A Appendix: Omitted Details of Some Derivations

A.1 Derivation of Derivatives in (43):

For simplicity of notation, define:

$$
\begin{aligned}
A_1(z;\phi_G) &\equiv t\left[\int_{\frac{t}{\phi_G}}^{\overline{\theta_G}}\int_{\underline{\theta_P}}^{\tau-t} f(\theta_G,\theta_P)\,d\theta_P d\theta_G\right],\\
B_1(z;\phi_G) &\equiv (p-c)\left[\int_{p}^{\overline{\theta_P}}\int_{\underline{\theta_G}}^{\frac{\tau-p}{\phi_G^*}} f(\theta_G,\theta_P)\,d\theta_G d\theta_P\right],\\
C_1(z;\phi_G) &\equiv (\tau-c)\left[\int_{\frac{\tau-p}{\phi_G}}^{\frac{t}{\phi_G}}\int_{\tau-\phi_G\theta_G}^{\overline{\theta_P}} f(\theta_G,\theta_P)\,d\theta_P d\theta_G+\int_{\frac{t}{\phi_G}}^{\overline{\theta_G}}\int_{\tau-t}^{\overline{\theta_P}} f(\theta_G,\theta_P)\,d\theta_P d\theta_G\right],
\end{aligned}
$$

so that

$$
G_1(z;\phi_G)=A_1(z;\phi_G)+B_1(z;\phi_G)+C_1(z;\phi_G)-K\phi_G.
$$

Differentiating with respect to t, we have:

$$
\begin{aligned}
\frac{\partial A_1(z;\phi_G)}{\partial t} &= \int_{\frac{t}{\phi_G}}^{\overline{\theta_G}}\int_{\underline{\theta_P}}^{\tau-t} f(\theta_G,\theta_P)\,d\theta_P d\theta_G - t\left[\int_{\underline{\theta_P}}^{\tau-t} f\left(\frac{t}{\phi_G},\theta_P\right)\frac{1}{\phi_G}d\theta_P+\int_{\frac{t}{\phi_G}}^{\overline{\theta_G}} f(\theta_G,\tau-t)\,d\theta_G\right],\\
\frac{\partial B_1(z;\phi_G)}{\partial t} &= 0,\\
\frac{\partial C_1(z;\phi_G)}{\partial t} &= (\tau-c)\left[\int_{\tau-t}^{\overline{\theta_P}} f\left(\frac{t}{\phi_G},\theta_P\right)\frac{1}{\phi_G}d\theta_P-\int_{\tau-t}^{\overline{\theta_P}} f\left(\frac{t}{\phi_G},\theta_P\right)\frac{1}{\phi_G}d\theta_P+\int_{\frac{t}{\phi_G}}^{\overline{\theta_G}} f(\theta_G,\tau-t)\,d\theta_G\right]\\
&= (\tau-c)\left[\int_{\frac{t}{\phi_G}}^{\overline{\theta_G}} f(\theta_G,\tau-t)\,d\theta_G\right].
\end{aligned}
$$

Thus,

$$
\begin{aligned}
\frac{\partial A_1(z^*;\phi_G^*)}{\partial t} &= \int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}}\int_{\underline{\theta_P}}^{p^*} f(\theta_G,\theta_P)\,d\theta_P d\theta_G - t^*\left[\int_{\underline{\theta_P}}^{p^*} f\left(\frac{t^*}{\phi_G^*},\theta_P\right)\frac{1}{\phi_G^*}d\theta_P+\int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}} f(\theta_G,p^*)\,d\theta_G\right],\\
\frac{\partial B_1(z^*;\phi_G^*)}{\partial t} &= 0,\\
\frac{\partial C_1(z^*;\phi_G^*)}{\partial t} &= (t^*+p^*-c)\left[\int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}} f(\theta_G,p^*)\,d\theta_G\right].
\end{aligned}
$$

So,

$$\frac{\partial G_1(z^*)}{\partial t} = \int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}} \left\{ \int_{\underline{\theta_P}}^{p^*} f(\theta_G, \theta_P)\, d\theta_P + (p^* - c) f(\theta_G, p^*) \right\} d\theta_G - t^* \int_{\underline{\theta_P}}^{p^*} f\left(\frac{t^*}{\phi_G^*}, \theta_P\right) \frac{1}{\phi_G^*} d\theta_P \tag{A1}$$

$$= \int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}} \left\{ \int_{\underline{\theta_P}}^{p^*} \underbrace{\frac{f(\theta_G, \theta_P)}{\int_{\underline{\theta_P}}^{\overline{\theta_P}} f(\theta_G, \theta_P)\, d\theta_P}}_{f_P(\theta_P|\theta_G)} d\theta_P + (p^* - c) \underbrace{\frac{f(\theta_G, p^*)}{\int_{\underline{\theta_P}}^{\overline{\theta_P}} f(\theta_G, \theta_P)\, d\theta_P}}_{f_P(p^*|\theta_G)} \right\} \underbrace{\left(\int_{\underline{\theta_P}}^{\overline{\theta_P}} f(\theta_G, \theta_P)\, d\theta_P \right)}_{f_G(\theta_G)} d\theta_G d\theta_G$$

$$-t^* \int_{\underline{\theta_P}}^{p^*} \underbrace{\frac{f\left(\frac{t^*}{\phi_G^*}, \theta_P\right)}{\int_{\underline{\theta_P}}^{\overline{\theta_P}} f\left(\frac{t^*}{\phi_G^*}, \theta_P\right) d\theta_P}}_{f_P\left(\theta_P | \frac{t^*}{\phi_G^*}\right)} \frac{1}{\phi_G^*} d\theta_P \underbrace{\int_{\underline{\theta_P}}^{\overline{\theta_P}} f\left(\frac{t^*}{\phi_G^*}, \theta_P\right) d\theta_P}_{f_G\left(\frac{t^*}{\phi_G^*}\right)}$$

$$= \int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}} \left\{ \int_{\underline{\theta_P}}^{p^*} f_P(\theta_P|\theta_G)\, d\theta_P + (p^* - c) f_P(p^*|\theta_G) \right\} f_G(\theta_G)\, d\theta_G - \frac{t^*}{\phi_G^*} \int_{\underline{\theta_P}}^{p^*} f_P\left(\theta_P | \frac{t^*}{\phi_G^*}\right) d\theta_P f_G\left(\frac{t^*}{\phi_G^*}\right)$$

$$= \int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}} \{F_P(p^*|\theta_G) + (p^* - c) f_P(p^*|\theta_G)\} f_G(\theta_G)\, d\theta_G - \frac{t^*}{\phi_G^*} F_P\left(p^* | \frac{t^*}{\phi_G^*}\right) f_G\left(\frac{t^*}{\phi_G^*}\right).$$

Differentiating with respect to τ yields:

$$\frac{dA_1(z;\phi_G)}{d\tau} = t\left[\int_{\frac{t}{\phi_G}}^{\overline{\theta_G}} f(\theta_G, \tau - t)\, d\theta_G\right]$$

$$\frac{\partial B_1(z;\phi_G)}{\partial \tau} = (p - c)\left[\int_p^{\overline{\theta_P}} f\left(\frac{\tau - p}{\phi_G^*}, \theta_P\right) \frac{1}{\phi_G^*} d\theta_P\right]$$

$$\frac{\partial C_1(z;\phi_G)}{\partial \tau} = \int_{\frac{\tau-p}{\phi_G}}^{\frac{t}{\phi_G}} \int_{\tau - \phi_G\theta_G}^{\overline{\theta_P}} f(\theta_G, \theta_P)\, d\theta_P d\theta_G + \int_{\frac{t}{\phi_G}}^{\overline{\theta_G}} \int_{\tau - t}^{\overline{\theta_P}} f(\theta_G, \theta_P)\, d\theta_P d\theta_G$$

$$-(\tau - c)\left[\int_p^{\overline{\theta_P}} f\left(\frac{\tau - p}{\phi_G}, \theta_P\right) \frac{1}{\phi_G} d\theta_P + \int_{\frac{\tau-p}{\phi_G}}^{\frac{t}{\phi_G}} f(\theta_G, \tau - \phi_G\theta_G)\, d\theta_G\right]$$

$$-(\tau - c)\left[\int_{\frac{t}{\phi_G}}^{\overline{\theta_G}} f(\theta_G, \tau - t)\, d\theta_G\right]$$

Hence,

$$\frac{dA_1(z^*;\phi_G^*)}{d\tau} = t^*\left[\int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}} f(\theta_G, \tau - t^*)\, d\theta_G\right]$$

$$\frac{\partial B_1(z^*;\phi_G^*)}{\partial \tau} = (p^* - c)\left[\int_{p^*}^{\overline{\theta_P}} f\left(\frac{t^*}{\phi_G^*}, \theta_P\right) \frac{1}{\phi_G^*} d\theta_P\right]$$

$$\frac{\partial C_1(z^*;\phi_G^*)}{\partial \tau} = \int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}} \int_{p^*}^{\overline{\theta_P}} f(\theta_G, \theta_P)\, d\theta_P d\theta_G - (p^* + t^* - c)\left\{\left[\int_{p^*}^{\overline{\theta_P}} f\left(\frac{t^*}{\phi_G^*}, \theta_P\right) \frac{1}{\phi_G^*} d\theta_P\right] + \left[\int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}} f(\theta_G, p^*)\, d\theta_G\right]\right\}.$$

It follows that

$$
\begin{aligned}
\frac{\partial G_1\left(z^*;\phi_G^*\right)}{\partial \tau} &= \int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}} \int_{p^*}^{\overline{\theta_P}} f\left(\theta_G,\theta_P\right) d\theta_P d\theta_G - t^* \left[\int_{p^*}^{\overline{\theta_P}} f\left(\frac{t^*}{\phi_G^*},\theta_P\right)\frac{1}{\phi_G^*} d\theta_P\right] - \left(p^*-c\right)\left[\int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}} f\left(\theta_G,p^*\right) d\theta_P d\theta_G\right] \\
&= \int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}} \left\{\int_{p^*}^{\overline{\theta_P}} f\left(\theta_G,\theta_P\right) d\theta_P - \left(p^*-c\right) f\left(\theta_G,p^*\right)\right\} d\theta_G - t^*\left[\int_{p^*}^{\overline{\theta_P}} f\left(\frac{t^*}{\phi_G^*},\theta_P\right)\frac{1}{\phi_G^*} d\theta_P\right] \\
\text{/same steps as in (A1)/} &= \int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}} \left\{\left[1-F_P\left(p^*|\theta_G\right)\right] - \left(p^*-c\right) f_P\left(p^*|\theta_G\right)\right\} f_G\left(\theta_G\right) d\theta_G - \frac{t^*}{\phi_G^*}\left[1-F_P\left(p^*|\frac{t^*}{\phi_G^*}\right)\right] f_G\left(\frac{t^*}{\phi_G^*}\right).
\end{aligned}
$$

Differentiating with respect to p yields:

$$
\begin{aligned}
\frac{\partial A_1\left(z;\phi_G\right)}{\partial p} &= 0, \\
\frac{\partial B_1\left(z;\phi_G\right)}{\partial p} &= \int_p^{\overline{\theta_P}} \int_{\underline{\theta}}^{\frac{\tau-p}{\phi_G}} f\left(\theta_G,\theta_P\right) d\theta_G d\theta_P - (p-c)\left[\int_{\underline{\theta}}^{\frac{\tau-p}{\phi_G}} f\left(\theta_G,p\right) d\theta_G + \int_p^{\overline{\theta_P}} f\left(\frac{\tau-p}{\phi_G},\theta_P\right)\frac{1}{\phi_G} d\theta_P\right], \\
\frac{\partial C_1\left(z;\phi_G\right)}{\partial p} &= (\tau-c)\left[\int_p^{\overline{\theta_P}} f\left(\frac{\tau-p}{\phi_G},\theta_P\right)\frac{1}{\phi_G} d\theta_P\right].
\end{aligned}
$$

Hence,

$$
\begin{aligned}
\frac{\partial A_1\left(z^*;\phi_G^*\right)}{\partial p} &= 0, \\
\frac{\partial B_1\left(z^*;\phi_G^*\right)}{\partial p} &= \int_{p^*}^{\overline{\theta_P}} \int_{\underline{\theta}}^{\frac{t^*}{\phi_G^*}} f\left(\theta_G,\theta_P\right) d\theta_G d\theta_P - \left(p^*-c\right)\left[\int_{\underline{\theta}}^{\frac{t^*}{\phi_G^*}} f\left(\theta_G,p^*\right) d\theta_G + \int_{p^*}^{\overline{\theta_P}} f\left(\frac{t^*}{\phi_G^*},\theta_P\right)\frac{1}{\phi_G^*} d\theta_P\right], \\
\frac{\partial C_1\left(z^*;\phi_G^*\right)}{\partial p} &= \left(t^*+p^*-c\right)\left[\int_{p^*}^{\overline{\theta_P}} f\left(\frac{t^*}{\phi_G^*},\theta_P\right)\frac{1}{\phi_G^*} d\theta_P\right],
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial G_1\left(z^*;\phi_G^*\right)}{\partial p} &= \int_{\underline{\theta}}^{\frac{t^*}{\phi_G^*}} \left[\int_{p^*}^{\overline{\theta_P}} f\left(\theta_G,\theta_P\right) d\theta_P - \left(p^*-c\right) f\left(\theta_G,p^*\right)\right] d\theta_G + t^*\left[\int_{p^*}^{\overline{\theta_P}} f\left(\frac{t^*}{\phi_G^*},\theta_P\right)\frac{1}{\phi_G^*} d\theta_P\right] \\
\text{/same steps as in (A1)/} &= \int_{\underline{\theta}}^{\frac{t^*}{\phi_G^*}} \left[\left\{\left(1-F_P\left(p^*|\theta_G\right)\right) - \left(p^*-c\right) f_P\left(p^*|\theta_G\right)\right\}\right] f_G\left(\theta_G\right) d\theta_G \\
&\quad + \frac{t^*}{\phi_G^*}\left(1-F_P\left(p^*|\frac{t^*}{\phi_G^*}\right)\right) f_G\left(\frac{t^*}{\phi_G^*}\right).
\end{aligned}
$$

A.2 Derivation of Derivatives in (44):

Let

$$
\begin{aligned}
D_1\left(z;\phi_G\right) &= \int_{\frac{t}{\phi_G}}^{\overline{\theta_G}} \int_{\underline{\theta_P}}^{\tau-t} \phi_G\theta_G f\left(\theta_G,\theta_P\right) d\theta_P d\theta_G, \\
E_1\left(z;\phi_G\right) &= \int_p^{\overline{\theta_P}} \int_{\underline{\theta}}^{\frac{\tau-p}{\phi_G}} \left(\theta_P-c\right) f\left(\theta_G,\theta_P\right) d\theta_G d\theta_P, \\
F_1\left(z;\phi_G\right) &= \int_{\frac{\tau-p}{\phi_G}}^{\frac{t}{\phi_G}} \int_{\tau-\phi_G\theta_G}^{\overline{\theta_P}} \left(\phi_G\theta_G+\theta_P-c\right) f\left(\theta_G,\theta_P\right) d\theta_P d\theta_G + \int_{\frac{t}{\phi_G}}^{\overline{\theta_G}} \int_{\tau-t}^{\overline{\theta_P}} \left(\phi_G\theta_G+\theta_P-c\right) f\left(\theta_G,\theta_P\right) d\theta_P d\theta_G.
\end{aligned}
$$

Hence,

$$\begin{aligned}
\frac{\partial D_1(z;\phi_G)}{\partial t} &= -\int_{\underline{\theta_P}}^{\tau-t} tf\left(\frac{t}{\phi_G},\theta_P\right)\frac{1}{\phi_G}d\theta_P - \int_{\frac{t}{\phi_G}}^{\overline{\theta_G}} \phi_G\theta_G f(\theta_G,\tau-t)\,d\theta_G \\
\frac{\partial D_1(z;\phi_G)}{\partial p} &= 0 \\
\frac{\partial D_1(z;\phi_G)}{\partial \tau} &= \int_{\frac{t}{\phi_G}}^{\overline{\theta_G}} \phi_G\theta_G f(\theta_G,\tau-t)\,d\theta_G
\end{aligned}$$

and thus,

$$\begin{aligned}
\frac{\partial D_1(z^*;\phi_G^*)}{\partial t} &= -\int_{\underline{\theta_P}}^{p^*} t^* f\left(\frac{t^*}{\phi_G^*},\theta_P\right)\frac{1}{\phi_G^*}d\theta_P - \int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}} \phi_G^*\theta_G f(\theta_G,p^*)\,d\theta_G \\
\frac{\partial D_1(z^*;\phi_G^*)}{\partial p} &= 0 \\
\frac{\partial D_1(z^*;\phi_G^*)}{\partial \tau} &= \int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}} \phi_G^*\theta_G f(\theta_G,p^*)\,d\theta_P d\theta_G.
\end{aligned} \tag{A2}$$

Similarly,

$$\begin{aligned}
\frac{\partial E_1(z;\phi_G)}{\partial t} &= 0, \\
\frac{\partial E_1(z;\phi_G)}{\partial p} &= -\int_{\underline{\theta}}^{\frac{\tau-p}{\phi_G}} (p-c)\,f(\theta_G,p)\,d\theta_G - \int_{p}^{\overline{\theta_P}} (\theta_P-c)\,f\left(\frac{\tau-p}{\phi_G},\theta_P\right)\frac{1}{\phi_G}d\theta_P, \\
\frac{\partial E_1(z;\phi_G)}{\partial \tau} &= \int_{p}^{\overline{\theta_P}} (\theta_P-c)\,f\left(\frac{\tau-p}{\phi_G},\theta_P\right)\frac{1}{\phi_G}d\theta_P,
\end{aligned}$$

thus,

$$\begin{aligned}
\frac{\partial E_1(z^*;\phi_G^*)}{\partial t} &= 0 \\
\frac{\partial E_1(z^*;\phi_G^*)}{\partial p} &= -\int_{\underline{\theta}}^{\frac{t^*}{\phi_G^*}} (p^*-c)\,f(\theta_G,p^*)\,d\theta_G - \int_{p^*}^{\overline{\theta_P}} (\theta_P-c)\,f\left(\frac{t^*}{\phi_G^*},\theta_P\right)\frac{1}{\phi_G^*}d\theta_P \\
\frac{\partial E_1(z^*;\phi_G^*)}{\partial \tau} &= \int_{p^*}^{\overline{\theta_P}} (\theta_P-c)\,f\left(\frac{t^*}{\phi_G^*},\theta_P\right)\frac{1}{\phi_G^*}d\theta_P.
\end{aligned} \tag{A3}$$

Finally,

$$\begin{aligned}
\frac{\partial F_1(z;\phi_G)}{\partial t} &= \int_{\tau-t}^{\overline{\theta_P}} (t+\theta_P-c)\, f\left(\frac{t}{\phi_G},\theta_P\right)\frac{1}{\phi_G}d\theta_P - \int_{\tau-t}^{\overline{\theta_P}} (t+\theta_P-c)\, f\left(\frac{t}{\phi_G},\theta_P\right)\frac{1}{\phi_G}d\theta_P \\
&\quad + \int_{\frac{t}{\phi_G}}^{\overline{\theta_G}} (\phi_G\theta_G+\tau-t-c)\, f(\theta_G,\tau-t)\, d\theta_G \\
&= \int_{\frac{t}{\phi_G}}^{\overline{\theta_G}} (\phi_G\theta_G+\tau-t-c)\, f(\theta_G,\tau-t) \\
\frac{\partial F_1(z;\phi_G)}{\partial p} &= \int_{p}^{\overline{\theta_P}} (\tau-p+\theta_P-c)\, f\left(\frac{\tau-p}{\phi_G},\theta_P\right)\frac{1}{\phi_G}d\theta_P \\
\frac{\partial F_1(z;\phi_G)}{\partial \tau} &= -\int_{p}^{\overline{\theta_P}} (\tau-p+\theta_P-c)\, f\left(\frac{\tau-p}{\phi_G},\theta_P\right)\frac{1}{\phi_G}d\theta_P - \int_{\frac{\tau-p}{\phi_G}}^{\frac{t}{\phi_G}} (\tau-c)\, f(\theta_G,\tau-\phi_G\theta_G)\, d\theta_G \\
&\quad - \int_{\frac{t}{\phi_G}}^{\overline{\theta_G}} (\phi_G\theta_G+\tau-t-c)\, f(\theta_G,\tau-t)\, d\theta_G
\end{aligned}$$

thus,

$$\begin{aligned}
\frac{\partial F_1(z^*;\phi_G^*)}{\partial t} &= \int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}} (\phi_G^*\theta_G+p^*-c)\, f(\theta_G,p^*)\, d\theta_G \qquad \text{(A4)}\\
\frac{\partial F_1(z^*;\phi_G^*)}{\partial p} &= \int_{p^*}^{\overline{\theta_P}} (t^*+\theta_P-c)\, f\left(\frac{t^*}{\phi_G^*},\theta_P\right)\frac{1}{\phi_G^*}d\theta_P \\
\frac{\partial F_1(z^*;\phi_G^*)}{\partial \tau} &= -\int_{p^*}^{\overline{\theta_P}} (t^*+\theta_P-c)\, f\left(\frac{t^*}{\phi_G^*},\theta_P\right)\frac{1}{\phi_G^*}d\theta_P - \int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}} (\phi_G^*\theta_G+p^*-c)\, f(\theta_G,p^*)\, d\theta_G.
\end{aligned}$$

Hence,

$$\begin{aligned}
\frac{\partial S_1(z^*;\phi_G^*)}{\partial t} &= \frac{\partial D_1(z^*;\phi_G^*)}{\partial t} + \frac{\partial E_1(z^*;\phi_G^*)}{\partial t} + \frac{\partial F_1(z^*;\phi_G^*)}{\partial t} \\
&= -\int_{\underline{\theta_P}}^{p^*} t^* f\left(\frac{t^*}{\phi_G^*},\theta_P\right)\frac{1}{\phi_G^*}d\theta_P - \int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}} \phi_G^*\theta_G f(\theta_G,p^*)\, d\theta_G + \int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}} (\phi_G^*\theta_G+p^*-c)\, f(\theta_G,p^*)\, d\theta_G \\
&= -\int_{\underline{\theta_P}}^{p^*} \frac{t^*}{\phi_G^*} f\left(\frac{t^*}{\phi_G^*},\theta_P\right) d\theta_P + \int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}} (p^*-c)\, f(\theta_G,p^*)\, d\theta_G \\
&= \int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}} (p^*-c)\, f_P(p^*|\theta_G)\, f_G(\theta_G)\, d\theta_G - \frac{t^*}{\phi_G^*} F_P\left(p^*|\frac{t^*}{\phi_G^*}\right) f_G\left(\frac{t^*}{\phi_G^*}\right)
\end{aligned}$$

and,

$$\begin{aligned}
\frac{\partial S_1\left(z^*;\phi_G^*\right)}{\partial p} &= \frac{\partial D_1\left(z^*;\phi_G^*\right)}{\partial p}+\frac{\partial E_1\left(z^*;\phi_G^*\right)}{\partial p}+\frac{\partial F_1\left(z^*;\phi_G^*\right)}{\partial p} \\
&= -\int_{\underline{\theta}}^{\frac{t^*}{\phi_G}}\left(p^*-c\right)f\left(\theta_G,p^*\right)d\theta_G-\int_{p^*}^{\overline{\theta_P}}\left(\theta_P-c\right)f\left(\frac{t^*}{\phi_G^*},\theta_P\right)\frac{1}{\phi_G^*}d\theta_P \\
&\quad +\int_{p^*}^{\overline{\theta_P}}\left(t^*+\theta_P-c\right)f\left(\frac{t^*}{\phi_G^*},\theta_P\right)\frac{1}{\phi_G^*}d\theta_P \\
&= -\int_{\underline{\theta}}^{\frac{t^*}{\phi_G}}\left(p^*-c\right)f\left(\theta_G,p^*\right)d\theta_G+\int_{p^*}^{\overline{\theta_P}}\frac{t^*}{\phi_G^*}f\left(\frac{t^*}{\phi_G^*},\theta_P\right)d\theta_P \\
&= -\int_{\underline{\theta}}^{\frac{t^*}{\phi_G}}\left(p^*-c\right)f_P\left(p^*|\theta_G\right)f_G\left(\theta_G\right)d\theta_G+\frac{t^*}{\phi_G^*}\left[1-F_P\left(p^*|\frac{t^*}{\phi_G^*}\right)\right]f_G\left(\frac{t^*}{\phi_G^*}\right),
\end{aligned}$$

and,

$$\begin{aligned}
\frac{\partial S_1\left(z^*;\phi_G^*\right)}{\partial \tau} &= \frac{\partial D_1\left(z^*;\phi_G^*\right)}{\partial \tau}+\frac{\partial E_1\left(z^*;\phi_G^*\right)}{\partial \tau}+\frac{\partial F_1\left(z^*;\phi_G^*\right)}{\partial \tau} \\
&= \int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}}\phi_G^*\theta_G f\left(\theta_G,p^*\right)d\theta_P d\theta_G+\int_{p^*}^{\overline{\theta_P}}\left(\theta_P-c\right)f\left(\frac{t^*}{\phi_G^*},\theta_P\right)\frac{1}{\phi_G^*}d\theta_P \\
&\quad -\int_{p^*}^{\overline{\theta_P}}\left(t^*+\theta_P-c\right)f\left(\frac{t^*}{\phi_G^*},\theta_P\right)\frac{1}{\phi_G^*}d\theta_P-\int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}}\left(\phi_G^*\theta_G+p^*-c\right)f\left(\theta_G,p^*\right)d\theta_G \\
&= -\int_{p^*}^{\overline{\theta_P}}\frac{t^*}{\phi_G^*}f\left(\frac{t^*}{\phi_G^*},\theta_P\right)d\theta_P-\int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}}\left(p^*-c\right)f\left(\theta_G,p^*\right)d\theta_G \\
&= -\int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}}\left(p^*-c\right)f_P\left(p^*|\theta_G\right)f_G\left(\theta_G\right)d\theta_G-\frac{t^*}{\phi_G^*}\left[1-F_P\left(p^*|\frac{t^*}{\phi_G^*}\right)\right]f_G\left(\frac{t^*}{\phi_G^*}\right).
\end{aligned}$$

A.3 Calculations in Proving Lemma 6

- **Part 1:** $DG_1\left(z^*;\phi_G^*\right)=DG_2\left(z^*;\phi_G^*\right)$:

Write $G_2\left(z;\phi_G\right)=A_2\left(z;\phi_G\right)+B_2\left(z;\phi_G\right)+C_2\left(z;\phi_G\right)-K\phi_G$ where:

$$\begin{aligned}
A_2\left(z;\phi_G\right) &\equiv t\left[\int_{\frac{t}{\phi_G}}^{\frac{\tau-p}{\phi_G}}\int_{\underline{\theta_P}}^{\phi_G\theta_G+p-t}f\left(\theta_G,\theta_P\right)d\theta_P d\theta_G+\int_{\frac{\tau-p}{\phi_G}}^{\overline{\theta_G}}\int_{\underline{\theta_P}}^{\tau-t}f\left(\theta_G,\theta_P\right)d\theta_P d\theta_G\right] \\
B_2\left(z;\phi_G\right) &\equiv (p-c)\left[\int_{p}^{\tau-t}\int_{\underline{\theta}}^{\frac{\theta_P+t-p}{\phi_G}}f\left(\theta_G,\theta_P\right)d\theta_G d\theta_P+\int_{\tau-t}^{\overline{\theta_P}}\int_{\underline{\theta}}^{\frac{\tau-p}{\phi_G}}f\left(\theta_G,\theta_P\right)d\theta_G d\theta_P\right] \\
C_2\left(z;\phi_G\right) &\equiv (\tau-c)\left[\int_{\frac{\tau-p}{\phi_G}}^{\overline{\theta_G}}\int_{\tau-t}^{\overline{\theta_P}}f\left(\theta_G,\theta_P\right)d\theta_P d\theta_G\right].
\end{aligned}$$

Differentiating A_2 we get:

$$
\begin{aligned}
\frac{\partial A_2(z;\phi_G)}{\partial t} &= \left[\int_{\frac{t}{\phi_G}}^{\frac{\tau-p}{\phi_G}}\int_{\underline{\theta_P}}^{\phi_G\theta_G+p-t} f(\theta_G,\theta_P)\,d\theta_P d\theta_G + \int_{\frac{\tau-p}{\phi_G}}^{\overline{\theta_G}}\int_{\underline{\theta_P}}^{\tau-t} f(\theta_G,\theta_P)\,d\theta_P d\theta_G\right] \\
&\quad -t\left[\int_{\underline{\theta_P}}^{p} f\left(\frac{t}{\phi_G},\theta_P\right)\frac{1}{\phi_G}d\theta_P + \int_{\frac{t}{\phi_G}}^{\frac{\tau-p}{\phi_G}} f(\theta_G,\phi_G\theta_G+p-t)\,d\theta_G + \int_{\frac{\tau-p}{\phi_G}}^{\overline{\theta_G}} f(\theta_G,\tau-t)\,d\theta_G\right] \\
\frac{\partial A_2(z;\phi_G)}{\partial p} &= -t\int_{\underline{\theta_P}}^{\tau-t} f\left(\frac{\tau-p}{\phi_G},\theta_P\right)\frac{1}{\phi_G}d\theta_P + t\int_{\frac{t}{\phi_G}}^{\frac{\tau-p}{\phi_G}} f(\theta_G,\phi_G\theta_G+p-t)\,d\theta_G + t\int_{\underline{\theta_P}}^{\tau-t} f\left(\frac{\tau-p}{\phi_G},\theta_P\right)\frac{1}{\phi_G}d\theta_P \\
&= t\int_{\frac{t}{\phi_G}}^{\frac{\tau-p}{\phi_G}} f(\theta_G,\phi_G\theta_G+p-t)\,d\theta_G \\
\frac{\partial A_2(z;\phi_G)}{\partial \tau} &= t\int_{\underline{\theta_P}}^{\tau-t} f\left(\frac{\tau-p}{\phi_G},\theta_P\right)\frac{1}{\phi_G}d\theta_P - t\int_{\underline{\theta_P}}^{\tau-t} f\left(\frac{\tau-p}{\phi_G},\theta_P\right)\frac{1}{\phi_G}d\theta_P + t\int_{\frac{\tau-p}{\phi_G}}^{\overline{\theta_G}} f(\theta_G,\tau-t)\,d\theta_G \\
&= t\int_{\frac{\tau-p}{\phi_G}}^{\overline{\theta_G}} f(\theta_G,\tau-t)\,d
\end{aligned}
$$

Hence,

$$
\begin{aligned}
\frac{\partial A_2(z^*;\phi_G^*)}{\partial t} &= \int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}}\int_{\underline{\theta_P}}^{p^*} f(\theta_G,\theta_P)\,d\theta_P d\theta_G - t^*\left[\int_{\underline{\theta_P}}^{p^*} f\left(\frac{t^*}{\phi_G^*},\theta_P\right)\frac{1}{\phi_G^*}d\theta_P + \int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}} f(\theta_G,p^*)\,d\theta_G\right] \\
\frac{\partial A_2(z^*;\phi_G^*)}{\partial p} &= 0 \\
\frac{\partial A_2(z^*;\phi_G^*)}{\partial \tau} &= t^*\int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}} f(\theta_G,p^*)\,d\theta_G.
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
\frac{\partial B_2(z;\phi_G)}{\partial t} &= (p-c)\left[-\int_{\underline{\theta}}^{\frac{\tau-p}{\phi_G}} f(\theta_G,\tau-t)\,d\theta_G + \int_{p}^{\tau-t} f\left(\frac{\theta_P+t-p}{\phi_G},\theta_P\right)\frac{1}{\phi_G}d\theta_P + \int_{\underline{\theta}}^{\frac{\tau-p}{\phi_G}} f(\theta_G,\tau-t)\,d\theta_G\right] \\
&= (p-c)\left[\int_{p}^{\tau-t} f\left(\frac{\theta_P+t-p}{\phi_G},\theta_P\right)\frac{1}{\phi_G}d\theta_P\right] \\
\frac{\partial B_2(z;\phi_G)}{\partial p} &= \int_{p}^{\tau-t}\int_{\underline{\theta}}^{\frac{\theta_P+t-p}{\phi_G}} f(\theta_G,\theta_P)\,d\theta_G d\theta_P + \int_{\tau-t}^{\overline{\theta_P}}\int_{\underline{\theta}}^{\frac{\tau-p}{\phi_G}} f(\theta_G,\theta_P)\,d\theta_P d\theta_G \\
&\quad -(p-c)\left[\int_{\underline{\theta}}^{\frac{t}{\phi_G}} f(\theta_G,p)\,d\theta_G + \int_{p}^{\tau-t} f\left(\frac{\theta_P+t-p}{\phi_G},\theta_P\right)\frac{1}{\phi_G}d\theta_P + \int_{\tau-t}^{\overline{\theta_P}} f\left(\frac{\tau-p}{\phi_G},\theta_P\right)d\theta_P\right] \\
\frac{\partial B_2(z;\phi_G)}{\partial \tau} &= (p-c)\left[\int_{\underline{\theta}}^{\frac{\tau-p}{\phi_G}} f(\theta_G,\tau-t)\,d\theta_G d\theta_P - \int_{\underline{\theta}}^{\frac{\tau-p}{\phi_G}} f(\theta_G,\tau-t)\,d\theta_G + \int_{\tau-t}^{\overline{\theta_P}} f\left(\frac{\tau-p}{\phi_G},\theta_P\right)\frac{1}{\phi_G}d\theta_G\right] \\
&= (p-c)\left[\int_{\tau-t}^{\overline{\theta_P}} f\left(\frac{\tau-p}{\phi_G},\theta_P\right)\frac{1}{\phi_G}d\theta_G\right].
\end{aligned}
$$

Hence,

$$\begin{aligned}
\frac{\partial B_2\left(z^*;\phi_G^*\right)}{\partial t} &= 0 \\
\frac{\partial B_2\left(z^*;\phi_G^*\right)}{\partial p} &= \int_{p^*}^{\overline{\theta_P}}\int_{\underline{\theta}}^{\frac{t^*}{\phi_G^*}} f\left(\theta_G,\theta_P\right)d\theta_P d\theta_G-\left(p^*-c\right)\left[\int_{\underline{\theta}}^{\frac{t^*}{\phi_G^*}} f\left(\theta_G,p^*\right)d\theta_G+\int_{p^*}^{\overline{\theta_P}} f\left(\frac{t^*}{\phi_G^*},\theta_P\right)d\theta_P\right] \\
\frac{\partial B_2\left(z^*;\phi_G^*\right)}{\partial \tau} &= \left(p^*-c\right)\left[\int_{p^*}^{\overline{\theta_P}} f\left(\frac{t^*}{\phi_G^*},\theta_P\right)\frac{1}{\phi_G^*}d\theta_G\right].
\end{aligned}$$

Similarly,

$$\begin{aligned}
\frac{\partial C_2\left(z;\phi_G\right)}{\partial t} &= \left(\tau-c\right)\left[\int_{\frac{\tau-p}{\phi_G}}^{\overline{\theta_G}} f\left(\theta_G,\tau-t\right)d\theta_G\right] \\
\frac{\partial C_2\left(z;\phi_G\right)}{\partial p} &= \left(\tau-c\right)\left[\int_{\tau-t}^{\overline{\theta_P}} f\left(\frac{\tau-p}{\phi_G},\theta_P\right)\frac{1}{\phi_G}d\theta_P\right] \\
\frac{\partial C_2\left(z;\phi_G\right)}{\partial \tau} &= \int_{\frac{\tau-p}{\phi_G}}^{\overline{\theta_G}}\int_{\tau-t}^{\overline{\theta_P}} f\left(\theta_G,\theta_P\right)d\theta_P d\theta_G-\left(\tau-c\right)\left[\int_{\tau-t}^{\overline{\theta_P}} f\left(\frac{\tau-p}{\phi_G},\theta_P\right)\frac{1}{\phi_G}d\theta_P+\int_{\frac{\tau-p}{\phi_G}}^{\overline{\theta_G}} f\left(\theta_G,\tau-t\right)d\theta_G\right].
\end{aligned}$$

Hence,

$$\begin{aligned}
\frac{\partial C_2\left(z^*;\phi_G^*\right)}{\partial t} &= \left(t^*+p^*-c\right)\left[\int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}} f\left(\theta_G,p^*\right)d\theta_G\right] \\
\frac{\partial C_2\left(z^*;\phi_G^*\right)}{\partial p} &= \left(t^*+p^*-c\right)\left[\int_{p^*}^{\overline{\theta_P}} f\left(\frac{t^*}{\phi_G^*},\theta_P\right)\frac{1}{\phi_G^*}d\theta_P\right] \\
\frac{\partial C_2\left(z^*;\phi_G^*\right)}{\partial \tau} &= \int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}}\int_{p^*}^{\overline{\theta_P}} f\left(\theta_G,\theta_P\right)d\theta_P d\theta_G-\left(t^*+p^*-c\right)\left[\int_{p^*}^{\overline{\theta_P}} f\left(\frac{t^*}{\phi_G^*},\theta_P\right)\frac{1}{\phi_G^*}d\theta_P+\int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}} f\left(\theta_G,p^*\right)d\theta_G\right]
\end{aligned}$$

Combining terms we get that

$$\begin{aligned}
\frac{\partial G_2\left(z^*;\phi_G^*\right)}{\partial t} &= \frac{\partial A_2\left(z^*\right)}{\partial t}+\frac{\partial B_2\left(z^*\right)}{\partial t}+\frac{\partial C_2\left(z^*\right)}{\partial t} \\
&= \int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}}\int_{\underline{\theta_P}}^{p^*} f\left(\theta_G,\theta_P\right)d\theta_P d\theta_G-t^*\left[\int_{\underline{\theta_P}}^{p^*} f\left(\frac{t^*}{\phi_G^*},\theta_P\right)\frac{1}{\phi_G^*}d\theta_P+\int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}} f\left(\theta_G,p^*\right)d\theta_G\right] \\
&\quad +\left(t^*+p^*-c\right)\left[\int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}} f\left(\theta_G,p^*\right)d\theta_G\right] \\
&= \int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}}\left\{\int_{\underline{\theta_P}}^{p^*} f\left(\theta_G,\theta_P\right)d\theta_P+\left(p^*-c\right)f\left(\theta_G,p^*\right)\right\}d\theta_G-t^*\int_{\underline{\theta_P}}^{p^*} f\left(\frac{t^*}{\phi_G^*},\theta_P\right)\frac{1}{\phi_G^*}d\theta_P \\
&= \int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}}\left\{\int_{\underline{\theta_P}}^{p^*} f_P\left(\theta_P|\theta_G\right)d\theta_P+\left(p^*-c\right)f_P\left(p^*|\theta_G\right)\right\}f_G\left(\theta_G\right)d\theta_G-\frac{t^*}{\phi_G^*}\int_{\underline{\theta_P}}^{p^*} f_P\left(\theta_P|\frac{t^*}{\phi_G^*}\right)d\theta_P f_G\left(\frac{t^*}{\phi_G^*}\right) \\
&= \int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}}\left\{F_P\left(p^*|\theta_G\right)+\left(p^*-c\right)f_P\left(p^*|\theta_G\right)\right\}f_G\left(\theta_G\right)d\theta_G-\frac{t^*}{\phi_G^*}F_P\left(p^*|\frac{t^*}{\phi_G^*}\right)f_G\left(\frac{t^*}{\phi_G^*}\right) \\
&= \frac{\partial G_1\left(z^*\right)}{\partial t},
\end{aligned}$$

and,

$$
\begin{aligned}
\frac{\partial G_2\left(z^*;\phi_G^*\right)}{\partial p} &= \frac{\partial A_2\left(z^*;\phi_G^*\right)}{\partial p}+\frac{\partial B_2\left(z^*;\phi_G^*\right)}{\partial p}+\frac{\partial C_2\left(z^*;\phi_G^*\right)}{\partial p} \\
&= \int_{p^*}^{\overline{\theta_P}}\int_{\underline{\theta}}^{\frac{t^*}{\phi_G^*}} f\left(\theta_G,\theta_P\right)d\theta_G d\theta_P-\left(p^*-c\right)\left[\int_{\underline{\theta}}^{\frac{t^*}{\phi_G^*}} f\left(\theta_G,p^*\right)d\theta_G+\int_{p^*}^{\overline{\theta_P}} f\left(\frac{t^*}{\phi_G^*},\theta_P\right)d\theta_P\right] \\
&\quad +\left(t^*+p^*-c\right)\left[\int_{p^*}^{\overline{\theta_P}} f\left(\frac{t^*}{\phi_G^*},\theta_P\right)\frac{1}{\phi_G^*}d\theta_P\right] \\
&= \int_{\underline{\theta}}^{\frac{t^*}{\phi_G^*}}\left\{\int_{p^*}^{\overline{\theta_P}} f\left(\theta_G,\theta_P\right)d\theta_P-\left(p^*-c\right)f\left(\theta_G,p^*\right)\right\}d\theta_G+t^*\left[\int_{p^*}^{\overline{\theta_P}} f\left(\frac{t^*}{\phi_G^*},\theta_P\right)\frac{1}{\phi_G^*}d\theta_P\right] \\
&= \int_{\underline{\theta}}^{\frac{t^*}{\phi_G^*}}\left\{1-F_P\left(p^*|\theta_G\right)-\left(p^*-c\right)f_P\left(p^*|\theta_G\right)\right\}f_G\left(\theta_G\right)d\theta_G+t^*\left[1-F_P\left(p^*|\frac{t^*}{\phi_G^*}\right)\right]f_G\left(\frac{t^*}{\phi_G^*}\right) \\
&= \frac{\partial G_1\left(z^*\right)}{\partial p}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial G_2\left(z^*;\phi_G^*\right)}{\partial \tau} &= \frac{\partial A_2\left(z^*;\phi_G^*\right)}{\partial \tau}+\frac{\partial B_2\left(z^*;\phi_G^*\right)}{\partial \tau}+\frac{\partial C_2\left(z^*;\phi_G^*\right)}{\partial \tau} \\
&= t^*\int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}} f\left(\theta_G,p^*\right)d\theta_G+\left(p^*-c\right)\left[\int_{p^*}^{\overline{\theta_P}} f\left(\frac{t^*}{\phi_G^*},\theta_P\right)\frac{1}{\phi_G^*}d\theta_G\right] \\
&\quad +\int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}}\int_{p^*}^{\overline{\theta_P}} f\left(\theta_G,\theta_P\right)d\theta_P d\theta_G-\left(t^*+p^*-c\right)\left[\int_{p^*}^{\overline{\theta_P}} f\left(\frac{t^*}{\phi_G^*},\theta_P\right)\frac{1}{\phi_G^*}d\theta_P+\int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}} f\left(\theta_G,p^*\right)d\theta_G\right] \\
&= \int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}}\left\{\int_{p^*}^{\overline{\theta_P}} f\left(\theta_G,\theta_P\right)d\theta_P-\left(p^*-c\right)f\left(\theta_G,p^*\right)\right\}d\theta_G-t^*\left[\int_{p^*}^{\overline{\theta_P}} f\left(\frac{t^*}{\phi_G^*},\theta_P\right)\frac{1}{\phi_G^*}d\theta_P\right] \\
&= \int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}}\left\{1-F_P\left(p^*|\theta_G\right)-\left(p^*-c\right)f_P\left(p^*|\theta_G\right)\right\}f_G\left(\theta_G\right)d\theta_G-\frac{t^*}{\phi_G^*}\left[1-F_P\left(p^*|\frac{t^*}{\phi_G^*}\right)f_G\left(\frac{t^*}{\phi_G^*}\right)\right] \\
&= \frac{\partial G_1\left(z^*\right)}{\partial \tau}
\end{aligned}
$$

- **Part 2:** $DS_1\left(z^*;\phi_G^*\right)=DS_2\left(z^*;\phi_G^*\right)$**:**

Write $S_2\left(z;\phi_G\right)=D_2\left(z;\phi_G\right)+E_2\left(z;\phi_G\right)+F_2\left(z;\phi_G\right)-K\phi_G$ where

$$
\begin{aligned}
D_2\left(z;\phi_G\right) &= \int_{\frac{t}{\phi_G}}^{\frac{\tau-p}{\phi_G}}\int_{\underline{\theta_P}}^{\phi_G\theta_G+p-t}\phi_G\theta_G f\left(\theta_G,\theta_P\right)d\theta_P d\theta_G+\int_{\frac{\tau-p}{\phi_G}}^{\overline{\theta_G}}\int_{\underline{\theta_P}}^{\tau-t}\phi_G\theta_G f\left(\theta_G,\theta_P\right)d\theta_P d\theta_G \\
E_2\left(z;\phi_G\right) &= \int_{p}^{\tau-t}\int_{\underline{\theta}}^{\frac{\theta_P+t-p}{\phi_G}}\left(\theta_P-c\right)f\left(\theta_G,\theta_P\right)d\theta_G d\theta_P+\int_{\tau-t}^{\overline{\theta_P}}\int_{\underline{\theta}}^{\frac{\tau-p}{\phi_G}}\left(\theta_P-c\right)f\left(\theta_G,\theta_P\right)d\theta_G d\theta_P \\
F_2\left(z;\phi_G\right) &= \int_{\frac{\tau-p}{\phi_G}}^{\overline{\theta_G}}\int_{\tau-t}^{\overline{\theta_P}}\left(\phi_G\theta_G+\theta_P-c\right)f\left(\theta_G,\theta_P\right)d\theta_P d\theta_G.
\end{aligned}
$$

Thus,

$$\begin{aligned}
\frac{\partial D_2\left(z;\phi_G\right)}{\partial t} &= -\int_{\underline{\theta_P}}^{p}\frac{t}{\phi_G}f\left(\frac{t}{\phi_G},\theta_P\right)d\theta_P-\int_{\frac{t}{\phi_G}}^{\frac{\tau-p}{\phi_G}}\phi_G\theta_G f\left(\theta_G,\phi_G\theta_G+p-t\right)d\theta_G-\int_{\frac{\tau-p}{\phi_G}}^{\overline{\theta_G}}\phi_G\theta_G f\left(\theta_G,\tau-t\right)d\theta_G \\
\frac{\partial D_2\left(z;\phi_G\right)}{\partial p} &= -\int_{\underline{\theta_P}}^{\tau-t}\frac{\tau-p}{\phi_G}f\left(\frac{\tau-p}{\phi_G},\theta_P\right)d\theta_P+\int_{\frac{t}{\phi_G}}^{\frac{\tau-p}{\phi_G}}\phi_G\theta_G f\left(\theta_G,\phi_G\theta_G+p-t\right)d\theta_P d\theta_G+\int_{\underline{\theta_P}}^{\tau-t}\frac{\tau-p}{\phi_G}f\left(\frac{\tau-p}{\phi_G},\theta_P\right)d \\
&= \int_{\frac{t}{\phi_G}}^{\frac{\tau-p}{\phi_G}}\phi_G\theta_G f\left(\theta_G,\phi_G\theta_G+p-t\right)d\theta_P d\theta_G \\
\frac{\partial D_2\left(z;\phi_G\right)}{\partial \tau} &= \int_{\underline{\theta_P}}^{\tau-t}\frac{\tau-p}{\phi_G}f\left(\frac{\tau-p}{\phi_G},\theta_P\right)d\theta_P-\int_{\underline{\theta_P}}^{\tau-t}\frac{\tau-p}{\phi_G}f\left(\frac{\tau-p}{\phi_G},\theta_P\right)d\theta_P+\int_{\frac{\tau-p}{\phi_G}}^{\overline{\theta_G}}\phi_G\theta_G f\left(\theta_G,\tau-t\right)d\theta_G \\
&= \int_{\frac{\tau-p}{\phi_G}}^{\overline{\theta_G}}\phi_G\theta_G f\left(\theta_G,\tau-t\right)d\theta_G,
\end{aligned}$$

so,

$$\begin{aligned}
\frac{\partial D_2\left(z^*;\phi_G^*\right)}{\partial t} &= -\int_{\underline{\theta_P}}^{p^*}\frac{t^*}{\phi_G^*}f\left(\frac{t^*}{\phi_G^*},\theta_P\right)d\theta_P-\int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}}\phi_G^*\theta_G f\left(\theta_G,p^*\right)d\theta_G \\
\frac{\partial D_2\left(z^*;\phi_G^*\right)}{\partial p} &= 0 \\
\frac{\partial D_2\left(z^*;\phi_G^*\right)}{\partial \tau} &= \int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}}\phi_G^*\theta_G f\left(\theta_G,p^*\right)d\theta_G,
\end{aligned}$$

which is the same as in (A2).

Similarly,

$$\begin{aligned}
\frac{\partial E_2\left(z;\phi_G\right)}{\partial t} &= -\int_{\underline{\theta}}^{\frac{\tau-p}{\phi_G}}\left(\tau-t-c\right)f\left(\theta_G,\tau-t\right)d\theta_G+\int_{p}^{\tau-t}\left(\theta_P-c\right)f\left(\frac{\theta_P+t-p}{\phi_G},\theta_P\right)\frac{1}{\phi_G}d\theta_P \\
&\quad+\int_{\underline{\theta}}^{\frac{\tau-p}{\phi_G}}\left(\tau-t-c\right)f\left(\theta_G,\tau-t\right)d\theta_G \\
&= \int_{p}^{\tau-t}\left(\theta_P-c\right)f\left(\frac{\theta_P+t-p}{\phi_G},\theta_P\right)\frac{1}{\phi_G}d\theta_P \\
\frac{\partial E_2\left(z;\phi_G\right)}{\partial p} &= -\int_{\underline{\theta}}^{\frac{t}{\phi_G}}\left(p-c\right)f\left(\theta_G,p\right)d\theta_G-\int_{p}^{\tau-t}\left(\theta_P-c\right)f\left(\frac{\theta_P+t-p}{\phi_G},\theta_P\right)\frac{1}{\phi_G}d\theta_P \\
&\quad-\int_{\tau-t}^{\overline{\theta_P}}\left(\theta_P-c\right)f\left(\frac{\tau-p}{\phi_G},\theta_P\right)\frac{1}{\phi_G}d\theta_P \\
\frac{\partial E_2\left(z;\phi_G\right)}{\partial \tau} &= \int_{\underline{\theta}}^{\frac{\tau-p}{\phi_G}}\left(\tau-t-c\right)f\left(\theta_G,\tau-t\right)d\theta_G-\int_{\underline{\theta}}^{\frac{\tau-p}{\phi_G}}\left(\tau-t-c\right)f\left(\theta_G,\tau-t\right)d\theta_G \\
&\quad+\int_{\tau-t}^{\overline{\theta_P}}\left(\theta_P-c\right)f\left(\frac{\tau-p}{\phi_G},\theta_P\right)\frac{1}{\phi_G}d\theta_P \\
&= \int_{\tau-t}^{\overline{\theta_P}}\left(\theta_P-c\right)f\left(\frac{\tau-p}{\phi_G},\theta_P\right)\frac{1}{\phi_G}d\theta_P,
\end{aligned}$$

so,

$$\begin{aligned}
\frac{\partial E_2\left(z^*;\phi_G^*\right)}{\partial t} &= 0 \\
\frac{\partial E_2\left(z^*;\phi_G^*\right)}{\partial p} &= -\int_{\underline{\theta}}^{\frac{t^*}{\phi_G^*}}\left(p^*-c\right)f\left(\theta_G,p^*\right)d\theta_G-\int_{p^*}^{\overline{\theta_P}}\left(\theta_P-c\right)f\left(\frac{t^*}{\phi_G^*},\theta_P\right)\frac{1}{\phi_G^*}d\theta_P \\
\frac{\partial E_2\left(z^*;\phi_G^*\right)}{\partial \tau} &= \int_{p^*}^{\overline{\theta_P}}\left(\theta_P-c\right)f\left(\frac{t^*}{\phi_G^*},\theta_P\right)\frac{1}{\phi_G^*}d\theta_P,
\end{aligned}$$

which is the same as the expressions in (A3).

Finally,

$$\begin{aligned}
\frac{\partial F_2\left(z;\phi_G\right)}{\partial t} &= \int_{\frac{\tau-p}{\phi_G}}^{\overline{\theta_G}}\left(\phi_G\theta_G+\tau-t-c\right)f\left(\theta_G,\tau-t\right)d\theta_G \\
\frac{\partial F_2\left(z;\phi_G\right)}{\partial p} &= \int_{\tau-t}^{\overline{\theta_P}}\left(\tau-p+\theta_P-c\right)f\left(\frac{\tau-p}{\phi_G},\theta_P\right)\frac{1}{\phi_G}d\theta_P \\
\frac{\partial F_2\left(z;\phi_G\right)}{\partial \tau} &= -\int_{\tau-t}^{\overline{\theta_P}}\left(\tau-p+\theta_P-c\right)f\left(\frac{\tau-p}{\phi_G},\theta_P\right)\frac{1}{\phi_G}d\theta_P-\int_{\frac{\tau-p}{\phi_G}}^{\overline{\theta_G}}\left(\phi_G\theta_G+\tau-t-c\right)f\left(\theta_G,\tau-t\right)d\theta_G,
\end{aligned}$$

so,

$$\begin{aligned}
\frac{\partial F_2\left(z^*;\phi_G^*\right)}{\partial t} &= \int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}}\left(\phi_G^*\theta_G+p^*-c\right)f\left(\theta_G,p^*\right)d\theta_G \\
\frac{\partial F_2\left(z^*;\phi_G^*\right)}{\partial p} &= \int_{p^*}^{\overline{\theta_P}}\left(t^*+\theta_P-c\right)f\left(\frac{t^*}{\phi_G^*},\theta_P\right)\frac{1}{\phi_G^*}d\theta_P \\
\frac{\partial F_2\left(z^*;\phi_G^*\right)}{\partial \tau} &= -\int_{p^*}^{\overline{\theta_P}}\left(t^*+\theta_P-c\right)f\left(\frac{t^*}{\phi_G^*},\theta_P\right)\frac{1}{\phi_G^*}d\theta_P-\int_{\frac{t^*}{\phi_G^*}}^{\overline{\theta_G}}\left[\phi_G^*\theta_G+p^*-c\right]f\left(\theta_G,p^*\right)d\theta_G
\end{aligned}$$

which is the same as the expressions in (A4). Since all the components are identical the result follows.