Economics 301
Problem Set 1 – Due Friday, Sept 15, 2006

1. Quasi-Linear Preferences. (Polak)
[This question looks at an example of quasi-linear preferences. Quasi-linear preferences are useful in economic applications because they allow us to 'ignore' wealth effects. This, in turn, allows us to treat problems using methods of partial (rather than general) equilibrium.]

Consider a consumer whose preferences over $\Re^3_+$ are represented by the utility function $x_1 + \alpha_2 \ln x_2 + \alpha_3 \ln x_3$ where α_2 and α_3 are both positive constants.

(a) Write down the consumer's utility maximization problem when **p**>>0 and w>0. Write down the Kuhn-Tucker first-order conditions for a solution **x***(**p**,w) taking care to allow for the possibility that the solution is at a corner.

(b) Argue that the budget constraint binds and that the constraints $x_2 \geq 0$ and $x_3 \geq 0$ are slack at the solution.

(c) Find the conditions under which $x_1^* = 0$ and write down the Walrasian demands for x_2 and x_3 in that case.

(d) Under the conditions such that $x_1^* > 0$, write down the Walrasian demands for all three goods.

(e) Describe the wealth elasticities of demand for all three goods? (You might find it helpful to draw the wealth expansion path for two of the three goods).

(f) For the case where $x_1^* > 0$ show that the indirect utility function can be written in the form $v(\mathbf{p},w) = w + \phi(\mathbf{p})$. (That is that the function $\phi(.)$ does not depend on w).

(g) Verify that this indirect utility function is increasing in w, non-increasing in **p**, quasi-convex in (**p**,w) and satisfies Roy's identity.

2. Homotheticity and Homogeneity of Degree One. (Mas-Collel et. al. Exercise 3.D.3)
[This question looks at homothetic preferences. These are often useful for aggregation because they allow us to treat a rich consumer as a 'blown up' version of a poor consumer.]

Assume that $u(\mathbf{x})$ is strictly quasiconcave and differentiable and that the Walrasian demand function $\mathbf{x}(\mathbf{p},w)$ is differentiable.

(a) Show that, if $u(\mathbf{x})$ is homogenous of degree one then preferences are homothetic and the Walrasian demand function $\boldsymbol{x}(\mathbf{p},w)$and the indirect utility function $v(\mathbf{p},w)$ are homogenous of degree one in w. Show that this means that wealth expansion paths are straight lines through the origin. What does this imply about the wealth elasticity of demand?

(b) Conversely, show that if $v(\mathbf{p},w)$ is homogenous of degree on in w than $u(x)$ must be homogenous of degree one.

3. <u>CES Utility Functions.</u> (Mas-Collel et. al. 3.C.6 and 3.D.5)
[These are the most commonly used family of homothetic preferences. Many other homothetic preferences are special cases of CES. It is good to get used to manipulating these preferences.]

Consider a consumer whose preferences over $\Re^2_+$ are represented by the utility function $u(x_1, x_2) = \left(\alpha_1 x_1^\rho + \alpha_2 x_2^\rho\right)^{1/\rho}$ where α_1 and α_2 are both positive constants, $\alpha_1 + \alpha_2 = 1$, and $\rho \leq 1$. This is known as the constant elasticity of substitution (CES) utility function.

(a) Show that CES preferences are homothetic. What does this imply about wealth expansion paths?

(b) Show that the indifference curves become linear (i.e., linear utility) when $\rho=1$ and become right angles (i.e., Leontief utility) as $\rho \to -\infty$.

(c) Show that these preferences become Cobb-Douglas as $\rho \to 0$ (you can restrict attention to **x**>>0. [hint: use L'Hopital's rule]

(d) Compute the Walrasian demands and indirect utility functions for this utility function when $\rho < 1$.

(e) Verify that the indirect utility function is quasi-convex and satisfies Roy's identity.

(f) Compute the Walrasian demands for linear and Leontief utility, and verify that the CES demands converge to these as $\rho \to 1$ and $\rho \to -\infty$ respectively.

4. <u>Properties of Preference Relations.</u> (Mas-Collel et. al. 3.B.2 and 3.C.1)

(a) The preference relation $\succeq$ defined on $\Re^L_+$ is said to be weakly monotone if and only if $x \geq y$ implies $x \succeq y$. Show that if $\succeq$ is transitive, local non-satiated, and weakly monotone, then it is monotone.

(b) Verify that the lexicographic preference ordering is complete, transitive, strongly monotone, and strictly convex.