH, IGARCH, BEKK, CCC, DCC.

Preface:

Rob Engle's seminal Nobel Prize winning 1982 *Econometrica* article on the AutoRegressive Conditional Heteroskedastic (ARCH) class of models spurred a virtual "arms race" into the development of new and better procedures for modeling and forecasting time-varying financial market volatility. Some of the most influential of these early papers were collected in Engle (1995). Numerous surveys of the burgeon ARCH literature also exist; e.g., Andersen and Bollerslev (1998), Andersen, Bollerslev, Christoffersen and Diebold (2006), Bauwens, Laurent and Rombouts (2006), Bera and Higgins (1993), Bollerslev, Chou and Kroner (1992), Bollerslev, Engle and Nelson (1994), Degiannakis and Xekalaki (2004), Diebold (2004), Diebold and Lopez (1995), Engle (2001, 2004), Engle and Patton (2001), Pagan (1996), Palm (1996), and Shephard (1996). Moreover, ARCH models have now become standard textbook material in econometrics and finance as exemplified by, e.g., Alexander (2001, 2008), Brooks (2002), Campbell, Lo and MacKinlay (1997), Chan (2002), Christoffersen (2003), Enders (2004), Franses and van Dijk (2000), Gourieroux and Jasiak (2001), Hamilton (1994), Mills (1993), Poon (2005), Singleton (2006), Stock and Watson (2005), Tsay (2002), and Taylor (2004). So, why another survey type paper?

Even a cursory glance at the many reviews and textbook treatments cited above reveals a perplexing "alphabet-soup" of acronyms and abbreviations used to describe the plethora of models and procedures that have been developed over the years. Hence, as a complement to these more traditional surveys, I have tried to provide an alternative and easy-to-use encyclopedic type reference guide to the long list of ARCH acronyms. Comparing the length of this list to the list of general Acronyms in Time Series Analysis (ATSA) compiled by Granger (1983) further underscores the scope of the research efforts and new developments that have occurred in the area following the introduction of the basic linear ARCH model in Engle (1982).

My definition of what constitutes an ARCH acronym is, of course, somewhat arbitrary and subjective. In addition to the obvious cases of acronyms association with specific parametric models, I have also included descriptions of some abbreviations association with more general procedures and ideas that figure especially prominently in the ARCH literature. With a few exceptions, I have restricted the list of acronyms to those which have appeared in already published studies. Following Granger (1983), I have purposely not included the names of specific computer programs or procedures, as these are often of limited availability and may also be sold commercially. Even though I have tried my best to be as comprehensive and inclusive as possible, I have almost surely omitted some abbreviations. To everyone responsible for an acronym that I have inadvertently left out, please accept my apology.

Lastly, let me make it clear that the mere compilation of this list does not mean that I endorse the practice of associating each and every ARCH formulation with it's own unique acronym. In fact, the sheer length of this list arguably suggests that the use of special names and abbreviations originally intended for easily telling different ARCH models apart might have reached a point of diminishing returns to scale.

AARCH (Augmented ARCH) The AARCH model of Bera, Higgins and Lee (1992) extends the linear ARCH(q) model (see ARCH) to allow the conditional variance to depend on cross-products of the lagged innovations. Defining the q×1 vector $e_{t-1} \equiv \{ \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots, \varepsilon_{t-q} \}$, the AARCH(q) model may be expressed as:

$$\sigma_t^2 = \omega + e_{t-1}^{\prime} A e_{t-1},$$

where A denotes a q×q symmetric positive definite matrix. If A is diagonal, the model reduces to the standard linear ARCH(q) model. The Generalized AARCH, or GAARCH model is obtained by including lagged conditional variances on the right-hand-side of the equation. The slightly more general GQARCH representation was proposed independently by Sentana (1995) (see GQARCH).

ACD (Autoregressive Conditional Duration) The ACD model of Engle and Russell (1998) was developed to describe dynamic dependencies in the durations between randomly occurring events. The model has found especially wide use in the analysis of high-frequency financial data and times between trades or quotes. Let $x_i \equiv t_i - t_{i-1}$ denote the time interval between the i^{th} and the $(i\text{-}1)^{th}$ event. The popular ACD(1,1) model then parameterizes the expected durations, $\psi_i = E(x_i \mid x_{i-1}, x_{i-2}, \ldots)$, analogous to the conditional variance in the GARCH(1,1) model (see GARCH),

$$\psi_i = \omega + \alpha x_{i-1} + \beta \psi_{i-1}.$$

Higher order ACD(p,q) models are defined in a similar manner. Quasi Maximum Likelihood Estimates (see QMLE) of the parameters in the ACD(p,q) model may be obtained by applying standard GARCH(p,q) estimation procedures to $y_i \equiv x_i^{1/2}$, with the conditional mean fixed at zero (see also ACH and MEM).

ACH[1] (Autoregessive Conditional Hazard) The ACH model of Hamilton and Jordá (2002) is designed to capture dynamic dependencies in hazard rates, or the probability for the occurrence of specific events. The basic ACH(p,q) model without any updating of the expected hazard rates between events is asymptotically equivalent to the ACD(p,q) model for the times between events (see ACD).

ACH[2] (Adaptive Conditional Heteroskedasticity) In parallel to the idea of allowing for time-varying variances in a sequence of normal distributions underlying the basic ARCH model (see ARCH), it is possible to allow the scale parameter in a sequence of Stable Paretian distributions to change over time. The ACH formulation for the scale parameter, c_t, first proposed by McCulluch (1985) postulates that the temporal variation may be described by an exponentially weighted moving average (see EWMA) of the form,

$$c_t = \alpha|\varepsilon_{t-1}| + (1-\alpha)c_{t-1}.$$

Many other more complicated Stable GARCH formulations have subsequently been proposed and analyzed in the literature (see SGARCH).

ACM (Autoregressive Conditional Multinomial) The ACM model of Engle and Russell (2005) involves an ARMA type representation for discrete-valued multinomial data, in which the conditional transition probabilities between the different values are guaranteed to lie between zero and one and sum to unity. The ACM and ACD models (see ACD) may be combined in modeling high-frequency financial price series and other irregularly spaced discrete data.

ADCC (Asymmetric Dynamic Conditional Correlations) The ADCC GARCH model of Cappiello, Engle and Sheppard (2006) extends the DCC model (see DCC) to allow for asymmetries in the time-varying conditional correlations based on a GJR threshold type formulation (see GJR).

AGARCH[1] (Asymmetric GARCH) The AGARCH model was introduced by Engle (1990) to allow for asymmetric effects of negative and positive innovations (see also EGARCH, GJR, NAGARCH, and VGARCH[1]). The AGARCH(1,1) model is defined by:

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1} + \beta\sigma_{t-1}^2,$$

where negative values of γ implies that positive shocks will result in smaller increases in future volatility than negative shocks of the same absolute magnitude. The model may alternative be expressed as:

$$\sigma_t^2 = \omega' + \alpha(\varepsilon_{t-1} + \gamma')^2 + \beta\sigma_{t-1}^2,$$

for which $\omega' > 0, \alpha \geq 0$ and $\beta \geq 0$ readily ensures that the conditional variance is positive almost surely.

AGARCH[2] (Absolute value GARCH) See TS-GARCH.

ANN-ARCH (Artificial Neural Network ARCH) Donaldson and Kamstra (1997) term the GJR model (see GJR) augmented with a logistic function, as commonly used in Neural Networks, the ANN-ARCH model.

ANST-GARCH (Asymmetric Nonlinear Smooth Transition GARCH) The ANST-GARCH(1,1) model of Nam, Pyun and Arize (2002) postulates that

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta_i\sigma_{t-1}^2 + [\kappa + \delta\varepsilon_{t-1}^2 + \rho\sigma_{t-1}^2]F(\varepsilon_{t-1},\gamma),$$

where $F(\cdot)$ denotes a smooth transition function. The model simplifies to the ST-GARCH(1,1) model of Gonzalez-Rivera (1998) for $\kappa = \rho = 0$ (see ST-GARCH) and the standard GARCH(1,1) model for $\kappa = \delta = \rho = 0$ (see GARCH).

APARCH (Asymmetric Power ARCH) The APARCH, or APGARCH, model of Ding, Granger and Engle (1993) nests several of the most popular univariate parameterizations. In particular, the APGARCH(p,q) model,

$$\sigma_t^\delta = \omega + \sum_{i=1}^{q} \alpha_i(|\varepsilon_{t-i}| - \gamma_i\varepsilon_{t-i})^\delta + \sum_{i=1}^{p} \beta_i\sigma_{t-i}^\delta,$$

reduces to the standard linear GARCH(p,q) model for $\delta = 2$ and $\gamma_i = 0$, the TS-GARCH(p,q) model for $\delta = 1$ and $\gamma_i = 0$, the NGARCH(p,q) model for $\gamma_i = 0$, the GJR-GARCH model for $\delta = 2$ and $0 \le \gamma_i \le 1$, the TGARCH(p,q) model for $\delta = 1$ and $0 \le \gamma_i \le 1$, while the log-GARCH(p,q) model is obtained as the limiting case of the model for $\delta \to 0$ and $\gamma_i = 0$ (see GARCH, TS-GARCH, NGARCH, GJR, TGARCH and log-GARCH).

ARCD (AutoRegressive Conditional Density) The ARCD class of models proposed by Hansen (1994) extends the basic ARCH class of models to allow for conditional dependencies beyond the mean and variance by postulating a specific non-normal distribution for the standardized innovations $z_t \equiv \varepsilon_t\sigma_t^{-1}$, explicitly parameterizing the shape parameters of this distribution as a function of lagged information. Most empirical applications of the ARCD model have relied on the standardized skewed Student-t distribution (see also GARCH-t and GED-GARCH). Specific examples of ARCD models include the GARCH with Skewness, or GARCHS, model of Harvey and Siddique (1999), in which the skewness is allowed to be time-varying. In particular, for the GARCHS(1,1,1) model,

$$s_t = \gamma_0 + \gamma_1 z_t^3 + \gamma_2 s_{t-1},$$

where $s_t \equiv E_{t-1}(z_t^3)$. Similarly, the GARCH with Skewness and Kurtosis, or GARCHSK, model of León, Rubio and Serna (2005), parameterizes the conditional kurtosis as:

$$k_t = \delta_0 + \delta_1 z_t^4 + \delta_2 k_{t-1},$$

where $k_t \equiv E_{t-1}(z_t^4)$.

ARCH (AutoRegressive Conditional Heteroskedastcity) The ARCH model was originally developed by Engle (1982) to describe U.K. inflationary uncertainty. However, the ARCH class of models has subsequently found especially wide use in characterizing time-varying financial market volatility. The ARCH regression model for y_t first analyzed in Engle (1982) is defined by:

$$y_t \,|\, \mathscr{F}_{t-1} \;\sim\; N(x_t'\beta, \sigma_t^2),$$

where $\mathscr{F}_{t-1}$ refers to the information set available at time *t-1*, and the conditional variance,

$$\sigma_t^2 \;=\; f(\varepsilon_{t-1}, \varepsilon_{t-2}, \dots, \varepsilon_{t-p}; \theta),$$

is an explicit function of the *p* lagged innovations, $\varepsilon_t \equiv y_t - x_t'\beta$. Using a standard prediction error decomposition type argument, the log-likelihood function for the ARCH model may be expressed as:

$$LogL(y_T, y_{t-1}, \dots, y_1; \beta, \theta) \;=\; -\frac{T}{2}\log(2\pi) \;-\; \frac{1}{2}\sum_{t=1}^{T}\,[\log(\sigma_t^2) \;+\; (y_t - x_t'\beta)\sigma_t^{-2}\,].$$

Even though analytical expressions for the Maximum Likelihood Estimates (see also QMLE) are not available in closed form, numerical procedures may easily be used to maximize the function. The *q*th-order linear ARCH(q) model suggested by Engle (1982) provides a particularly convenient and natural parameterizarion for capturing the tendency for large (small) variances to be followed by other large (small) variances,

$$\sigma_t^2 \;=\; \omega \;+\; \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2,$$

where for the conditional variance to non-negative and the model well-defined ω has to be positive and all of the α_i's non-negative. Most of the early empirical applications of ARCH models, including Engle (1982), were based on the linear ARCH(q) model with the additional constraint that the α_i's decline linearly with the lag,

$$\sigma_t^2 \;=\; \omega \;+\; \alpha \sum_{i=1}^{q} (q+1-i)\varepsilon_{t-i}^2,$$

in turn requiring the estimation of only a single α parameter irrespective of the value of q. More generally, any non-trivial measurable function of the time *t-1* information set, σ_t^2, such that

$$\varepsilon_t \;=\; \sigma_t z_t,$$

where z_t is a sequence of independent random variables with mean zero and unit variance, is now commonly referred to as an ARCH model.

ARCH-Filters ARCH and GARCH models may alternatively be given a non-parametric interpretation as discrete-time filters designed to extract information about some underlying, possibly latent continuous-time, stochastic volatility process. Issues related to the design of consistent and asymptotically optimal ARCH-Filters have been studied extensively by Nelson (1992, 1996a) and Nelson and Foster (1994). For instance, the asymptotically efficient filter (in a mean-square-error sense for increasingly finer sample observations) for the instantaneous volatility in the GARCH diffusion model (see GARCH Diffusion) is given by the discrete-time GARCH(1,1) model (see also ARCH-Smoothers).

ARCH-NNH (ARCH Nonstationary Nonlinear Heteroskedasticity) The ARCH-NNH model of Han and Park (2008) includes a nonlinear function of a near or exact unit root process, x_t, in the conditional variance of the ARCH(1) model,

$$\sigma_t^2 = \alpha \varepsilon_{t-1}^2 + f(x_t).$$

The model is designed to capture slowly decaying stochastic long-run volatility dependencies (see also CGARCH[1], FIGARCH, IGARCH).

ARCH-M (ARCH-in-Mean) The ARCH-M model was first introduced by Engle, Lilien and Robins (1987) for modelling risk-return tradeoffs in the term structure of U.S. interest rates. The model extends the ARCH regression model in Engle (1982) (see ARCH) by allowing the conditional mean to depend directly on the conditional variance,

$$y_t \,|\, \mathscr{F}_{t-1} \sim N(x_t'\beta + \delta\sigma_t^2, \sigma_t^2).$$

This breaks the block-diagonality between the parameters in the conditional mean and the parameters in the conditional variance, so that the two sets of parameters must be estimated jointly to achieve asymptotic efficiency. Non-linear functions of the conditional variance may be included in the conditional mean in a similar fashion. The final preferred model estimated in Engle, Lilien and Robins (1987) parameterizes the conditional mean as a function of $\log(\sigma_t^2)$. Multivariate extensions of the ARCH-M model were first analyzed and estimated by Bollerslev, Engle and Wooldridge (1988) (see also MGARCH[1]).

ARCH-SM (ARCH Stochastic Mean) The ARCH-SM acronym was coined by Lee and Taniguchi (2005) to distinguish ARCH models in which $\varepsilon_t \equiv y_t - E_{t-1}(y_t) \neq y_t - E(y_t)$ (see ARCH).

ARCH-Smoothers ARCH-Smoothers, first developed by Nelson (1996b) and Foster and Nelson (1996), extend the ARCH and GARCH models and corresponding ARCH-Filters based solely on past observations (see ARCH-Filters) to allow for the use of both current and future observations in the estimation of the latent volatility.

ATGARCH (Asymmetric Threshold GARCH) The ATGARCH(1,1) model of Crouhy and Rockinger (1997) combines and extends the TS-GARCH(1,1) and GJR(1,1) models (see TS-GARCH and GJR) by allowing the threshold used in characterizing the asymmetric response to differ from zero,

$$\sigma_t = \omega + \alpha|\varepsilon_{t-1}|I(\varepsilon_{t-1} \geq \gamma) + \delta|\varepsilon_{t-1}|I(\varepsilon_{t-1} < \gamma) + \beta\sigma_{t-1}.$$

Higher order ATGARCH(p,q) models may be defined analogously (see also AGARCH and TGARCH).

Aug-GARCH (Augmented GARCH) The Aug-GARCH model developed by Duan (1997) nests most of the popular univariate parameterizations, including the standard linear GARCH model, the Multiplicative GARCH model, the Exponential GARCH model, the GJR-GARCH model, the Threshold GARCH model, the Nonlinear GARCH model, the Taylor-Schwert GARCH model, and the VGARCH model (see GARCH, MGARCH[2], EGARCH, GJR, TGARCH, NGARCH, TS-GARCH and VGARCH[1]). The Aug-GARCH(1,1) model may be expressed as:

$$\sigma_t^2 = |\lambda\varphi_t - \lambda + 1|I(\lambda \neq 0) + \exp(\varphi_t - 1)I(\lambda = 0),$$

where

$$\varphi_t = \omega + \alpha_1|z_{t-1} - \kappa|^\delta\varphi_{t-1} + \alpha_2\max(0, \kappa - z_{t-1})^\delta\varphi_{t-1} + \alpha_3(|z_{t-1} - \kappa|^\delta - 1)/\delta$$

$$+ \alpha_4(\max(0, \kappa - z_{t-1})^\delta - 1)/\delta + \beta\varphi_{t-1},$$

and $z_t \equiv \varepsilon_t\sigma_t^{-1}$ denotes the corresponding standardized innovations. The basic GARCH(1,1) model is obtained by fixing $\lambda = 1, \kappa = 0, \delta = 2$ and $\alpha_2 = \alpha_3 = \alpha_4 = 0$, while the EGARCH model corresponds to $\lambda = 0, \kappa = 0, \delta = 1$ and $\alpha_1 = \alpha_2 = 0$ (see also HGARCH).

AVGARCH (Absolute Value GARCH) See TS-GARCH.

β-ARCH (Beta ARCH) The β-ARCH(1) model of Guégan and Diebolt (1994) allows the conditional variance to depend asymmetrically on positive and negative lagged innovations,

$$\sigma_t^2 = \omega + [\alpha I(\varepsilon_{t-1} > 0) + \gamma I(\varepsilon_{t-1} < 0)]\varepsilon_{t-1}^{2\cdot\beta},$$

where *I(·)* denotes the indicator function. For $\alpha = \gamma$ and $\beta = 1$ the model reduces to the standard linear ARCH(1) model. More general β-ARCH(q) and β-GARCH(p,q) models may be defined in a similar fashion (see also GJR, TGARCH, and VGARCH[1]).

BEKK (Baba, Engle, Kraft and Kroner) The BEKK acronym refers to a specific parameteriztion of the multivariate GARCH model (see MGARCH[1]) developed in Engle and Kroner (1995). The simplest BEKK representation for the $N \times N$ conditional covariance matrix Ω_t takes the form:

$$\Omega_t = C'C + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'\Omega_{t-1}B,$$

where C denotes an upper triangular $N \times N$ matrix, and A and B are both unrestricted $N \times N$ matrices. This quadratic representation automatically guarantees that Ω_t is positive definite. The reference to Y. Baba and D. Kraft in the acronym stems from an earlier unpublished four-authored working paper.

BGARCH (Bivariate GARCH) See MGARCH[1].

CARR (Conditional AutoRegressive Range) The CARR(p,q) model proposed by Chou (2005) postulates a GARCH(p,q) structure (see GARCH) for the dynamic dependencies in time series of high-low asset prices over some fixed time interval. The model is essentially analogous to the ACD model (see ACD) for the times between randomly occurring events (see also REGARCH).

CAViaR (Conditional Autoregressive Value at Risk) The CAViaR model of Engle and Manganelli (2004) specifies the evolution of a particular conditional quantile of a times series, say f_t where $P_{t-1}(y_t \leq f_t) = p$ for some pre-specified fixed level p, as an autoregressive process. The indirect GARCH(1,1) model parameterizes the conditional quantiles as:

$$f_t = (\omega + \alpha y_{t-1}^2 + \beta f_{t-1}^2)^{1/2}.$$

This formulation would be correctly specified if the underlying process for y_t follows a GARCH(1,1) model with *i.i.d.* standardized innovations (see GARCH). Alternative models allowing for asymmetries may be specified in a similar manner. The CAViaR model was explicitly developed for predicting quantiles in financial asset return distributions, or so-called Value-at-Risk.

CCC (Constant Conditional Correlations) The $N \times N$ conditional covariance matrix for the $N \times 1$ vector process ε_t, say Ω_t, may always be decomposed as:

$$\Omega_t = R_t D_t R_t,$$

where R_t denotes the $N \times N$ matrix of conditional correlations with typical element

$$\rho_{ijt} = \frac{Cov_{t-1}(\varepsilon_{it}, \varepsilon_{jt})}{Var_{t-1}(\varepsilon_{it})^{1/2}\, Var_{t-1}(\varepsilon_{jt})^{1/2}},$$

and D_t denotes the $N \times N$ diagonal matrix with typical element $Var_{t-1}(\varepsilon_{it})$. The CCC GARCH model of Bollerslev (1990) assumes that the conditional correlations are constant $\rho_{ijt} = \rho_{ij}$, so that the temporal variation in Ω_t is determined solely by the time-varying conditional variances for each of the elements in ε_t. This assumption greatly simplifies the inference, requiring only the non-linear estimation of N univariate GARCH models, while $R_t = R$ may be estimated by the sample correlations of the corresponding standardized residuals. Moreover, as long as each of the conditional variances are positive, the CCC model guarantees that the resulting conditional covariance matrices are positive definite (see also DCC and MGARCH[1]).

Censored-GARCH See Tobit-GARCH.

CGARCH[1] (Component GARCH) The component GARCH model of Engle and Lee (1999) was designed to better account for long-run volatility dependencies. Rewriting the GARCH(1,1) model as:

$$(\sigma_t^2 - \sigma^2) = \alpha(\varepsilon_{t-1}^2 - \sigma^2) + \beta(\sigma_{t-1}^2 - \sigma^2),$$

where $\sigma^2 \equiv \omega/(1-\alpha-\beta)$ refers to the unconditional variance, the CGARCH model is obtained by relaxing the assumption of a constant σ^2. Specifically,

$$(\sigma_t^2 - \zeta_t^2) = \alpha(\varepsilon_{t-1}^2 - \zeta_{t-1}^2) + \beta(\sigma_{t-1}^2 - \zeta_{t-1}^2),$$

with the corresponding long-run variance parameterized by the separate equation,

$$\zeta_t^2 = \omega + \rho\zeta_{t-1}^2 + \varphi(\varepsilon_{t-1}^2 - \sigma_{t-1}^2).$$

Substituting this expression for ζ_t^2 into the former equation, the CGARCH model may alternatively be expressed as a restricted GARCH(2,2) model (see also FIGARCH).

CGARCH[2] (Composite GARCH) The CGARCH model of den Hertog (1994) represents ε_t^2 as the sum of a latent permanent random walk component and another latent AR(1) component.

COGARCH (Continuous GARCH) The continuous-time COGARCH(1,1) model proposed by Klüppelberg, Lindner and Maller (2004) may be expressed as,

$$dy(t) = \sigma(t)dL(t),$$

and

$$\sigma^2(t) = [\sigma^2(0) - \omega\int_0^t \exp(x(s))ds]\exp(-x(t_-)),$$

where

$$x(t) = -\log\beta - \sum_{0<s\leq t} \log[1 + \alpha\exp(-\log\beta)\,\Delta L(s)^2].$$

The model is obtained by backward solution of the difference equation defining the discrete-time GARCH(1,1) model (see GARCH), replacing the standardized innovations by the increments to the Lévy process, $L(t)$. In contrast to the GARCH diffusion model of Nelson (1990b) (see GARCH Diffusion), which involves two independent Brownian motions, the COGARCH model is driven by a single innovation process. Higher order COGARC(p,q) processes have been developed by Brockwell, Chadraa and Lindner (2006) (see also ECOGARCH).

Copula GARCH Any joint distribution function may be expressed in terms of its marginal distribution functions and a copula function linking these. The class of copula GARCH models builds on this idea in the formulation of multivariate GARCH models (see MGARCH[1]) by linking univariate GARCH models through a sequence of possibly time-varying conditional copulas. For further discussion of estimation and inference in copula GARCH models, see, e.g., Jondeau and Rockinger (2006) and Patton (2006) (see also CCC and DCC).

CorrARCH (Correlated ARCH) The bivariate CorrARCH model of Christodoulakis and Satchell (2002) parameterizes the time-varying conditional correlations as a distributed lag of the product of the standardized innovations from univariate GARCH models for each of the two series. A Fisher transform is used to ensure that the resulting correlations always lie between -1 and 1 (see also CCC, DCC and MGARCH[1]).

DAGARCH (Dynamic Asymmetric GARCH) The DAGARCH model of Caporin and McAleer (2006) extends the GJR-GARCH model (see GJR) to allow for multiple thresholds and time-varying asymmetric effects (see also AGARCH, ATGARCH and TGARCH).

DCC (Dynamic Conditional Correlations) The multivariate DCC-GARCH model of Engle (2002a) extends the CCC model (see CCC) by allowing the conditional correlations to be time-varying. To facilitate the analysis of large dimensional systems, the basic DCC model postulates that the temporal variation in the conditional correlations may be described by exponential smoothing (see EWMA) so that

$$\rho_{ijt} = \frac{q_{ijt}}{q_{iit}^{1/2}\, q_{jjt}^{1/2}},$$

where

$$q_{ijt} = (1-\lambda)\varepsilon_{it-1}\varepsilon_{jt-1} + \lambda q_{ijt-1},$$

and $\boldsymbol{\varepsilon}_t$ denotes the $N\times 1$ vector innovation process. A closely related formulation was proposed independently by Tse and Tsui (2002), who refer to their approach as a Varying Conditional Correlation, or VCC-MGARCH model (see also ADCC, CorrARCH, FDCC and MGARCH[1]).

diag MGARCH (diagonal GARCH) The diag MGARCH model refers to the simplification of the vech GARCH model (see vech GARCH) in which each of the elements in the conditional covariance matrix depends on its own past values and the products of the corresponding elements in the innovation vector only. The model is conveniently expressed in terms of Hadamard products, or matrix element-by-element multiplication. In particular, for the diag MGARCH(1,1) model,

$$\Omega_t = C^o + A^o \odot \varepsilon_{t-1}\varepsilon_{t-1} + B^o \odot \Omega_{t-1}.$$

It follows (see Attanasio, 1991) that if each of the three $N \times N$ matrices C^o, A^o and B^o are positive definite, the conditional covariance matrix will also be positive definite (see also MGARCH[1]).

DTARCH (Double Threshold ARCH) The DTARCH model of Li and Li (1996) allows the parameters in both the conditional mean and the conditional variance to change across regimes, with the *m* different regimes determined by a set of threshold parameters for some lag $k \geq 1$ of the observed y_t process, say $r_{j-1} < y_{t-k} \leq r_j$, where $-\infty = r_0 < r_1 < \ldots < r_m = \infty$ (see also TGARCH).

DVEC-GARCH (Diagonal VECtorized GARCH) See diag MGARCH.

ECOGARCH (Exponential Continuous GARCH) The continuous-time ECOGARCH model developed by Haug and Czado (2007) extends the Lévy driven COGARCH model of Klüppelberg, Lindner and Maller (2004) (see COGARCH) to allow for different impact of positive and negative jump innovations, or so-called leverage effects. The model may be seen as a continuous-time analog of the discrete-time EGARCH model (see also EGARCH, GJR and TGARCH).

EGARCH (Exponential GARCH) The EGARCH model was developed by Nelson (1991). The model explicitly allows for asymmetries in the relationship between return and volatility (see also GJR and TGARCH). In particular, let $z_t \equiv \varepsilon_t \sigma_t^{-1}$ denote the standardized innovations. The EGARCH(1,1) model may then be expressed as:

$$\log(\sigma_t^2) = \omega + \alpha(|z_{t-1}| - E(|z_{t-1}|)) + \gamma z_{t-1} + \beta \log(\sigma_{t-1}^2).$$

For $\gamma < 0$ negative shocks will obviously have a bigger impact on future volatility than positive shocks of the same magnitude. This effect, which is typically observed empirically with equity index returns, is often referred to as a "leverage effect," although it is now widely agreed that the apparent asymmetry has little to do with actual financial leverage. By parameterizing the logarithm of the conditional variance as opposed to the conditional variance, the EGARCH model also avoids complications from having to ensure that the process remains positive. This is especially useful when conditioning on other explanatory variables. Meanwhile, the logarithmic transformation complicates the construction of unbiased forecasts for the level of future variances (see also GARCH and log-GARCH).

EVT-GARCH (Extreme Value Theory GARCH) The EVT-GARCH approach pioneered by McNeil and Frey (2000), relies on extreme value theory for *i.i.d.* random variables and corresponding generalized Pareto distributions for more accurately characterizing the tails of the distributions of the standardized innovations from GARCH models. This idea may be used in the calculation of low-probability quantile, or Value-at-Risk, type predictions (see also CAViaR, GARCH-t and GED-GARCH).

EWMA (Exponentially Weighted Moving Average) EWMA variance measures are defined by the recursion,

$$\sigma_t^2 = (1-\lambda)\varepsilon_{t-1}^2 + \lambda\sigma_{t-1}^2 .$$

EWMA may be seen as a special case of the GARCH(1,1), or IGARCH(1,1), model in which $\omega \equiv 0, \alpha \equiv 1-\lambda$ and $\beta \equiv \lambda$ (see GARCH and IGARCH). EWMA covariance measures are readily defined in a similar manner. The EWMA approach to variance estimation was popularized by RiskMetrics, advocating the use of $\lambda = 0.94$ with daily financial returns.

F-ARCH (Factor ARCH) The multivariate factor ARCH model developed by Diebold and Nerlove (1989) (see also Latent GARCH) and the factor GARCH model of Engle, Ng and Rothschild (1990) assumes that the temporal variation in the $N\times N$ conditional covariance matrix for a set of N returns can be described by univariate GARCH models for smaller set of $K<N$ portfolios,

$$\Omega_t = \Omega + \sum_{k=1}^{K} \lambda_k \lambda_k' \sigma_{kt}^2 ,$$

where λ_k and σ_{kt}^2 refer to the time invariant $N\times 1$ vector of factor loadings and time t conditional variance for the kth factor, respectively. More specifically, the F-GARCH(1,1) model may be expressed as:

$$\Omega_t = \Omega + \lambda\lambda'[\beta w'\Omega_{t-1}w + \alpha(w'\varepsilon_{t-1})^2]$$

where w denotes an $N\times 1$ vector, and α and β are both scalar parameters (see also OGARCH and MGARCH[1]).

FCGARCH (Flexible Coefficient GARCH) The FCGARCH model of Medeiros and Veiga (2008) defines the conditional variance as a linear combination of standard GARCH type models, with the weights assigned to each model determined by a set of logistic functions. The model nests several alternative smooth transition and asymmetric GARCH models as special limiting cases, including the DTARCH, GJR, STGARCH, TGARCH, and VSGARCH models.

FDCC (Flexible Dynamic Conditional Correlations) The FDCC-GARCH model of Billio, Caporin and Gobbo (2006) generalizes the basic DCC model (see DCC) to allow for different dynamic dependencies in the time-varying conditional correlations (see also ADCC).

FGARCH (Factor GARCH) See F-ARCH.

FIAPARCH (Fractionally Integrated Power ARCH) The FIAPARCH(p,d,q) model of Tse (1998) combines the FIGARCH(p,d,q) and the APARCH(p,q) models in parameterizing σ_t^δ as a fractionally integrated distributed lag of $(|\varepsilon_t| - \gamma\varepsilon_t)^\delta$ (see FIGARCH and APARCH).

FIEGARCH (Fractionally Integrated EGARCH) The FIEGARCH model of Bollerslev and Mikkelsen (1996) imposes a fractional unit root in the autoregressive polynomial in the ARMA representation of the EGARCH model (see EGARCH). In particular, the FIEGARCH(1,d,1) model may be conveniently expressed as:

$$(1 - \beta L)(1 - L)^d \log(\sigma_t^2) = \omega + \alpha(|z_{t-1}| - E(|z_{t-1}|)) + \gamma z_{t-1}.$$

For *0<d<1* this representation implies fractional integrated slowly decaying hyperbolic dependencies in $\log(\sigma_t^2)$ (see also FIGARCH, HYGARCH and LMGARCH).

FIGARCH (Fractionally Integrated GARCH) The FIGARCH model proposed by Baillie, Bollerslev and Mikkelsen (1996) relies on an ARFIMA type representation to better capture the long-run dynamic dependencies in the conditional variance. The model may be seen as natural extension of the IGARCH model (see IGARCH), allowing for fractional orders of integration in the autoregressive polynomial in the corresponding ARMA representation,

$$\varphi(L)(1 - L)^d \varepsilon_t^2 = \omega + (1 - \beta(L))\nu_t,$$

where $\nu_t \equiv \varepsilon_t^2 - \sigma_t^2$, *0<d<1*, and the roots of $\varphi(z) = 0$ and $\beta(z) = 1$ are all outside the unit circle. For values of *0<d<½* the model implies an eventual slow hyperbolic decay in the autocorrelations for σ_t^2 (see also FIEGARCH, HYGARCH and LMGARCH).

FIREGARCH (Fractionally Integrated Range EGARCH) See REGARCH.

Flex-GARCH (Flexible GARCH) The multivariate Flex-GARCH model of Ledoit, Santa-Clara and Wolf (2003) is designed to reduce the computational burden involved in the estimation of multivariate diagonal MGARCH models (see diag MGARCH). This is accomplished by estimating a set bivariate MGARCH models for each of the *N(N+1)/2* possible different pairwise combinations of the *N* variables, and then subsequently "paste" together the parameter estimates subject to the constraint that the resulting parameter matrices for the full *N*-dimensional MGARCH model guarantee positive semidefinite conditional covariance matrices.

GAARCH (Generalized Augmented ARCH) See AARCH.

GARCH (Generalized AutoRegressive Conditional Heteroskedasticity) The GARCH(p,q) model of Bollerslev (1986) includes *p* lags of the conditional variance in the linear ARCH(q) (see ARCH) conditional variance equation,

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 .$$

Conditions on the parameters to ensure that the GARCH(p,q) conditional variance is always positive are given in Nelson and Cao (1992). The GARCH(p,q) model may alternative be represented as an ARMA(max{p,q},p) model for the squared innovation:

$$\varepsilon_t^2 = \omega + \sum_{i=1}^{\max\{p,q\}} (\alpha_i + \beta_i) \varepsilon_{t-i}^2 - \sum_{i=1}^{p} \beta_i \nu_{t-i} ,$$

where $\nu_t \equiv \varepsilon_t^2 - \sigma_t^2$, so that by definition $E_{t-1}(\nu_t) = 0$. The relatively simple GARCH(1,1) model,

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 ,$$

often provides a good fit in empirical applications. This particular parameterization was also proposed independently by Taylor (1986). The GARCH(1,1) model is well-defined and the conditional variance positive almost surely provided that $\omega > 0, \alpha \geq 0$ and $\beta \geq 0$. The GARCH(1,1) model may alternatively be express as an ARCH(∞) model,

$$\sigma_t^2 = \omega(1 - \beta)^{-1} + \alpha \sum_{i=1}^{\infty} \beta^{i-1} \varepsilon_{t-i}^2 ,$$

provided that $\beta < 1$. If $\alpha + \beta < 1$ the model is covariance stationary and the unconditional variance equals $\sigma^2 \equiv \omega/(1 - \alpha - \beta)$. Multi-period conditional variance forecasts from the GARCH(1,1) model may readily be calculated as:

$$\sigma_{t+h|t}^2 = \sigma^2 + (\alpha + \beta)^{h-1} (\sigma_{t+1}^2 - \sigma^2) ,$$

where $h \geq 2$ denotes the horizon of the forecast.

GARCH-Δ (GARCH Delta) See GARCH-Γ.

GARCH Diffusion The continuous-time GARCH diffusion model is defined by:

$$dy(t) = \sigma(t)dW_1(t),$$

and

$$d\sigma^2(t) = (\omega - \theta\sigma^2(t))dt + \sqrt{2}\alpha\sigma^2(t)dW_2(t),$$

where the two Wiener processes, $W_1(t)$ and $W_2(t)$, that drive the observable $y(t)$ process and the instantaneous latent volatility process, $\sigma^2(t)$, are assumed to be independent. As shown by Nelson (1990b), the sequence of GARCH(1,1) models defined over discrete time intervals of length $1/n$,

$$\sigma_{t,n}^2 = (\omega/n) + (\alpha/n^{1/2})\varepsilon_{t-1/n,n}^2 + (1 - \alpha/n^{1/2} - \theta/n)\sigma_{t-1/n,n}^2,$$

where $\varepsilon_{t,n} \equiv y(t) - y(t-1/n)$, converges weakly to a GARCH diffusion model for $n \to \infty$ (see also COGARCH and ARCH-Filters).

GARCH-EAR (GARCH Exponential AutoRegression) The GARCH-EAR model of LeBaron (1992) allows the first order serial correlation of the underlying process to depend directly on the conditional variance,

$$y_t = \varphi_0 + [\varphi_1 + \varphi_2\exp(-\sigma_t^2/\varphi_3)]y_{t-1} + \varepsilon_t.$$

For $\varphi_2 = 0$ the model reduces to a standard AR(1) model, but for $\varphi_2 > 0$ and $\varphi_3 > 0$ the magnitude of the serial correlation in the mean will be a decreasing function of the conditional variance (see also ARCH-M).

GARCH-Γ (GARCH Gamma) The gamma of an option is defined as the second derivative of the option price with respect to the price of the underlying asset. Options gamma play an important role in hedging volatility risk embedded in options positions. GARCH-Γ refers to the gamma obtained under the assumption that the return on the underlying asset follows a GARCH process. Engle and Rosenberg (1995) find that GARCH-Γ's are typically much higher than conventional Black-Scholes gammas. Meanwhile, GARCH-Δ's, or the first derivative of the option price with respect to the price of the underlying asset, tend to be fairly close to their Black-Scholes counterparts.

GARCH-M (GARCH in Mean) See ARCH-M.

GARCHS (GARCH with Skewness) See ARCD.

GARCHSK (GARCH with Skewness and Kurtosis) See ARCD.

GARCH-t (GARCH t-distribution) ARCH models are typically estimated by maximum likelihood under the assumption that the errors are conditionally normally distributed (see ARCH). However, in many empirical applications the standardized residuals, $\hat{\varepsilon}_t \hat{\sigma}_t^{-1}$, appear to have fatter tails than the normal distribution. The GARCH-t model of Bollerslev (1987) relaxes the assumption of conditional normality by instead assuming that the standardized innovations follow a standardized Student t-distribution. The corresponding log Likelihood function may be expressed as:

$$LogL(\theta) = \sum_{t=1}^{T} \log\left(\Gamma\left(\frac{\nu+1}{2}\right) \Gamma\left(\frac{\nu}{2}\right)^{-1} ((\nu-2)\sigma_t^2)^{-1/2} (1 + (\nu-2)^{-1} \sigma_t^{-2} \varepsilon_t^2)^{-(\nu+1)/2} \right),$$

where $\nu > 2$ denotes the degrees of freedom to be estimated along with the parameters in the conditional variance equation (see also GED-GARCH, QMLE and SPARCH).

GARCH-X[1] The multivariate GARCH-X model of Lee (1994) includes the error correction term from a cointegrating type relationship for the underlying vector process $y_t \sim I(1)$, say $z_{t-1} = b' y_{t-1} \sim I(0)$, as an explanatory variable in the conditional covariance matrix (see also MGARCH[1])

GARCH-X[2] The GARCH-X model proposed by Brenner, Harjes and Kroner (1996) for modeling short term interest rates includes the lagged interest rate raised to some power, say δr_{t-1}^{γ}, as an explanatory variable in the GARCH conditional variance equation (see GARCH).

GARCHX The GARCHX model proposed by Hwang and Satchell (2005) for modeling aggregate stock market return volatility includes a measure of the lagged cross-sectional return variation as an explanatory variable in the GARCH conditional variance equation (see GARCH).

GARJI Maheu and McCurdy (2004) refer to the standard GARCH model (see GARCH) augmented with occasional Poisson distributed "jumps" or large moves, where the time-varying jump intensity is determined by a separate autoregressive process as a GARJI model.

GDCC (Generalized Dynamic Conditional Correlations) The multivariate GDCC-GARCH model of Cappiello, Engle and Sheppard (2006) utilizes a more flexible BEKK type parameterization (see BEKK) for the dynamic conditional correlations (see DCC). Combining the ADCC (see ADCC) and the GDCC models results in an AGDCC model (see also FDCC).

GED-GARCH (Generalized Error Distribution GARCH) The GED-GARCH model of Nelson (1991) replaces the assumption of conditionally normal errors traditional used in the estimation of ARCH models with the assumption that the standardized innovations follow a generalized error distribution, or what is also sometimes referred to as an exponential power distribution (see also GARCH-t).

GJR (Glosten, Jagannathan and Runkle GARCH) The GJR-GARCH, or just GJR, model of Glosten, Jagannathan and Runkle (1993) allows the conditional variance to respond differently to the past negative and positive innovations. The GJR(1,1) model may be expressed as:

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1}^2 I(\varepsilon_{t-1}<0) + \beta\sigma_{t-1}^2,$$

where $I(\cdot)$ denotes the indicator function. The model is also sometimes referred to as a Sign-GARCH model. The GJR formulation is closely related to the Threshold GARCH, or TGARCH, model proposed independently by Zakoian (1994) (see TGARCH), and the Asymmetric GARCH, or AGARCH, model of Engle (1990) (see AGARCH). When estimating the GJR model with equity index returns, γ is typically found to be positive, so that the volatility increases proportionally more following negative than positive shocks. This asymmetry is sometimes referred to in the literature as a "leverage effect," although it is now widely agreed that it has little to do with actual financial leverage (see also EGARCH).

GO-GARCH (Generalized Orthogonal GARCH) The multivariate GO-GARCH model of van der Weide (2002) assumes that the temporal variation in the $N \times N$ conditional covariance matrix may be expressed in terms of N conditionally uncorrelated components,

$$\Omega_t = X D_t X',$$

where X denotes a $N \times N$ matrix, and D_t is diagonal with the conditional variances for each of the components along the diagonal. This formulation permits estimation by a relatively easy-to-implement two-step procedure (see also F-ARCH, GO-GARCH and MGARCH[1]).

GQARCH (Generalized Quadratic ARCH) The GQARCH(p,q) model of Sentana (1995) is defined by:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \psi_i \varepsilon_{t-i} + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + 2\sum_{i=1}^{q} \sum_{j=i+1}^{q} \alpha_{ij} \varepsilon_{t-i} \varepsilon_{t-j} + \sum_{i=1}^{q} \beta_i \sigma_{t-i}^2.$$

The model simplifies to the linear GARCH(p,q) model if all of the ψ_i's and the α_{ij}'s are equal to zero. Defining the $q \times 1$ vector $e_{t-1} \equiv \{\varepsilon_{t-1}, \varepsilon_{t-2}, \ldots, \varepsilon_{t-q}\}$, the model may alternatively be expressed as:

$$\sigma_t^2 = \omega + \Psi' e_{t-1} + e_{t-1}' A e_{t-1} + \sum_{i=1}^{q} \beta_i \sigma_{t-i}^2,$$

where Ψ denotes the $q \times 1$ vector of ψ_i coefficients and A refer to the $q \times q$ symmetric matrix of α_i and α_{ij} coefficients. Conditions on the parameters for the conditional variance to be positive almost surely and the model well-defined are given in Sentana (1995) (see also AARCH).

GQTARCH (Generalized Qualitative Threshold ARCH) See QTARCH.

GRS-GARCH (Generalized Regime-Switching GARCH) The RGS-GARCH model proposed by Gray (1996) allows the parameters in the GARCH model to depend upon an unobservable latent state variable governed by a first-order Markov process. By aggregating the conditional variances over all of the possible states at each point in time, the model is formulated in such a way that it breaks the path-dependence which complicates the estimation of the SWARCH model of Cai (1994) and Hamilton and Susmel (1994) (see SWARCH).

HARCH (Heterogeneous ARCH) The HARCH(n) model of Müller, Dacorogna, Davé, Olsen, Puctet and Weizsäcker (1997) parameterizes the conditional variance as a function of the square of the sum of lagged innovations, or the squared lagged returns, over different horizons,

$$\sigma_t^2 = \omega + \sum_{i=1}^{n} \gamma_i \left(\sum_{j=1}^{i} \varepsilon_{t-j} \right)^2 .$$

The model is motivated as arising from the interaction of traders with different investment horizons. The HARCH model may been interpreted as a restricted QARCH model (see GQARCH).

Heston GARCH See SQR-GARCH.

HGARCH (Hentschel GARCH) The HGARCH model of Hentschel (1995) is based on a Box-Cox transform of the conditional standard deviation. It is explicitly designed to nest some of the most popular univariate parameterizations. The HGARCH(1,1) model may be expressed as:

$$\sigma_t^\delta = \omega + \alpha\delta\sigma_{t-1}^\delta (|\varepsilon_{t-1}\sigma_{t-1}^{-1} - \kappa| - \gamma(\varepsilon_{t-1}\sigma_{t-1}^{-1} - \kappa))^\nu + \beta\sigma_{t-1}^\delta .$$

The model obviously reduces to the standard linear GARCH(1,1) model for $\delta = 2, \nu = 2, \kappa = 0$ and $\gamma = 0$, but it also nests the APARCH, AGARCH[1], EGARCH, GJR, NGARCH, TGARCH, and TS-GARCH models as special cases (see also Aug-GARCH).

HYGARCH (Hyperbolic GARCH) The HYGARCH model proposed by Davidson (2004) nests the GARCH, IGARCH and FIGARCH models (see GARCH, IGARCH and FIGARCH). The model is defined in terms of the ARCH(∞) representation (see also LARCH),

$$\sigma_t^2 = \omega + \sum_{i=1}^{\infty} \alpha_i \varepsilon_{t-1}^2 \equiv \omega + [1 - \frac{\delta(L)}{\beta(L)}(1 + \alpha((1-L)^d - 1))]\varepsilon_{t-1}^2.$$

The standard GARCH and FIGARCH models correspond to $\alpha = 0$, and $\alpha = 1$ and $0 < d < 1$, respectively. For $d = 1$ the HYGARCH model reduces to a standard GARCH or an IGARCH model depending upon whether $\alpha < 1$ or $\alpha = 1$.

IGARCH (Integrated GARCH) Estimates of the standard linear GARCH(p,q) model (see GARCH) often results in the sum of the estimated α_i and β_i coefficients being close to unity. Rewriting the GARCH(p,q) model as an ARMA(max{p,q},p) model for the squared innovations,

$$(1 - \alpha(L) - \beta(L))\varepsilon_t^2 = \omega + (1 - \beta(L))\nu_t$$

where $\nu_t \equiv \varepsilon_t^2 - \sigma_t^2$, and $\alpha(L)$ and $\beta(L)$ denote appropriately defined lag polynomials, the IGARCH model of Engle and Bollerslev (1986) imposes an exact unit root in the corresponding autoregressive polynomial, $(1 - \alpha(L) - \beta(L)) = \varphi(L)(1 - L)$, so that the model may be written as:

$$\varphi(L)(1 - L)\varepsilon_t^2 = \omega + (1 - \beta(L))\nu_t.$$

Even though the IGARCH model is not covariance stationary, it is still strictly stationary with a well defined non-degenerate limiting distribution; see Nelson (1990a). Also, as shown by Lee and Hansen (1994) and Lumsdaine (1996), standard inference procedures may be applied in testing the hypothesis of a unit root, or $\alpha(1) + \beta(1) = 1$ (see also FIGARCH).

IV (Implied Volatility) Implied volatility refer to the volatility that would equate the theoretical price of an option according to some valuation model, typically Black-Scholes, to that of the actual market price of the option.

LARCH (Linear ARCH) The ARCH(∞) representation,

$$\sigma_t^2 = \omega + \sum_{i=1}^{\infty} \alpha_i \varepsilon_{t-1}^2,$$

is sometimes referred to as an LARCH model. This representation was first used by Robinson (1991) in the derivation of general tests for conditional heteroskedasticity.

Latent GARCH Models formulated in terms of latent variables that adhere to GARCH structures are sometimes referred to as latent GARCH, or unobserved GARCH, models. A leading example is the *N*-dimensional factor ARCH model of Diebold and Nerlove (1989),

$\varepsilon_t = \lambda f_t + \eta_t$, where λ and η_t denote $N \times 1$ vectors of factor loadings and *i.i.d.* innovations respectively, and the conditional variance of f_t is determined by an ARCH model in lagged squared values of the latent factor (see also F-ARCH). Models in which the innovations are subject to censoring is another example (see Tobit-GARCH). In contrast to standard ARCH and GARCH models, for which the likelihood functions are readily available through a prediction error decomposition type argument (see ARCH), the likelihood functions for latent GARCH models are generally not available in closed form. General estimation and inference procedures for latent GARCH models based on Markov Chain Monte Carlo methods have been developed by Fiorentini, Sentana and Shephard (2004) (see also SV).

Level-GARCH The Level-GARCH model proposed by Brenner, Harjes and Kroner (1996) for modeling the conditional variance of short term interest rates postulates that

$$\sigma_t^2 = \psi_t^2 r_{t-1}^{2\gamma},$$

where ψ_t follows a GARCH(1,1) structure,

$$\psi_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \psi_{t-1}^2.$$

For $\gamma = 0$ the model obviously reduces to a standard GARCH(1,1) model. The Level-GARCH model is also sometimes referred to as Time-Varying Parameter Level, or TVP-Level, model (see also GARCH and GARCH-X[2]).

LGARCH[1] (Leverage GARCH) The GJR model is sometimes referred to as an LGARCH model (see GJR).

LGARCH[2] (Linear GARCH) The standard GARCH(p,q) model (see GARCH) in which the conditional variance is a linear function of p own lags and q lagged squared innovations is sometimes refereed to as an LGARCH model.

LMGARCH (Long Memory GARCH) The LMGARCH(p,d,q) model is defined by,

$$\sigma_t^2 = \omega + [\beta(L)\varphi(L)^{-1}(1-L)^{-d} - 1]\nu_t,$$

where $\nu_t \equiv \varepsilon_t^2 - \sigma_t^2$, and $0<d<0.5$. Provided that the fourth order moment exists, the resulting process for ε_t^2 is covariance stationary and exhibits long memory. For further discussion comparisons with the FIGARCH model see Conrad and Karanasos (2006) (see also FIGARCH and HYGARCH).

log-GARCH (Logarithmic GARCH) The log-GARCH(p,q) model, which was suggested independently in slightly different forms by Geweke (1986), Pantula (1986) and Milhøj (1987), parameterizes the logarithmic conditional variance as a function of the lagged logarithmic variances and the lagged logarithmic squared innovations,

$$\log(\sigma_t^2) = \omega + \sum_{i=1}^{q} \alpha_i \log(\varepsilon_{t-i}^2) + \sum_{i=1}^{p} \beta_i \log(\sigma_{t-i}^2).$$

The model may alternatively be expressed as:

$$\sigma_t^2 = \exp(\omega) \prod_{i=1}^{q} (\varepsilon_{t-i}^2)^{\alpha_i} \prod_{i=1}^{p} (\sigma_{t-i}^2)^{\beta_i}.$$

In light of this alternative representation, the model is also sometimes referred to as a Multiplicative GARCH, or MGARCH, model.

MACH (Moving Average Conditional Heteroskedastic) The MACH(p) class of models proposed by Yang and Bewley (1995) is formally by the condition:

$$E_t(\sigma_{t+i}^2) = E(\sigma_{t+i}^2) \qquad i > p,$$

so that the effect of a shock to the conditional variance lasts for at most *p* periods. More specifically, the Linear MACH(1), or L-MACH(1), model is defined by $\sigma_t^2 = \omega + \alpha(\varepsilon_{t-1}/\sigma_{t-1})^2$. Higher order L-MACH(p) models, Exponential MACH(p), or E-MACH(p), models, Quadratic MACH(p), or Q-MACH(p), models, may be defined in a similar manner (see also EGARCH and GQARCH). The standard linear ARCH(1) model, $\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2$, is not a MACH(1) process.

MAR-ARCH (Mixture AutoRegressive ARCH) See MGARCH[3].

MARCH[1] (Modified ARCH) Friedman, Laibson and Minsky (1989) denote the class of GARCH(1,1) models in which the conditional variance depends non-linearly on the lagged squared innovations as Modified ARCH models,

$$\sigma_t^2 = \omega + \alpha F(\varepsilon_{t-1}^2) + \beta\sigma_{t-1}^2,$$

where $F(\cdot)$ denotes a positive valued function. In their estimation of the model Friedman, Laibson and Minsky (1989) use the function $F(x) = \sin(\theta x) \cdot I(\theta x < \pi/2) + 1 \cdot I(\theta x \geq \pi/2)$ (see also NGARCH).

MARCH[2] (Multiplicative ARCH) See MGARCH[2].

Matrix EGARCH The multivariate matrix exponential GARCH model of Kawakatsu (2006) (see also EGARCH and MGARCH[1]) specifies the second moment dynamics in terms of the matrix logarithm of the conditional covariance matrix. More specifically, let $h_t = vech(\log\Omega_t)$ denote the $N(N+1)/2\times 1$ vector of unique elements in $\log\Omega_t$, where the logarithm of a matrix is defined by the inverse of the power series expansion used in defining the matrix exponential. A simple multivariate matrix EGARCH extension of the univariate EGARCH(1,1) model may then be expressed as:

$$h_t = \Omega + A(|\varepsilon_{t-1}| - E(|\varepsilon_{t-1}|)) + \Gamma\varepsilon_{t-1} + Bh_{t-1},$$

for appropriately dimensioned matrices Ω, A, Γ and B. By parameterizing only the unique elements of the logarithmic conditional covariance matrix, the matrix EGARCH model automatically guarantees that $\Omega_t \equiv \exp(h_t)$ is positive definite.

MDH (Mixture of Distributions Hypothesis) The MDH first developed by Clark (1973) postulates that financial returns over non-trivial time intervals, say one day, represent the accumulated effect of numerous within period, or intraday, news arrivals and corresponding price changes. The MDH coupled with the assumption of serially correlated news arrivals is often used to rationalize the apparent volatility clustering, or ARCH effects, in asset returns. More advanced versions of the MDH, relating the time-deformation to various financial market activity variables, such as the number of trades, the cumulative trading volume or the number of quotes, have been developed and explored empirically by Tauchen and Pitts (1983) and Andersen (1996) among many others.

MEM (Multiplicative Error Model) The Multiplicative Error class of Models (MEM) was proposed by Engle (2002b) as a general framework for modeling non-negative valued time series. The MEM may be expressed as,

$$x_t = \mu_t\eta_t,$$

where $x_t \geq 0$ denotes the time series of interest, μ_t refers to its conditional mean, and η_t is a non-negative *i.i.d.* process with unit mean. The conditional mean is natural parameterized as,

$$\mu_t = \omega + \sum_{i=1}^{q} \alpha_i x_{t-i} + \sum_{i=1}^{p} \beta_i \mu_{t-i},$$

where conditions on the parameters for μ_t to be positive follows from the corresponding conditions for the GARCH(p,q) model (see GARCH). Defining $x_t \equiv \varepsilon_t^2$ and $\mu_t \equiv \sigma_t^2$, the MEM class of models encompasses all ARCH and GARCH models, and specific formulations are readily estimated by the corresponding software for GARCH models. The ACD model for durations may also be interpreted as a MEM (see ACD).

MGARCH[1] (Multivariate GARCH) Multivariate GARCH models were first analyzed and estimated empirically by Bollerslev, Engle and Wooldridge (1988). The unrestricted linear MGARCH(p,q) model is defined by:

$$vech(\Omega_t) = \Omega + \sum_{i=1}^{q} A_i vech(\varepsilon_{t-i}\varepsilon_{t-i}^{\prime}) + \sum_{i=1}^{p} B_i vech(\Omega_{t-i}),$$

where $vech(\cdot)$ denotes the operator that stacks the lower triangular portion of a symmetric $N\times N$ matrix into an $N(N+1)/2\times 1$ vector of the corresponding unique elements, and the A_i and B_i matrices are all of compatible dimension $N(N+1)/2\times N(N+1)/2$. This vectorized representation is also sometime referred to as a VECH GARCH model. The general vech representation does not guarantee that the resulting conditional covariance matrices Ω_t are positive definite. Also, the model involves a total of $N(N+1)/2 + (p+q)(N^4 + 2N^3 + N^2)/4$ parameters, which becomes prohibitively expensive from a practical computational point of view for anything but the bivariate case, or $N=2$. Much of the research on multivariate GARCH models has been concerned with the development of alternative more parsimonious, yet empirically realistic, representations, that easily ensure the conditional covariance matrices are positive definite. The trivariate vech MGARCH(1,1) model estimated in Bollerslev, Engle and Wooldridge (1988) assumes that the A_1 and B_1 matrices are both diagonal, so that each element in Ω_t depends exclusively on its own lagged value and the product of the corresponding shocks. This diagonal simplification, resulting in "only" $(1+p+q)(N^2 + N)/2$ parameters to be estimated, is often denoted as a diag MGARCH model (see also diag MGARCH).

MGARCH[2] (Multiplicative GARCH) Slightly different versions of the univariate Multiplicative GARCH model was proposed independently by Geweke (1986), Pantula (1986) and Milhøj (1987). The model is more commonly refereed to as the log-GARCH model (see log-GARCH).

MGARCH[3] (Mixture GARCH) The MAR-ARCH model of Wong and Li (2001) and the MGARCH model Zhang, Li and Yuen (2006) postulates that the time t conditional variance is given by a time-invariant mixture of different GARCH models (see also GRS-GARCH, NM-GARCH and SWARCH).

MS-GARCH (Markov Switching GARCH) See SWARCH.

MV-GARCH (MultiVariate GARCH) The MV-GARCH, MGARCH and VGARCH acronyms are used interchangeably (see MGARCH[1]).

NAGARCH (Nonlinear Asymmetric GARCH) The NAGARCH(1,1) model of Engle and Ng (1993) is defined by:

$$\sigma_t^2 = \omega + \alpha(\varepsilon_{t-1}\sigma_{t-1}^{-1} + \gamma)^2 + \beta\sigma_{t-1}^2.$$

Higher order NAGARCH(p,q) models may be defined similarly (see also AGARCH[1] and VGARCH[1]).

NGARCH (Nonlinear GARCH) The NGARCH(p,q) model proposed by Higgins and Bera (1992) parameterizes the conditional standard deviation raised to the power δ as a function of the lagged conditional standard deviations and the lagged absolute innovations raised to the same power,

$$\sigma_t^{\delta} = \omega + \sum_{i=1}^{q} \alpha_i |\varepsilon_{t-i}|^{\delta} + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^{\delta}.$$

This formulation obviously reduces to the standard GARCH(p,q) model for $\delta=2$ (see GARCH). The NGARCH model is also sometimes referred to as a Power ARCH or Power GARCH model, or PARCH or PGARCH model. A slightly different version of the NGARCH model was originally estimated by Engle and Bollerslev (1986),

$$\sigma_t^2 = \omega + \alpha |\varepsilon_{t-1}|^{\delta} + \beta \sigma_{t-1}^2.$$

With most financial rates of returns, the estimates for δ are found to be less than two, although not always significantly so (see also APARCH and TS-GARCH).

NL-GARCH (Non-Linear GARCH) The NL-GARCH acronym is sometimes used to describe all parameterizations different from the benchmark linear GARCH(p,q) representation (see GARCH).

NM-GARCH (Normal Mixture GARCH) The NM-GARCH model postulates that the distribution of the standardized innovations $\varepsilon_t \sigma_t^{-1}$ is determined by a mixture of two or more normal distributions. The statistical properties of the NM-GARCH(1,1) model have been studied extensively by Alexander and Lazar (2006) (see also GARCH-t, GED-GARCH and SWARCH).

OGARCH (Orthogonal GARCH) The multivariate OGARCH model assumes that the $N\times 1$ vector process $\boldsymbol{\varepsilon}_t$ may be represented as $\boldsymbol{\varepsilon}_t = \Gamma f_t$, where the columns of the $N\times m$ matrix Γ are mutually orthogonal, and the m elements in the $m\times 1$ f_t vector process are conditionally uncorrelated with GARCH conditional variances. Consequently, the conditional covariance matrix for $\boldsymbol{\varepsilon}_t$ may be expressed as:

$$\Omega_t = \Gamma D_t \Gamma',$$

where D_t denotes the $m\times m$ diagonal matrix with the conditional factor variances along the diagonal. Estimation and inference in the OGARCH model are discussed in detail in Alexander (2001, 2008). The OGARCH model is also sometimes referred to as a principal component

MGARCH model. The approach is related to but formally different from the PC-GARCH model of Burns (2005) (see also F-ARCH, GO-GARCH, MGARCH[1] and PC-GARCH).

PARCH (Power ARCH) See NGARCH.

PC-GARCH (Principal Component GARCH) The multivariate PC-GARCH model of Burns (2005) is based on the estimation of univariate GARCH models to the principal components defined by the covariance matrix for the standardized residuals from a first stage estimation of univariate GARCH models for each of the individual series (see also OGARCH).

PGARCH[1] (Periodic GARCH) The PGARCH model of Bollerslev and Ghysels (1996) was designed to account for periodic dependencies in the conditional variance by allowing the parameters of the model to vary over the cycle. In particular, the PGARCH(1,1) model is defined by:

$$\sigma_t^2 = \omega_{s(t)} + \alpha_{s(t)}\varepsilon_{t-1}^2 + \beta_{s(t)}\sigma_{t-1}^2,$$

where $s(t)$ refers to the stage of the periodic cycle at time t, and $\omega_{s(t)}, \alpha_{s(t)}$ and $\beta_{s(t)}$ denote the different GARCH(1,1) parameter values for $s(t) = 1, 2, \ldots, P$.

PGARCH[2] (Power GARCH) See NGARCH.

PNP-ARCH (Partially Non-Parametric ARCH) The PNP-ARCH model estimated by Engle and Ng (1993) allows the conditional variance to be a partially linear function of the lagged innovations and the lagged conditional variance,

$$\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \sum_{i=-m}^{m} \theta_i(\varepsilon_{t-1} - i\cdot\sigma)I(\varepsilon_{t-1} < i\cdot\sigma),$$

where σ denotes the unconditional standard deviation of the process, and m is an integer. The PNP-ARCH model was used by Engle and Ng (1993) in the construction of so-called news impact curves, reflecting how the conditional variance responds to different sized shocks (see also GJR and TGARCH).

QARCH (Quadratic ARCH) See GQARCH.

QMLE (Quasi Maximum Likelihood Estimation) ARCH models are typically estimated under the assumption of conditional normality (see ARCH). Even if the assumption of conditional normality is violated (see also GARCH-t, GED-GARCH and SPARCH), the parameter estimates generally remain consistent and asymptotically normally distributed, as long as the first two conditional moments of the model are correctly specified; i.e, $E_{t-1}(\varepsilon_t) = 0$ and $E_{t-1}(\varepsilon_t^2) = \sigma_t^2$. A robust covariance matrix for the resulting Quasi MLE parameter estimates may be obtained by post- and pre-multiplying the matrix of outer products of the gradients with an estimate of Fisher's Information matrix. A relatively simple-to-compute expression for this matrix involving only first derivatives was derived in Bollerslev and Wooldridge (1992). The corresponding robust standard errors is sometimes referred to in the ARCH literature as Bollerslev-Wooldridge standard errors.

QTARCH (Qualitative Threshold ARCH) The QTARCH(q) model of Gourieroux and Monfort (1992) assumes that the conditional variance may be represented by a sum of step functions:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \sum_{j=1}^{J} \alpha_{ij} I_j(\varepsilon_{t-i}),$$

where the $I_j(\cdot)$ functions partitions the real line into J sub-intervals, so that $I_j(\varepsilon_{t-i})$ equals unity if ε_{t-i} falls in the j[th] sub-interval and zero otherwise. The Generalized QTARCH, or GQTARCH(p,q), model is readily defined by including p lagged conditional variances on the right-hand-side of the equation.

REGARCH (Range EGARCH) The REGARCH model of Brandt and Jones (2006) postulates an EGARCH type formulation for the conditional mean of the demeaned standardized logarithmic range. The FIREGARCH model allows for long-memory dependencies (see EGARCH and FIEGARCH).

RGARCH[1] (Randomized GARCH) The R-GARCH(r,p,q) model of Nowicka-Zagrajek and Weron (2001) replaces the intercept in the standard GARCH(p,q) model with a sum of r positive *i.i.d.* stable random variables, $\eta_{t-i}, i = 1, 2, \ldots, r$,

$$\sigma_t^2 = \sum_{i=1}^{r} c_i \eta_{t-i} + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2,$$

where $c_i \geq 0$.

RGARCH[2] (Robust GARCH) The robust GARCH model of Park (2002) is designed to minimize the impact of outliers by parameterizing the conditional variance as a TS-GARCH model (see TS-GARCH) with the parameters estimated by least absolute deviations, or LAD.

RGARCH[3] (Root GARCH) The multivariate RGARCH model (see also MGARCH[1] and Stdev-ARCH) of Gallant and Tauchen (1998) is formulated in terms of the lower triangular $N \times N$ matrix R_t, where by definition,

$$\Omega_t = R_t R_t^{\prime}.$$

By parameterizing R_t instead of Ω_t, the RGARCH formulation automatically guarantees that the resulting conditional covariance matrices are positive definite. However, the formulation complicates the inclusion of asymmetries or "leverage effects" in the conditional covariance matrix.

RS-GARCH (Regime Switching GARCH) See SWARCH.

RV (Realized Volatility) The term realized volatility, or realized variation, is commonly used in the ARCH literature to denote ex-post variation measures defined by the summation of within period squared or absolute returns over some non-trivial time interval. A rapidly growing recent literature has been concerned with the use of such measures and the development of new and refined procedures in light of various data complications. Many new empirical insights afforded by the use of daily realized volatility measures constructed from high-frequency intraday returns have also recently been reported in the literature; see, e.g., the review in Andersen, Bollerslev and Diebold (2009).

SARV (Stochastic AutoRegressive Volatility) See SV.

SGARCH (Stable GARCH) Let $\varepsilon_t \equiv z_t c_t$, where z_t is independent and identically distributed over time as a standard Stable Paretian distribution. The Stable GARCH model for ε_t of Liu and Brorsen (1995) is then defined by:

$$c_t^{\lambda} = \omega + \alpha |\varepsilon_{t-1}|^{\lambda} + \beta c_{t-1}^{\lambda}.$$

The SGARCH model nests the ACH model (see ACH^2) of McCulluch (1985) as a special case for $\lambda = 1$, $\omega = 0$ and $\beta = 1 - \alpha$ (see also GARCH-t, GED-GARCH and NGARCH).

S-GARCH (Simplified GARCH) The simplified multivariate GARCH (see MGARCH^1) approach of Harris, Stoja and Tucker (2007) infers the conditional covariances through the estimation of auxiliary univariate GARCH models for the linear combinations in the identity,

$$Cov_{t-1}(\varepsilon_{it}, \varepsilon_{jt}) = (1/4)\cdot[Var_{t-1}(\varepsilon_{it} + \varepsilon_{jt}) + Var_{t-1}(\varepsilon_{it} - \varepsilon_{jt})].$$

Nothing guarantees that the resulting $N\times N$ conditional covariance matrix is positive definite (see also CCC and Flex-GARCH).

Sign-GARCH See GJR.

SPARCH (SemiParametric ARCH) To allow for non-normal standardized residuals, as commonly found in the estimation of ARCH models (see also GARCH-t, GED-GARCH and QMLE), Engle and Gonzalez-Rivera (1991) suggest estimating the distribution of $\hat{\varepsilon}_t\hat{\sigma}_t^{-1}$ through non-parametric density estimation techniques. Although Engle and Gonzalez-Rivera (1991) do not explicitly use the name SPARCH, the approach has subsequently been referred to as such by several other authors in the literature.

Spline-GARCH The Spline-GARCH model of Engle and Rangel (2008) specifies the conditional variance of ε_t as the product of a standardized unit GARCH(1,1) model,

$$\sigma_t^2 = (1-\alpha-\beta)\omega + \alpha(\varepsilon_{t-1}^2/\tau_t) + \beta\sigma_{t-1}^2,$$

and a deterministic component represented by an exponential spline function of time,

$$\tau_t = c\cdot\exp[\omega_0 t + \omega_1((t-t_0)_+)^2 + \omega_2((t-t_1)_+)^2 + \ldots + \omega_k((t-t_{k-1})_+)^2],$$

where $(t-t_i)_+$ is equal to $(t-t_i)$ for $t>t_i$ and 0 otherwise, and $0 = t_0 < t_1 < \ldots < t_k = T$ defines a partition of the full sample into k equally spaced time intervals. Other exogenous explanatory variables may also be include in the equation for τ_t. The Spline GARCH model was explicitly designed to investigate macroeconomic causes of slowly moving, or low frequency volatility components (see also CGARCH[1]).

SQR-GARCH (Square-Root GARCH) The discrete-time SQR-GARCH model of Heston and Nandi (2000),

$$\sigma_t^2 = \omega + \alpha(\varepsilon_{t-1}\sigma_{t-1}^{-1} - \gamma\sigma_{t-1})^2 + \beta\sigma_{t-1}^2,$$

is closely related to the VGARCH model of Engle and Ng (1993) (see VGARCH[1]). In contrast to the standard GARCH(1,1) model, the SQR-GARCH formulation allows for closed form option pricing under reasonable auxiliary assumptions. When defined over increasingly finer sampling intervals, the SQR-GARCH model converges weakly to the continuous-time affine, or square-root, diffusion analyzed by Heston (1993),

$$d\sigma^2(t) = \kappa(\theta - \sigma^2(t))dt + \nu\sigma(t)dW(t).$$

The SQR-GARCH model is also sometime referred to as the Heston GARCH or the Heston-Nandi GARCH model (see also GARCH diffusion).

STARCH (Structural ARCH) An unobserved component, or "structural," time series model in which one or more of the disturbances follow an ARCH model was dubbed a STARCH model by

Harvey, Ruiz and Sentana (1992).

Stdev-ARCH (Standard deviation ARCH) The Stdev-ARCH(q) model first estimated by Schwert (1990) takes the form,

$$\sigma_t^2 = (\omega + \sum_{i=1}^{q} \alpha_i |\varepsilon_{t-i}|)^2 .$$

This formulation obviously ensures that the conditional variance is positive. However, the non-linearity complicates the construction of forecasts from the model (see also AARCH).

STGARCH (Smooth Transition GARCH) The ST-GARCH(1,1) model of Gonzalez-Rivera (1998) allows the impact of the past squared innovations to depend upon both the sign and the magnitude of ε_{t-1} through a smooth transition function,

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \delta \varepsilon_{t-i}^2 F(\varepsilon_{t-1}, \gamma) + \beta_i \sigma_{t-1}^2 ,$$

where

$$F(\varepsilon_{t-1}, \gamma) = (1 + \exp(\gamma \varepsilon_{t-1}))^{-1} ,$$

so that the value of the function is bounded between 0 and 1 (see also ANST-GARCH, GJR and TGARCH).

Structural GARCH The Structural GARCH approach named by Rigobon (2002) relies on a multivariate GARCH model for the innovations in an otherwise unidentified structural VAR to identify the parameters through time-varying conditional heteroskedasticity. Closely related ideas and models have been explored by Sentana and Fiorentine (2001) among others.

Strong GARCH GARCH models in which the standardized innovations, $z_t = \varepsilon_t \sigma_t^{-1}$, are assumed to be *i.i.d.* through time are referred to as a strong GARCH models (see also Weak GARCH).

SV (Stochastic Volatility) The term stochastic volatility, or SV model, refers to formulations in which σ_t^2 is specified as a non-measurable, or stochastic, function of the observable information set. To facilitate estimation and inference via linear state-space representations, discrete-time SV models are often formulated in terms of time series models for $\log(\sigma_t^2)$, as exemplified by the simple SARV(1) model,

$$\log(\sigma_t^2) = \mu + \varphi \log(\sigma_{t-1}^2) + \sigma_u u_t ,$$

where u_t is *i.i.d.* with mean zero and variance one. Meanwhile, the SV approach has proven especially useful in the formulation of empirically realistic continuous-time volatility models of the form,

$$dy(t) = \mu(t)dt + \sigma(t)dW(t),$$

where $\mu(t)$ denotes the drift, $W(t)$ refers to a standard Brownian Motion, and the diffusive volatility coefficient $\sigma(t)$ is determined by a separate stochastic process (see also GARCH Diffusion).

SVJ (Stochastic Volatility Jump) The SVJ acronym is commonly used to describe continues-time stochastic volatility models in which the sample paths may be dis-continuous, or exhibit jumps (see also SV and GARJI).

SWARCH (regime SWitching ARCH) The SWARCH model proposed independently by Cai (1994) and Hamilton and Susmel (1994) extends the standard linear ARCH(q) model (see ARCH) by allowing the intercept, $\omega_{s(t)}$, and/or the magnitude of the squared innovations, $\varepsilon_{t-i}^2/s(t-i)$, entering the conditional variance equation to depend upon some latent state variable, $s(t)$, with the transition between the different states governed by a Markov chain. Regime switching GARCH models were first developed by Gray (1996) (see GRS-GARCH). Different variants of these models are also sometimes refereed to in the literature as Markov Switching GARCH, or MS-GARCH, Regime Switching GARCH, or RS-GARCH, or Mixture GARCH, or MGARCH, models.

TGARCH (Threshold GARCH) The TGARCH(p,q) model proposed by Zakoian (1994) extends the TS-GARCH(p,q) model (see TS-GARCH) to allow the conditional standard deviation to depend upon the sign of the lagged innovations. In particular, the TGARCH(1,1) model may be expressed as:

$$\sigma_t = \omega + \alpha|\varepsilon_{t-1}| + \gamma|\varepsilon_{t-1}|I(\varepsilon_{t-1}<0) + \beta\sigma_{t-1}.$$

The TGARCH model is also sometimes referred to as the ZARCH, or ZGARCH, model. The basic idea behind the model is closely related to that of the GJR-GARCH model developed independently by Glosten, Jagannathan and Runkle (1993) (see GJR).

t-GARCH (t-distributed GARCH) See GARCH-t.

Tobit-GARCH The Tobit-GARCH model, first proposed by Kodres (1993) for analyzing futures prices, extends the standard GARCH model (see GARCH) to allow for the possibility of censored observations on the ε_t's , or the underlying y_t's. More general formulations allowing for multi-period censoring and related inference procedures been developed by Lee (1999), Morgan and Trevor (1999) and Wei (2002).

TS-GARCH (Taylor-Schwert GARCH) The TS-GARCH(p,q) model of Taylor (1986) and Schwert (1989) parameterizes the conditional standard deviation as a distributed lag of the absolute innovations and the lagged conditional standard deviations,

$$\sigma_t = \omega + \sum_{i=1}^{q} \alpha_i |\varepsilon_{t-i}| + \sum_{i=1}^{p} \beta_i \sigma_{t-i}.$$

This formulation mitigates the influence of large, in an absolute sense, observations relative to the traditional GARCH(p,q) model (see GARCH). The TS-GARCH model is also sometimes referred to as an Absolute Value GARCH, or AVGARCH, model, or simply an AGARCH model. It is a special case of the more general Power GARCH, or NGARCH, formulation (see NGARCH).

TVP-Level (Time-Varying Parameter Level) See Level-GARCH.

UGARCH (Univariate GARCH) See GARCH.

Unobserved GARCH See Latent GARCH.

Variance Targeting The use of variance targeting in GARCH models was first suggested by Engle and Mezrich (1996). To illustrate, consider the GARCH(1,1) model (see GARCH),

$$\sigma_t^2 = (1 - \alpha - \beta)\bar{\sigma}^2 + \alpha\varepsilon_{t-1} + \beta\sigma_{t-1}^2,$$

where $\bar{\sigma}^2 = \omega(1 - \alpha - \beta)^{-1}$. Fixing $\bar{\sigma}^2$ at some pre-set value ensures that the long-run variance forecasts from the model converges to $\bar{\sigma}^2$. Variance targeting has proven especially useful in multivariate GARCH modeling (see MGARCH[1]).

VCC (Varying Conditional Correlations) See DCC.

vech GARCH (vectorized GARCH) See MGARCH[1].

VGARCH[1] Following Engle and Ng (1993), the VGARCH(1,1) model refers to the parameterization,

$$\sigma_t^2 = \omega + \alpha(\varepsilon_{t-1}\sigma_{t-1}^{-1} + \gamma)^2 + \beta\sigma_{t-1}^2,$$

in which the impact of the innovations for the conditional variance is symmetric and centered at $-\gamma\sigma_{t-1}$. Higher order VGARCH(p,q) models may be defined in a similar manner (see also AGARCH[1] and NAGARCH).

VGARCH[2] (Vector GARCH) The VGARCH, MGARCH and MV-GARCH acronyms are used interchangeably (see MGARCH[1]).

VSGARCH (Volatility Switching GARCH) The VSGARCH(1,1) model of Fornari and Mele (1996) directly mirrors the GJR model (see GJR),

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \gamma(\varepsilon_{t-1}^2/\sigma_{t-1}^2)I(\varepsilon_{t-1}<0) + \beta\sigma_{t-1}^2,$$

except that the asymmetric impact of the lagged squared negative innovations is scaled by the corresponding lagged conditional variance.

Weak GARCH The weak GARCH class of models, or concept, was first developed by Drost and Nijman (1993). In the weak GARCH class of models σ_t^2 is defined as the linear projection of ε_t^2 on the space spanned by $\{1, \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots, \varepsilon_{t-1}^2, \varepsilon_{t-2}^2, \ldots\}$ as opposed to the conditional expectation of ε_t^2, or $E_{t-1}(\varepsilon_t^2)$ (see also ARCH and GARCH). In contrast to the standard GARCH(p,q) class of models, which is not formally closed under temporal aggregation, the sum of successive observations from a weak GARCH(p,q) model remains a weak GARCH(p',q') model, albeit with different orders p' and q'. Similarly, as shown by Nijman and Sentana (1996) the unrestricted multivariate linear weak MGARCH(p,q) model (see MGARCH[1]) defined in terms of linear projections as opposed to conditional expectations is closed under contemporaneous aggregation, or portfolio formation (see also Strong GARCH).

ZARCH (Zakoian ARCH) See TGARCH.

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