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Economics 153

Midterm Exam

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Point values are indicated for each question. The exam consists of a total of 60 points, and you have 60 minutes.

1. (20 points) Consider the Cagan model of money demand

$$\log M_t - \log P_t = -\eta E_t[\log P_{t+1} - \log P_t], \quad \eta > 0.$$

The attached figure (taken from Cagan's original (1956) paper) plots with a solid line actual observed values of real balances ($\log M_t - \log P_t$) during the German hyperinflation of 1923 and with a broken line values of real balances predicted by the Cagan model, that is, $-\eta E_t[\log P_{t+1} - \log P_t]$. To produce the broken line Cagan used past observations of inflation to construct his measure of expected inflation $E_t[\log P_{t+1} - \log P_t]$. The figure shows that for most of the German hyperinflation the level of real balances predicted by Cagan tracks actually observed values of real balances closely. However, towards the end of the hyperinflation Cagan's model breaks down because it predicts further declines in real balances at a time when the data shows that real balances actually started increasing. Provide an explanation for why the model and data diverge at that point.

2. (40 points) Consider an Overlapping Generations model of money demand. Assume that each generation lives for two periods and is endowed with y units of consumption when young. Agents have preferences over consumption when young, $C_{1,t}$ and consumption when old $C_{2,t+1}$. Agents can acquire money balances when young to purchase consumption in the second period of life, when their endowment is assumed to be nil. Assume that the population is constant over time, that the government pegs the growth rate of the money supply at the rate $\mu \geq 0$ such that

$$M_t = (1 + \mu)M_{t-1}$$

for all $t \geq 0$, and that the government rebates any seignorage income to the old generation in each period. Specifically, the budget constraint of the government is given by:

$$M_t = M_{t-1} + P_t a_t$$

where a_t denotes real transfers to the old in period t.

(a) Limit attention to an equilibrium in which real balances is constant over time, that is, an equilibrium with $m_t = m_{t+1} = m$ for all $t \geq 0$. What is the gross rate of inflation in such an equilibrium.

(b) Find the equilibrium level of transfers as a function of the growth rate of the money supply and of the level of real balances.

(c) State the budget constraint of a household born in period $t \geq 0$ for each period the household is alive, that is, for periods t and $t+1$. Find the equilibrium level of $C_{1,t}$ and $C_{2,t+1}$ as a function of the endowment y and real balances m.

(d) Find the equilibrium level of real balances, m, as a function of μ and y under the assumption that in equilibrium

$$C_1 = (1+\mu)^2 C_2.$$

(If you wonder where this relationship comes from: This relation can, for example, be derived from the restriction that in equilibrium the MRS substitution must equal the inverse of the rate of inflation and by assuming that the utility function of a generation born at time $t \geq 0$ is given by $U(C_{1,t}C_{2,t+1}) = \sqrt{C_{1,t}} + \sqrt{C_{2,t+1}}$. But for now just take this relationship as given.)

(e) Compare real balances in the following two economies: one in which the money supply is constant and one in which the money supply doubles each period. Give an intuitive interpretation of your result.

(f) Now assume that the government does not transfer the seignorage income to the currently old but rather to the young. First, discuss intuitively how this change in policy alters the agents incentives to save in the first period of life and hence how this change should affect the demand for real balances by the young. Then answer questions (c)-(e) limiting again attention to an equilibrium with constant real balances. In which case (seignorage rebated to the old or seignorage rebated to the young) will the increase in money growth lead to a larger decline in real balances and why.