Dynamic Voluntary Provision of Public Goods

- Thomas Schelling:

  "If each party agrees to send a million dollar to the red cross on condition that the other does, each may be tempted to cheat if the other contributes first, and each one's anticipation of the other cheating will inhibit agreement. But if the contributions are divided into consecutive small contributions, each can try the other's good faith at a small price. Furthermore, since each can keep the other on short tether to finish, no one ever need risk more than a small contribution at a time."

- Idea: Dividing the contributions into small sums may help.

- Admati and Perry (1991) and Varian (1994) showed the limitation of Shelling's intuition; Marx and Matthews (2000) partially confirmed Schelling's intuition.

- The main difference between AP and Varian lies in the nature of the public good:
  - In AP, the public good is either completed or not and it is useless unless completed.
  - In Varian, public good is continuous and agents can benefit from partial completion.

- Marx and Matthews (2000) set up is closer to AP:
  - In AP, two agents and they take turns;
  - In MM, n agents and all agents are allowed to contribute at any round.

1 Varian (JPubE, 1994)

- Consier two agents, $i = 1, 2$.

- w_i : Initial wealth of agent i.

- x_i : $i's$ private consumption;

- g_i : $i's$ contribution to public goods.

- $G = g_1 + g_2$: Total amount of public good.

- Agent i's utility function is quasi-linear: $v_i(G) + x_i$.

1.1 Reaction Function

- Let $\bar{g}_i$ be the optimal amount of contribution by agent i if agent j contributes zero, i.e.

$$\bar{g}_i = \arg \max_{g_i \in [0, w_i]} v_i\left(g_i\right) + \left(w - g_i\right)$$

  That is $v_i\left(\bar{g}_i\right) = 1$.

- Assume that $w_i > \bar{g}_i$.

- The reaction function of agent i, when agent j contributes g_j is:

$$G_i\left(g_j\right) = \max\left\{\bar{g}_i - g_j, 0\right\}.$$

1.2 Simultaneous Nash Equilibrium

- Simultaneous Nash equilibrium is a pair of contributions (g_1, g_2) such that

$$g_1 = G_1(g_2), g_2 = G_2(g_1).$$

- If $\bar{g}_1 > \bar{g}_2$, the unique Nash equilibrium public good level is $\bar{g}_1$ and agent 2 free rides.

1.3 Stackleberg Equilibrium

- Suppose that agent 1 moves first. Agent 1 maximizes

$$\begin{aligned} V_1(g_1) &\equiv v_1(g_1 + G_2(g_1)) - g_1 + w_1 \\ &= v_1(g_1 + \max\{\bar{g}_2 - g_1, 0\}) - g_1 + w_1 \\ &= \begin{cases} v_1(g_2) - g_1 + w_1 \text{ if } g_1 \leq \bar{g}_2 \\ v_1(g_1) - g_1 + w_1 \text{ if } g_1 \geq \bar{g}_2. \end{cases} \end{aligned}$$

- Thus agent 1 has two possible optima: either contributes zero and achieves utility $v_1(\bar{g}_2) + w_1$ or contributes $\bar{g}_1$ and achieves utlity $v_1(\bar{g}_1) + (w_1 - \bar{g}_1)$.

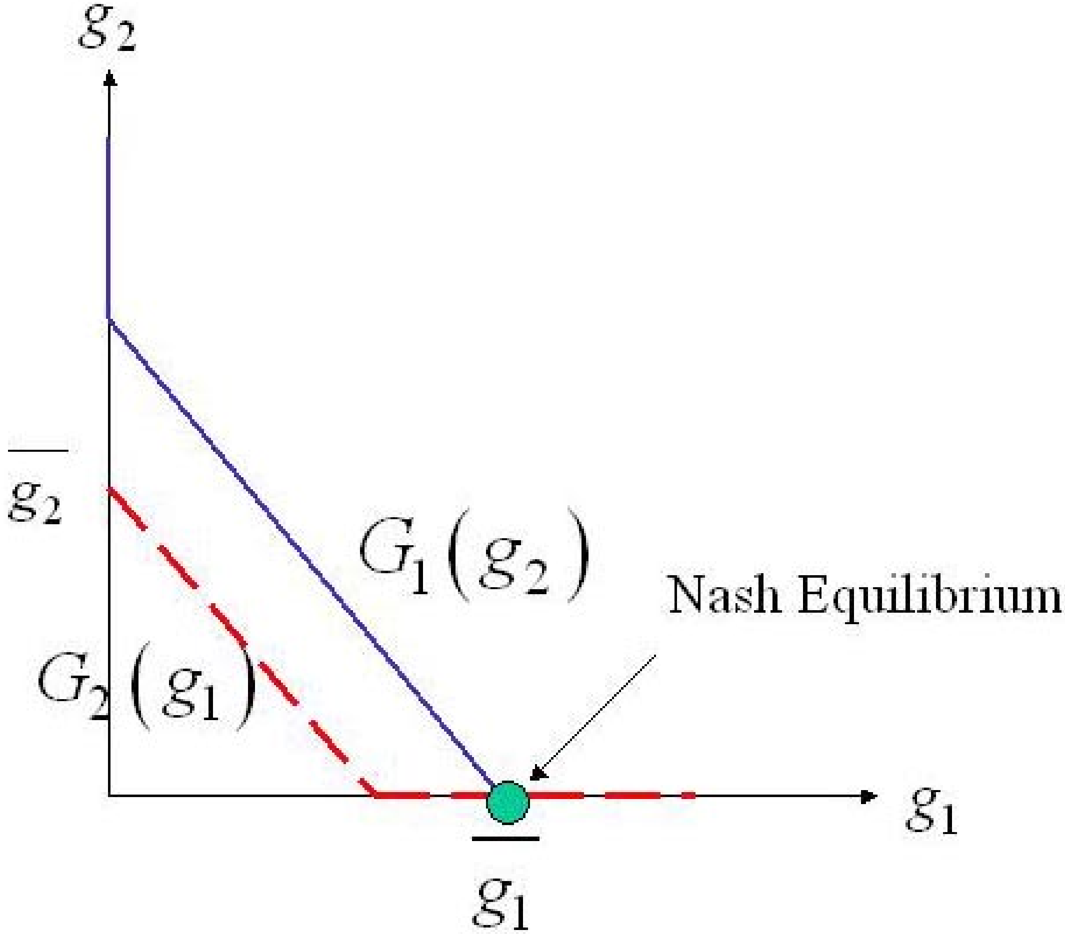

Figure 1: Nash Equilibrium.

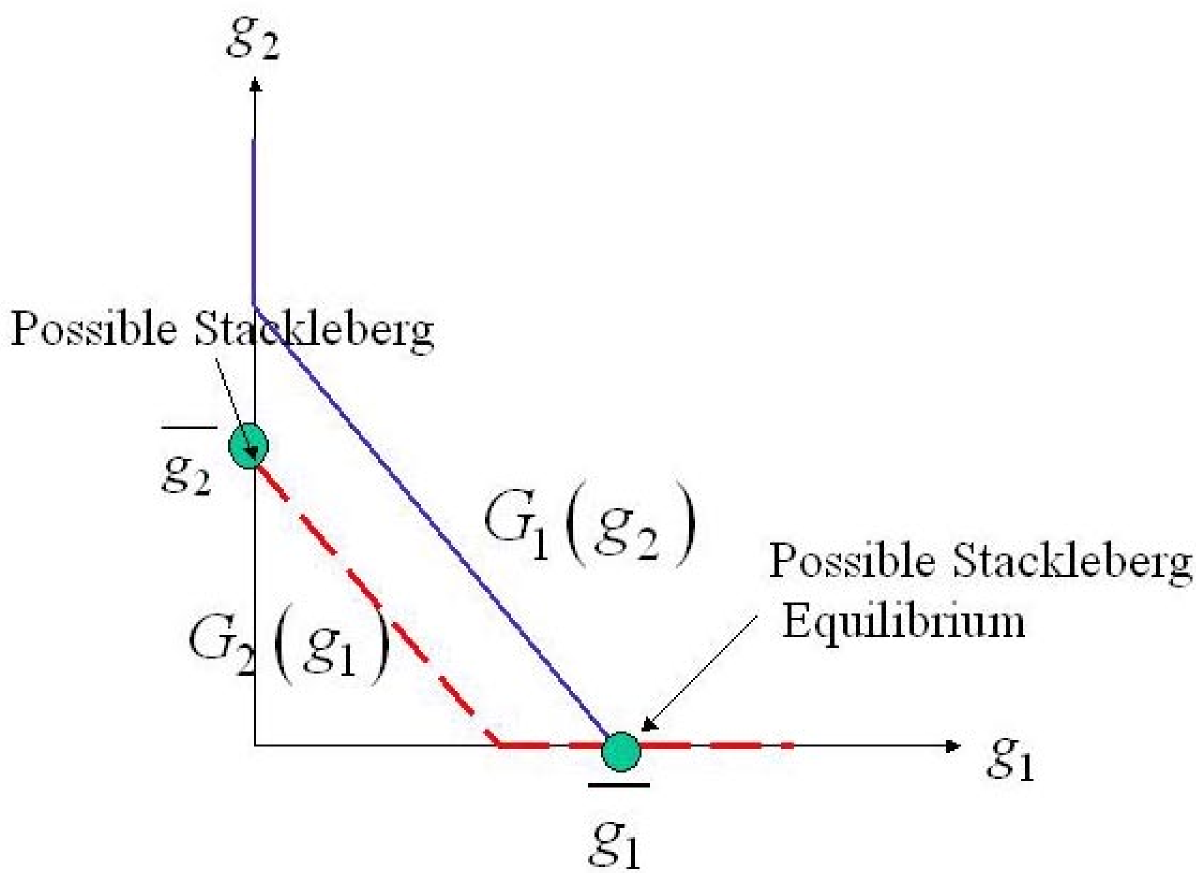

Figure 2: Stackleberg Equilibrium Outcomes.