Journal of Monetary Economics 117 (2021) 441–459

Contents lists available at ScienceDirect

Journal of Monetary Economics

journal homepage: www.elsevier.com/locate/jmoneco

Wealth creation, wealth dilution and demography[☆]

Christa N. Brunnschweiler[a], Pietro F. Peretto[b], Simone Valente[a,*]

[a] University of East Anglia, United Kingdom
[b] Duke University, NC, United States

ARTICLE INFO

Article history:
Received 11 December 2018
Revised 10 February 2020
Accepted 17 February 2020
Available online 19 February 2020

JEL classification:
O41
J11
E25

Keywords:
R&D-based growth
Overlapping generations
Endogenous fertility
Population level
Wealth dilution

ABSTRACT

Demographic forces are crucial drivers of macroeconomic performance. Yet, existing theories do not allow demography to respond to fundamentals and policies while determining key macroeconomic variables. We build a model of endogenous interactions between fertility and innovation-led productivity growth that delivers empirically consistent co-movements of population, income and wealth. Wealth dilution and wage dynamics stabilize population through non-Malthusian forces; demography determines the ratios of labor income and consumption to financial wealth. Shocks that reduce population size, like immigration barriers, reduce permanently the labor share and the mass of firms, creating prolonged stagnation and substantial intergenerational redistribution of income and welfare.

1. Introduction

Demographic forces (fertility decline, migration, ageing) challenge advanced economies with fundamental questions concerning aggregate phenomena such as the productivity slowdown, international imbalances, post-crisis stagnation and low real interest rates.[1] Further questions arise from the policy-making arena, where proposed interventions such as barriers to migration and reform of the welfare state call for assessing the effects of demographic and policy shocks on economic performance. Studying such questions requires empirically consistent models where demographic forces drive the determination of key macroeconomic variables, including long-run growth.

Existing theories are not yet satisfactory. A first reason is that they typically predict that population grows at a constant exponential rate in the long run.[2] That is, they are not theories of the population *level*. This is not a mere technical point:

☆ We thank Sjak Smulders and Ragnar Torvik for insightful discussions, and participants to the Royal Economic Society Conference 2018 and SURED Monte Veritá 2018 for comments and suggestions. The usual disclaimer applies.

* Corresponding author.
E-mail address: s.valente@uea.ac.uk (S. Valente).

[1] See e.g. Backus et al. (2014), Carvalho et al. (2016), Cooley and Henriksen (2018), Gordon (2018).

[2] This class of models is quite large (see Ehrlich and Lui (1997)) and encompasses all the well-established specifications of the supply side, from neoclassical technologies (e.g., Barro and Becker (1989)) to endogenous growth frameworks (e.g., Chu et al. (2013)). The distinctive prediction of semi-endogenous growth models Jones (1995) that sustained output growth requires strictly positive population growth excludes by construction equilibria with constant population.

https://doi.org/10.1016/j.jmoneco.2020.02.002
0304-3932/

Given the fertility decline in the industrialized world, applied research needs models that explain not only how fertility falls as the economy develops, but also how it converges to equality with the mortality rate. Standard balanced-growth models that predict constant population growth are at odds with the demographers' view of the long run and the zero population growth so evident in the data. A second reason for dissatisfaction is that the traditional workhorse-models of macroeconomics, the Solow–Ramsey model and its overlapping-generations variants, do not account for the interaction of population and productivity. Modeling productivity growth as an exogenous process, that is therefore orthogonal to population growth, itself exogenous, drastically limits the role of demography despite the massive evidence that it matters (Jones and Tertilt, 2006; Madsen, 2010). Allowing for endogenous demography-technology interactions is challenging but necessary to properly inform empirics and policy analysis.

In this paper we build a tractable general equilibrium model where fertility interacts with innovation-led productivity growth. The model produces a steady state with positive growth of income per capita associated to *constant* population. It also produces transitional dynamics consistent with the empirical evidence. Two results are especially novel. First, the population stabilizes because as it grows it dilutes financial wealth per capita and yields a decline in fertility. This negative feedback has not been investigated before and abstracts from Malthusian forces (more on this below). Second, the model predicts that in the long run the ratios to GDP of key macroeconomic variables – consumption, labor income, financial wealth – are exclusively determined by demographic and preference parameters. Shocks like barriers to migration or exogenous changes in child-rearing costs have first-order effects on the functional distribution of income, consumption and welfare that we can characterize analytically and assess numerically.

The two building blocks of our model are the Yaari–Blanchard overlapping-generations demographic structure (Blanchard, 1985; Yaari, 1965) and the Schumpeterian theory of endogenous growth with endogenous market structure (Peretto, 1998; Peretto and Connolly, 2007). We extend the former to include endogenous fertility: individuals maximize lifetime utility facing a positive probability of death and choosing, in addition to consumption, the mass of children subject to a time cost of reproduction. Differently from altruistic models where the head of the dynasty maximizes collective utility over an infinite horizon, each cohort enters the economy with zero financial assets and pursues independent consumption and reproduction plans. In this framework, population growth tends to reduce consumption via *wealth dilution*: the arrival of new disconnected generations reduces financial wealth per capita, which in turn reduces consumption per capita.[3] Moreover, because it reduces consumption per capita, the dilution of financial wealth reduces the mass of children that each households decides to have. This is the first component of the general-equilibrium mechanism driving our model dynamics: Holding aggregate wealth constant, population growth lowers the fertility rate.

The second component is *wealth creation*, i.e., the process driving the value of aggregate wealth. Financial assets represent ownership of firms. The key hypothesis is that the mass of firms and the profitability of each firm evolve as the result of different R&D activities (Peretto, 1998; Peretto and Connolly, 2007): The total value of firms grows as a result of both vertical innovations (i.e., each individual firm invests in R&D that raises internal productivity) and horizontal innovations (i.e., new firms enter the market). Both activities compete for homogeneous labor and, in free-entry equilibrium, generate aggregate wealth that is less than proportional to population. Therefore, the ratio between wage and wealth per capita is *increasing* in the mass of workers, which means that as population grows, the individual wage-to-wealth ratio rises and households reduce fertility because the opportunity cost of reproduction is higher. The net general-equilibrium feedback effect of wealth dilution and wealth creation is thus negative and stabilizes the population in the long run. We show that the model produces transitional co-movements of fertility, population and consumption per capita relative to financial assets consistent with panel data for OECD economies. Moreover, the model features co-movements of these demographic variables with the mass of firms, firm size, firm-specific innovation rate and entry rate that make our contribution relevant to the literature on "business dynamism" that recently has considered demographic forces as a potential explanation for the trends displayed by many advanced economies (see, e.g., Decker et al. 2016 and, especially, Karahan et al. 2019).

Our results are in stark contrast with those of the existing frameworks. In particular, we obtain a novel theory of the population level that is (i) non-Malthusian and in which (ii) demography, rather than technology, is the fundamental determinant of key macroeconomic variables in the long run. As noted above, most existing models predict steady-state exponential population growth. The only theories capable of producing a stable population are Malthusian, that is, they are built on the idea that the size of the population is bounded by essential factors in fixed supply.[4] The recent vintage of such theories (see, e.g., Galor (2011)) seeks to explain the escape of modern economies from the pre-industrial Malthusian trap of the past. In contrast, our model fully abstracts from Malthusian forces: wealth consists of accumulable factors – labor and knowledge – and there are no fixed factors. What stabilizes population in the long run is not natural resource scarcity but the dilution of financial wealth.

[3] See Barro and Sala-i Martin (2004: p.183). Buiter (1988) and Weil (1989) provide an early recognition of the wealth dilution effect in the Blanchard–Yaari framework with exogenous population growth.

[4] Malthusian forces may take various forms: decreasing returns to scale in Eckstein et al. (1988) land scarcity combined with subsistence requirements in Galor and Weil (2000) open-access resources in Brander and Taylor (1998). Two borderline cases are Strulik and Weisdorf (2008) and Peretto and Valente (2015), where the fixed factor is a marketed input and its relative scarcity creates price effects that tend to reduce fertility through higher cost of living and/or lower real income. In Strulik and Weisdorf (2008) scarcity raises the relative price of food and thereby the private cost of reproduction, resulting in fertility decline.

Our second key result is in contrast with traditional balanced-growth models, where the ratios of aggregate consumption and aggregate labor income to aggregate financial wealth are determined by the production technology. In our model, instead, the same grand ratios are determined by demography and preference parameters. A major consequence is that negative demographic shocks caused by immigration barriers or higher reproduction costs yield a permanent reduction in the labor income share, while they have the opposite effect on the growth rate: productivity may grow faster in the long run, but the transition exhibits prolonged stagnation and lower wage. This phenomenon is due to the positive co-movement of population and the mass of firms, which drives down aggregate output as population shrinks. The consequences for intergenerational welfare can be substantial: our numerical simulations show that a long sequence of cohorts entering the economy after the shock experience welfare losses due to the combined effects of permanently lower labor income and slow transitional growth.

2. The demographic model

This section describes the demographic structure and the intertemporal choices of households abstracting from the production side of the economy. This allows us to formalize the concept of financial wealth dilution and highlight its general implications in isolation from the functioning of labor and product markets.[5]

2.1. Households

The economy is populated by overlapping generations (cohorts) of single-individual families facing a constant probability of death (Blanchard, 1985; Yaari, 1965). We extend the structure by assuming that individuals derive utility from the mass of children they rear subject to a pure time cost of reproduction. Individual variables take the form $x_j(t)$, where $j \in (-\infty, t)$ is the cohort index representing the birth date, and $t \in (-\infty, \infty)$ is continuous calendar time.[6] In particular, $c_j(t)$ denotes consumption at time t of an individual born at time $j < t$, and $b_j(t)$ denotes the mass of children reared at time t by an agent who belongs to cohort j. The expected lifetime utility of an individual born at time j is

$$U_j^E = \int_j^\infty \left[\ln c_j(t) + \psi \ln b_j(t)\right] e^{-(\rho+\delta)(t-j)} dt, \tag{1}$$

where $\psi > 0$ is the weight attached to the utility from rearing $b_j(t)$ children, $\rho > 0$ is the rate of time preference and $\delta > 0$ is the constant instantaneous probability of death.[7] Differently from dynastic models (e.g., Barro and Becker, 1989), individuals do not internalize the lifetime utility of their descendants: children leave the family after birth, enter the economy as workers owning zero assets and make saving plans independently from their predecessors. Individuals accumulate assets and allocate one unit of time between working and child rearing. The individual budget constraint is

$$\dot{a}_j(t) = (r(t) + \delta)a_j(t) + \left(1 - \gamma b_j(t)\right)w(t) - p(t)c_j(t), \tag{2}$$

where a_j is individual asset holdings, r is the rate of return on assets, w is the wage rate, p is the price of the consumption good and $\gamma > 0$ is the time cost of child rearing per child. The term $\left(1 - \gamma b_j\right)$ thus is individual labor supply and the term $\gamma b_j w$ is the cost of fertility in terms of foregone labor income. This structure of fertility costs is not crucial for our main results.[8] What really drives the wealth dilution mechanism is, instead, the hypothesis of finite individual horizons which makes the accumulation plans of different cohorts disconnected.[9]

An individual born at time j maximizes (1) subject to (2), taking the paths of all prices as given. Necessary conditions for utility maximization are the individual Euler equation for consumption

$$\frac{\dot{c}_j(t)}{c_j(t)} + \frac{\dot{p}(t)}{p(t)} = r(t) - \rho, \tag{3}$$

and the condition equating the marginal rate of substitution between consumption and child-rearing to the ratio of the respective marginal costs,

$$\frac{1/c_j(t)}{\psi/b_j(t)} = \frac{p(t)}{\gamma w(t)}, \qquad \text{or} \qquad b_j(t) = \frac{\psi}{\gamma} \cdot \frac{p(t)c_j(t)}{w(t)}, \tag{4}$$

[5] To preserve expositional clarity, we collect all the derivations and proofs of the propositions in a separate online Appendix.

[6] Using standard notation, the time-derivative of variable $x_j(t)$ is $\dot{x}_j(t) \equiv \mathrm{d}x_j(t)/\mathrm{d}t$.

[7] Assuming log-separability of the instantaneous utility function is necessary to aggregate wealth constraints across cohorts in the Yaari–Blanchard framework (see Blanchard, 1985). With non-separable functions – e.g., imposing strict complementarity/substitutability between consumption and fertility – expenditure shares become dependent on shadow prices and make analytical aggregation unfeasible.

[8] There are many competing theories of fertility in the literature. Our specification emphasizes the trade-off between reproduction and labor participation, which is indeed an empirically relevant phenomenon (Attanasio et al., 2008). Alternative specifications (e.g., Peretto and Valente, 2015) in which child-rearing costs take the form of additional consumption expenditures – i.e., a form of *inter vivos* transfers during childhood – would not change our conclusions.

[9] Conceptually, the source of 'disconnected accumulation plans' is not the absence of bequests as such, but rather the absence of a mechanism that would maximize *all* descendants' utilities over an *infinite* time horizon. In fact, expression (1) can be reinterpreted as the objective function of a myopic dynasty where altruism exists but only operates over a limited time horizon. This is indeed a popular, alternative interpretation of the Yaari–Blanchard model.

where γw is the marginal cost of reproduction in terms of foregone labor income. The second expression in (4) emphasizes that individual fertility is proportional to the ratio between consumption expenditure and the wage.

2.2. Aggregation and population dynamics

Denoting by $k_j(t)$ the size of cohort j at time t, adult population L and total births B at time t equal $L(t) \equiv \int_{-\infty}^{t} k_j(t)dj$ and $B(t) \equiv \int_{-\infty}^{t} k_j(t)b_j(t)dj$, respectively. Similarly, total assets A and aggregate consumption C equal $A(t) \equiv \int_{-\infty}^{t} k_j(t)a_j(t)dj$ and $C(t) \equiv \int_{-\infty}^{t} k_j(t)c_j(t)dj$. We define *per capita* variables by referring to L as the economy's population: births, assets, and consumption per capita are respectively $b \equiv B/L$, $a \equiv A/L$, and $c \equiv C/L$. Since individuals are homogeneous within cohorts, the size of each cohort declines over time at rate δ, which represents the economy's mortality rate. Population evolves according to the demographic law

$$\dot{L}(t) = B(t) - \delta L(t). \tag{5}$$

Aggregating the individual fertility decision (4) across cohorts, we obtain the following equilibrium relationship between the economy's gross fertility, consumption expenditure and the wage:

$$B(t) = \frac{\psi}{\gamma} \cdot \frac{p(t)\int_{-\infty}^{t} k_j(t)c_j(t)dj}{w(t)} = \frac{\psi}{\gamma} \cdot \frac{p(t)C(t)}{w(t)}. \tag{6}$$

Aggregating the individual budget (2) across cohorts yields the following expression for the growth rate of aggregate wealth:

$$\frac{\dot{A}(t)}{A(t)} = r(t) + \frac{w(t)(L(t) - \gamma B(t))}{A(t)} - \frac{p(t)C(t)}{A(t)}, \tag{7}$$

where the term $L - \gamma B$ is aggregate labor supply.

2.3. Consumption and wealth dilution

We define human wealth as

$$h(t) \equiv \int_{t}^{\infty} w(s) \cdot e^{-\int_t^s (r(v)+\delta)dv} ds. \tag{8}$$

Combining the fertility Eq. (4) with the budget constraint (2), we obtain the expenditure of an individual born at time j as

$$p(t)c_j(t) = \frac{\rho + \delta}{1 + \psi} \cdot \left[a_j(t) + h(t)\right]. \tag{9}$$

This expression shows that individual expenditure is proportional to individual wealth, the sum of financial and human wealth. The preference for children, ψ, reduces the individual propensity to consume. Integrating individual expenditures across cohorts and dividing by the population level, we write consumption expenditure per capita as

$$p(t)c(t) = \frac{\rho + \delta}{1 + \psi} \cdot [a(t) + h(t)]. \tag{10}$$

Despite their apparent similarity, expressions (9) and (10) represent different objects. In the individual expenditure function, both $c_j(t)$ and $a_j(t)$ are chosen by individuals given $a_j(j) = 0$. The per capita variables $c(t)$ and $a(t)$, instead, are averages determined by the age structure of the population. This distinction is important when computing growth rates. Time-differentiation of (10) yields

$$\frac{\dot{c}(t)}{c(t)} + \frac{\dot{p}(t)}{p(t)} = r(t) - \rho - \underbrace{\frac{\psi(\rho + \delta)}{\gamma(1 + \psi)} \cdot \frac{a(t)}{w(t)}}_{\text{Financial wealth dilution}}. \tag{11}$$

Comparing this expression to the individual Euler Eq. (3), we see that the growth rates of individual and per capita consumption expenditure differ by the last term in (11). This term is the rate of *financial wealth dilution* due to fertility, i.e., the decline in wealth per capita caused by the arrival of a new cohort with B members born with zero assets. Combining Eqs. (10) and (6), we have

$$\underbrace{\frac{\psi(\rho + \delta)}{\gamma(1 + \psi)} \cdot \frac{a(t)}{w(t)}}_{\text{Financial wealth dilution}} = \frac{A(t)/L(t)}{h(t) + A(t)/L(t)} \cdot \frac{B(t)}{L(t)}. \tag{12}$$

Financial wealth dilution affects per capita consumption growth because generations are disconnected: new cohorts enter the economy with zero assets and start pursuing their own accumulation and fertility plans independently from their predecessors. This process makes the consumption possibilities of each generation subject to the accumulation and fertility decisions of subsequent generations, creating a form of wealth dilution that does not arise in models with perfect dynastic altruism, where the head of the dynasty maximizes dynastic utility over an infinite horizon. While these general characteristics of the wealth dilution mechanism have long been recognized in the literature (Buiter, 1988; Weil, 1989), our analysis adds an important insight: in our model wealth dilution interacts with fertility choice and is thus both a consequence and a determinant of population dynamics. More precisely, financial wealth dilution reduces the economy's fertility rate by reducing individual consumption, as we show next.

2.4. Fertility dynamics: Expenditure and wage channels

To gain insight on the population-fertility feedback, consider how the fertility rate, b, responds to a change in population, L, for given aggregate financial wealth, A, and individual human wealth, h. From (6) and (10), the fertility rate equals

$$b(t) = \frac{\psi}{\gamma} \frac{\overbrace{p(t)c(t)}^{\text{Expenditure channel}}}{\underbrace{w(t)}_{\text{Wage channel}}} = \frac{\psi}{\gamma}\frac{\rho+\delta}{1+\psi}\left[\frac{A(t)}{L(t)} + h(t)\right]\frac{1}{w(t)} \tag{13}$$

This expression shows that changes in population size affect the fertility rate through two channels: the *expenditure channel* and the *wage channel*. The former incorporates the mechanism of wealth dilution discussed in the previous subsection: accounting for the expenditure decision, we see that given A an increase in L reduces assets per capita, $a = A/L$, and thereby consumption expenditure per capita. Hence, a growing population tends to reduce fertility through the dilution of financial wealth. The wage channel, instead, operates through the effect of population on the equilibrium wage, which is the opportunity cost of reproduction. The sign and strength of the wage channel are determined in the supply side of the economy, which we have not modeled yet. We can nevertheless extract the main insight of this subsection by deriving the general dynamic equation that governs the growth rate of the fertility rate.

Time-differentiating Eq. (13), and substituting both the Euler Eq. (11) for consumption growth and the dynamic wealth constraint (7) in per capita terms, we obtain

$$\frac{\dot{b}(t)}{b(t)} = b(t)\left(1+\gamma\frac{1+\psi}{\psi}\cdot\frac{w(t)}{a(t)}\right) - \rho - \delta - \frac{w(t)}{a(t)} + \frac{\dot{a}(t)}{a(t)} - \frac{\dot{w}(t)}{w(t)} - \frac{\psi(\rho+\delta)}{\gamma(1+\psi)}\cdot\frac{a(t)}{w(t)}, \tag{14}$$

where the last term is the rate of financial wealth dilution. Equation (14) provides fundamental information: it consolidates the aggregation of all households' intertemporal decisions concerning fertility and consumption choices into a single expression that contains only two variables, b and a/w, and their respective growth rates. Therefore, combining (14) with a model of the supply side that determines the a/w ratio and the dynamics of the wage, w, allows us to characterize the equilibrium dynamics as a reduced system in three core variables: population, L, fertility, b, and the asset-wage ratio, a/w.

It should be clear that different specifications of the supply side deliver different predictions. In Appendix we derive the fertility dynamics implied by four alternative production structures – two models with neoclassical technology and two models of endogenous growth – obtaining two main insights. First, neoclassical models tend to generate exponential population growth because they neutralize the role of wealth dilution as a potential stabilizer of the population level. When population expands, the declining capital per worker and the falling wage perfectly offset each other and yield a constant rate of population growth. Second, models of endogenous growth with simultaneous vertical and horizontal innovations (Peretto, 1998; Peretto and Connolly, 2007) can provide a radically different theory of population and fertility dynamics because their core mechanism of wealth creation – the accumulation of intangible assets raising the mass of firms *and* each firm's profitability – may reinforce, instead of neutralize, the wealth dilution effect. We investigate this point by incorporating vertical and horizontal innovations in the production side of our model.

3. The production side

The model of the production side draws on Peretto and Connolly (2007). The final sector produces the consumption good by means of differentiated intermediates supplied by monopolistic firms. Productivity growth is driven by both vertical and horizontal innovations in the intermediate sector: incumbent firms pursue vertical R&D to raise internal productivity; outside entrepreneurs create new firms to enter the market. This setup yields a transparent equilibrium relationship that links the total value of firms to population size and the wage. As mentioned, it also makes our contribution relevant to the recent literature on the decline of "business dynamism" (see, e.g., Decker et al., 2016 and, especially, Karahan et al., 2019).

3.1. Final sector

A representative competitive firm produces the final consumption good by assembling differentiated intermediate products according to the technology

$$C(t) = N(t)^{\chi-\frac{\epsilon}{\epsilon-1}} \cdot \left(\int_0^{N(t)} x_i(t)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}, \tag{15}$$

where N is mass of intermediates, x_i is the quantity of the i-th intermediate good, $\epsilon > 1$ is the elasticity of substitution between pairs of intermediates and $\chi > 1$ is the degree of increasing returns to specialization. Profit maximization taking the mass of goods and the price p_{xi} of each good i as given yields the final producer's demand schedule.

3.2. Intermediate producers: Incumbents

The typical intermediate firm produces according to the technology

$$x_i(t) = z_i(t)^{\theta} \cdot (\ell_{xi}(t) - \varphi), \tag{16}$$

where z_i is firm-specific knowledge, $\theta \in (0, 1)$ is the elasticity of labor productivity with respect to knowledge, ℓ_{xi} is labor employed in production and $\varphi > 0$ is overhead labor. The firm accumulates knowledge according to

$$\dot{z}_i(t) = \omega Z(t) \cdot \ell_{zi}(t), \tag{17}$$

where ℓ_{zi} is labor employed in vertical R&D. The productivity of R&D employment is given by parameter $\omega > 0$ times the economy's stock of public knowledge

$$Z(t) = \frac{1}{N(t)} \int_0^{N(t)} z_j(t) dj. \tag{18}$$

This expression posits that public knowledge is a weighted sum of the firm-specific stocks of knowledge, z_j. The intuition is that firms cross-fertilize each other: when firm j develops a more efficient process to produce its own differentiated good, it also generates non-excludable knowledge that flows into the public domain. The mass of firms affects the intensity of such spillovers since the impact of any given stock of firm-specific knowledge, z_j, on public knowledge, Z, becomes weaker as the mass of firms increases (Peretto and Smulders, 2002).

Each intermediate firm faces a constant probability $\mu > 0$ of disappearing.[10] Therefore, the incumbent monopolist at time t chooses the time paths $\{p_{xi}, x_i, \ell_{xi}, \ell_{zi}\}$ that maximize the present-value of the expected profit stream

$$V_i(t) = \int_t^{\infty} [p_{xi}(t)x_i(t) - w(t)\ell_{xi}(t) - w(t)\ell_{zi}(t)] e^{-\int_t^{v}(r(s)+\mu)ds} dv, \tag{19}$$

subject to the technologies (16) and (17) and to the demand schedule of the final producer. The solution to this problem yields the standard mark-up pricing rule (see Appendix) and the dynamic no-arbitrage condition

$$r(t) = \left[\theta \cdot \frac{\epsilon - 1}{\epsilon} \cdot \frac{p_{xi}(t)x_i(t)}{z_i(t)} \cdot \frac{\omega Z(t)}{w(t)}\right] + \frac{\dot{w}(t)}{w(t)} - \frac{\dot{Z}(t)}{Z(t)} - \mu. \tag{20}$$

This expression equates the interest rate, r, to the firm's rate of return from knowledge accumulation, where the term in square brackets is the marginal profit from increasing firm-specific knowledge, z_i.

3.3. Intermediate producers: Entrants

Agents allocate labor to developing new intermediate goods, designing the associated production processes and setting up firms to serve the market. This process of horizontal innovation or, equivalently, entrepreneurship, expands the mass of firms, N. At time t, an entrant, denoted i without loss of generality, correctly anticipates the value $V_i(t)$ that the new firm creates. Recalling that a constant fraction $\mu > 0$ of the existing firms disappears in each instant, the net increase in the mass of firms is

$$\dot{N}(t) = \eta \frac{N(t)}{L(t)^{\varkappa}} \ell_N(t) - \mu N(t), \qquad 0 \leqslant \varkappa < 1, \tag{21}$$

where ℓ_N is labor employed in entry.[11] The productivity of labor in entry depends on the exogenous parameter $\eta > 0$ and on two endogenous variables, the mass of firms and population size. The positive effect of the mass of firms, N, captures the intertemporal spillovers characteristic of the first-generation models of endogenous growth (Romer, 1990). The negative effect of population size, represented by the term $1/L^{\varkappa}$, captures the notion that entering large markets requires more effort (Peretto and Connolly, 2007): parameter $\varkappa$ regulates the intensity of this effect. Our results remain valid if we set $\varkappa = 0$. Given technology (21), the free-entry condition reads[12]

$$V_i(t) = \frac{w(t)L(t)^{\varkappa}}{\eta N(t)}. \tag{22}$$

[10] Parameter μ is the average death rate of intermediate firms. In the main text, we refer to μ as to the rate of product obsolescence in order to avoid confusion with the households' death rate δ.

[11] The hypothesis that gross firm creation $\dot{N} + \mu N$ is linear in one type of R&D labor, ℓ_N, simplifies the analysis, but is not strictly necessary to obtain our main results: see footnote [13].

[12] Given the entry technology (21), the free entry condition (22) establishes that the total value of new firms, $\int_0^{\dot{N}+\mu N} V_i di$, matches the total cost of their creation, $w\ell_N$.

This expression states that the financial market prices firms at their cost of creation. Wealth creation thus has two dimensions: while incumbent firms accumulate knowledge to raise their market valuation, free-entry pins down the market valuation of firms from the cost of creation side. Since both in-house R&D and entrepreneurship employ labor, the wage and the value of firms are jointly determined by both activities in equilibrium.

3.4. Knowledge, wage and assets

The model exhibits a symmetric equilibrium where firms make identical decisions. The labor market clearing condition reads

$$\ell_X(t) + \ell_Z(t) + \ell_N(t) = L(t) - \gamma B(t), \tag{23}$$

where $\ell_X \equiv N\ell_{xi}$ and $\ell_Z \equiv N\ell_{zi}$ are aggregate employment in intermediate production and in vertical R&D, respectively, and the right-hand side is total labor supply. Combining (23) with the profit-maximizing conditions of intermediate producers, we obtain the equilibrium real wage

$$\frac{w(t)}{p(t)} = \frac{\epsilon - 1}{\epsilon} Z(t)^{\theta} N(t)^{\chi - 1}. \tag{24}$$

This expression shows that the real wage is a function of both dimensions of technology, namely, average firm-specific knowledge, Z, and the aggregate stock of knowledge accumulated through horizontal innovation, N.

Equilibrium of the financial market requires $A = NV$ so that the free-entry condition (22) yields

$$A(t) = N(t)V(t) = \frac{w(t)L(t)^{\varkappa}}{\eta}. \tag{25}$$

Combining (25) with (24), we can write

$$\frac{A(t)}{p(t)} = \frac{\epsilon - 1}{\epsilon \eta} \cdot \frac{Z(t)^{\theta} N(t)^{\chi - 1}}{L(t)^{\varkappa}}. \tag{26}$$

This expression shows that the economy's *real* aggregate wealth has three fundamental determinants: average firm-specific knowledge, Z; mass of firms, N; population, L.

4. General equilibrium

This section merges the demographic block of the model (Section 2) with the supply side (Section 3) and characterizes the resulting equilibrium dynamics. We show that the combination of wealth creation and wealth dilution generates a steady state in which a constant endogenous population *level* coexists with constant endogenous *growth* of income per capita. We take the final good as our numeraire and set $p(t) = 1$.

4.1. The dynamic system

Our discussion of intertemporal choices (Section 2.4) showed that the equilibrium dynamics reduce to a system comprising three elements: the demographic law (5), the fertility Eq. (14) and the supply side of the economy. The key ingredient coming from the supply side is the equilibrium relationship (25), which links the total value of firms to labor productivity in firm creation. Dividing both sides of (25) by population size, we obtain

$$\frac{a(t)}{w(t)} = \frac{1}{\eta L(t)^{1-\varkappa}}. \tag{27}$$

Eq. (27) says that the equilibrium value of the asset-wage ratio is strictly decreasing in population, L, even when we set $\varkappa = 0$. The intuition is that when population increases, the wage response to the change in labor supply does not offset the wealth dilution effect: the larger population L causes a drop in financial wealth per capita $a = A/L$ that outweighs the decline in the wage rate w. This result originates in the free-entry condition (22) whereby the value of firms matches the cost of firms creation.[13]

The negative relationship between a/w and L has crucial consequences for fertility dynamics because from (13) the fertility rate is positively related to assets per capita and negatively related to the wage. In fact, condition (27) turns out to be essential to obtain a *negative feedback* of population on fertility along the equilibrium path and, hence, to produce a theory

[13] Result (27) incorporates the entry technology (21), which postulates linear returns to R&D labor ℓ_N. Alternative entry technologies where ℓ_N exhibits diminishing marginal returns – possibly including further rival inputs – are also compatible with the negative relationship between a/w and L and would not affect its steady-state properties, but would substantially complicate the analysis of the dynamics by adding further labor re-allocation effects during the transition.

of finite population. Using the demographic law (5) and using (27) to substitute a/w in the fertility Eq. (14), we obtain the autonomous dynamic system:

$$\frac{\dot{L}(t)}{L(t)} = b(t) - \delta; \tag{28}$$

$$\frac{\dot{b}(t)}{b(t)} = \frac{\gamma(1+\psi)b(t) - \psi}{\psi}\eta L(t)^{1-\varkappa} - \rho + \varkappa(b(t) - \delta) - \frac{\psi(\rho+\delta)}{\gamma(1+\psi)}\frac{1}{\eta L(t)^{1-\varkappa}}. \tag{29}$$

Eq. (29) delivers the complete picture of the feedback effects of population on fertility along the equilibrium path: larger population reduces assets per capita relative to the wage, a/w, and this reduces fertility via financial wealth dilution – the last term in (29) – and via changes in the rate of return to assets, which modifies the agents' consumption possibilities and their willingness to rear children. System (28)-(29) determines the dynamics of population and fertility rates. The stationary loci are:

$$\dot{L} = 0 \qquad \Rightarrow \qquad b = \delta; \tag{30}$$

$$\dot{b} = 0 \qquad \Rightarrow \qquad \bar{b}(L) = \frac{\varkappa\delta + \eta L^{1-\varkappa}}{\varkappa + \gamma\frac{1+\psi}{\psi}\eta L^{1-\varkappa}} + \frac{\rho\eta L^{1-\varkappa} + \frac{\psi}{\gamma}\frac{\rho+\delta}{1+\psi}}{\varkappa\eta L^{1-\varkappa} + \gamma\frac{1+\psi}{\psi}\left(\eta L^{1-\varkappa}\right)^2}. \tag{31}$$

The $\dot{L} = 0$ locus establishes that population is constant when the fertility rate, b, equals the mortality rate. The $\dot{b} = 0$ locus is a negative relationship between fertility and population, $\bar{b}(L)$, displaying the properties (see Appendix):

$$\partial\bar{b}(L)/\partial L < 0; \qquad \lim_{L\to 0}\bar{b}(L) = +\infty; \qquad \lim_{L\to\infty}\bar{b}(L) = \frac{\psi}{\gamma(1+\psi)}. \tag{32}$$

These properties ensure the existence of a steady state (L_{ss}, b_{ss}) where fertility is at replacement and population is constant. The phase diagram in Fig. 1, graph (a), shows that such steady state exists when the $\dot{L} = 0$ locus lies strictly above the horizontal asymptote of the $\dot{b} = 0$ locus, given by the second limit appearing in (32). Consequently, the steady state (L_{ss}, b_{ss}) exists and is unique if parameters satisfy

$$\gamma\delta(1+\psi) > \psi. \tag{33}$$

The intuition behind this condition is that the negative feedback of population on fertility brings population growth to a halt when the marginal cost of child-bearing γ is high relative to the preference for children ψ, given the probability of death, δ. In the remainder of the analysis, we assume that (33) holds (see Appendix for further details on existence).

4.2. The steady state with constant population

When the steady state (L_{ss}, b_{ss}) exists, the model delivers a non-Malthusian theory of the *population level*. The phase diagram in Fig. 1, graph (a), shows that given the initial population $L(0)$, the economy jumps onto the saddle path by selecting initial fertility $b(0)$ and then converges to the steady state.[14] The trajectory that starts from $L(0) < L_{ss}$ represents the case which is empirically relevant for most developed countries: population grows during the transition, but the fertility rate declines and eventually becomes equal to the mortality rate, δ. The following proposition formalizes the result.

Proposition 1. *If parameters satisfy $\gamma\delta(1+\psi) > \psi$, the steady state (L_{ss}, b_{ss}) is saddle-point stable and represents the long-run equilibrium of the economy:*

$$\lim_{t\to\infty} b(t) = b_{ss} \equiv \delta; \tag{34}$$

$$\lim_{t\to\infty} L(t) = L_{ss} \equiv \left[\frac{\psi}{\eta 2}\cdot\frac{\rho + \sqrt{\rho^2 + 4(\rho+\delta)\left(\delta - \frac{\psi}{\gamma(1+\psi)}\right)}}{\gamma(1+\psi)\delta - \psi}\right]^{\frac{1}{1-\varkappa}}; \tag{35}$$

$$\lim_{t\to\infty}\frac{a(t)}{w(t)} = \left(\frac{a}{w}\right)_{ss} \equiv \frac{1}{\eta L_{ss}^{1-\varkappa}} = \frac{2}{\psi}\cdot\frac{\gamma(1+\psi)\delta - \psi}{\rho + \sqrt{\rho^2 + 4(\rho+\delta)\left(\delta - \frac{\psi}{\gamma(1+\psi)}\right)}}. \tag{36}$$

The most striking result in Proposition 1 is that in the long run the ratio between assets per capita and the wage depends exclusively on demographic factors and preferences: expression (36) shows that a/w converges to the steady-state

[14] The diverging trajectories in Fig. 1, graph (a), can be ruled out by standard arguments (i.e., they would imply explosive dynamics in $b(t)$ violating the conditions for utility maximization).

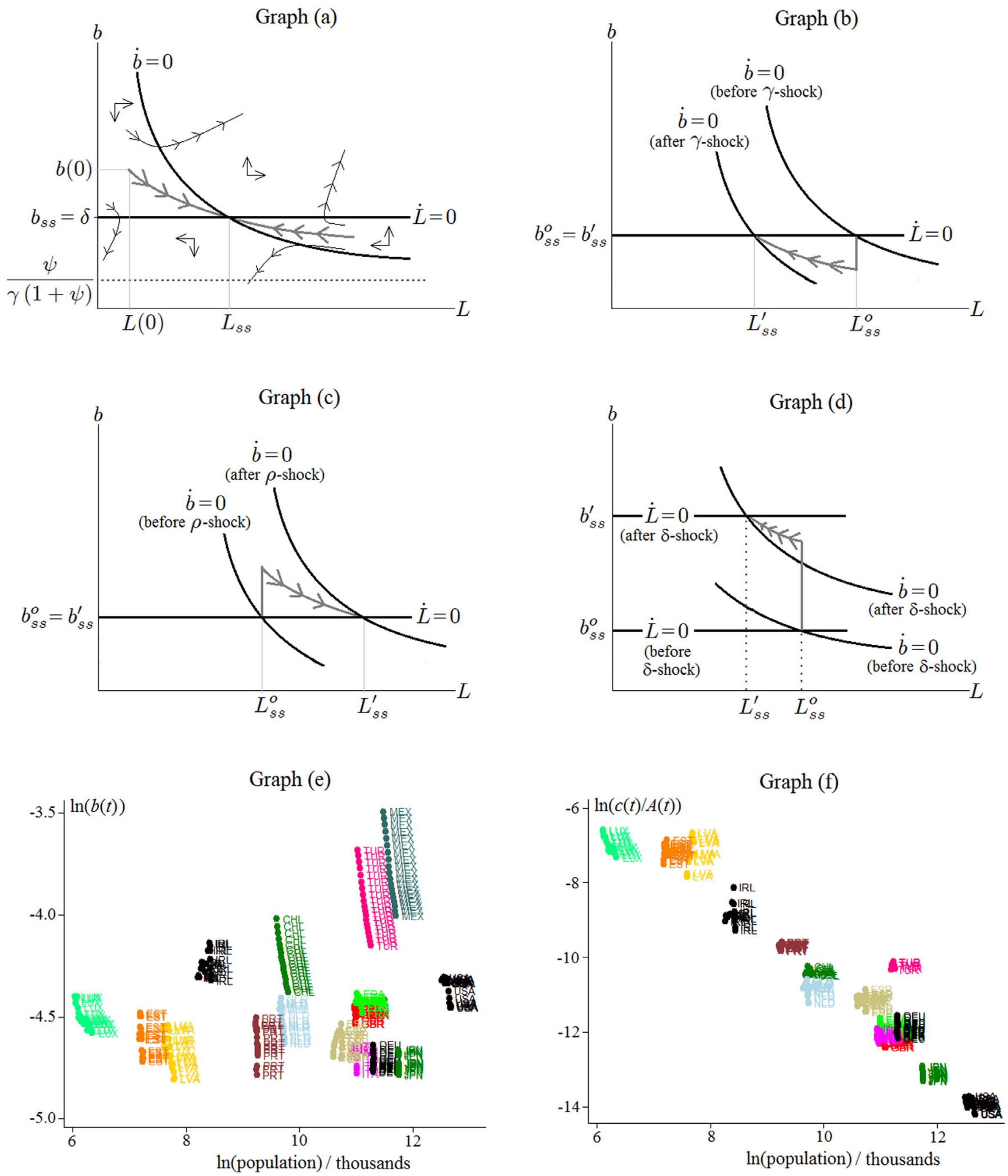

Fig. 1. Graph (a): Phase diagram of system (28)-(29). Graph (b): Effects of an increase in γ. Graph (c): Effects of an increase in ρ. Graph (d): Effects of an increase in δ. Graph (e): Scatter diagram in logarithmic scale of population versus fertility rates in OECD countries. Graph (f): Scatter diagram in logarithmic scale of population versus consumption per capita divided by financial wealth in OECD countries.

value $(a/w)_{ss}$ that does not depend on technology parameters. Nonetheless, the entry technology (21) affects steady-state population: (35) shows that the steady-state value L_{ss} contains the parameters η and $\varkappa$. The reason for these results is that the dominant feedback of population on fertility comes from financial wealth dilution, which originates in the economy's demographic structure and process of wealth creation.[15] As the economy grows, households adjust fertility until they achieve the specific wealth-to-wage ratio $(a/w)_{ss}$ that stabilizes their marginal rate of substitution between consumption and child-rearing, at which point the economy is in the steady state. Although the specific level $(a/w)_{ss}$ depends only on

[15] The fact that the core mechanism stabilizing population is wealth dilution is confirmed by condition (33), which establishes that the existence of the steady state (L_{ss}, b_{ss}) only depends on demographic and preference parameters, $(\delta, \gamma, \psi, \rho)$.

Table 1
Fixed-effects panel regression results for OECD countries, 1995–2016.

	(1) ln(b)	(2) ln(b)	(3) ln(c/A)	(4) ln(c/A)	(5) ln(c/A)
ln(L)	-0.503***	-0.421***	-1.554***	-0.512*	-0.401
	(0.0963)	(0.0831)	(0.439)	(0.258)	(0.313)
Trend		-0.00107		-0.0110***	-0.00891*
		(0.00113)		(0.00379)	(0.00484)
ln(b)					0.448*
					(0.253)
Constant	3.782**	2.444*	15.40**	-1.629	-1.443
	(1.572)	(1.354)	(7.199)	(4.230)	(4.394)
Observations	665	665	606	606	549
Number of groups	35	35	33	33	33
R^2	0.118	0.121	0.115	0.177	0.148

Notes: All estimations are panel fixed effects. Driscoll–Kraay robust standard errors are shown in parentheses; these correct for cross-sectional dependencies in our sample, as well as heteroskedasticity and within-country autocorrelation. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

demographic and preference parameters, population in the long run still depends on technology because the steady-state population size, L_{ss}, that is compatible with $(a/w)_{ss}$ depends on the response of the wage to population size through the entry technology.

Three remarks on the transitional dynamics are in order. First, the dynamic system (28) and (29) determines the equilibrium path of the *consumption-assets* ratio. Combining (13) with (27), we obtain

$$\frac{C(t)}{A(t)} = \frac{\gamma}{\psi} \cdot \frac{b(t)w(t)}{a(t)} = \frac{\gamma}{\psi} \cdot b(t)\eta L(t)^{1-\varkappa}. \tag{37}$$

In the long run,

$$\lim_{t\to\infty} \frac{C(t)}{A(t)} = \left(\frac{C}{A}\right)_{ss} = \frac{\gamma}{\psi} \cdot \frac{b_{ss}}{(a/w)_{ss}} \tag{38}$$

which, by Proposition 1, depends exclusively on demography and preference parameters. The property that demographic forces determine both a/w and C/A implies that demography is a major driver of the functional distribution of income, an important result that we discuss in Section 5.

The second remark concerns the nature of the steady state. Eq. (35) says that steady-state population size depends on preference parameters, fertility costs and the productivity of labor in creating new firms. It does not depend on fixed factors (e.g., natural resources) as we purposefully omitted them from the model. In other words, the steady state (L_{ss}, b_{ss}) is *non-Malthusian*: Human population is not limited by binding physical constraints set by finite natural resources. This is a distinctive result of our model because the existing literature predicts that a finite endogenous population size is the outcome of Malthusian mechanisms. The idea that constant population results from the dilution of *financial* wealth – where assets represent ownership of firms created by labor – is an original insight of our analysis that deserves empirical scrutiny.

The third remark concerns the transitional co-movements of fertility and consumption. The time path of C/A is not necessarily monotonic because b and L move in opposite directions over time. However, Eq. (37) says that the path of the ratio between consumption *per capita* and *total* assets is monotonic because c/A is increasing in the fertility rate and decreasing in population. In particular, starting from $L(0) < L_{ss}$, the transition features (i) declining fertility associated to (ii) population growth and (iii) declining c/A ratio. These equilibrium co-movements are empirically plausible. Interpreting b as the annual crude birth rate and c as household final consumption divided by total population L, we can calculate the empirical counterpart of the c/A ratio for all OECD countries by identifying A with financial net of worth of households as reported in OECD (2017). The overall panel dataset covers the 1995–2016 period and only excludes Mexico and New Zealand due to lack of data on wealth for these countries.[16] Table 1 reports results from panel estimations including country fixed-effects and country-specific time trends. Columns [1]-[2] report a strong negative fertility-population relationship, while columns [3]-[5] report an inverse relationship between c/A and population. The scatter plots reported in Fig. 1, Graphs (e)-(f), which refer to a sub-sample of 16 countries for the sake of clarity, confirm that the shape of the saddle path predicted by our model is consistent with panel data for OECD economies.

[16] See the Appendix for further details on data sources and list of countries.

4.3. Wealth creation and output growth

The economy's rate of wealth creation depends on both horizontal and vertical innovations. Time-differentiating (26), the growth rate of wealth is

$$\frac{\dot{A}(t)}{A(t)} = \theta\frac{\dot{Z}(t)}{Z(t)} + (\chi - 1)\frac{\dot{N}(t)}{N(t)} - \varkappa\frac{\dot{L}(t)}{L(t)}. \tag{39}$$

Provided that certain restrictions hold, both types of R&D are active along the equilibrium path (see Appendix for details). The rates of vertical and horizontal innovation rates are, respectively:[17]

$$\frac{\dot{Z}(t)}{Z(t)} = (1-\gamma b(t))\frac{w(t)}{a(t)} + \left[\left(\frac{\epsilon-1}{\epsilon}\right)\frac{\omega\theta}{\eta}\frac{L(t)^{\varkappa}}{N(t)} - 1\right]\frac{c(t)}{a(t)} - \varkappa\frac{\dot{L}(t)}{L(t)} - \mu; \tag{40}$$

$$\frac{\dot{N}(t)}{N(t)} = (1-\gamma b(t))\frac{w(t)}{a(t)} - \frac{\epsilon-1}{\epsilon L(t)^{2\varkappa}}\frac{c(t)}{a(t)} - \mu - \left[\frac{\eta}{L(t)^{\varkappa}}\left(\varphi + \frac{1}{\omega}\frac{\dot{Z}(t)}{Z(t)}\right)\right]N(t). \tag{41}$$

The time paths of w/a and c/a are determined by the dynamic system studied in the previous subsection. The central message of (40) and (41) is that the growth rates of firm-specific knowledge and of the mass of firms exhibit negative co-movement over time. While the entry of new firms reduces the profitability of each individual firm through market fragmentation, and thereby the incentive to invest in R&D in-house, investment in firm-specific knowledge slows down entry by diverting labor away from horizontal R&D activity.[18] Importantly, these co-movements guide the economy towards a long-run equilibrium in which vertical R&D generates sustained income per capita growth whereas the mass of firms converges to a constant level.

Proposition 2. *In the steady state (L_{ss}, b_{ss}), the mass of firms is constant and finite, $\lim_{t\to\infty} N(t) = N_{ss} > 0$. During the transition, the mass of firms follows a logistic process of the form*

$$\frac{\dot{N}(t)}{N(t)} = q_1(b(t), L(t)) - q_2(b(t), L(t)) \cdot N(t), \tag{42}$$

where $q_1(b, L)$ and $q_2(b, L)$ converge to the finite constant values $q_1(b_{ss}, L_{ss}) > 0$ and $q_2(b_{ss}, L_{ss}) > 0$ in the long run. With active vertical R&D, the long-run mass of firms equals

$$\lim_{t\to\infty} N(t) = N_{ss} \equiv \frac{\eta L_{ss}^{1-\varkappa}\left[1 - \gamma b_{ss} - \left(\theta + \frac{1}{L_{ss}^{2\varkappa}}\right)\frac{\epsilon-1}{\epsilon}\frac{\gamma}{\psi}b_{ss}\right] - \mu}{\varphi - \frac{1}{\omega}\left[\frac{(1+\psi)\gamma b_{ss} - \psi}{\psi}\eta L_{ss}^{1-\varkappa} + \mu\right]} \cdot \frac{L_{ss}^{\varkappa}}{\eta} > 0 \tag{43}$$

which exhibits $dN_{ss}/dL_{ss} > 0$ for any $\varkappa \in [0, 1)$.

Proposition 2 establishes that the process of firms' entry eventually stops, a general result that holds regardless of whether vertical R&D is operative. The intuition is that entrepreneurs create new firms as long as their anticipated market share yields the desired rate of return but, as new firms join the intermediate sector, each firm's market share declines and the profitability of entry eventually vanishes due to the competing use of labor in the production of intermediates – which is subject to the fixed operating cost $\varphi > 0$ – and in vertical R&D activities if operative.[19] When the mass of firms approaches the steady-state N_{ss}, further product creation is not profitable given labor supply and aggregate consumption expenditure. In other words, the profitability of entry adjusts to the endogenous values (b_{ss}, L_{ss}). This process explains why the long-run mass of firms N_{ss} is increasing in population size L_{ss}: a larger population increases the number of firms that the market for intermediate goods accommodates with profitability commensurate with the rate of return demanded by savers.

In steady state population and the mass of firms are constant and the source of productivity growth is vertical R&D. Eq. (40) yields

$$\lim_{t\to\infty}\frac{\dot{Z}(t)}{Z(t)} = g_Z^{ss} \equiv \frac{\epsilon-1}{\epsilon}\cdot\frac{\omega\theta}{\eta}\frac{L_{ss}^{\varkappa}}{N_{ss}}\left(\frac{c}{a}\right)_{ss} + \underbrace{(1-\gamma b_{ss})\left(\frac{w}{a}\right)_{ss} - \left(\frac{c}{a}\right)_{ss} - \mu}_{\frac{\dot{A}(t)}{A(t)} - r(t) - \mu}, \tag{44}$$

[17] Eq. (40) follows from aggregating the return to firm-specific knowledge (20) across firms. Eq. (41) follows from the entry technology (21) and the labor market clearing condition (23).

[18] The market-fragmentation effect is captured by the term in square brackets in (40): an increase in N reduces $\dot{Z}/Z$ by squeezing the marginal profit that each firm gains from investing in own knowledge. The labor-reallocation mechanism that negatively affects horizontal R&D is captured by the last term in (41).

[19] In the logistic process (42), the term $q_1(b, L)$ represents the incentive to create a new firm, given by the market share anticipated by individual entrepreneurs, whereas $q_2(b, L)$ measures the decreased profitability of entry induced by market crowding. See Appendix for detailed derivations.

which is strictly positive as long as the mass of firms N_{ss} is not too large. In the right hand side of (44), the first term captures an intratemporal gain, namely, the increase in firms' profitability given by a marginal increase in firm-specific knowledge, which depends on the ratio between sales and firm value and is thus positively related to (c/a). The second and third terms capture the inter-temporal net gains from R&D investment given by the gap between wealth creation, $\dot{A}/A$, and the effective discount rate, $r+\mu$.

The economy's rate of wealth creation obeys equation (39). Since the mass of firms is asymptotically constant, $\dot{N}/N \to 0$, the growth rate of assets in the long run is proportional to that of knowledge, $\dot{A}/A \to \theta \cdot g_Z^{ss}$, and the same growth rate applies to final output in view of stationarity of the consumption-wealth ratio. The economy's long-run growth rate thus equals[20]

$$\lim_{t\to\infty}\frac{\dot{A}(t)}{A(t)}=\lim_{t\to\infty}\frac{\dot{C}(t)}{C(t)}=\theta g_Z^{ss}=\theta\left[1-\gamma b_{ss}+\left(\omega\theta\frac{\epsilon-1}{\eta\epsilon}L_{ss}^{\varkappa}-N_{ss}\right)\frac{\gamma}{\psi}\frac{b_{ss}}{N_{ss}}\right]\eta L_{ss}^{1-\varkappa}-\theta\mu. \tag{45}$$

This expression shows that both technology and demography affect the pace of knowledge accumulation and, hence, economic growth in the long run. In particular, demography affects economic growth by modifying the composition of R&D investment: a higher steady-state population L_{ss} tends to boost horizontal innovations, yielding a larger steady-state mass of firms N_{ss} (see Proposition 2). This mechanism plays a central role in determining the welfare consequences of demographic shocks, a point that we address in the quantitative analysis of Section 6.

5. Demography, grand ratios and migration

As mentioned, our model delivers predictions for macroeconomic grand ratios that are in stark contrast with most traditional growth models. This section discusses these and related results by extending the model to include migration.

5.1. Exogenous shocks

The following Proposition summarizes the effects of changes in the time cost of reproduction, the time preference rate and the probability of death.

Proposition 3. *Increases in γ, ρ, and δ modify the steady-state as follows:*

$$\begin{aligned} \mathrm{d}b_{ss}/\mathrm{d}\gamma &= 0 \quad \text{and} \quad \mathrm{d}L_{ss}/\mathrm{d}\gamma < 0;\\ \mathrm{d}b_{ss}/\mathrm{d}\rho &= 0 \quad \text{and} \quad \mathrm{d}L_{ss}/\mathrm{d}\rho > 0;\\ \mathrm{d}b_{ss}/\mathrm{d}\delta &> 0 \quad \text{and} \quad \mathrm{d}L_{ss}/\mathrm{d}\delta < 0. \end{aligned}$$

Fig. 1 describes these results in three phase diagrams where the economy is initially in the steady state (L_{ss}^o, b_{ss}^o) and then moves to the steady state (L_{ss}', b_{ss}'). An increase in γ reduces steady-state population but does not affect steady-state fertility: while higher reproduction costs prompt workers to have fewer children during the transition, the fertility rate b_{ss} reverts to δ. An increase in ρ raises the propensity to consume out of wealth and yields higher consumption and fertility at earlier dates over the life-cycle. This 'discounting effect' yields higher fertility during the transition and, hence, larger steady-state population, L_{ss}. The result d $L_{ss}/\mathrm{d}\delta<0$ arises from two contrasting effects. On the one hand, a higher δ affects intertemporal choices in the same way as a higher time-preference rate. This *discounting effect* of δ tends to increase L_{ss} via the same mechanism as the increase in ρ. On the other hand, a higher mortality rate lowers population growth and this *mortality effect* tends to reduce L_{ss} and increase b_{ss}. In the proof of Proposition 3, we establish that the mortality effect dominates the discounting effect so that the higher probability of death reduces steady-state population.[21] An interesting consequence is that an increase in life expectancy – the reciprocal of the probability of death, $1/\delta$ – affects the wage-to-wealth ratio in the same way as an increase in the impatience rate, ρ. This prediction is opposite to Blanchard (1985) neoclassical model, where changes in the probability of death have only discounting effects.[22]

5.2. The grand ratios: Labor income, consumption and wealth

In the steady state (L_{ss}, b_{ss}) the determinants of many macroeconomic variables are qualitatively different from those of traditional growth models. A useful benchmark for comparison is Blanchard (1985), which combines (Yaari, 1965) demographic structure with the neoclassical supply side. In that framework diminishing returns yield that capital and population grow at the same rate in steady state. This notion of balanced growth also applies to neoclassical models with endogenous fertility (e.g., Barro and Becker (1989)). We can summarize the main differences between our predictions and the traditional

[20] The last term in (45) is obtained from (44) by substituting $(w/a)_{ss}$ and $(c/a)_{ss}$ with the steady-state values reported in (36) and (38).

[21] In Fig. 1, graph (d), the upward shift in the $\dot{b}=0$ locus represents the discounting effect whereas the upward shift in the $\dot{L}=0$ locus represents the mortality effect. The initial and final steady states, respectively denoted by (L_{ss}^o, b_{ss}^o) and (L_{ss}', b_{ss}'), can be immediately compared to those generated by the time-preference shock described in graph (c).

[22] We prove this result in Appendix, below the proof of Proposition 4.

ones by considering three variables, namely, the ratio of labor income to wealth, the consumption-wealth ratio and the ratio of labor income to consumption:

$$\frac{w(t)L(t)(1-\gamma b(t))}{A(t)};\qquad \frac{C(t)}{A(t)};\qquad \frac{w(t)L(t)(1-\gamma b(t))}{C(t)}.$$

Both the traditional framework and our model predict that these grand ratios are stationary but the underlying mechanisms are totally different. Traditional balanced growth hinges on diminishing returns to capital and labor that stabilize the capital-labor ratio: as the growth rate of capital adjusts to that of labor, financial wealth grows at the same rate as labor income while population grows at a constant rate. The key hypothesis is that labor is only used in final production and is combined with capital under constant returns to scale, so that capital-labor dynamics drive the convergence process and technology determines the grand ratios. In our model, instead, demography drives convergence to the steady state because (i) population dilutes wealth and (ii) labor produces not only goods and knowledge, but also new firms the value of which is tied to the cost of creation. As population grows, wealth dilution and the decline in the a/w ratio induced by the entry technology – see (27) – affect households' choices and drive net fertility to zero. In the steady state, population is constant but wage and wealth grow at the same rate because the value of firms depends on the labor cost of firm creation. Since the convergence process hinges on the response of fertility to population, demography and preferences are the fundamental determinants of the grand ratios:

Proposition 4. *In the steady state* (L_{ss}, b_{ss}), *the ratios*

$$\lim_{t\to\infty}\frac{w(t)L(t)(1-\gamma b(t))}{A(t)}=\frac{1-\gamma\delta}{(a/w)_{ss}},\quad \lim_{t\to\infty}\frac{C(t)}{A(t)}=\frac{\gamma}{\psi}\cdot\frac{\delta}{(a/w)_{ss}}\ \text{and}$$
$$\lim_{t\to\infty}\frac{w(t)L(t)(1-\gamma b(t))}{C(t)}=\psi\cdot\frac{1-\gamma\delta}{\gamma\delta}$$

are exclusively determined by demographic and preference parameters, with $(a/w)_{ss}$ *given by (36).*

A major consequence of this property is that in our theory, shocks to demographic or preference parameters – and by extension, public policies affecting reproduction costs or life expectancy – have a first-order effect on the functional distribution of income, individual welfare and economic growth. The quantitative analysis in Section 6 provides an in-depth discussion of this point.

5.3. Migration

Introducing migration is a natural extension of this model. First, as noted by Weil (1989), immigrants are by definition disconnected generations that reinforce wealth dilution. Second, inflows of people affect wealth creation because a larger population attracts entry and results in a larger mass of firms. We assess these mechanisms analytically and quantitatively by making two assumptions that preserve the model's tractability. First, migrants enter or leave the economy exclusively at the beginning of their working age. Second, immigrants have the same preferences and life expectancy as domestic residents.[23]

In the following analysis, $B(t)$ denotes domestic births and $M(t)$ denotes migration. The size of the cohort entering the economy at time j thus is $k(j,j)=B(j)+M(j)$. To amend the model, we modify a few equations from Section 2 and Sections 4.1–4.2 (see Appendix for the details). First, the demographic law (5) becomes

$$\dot{L}(t)=B(t)+M(t)-\delta L(t). \tag{46}$$

Second, immigration boosts wealth dilution: the arrival of further disconnected generations, in addition to domestic births, affects the growth rate of consumption per capita and thereby the dynamics of the fertility rate. Formally, we have the augmented term

$$\underbrace{\frac{\psi(\rho+\delta)}{\gamma(1+\psi)}\cdot\frac{B(t)+M(t)}{B(t)}\cdot\frac{a(t)}{w(t)}}_{\text{Augmented financial wealth dilution}}=\frac{A(t)/L(t)}{h(t)+A(t)/L(t)}\cdot\frac{B(t)+M(t)}{L(t)}. \tag{47}$$

Third, migration modifies the system (28) and (29) and its properties depending on how we specify the behavior of the flow $M(t)$ or, alternatively, of the *net migration rate* defined as $m(t)\equiv M(t)/L(t)$. If we focus on immigration, we can consider two alternatives: a constant inflow, $M(t)=\bar{M}$, or a constant immigration rate, $m(t)=\bar{m}$. In the first case, the immigration rate $m(t)$ is time-varying and subject to the dynamics of population. In the second case, the constant immigration rate $\bar{m}$ implies a time-varying mass of immigrants. Which specification is better depends on the purpose of the analysis. In Section 6 we perform numerical simulations assuming $M(t)=\bar{M}$ in order to assess the effects of immigration restrictions where the policy target is the number of immigrants. Nonetheless, both specifications support our main conclusions and expand our notion

[23] The role of these two hypotheses is merely that of avoiding that migration introduce heterogeneities in preferences or in the age-composition of the population.

of non-Malthusian steady state. In Appendix, we modify the dynamic system (28)-(29) to include migration and we prove the following result.

Proposition 5. *(Steady state with migration) Assuming either $M(t) = \bar{M}$ or $m(t) = \bar{m}$, the equilibrium dynamics of ($L(t)$, $b(t)$, $m(t)$) exhibit a stable steady state (L_{ss}, b_{ss}, m_{ss}) where:*

$$\lim_{t\to\infty} b(t) = b_{ss} \equiv \delta - m_{ss}; \tag{48}$$

$$\lim_{t\to\infty} L(t) = L_{ss} \equiv \left[\frac{\psi}{\eta 2} \cdot \frac{\rho + \sqrt{\rho^2 + 4\delta(\rho+\delta)\left(1 - \frac{\psi}{\gamma(1+\psi)(\delta - m_{ss})}\right)}}{\gamma(1+\psi)(\delta - m_{ss}) - \psi} \right]^{\frac{1}{1-\varkappa}}. \tag{49}$$

Such steady state exists provided that $\gamma(\delta - m_{ss})(1+\psi) > \psi$.

The long-run immigration rate, $\lim_{t\to\infty} m(t) = m_{ss}$, depends on how the immigration process is specified. Assuming $m(t) = \bar{m}$, the immigration rate is exogenous and our previous analysis of demographic shocks is virtually unchanged. Assuming $M(t) = \bar{M}$, the migration rate is endogenous and demographic shocks have richer effects than those described in Proposition 3, because changes in steady-state population L_{ss} also induce changes in steady-state fertility b_{ss} via the immigration rate $m_{ss} = \bar{M}/L_{ss}$. Aside from these second-order effects, both specifications of migration flows yield the same general insights. The most important is that the fertility rate b_{ss} adjusts to the turnover rate $\delta - m_{ss}$ and is therefore decreasing in the (asymptotic) immigration rate.[24] Moreover, the immigration rate becomes a determinant of the grand ratios previously discussed: as we show in Appendix, all expressions appearing in Proposition 4 hold with δ replaced by $\delta - m_{ss}$. In particular, the ratio of labor income to consumption equals

$$\lim_{t\to\infty} \frac{w(t)L(t)(1-\gamma b(t))}{C(t)} = \psi \cdot \frac{1-\gamma(\delta - m_{ss})}{\gamma(\delta - m_{ss})}, \tag{50}$$

so that the wage bill relative to consumption is *strictly increasing* in the immigration rate. This result drives the welfare consequences of immigration barriers in the quantitative analysis presented below.

6. Quantitative analysis

Immigration barriers and public policies affecting reproduction costs are widely debated at the global level. These interventions may induce substantial demographic shocks affecting intergenerational welfare in non-trivial ways. We investigate this point by means of numerical simulations that evaluate the transitional and the long-run effects of (i) a permanent rise in the time cost of reproduction and of (ii) a permanent reduction in total immigration according to the specification $M(t) = \bar{M}$.

6.1. Baseline parameters

The parametrization assumes an economy in steady state the key target variables match the average values observed across OECD countries.[25] Panel A in Table 2 lists six *endogenous* variables for which we calculate target values from available data (OECD, 2017) or empirical evidence: population size L, the propensity to consume out of total wealth $c/(a+h)$, the consumption-assets ratio (C/A), the mass of firms relative to population N/L, the rate of wealth creation ($\dot{A}/A$), and the share of GDP invested in R&D. Panel B lists our *preset parameters* reflecting available data or empirical estimates: Death probability δ, the long-run migration rate m_{ss}, the elasticity of substitution across intermediates ϵ, the rates of time preference and product obsolescence, ρ and μ, and the elasticity of productivity to the mass of intermediate goods $\chi - 1$. For parameter $\varkappa$, we set a baseline value of zero and then check the robustness of our results under alternative values. The remaining six parameters are set so as to match the six target values of the endogenous variables listed in Panel A. This procedure, which distinguishes between the demographic and the production side of the model (see Appendix), yields the values of the parameters reported in Table 2, Panel C.

6.2. Steady state results

The first row of panel D in Table 2 reports steady-state values of the main variables under the baseline parametrization. The gross fertility rate $b_{ss} = 1.37\%$ and the ratio of total labor incomes to assets $(w\tilde{L}/A)_{ss} = 0.62$ are empirically plausible. The same panel considers six alternative parametrizations showing how the steady state changes in response to small *ceteris paribus* variations. The results for higher mortality and stronger impatience confirm and extend our analytical findings

[24] Satisfying the existence condition $\gamma(1+\psi)(\delta - m_{ss}) > \psi$ requires $\delta - m_{ss} > 0$, which is intuitive: constant population with constant gross fertility requires a positive rate of population turnover.

[25] Sources and identification methods are discussed in detail in Appendix.

Table 2
Calibration of baseline parameters, steady state results and variations in parameters.

A. Targeted variables		Population L_{ss}	Cons.Propensity $c/(a+h)$	Cons./Assets $(C/A)_{ss}$	Firms/Population N_{ss}/L_{ss}	Long-term Growth $(\dot{A}/A)_{ss}$	R&D Propensity $w(L_N+L_Z)/GDP$
Target values		36,525,680	0.03	0.64	0.0327	0.0140	0.022
(sources of target values)		(OECD data)	(OECD evidence)	(OECD data)	(OECD data)	(OECD forecast)	(OECD data)
Baseline simulation results		36,525,680	0.03	0.64	0.0327	0.014	0.022
B. Preset parameters		δ	m_{ss}	ϵ	ρ	μ	χ
Baseline simulation parameters		0.016	0.0023	4.3	0.015	0.01	1.05
(identification)		(adult life exp.)	(migration rates)	(mark-up)	(utility disc.)	(profit disc.)	(variety gains)
C. Calibrated parameters		γ	ψ	η	ϕ	ω	θ
Baseline simulation parameters		2.412	0.033	$1.76 \cdot 10^8$	5.602	6.127	0.01
(identification)		(cons./assets)	(cons.propens.)	(wage/assets)	(firms/population)	(long-term growth)	(R&D propens.)
D. Steady state values		L_{ss}	b_{ss}	$(w\tilde{L}/A)_{ss}$	N_{ss}	$(\dot{A}/A)_{ss}$	r_{ss}
Baseline simulation		36,525,680	1.370%	0.624	1,194,390	1.400%	2.975%
Parameter variations:							
Higher mortality	(1% increase in δ)	34,443,479	1.372%	0.589	1,112,998	1.419%	3.000%
Stronger impatience	(1% increase in ρ)	36,569,637	1.370%	0.625	1,194,898	1.401%	2.992%
Reduced immigration	(1% decrease in $\bar{M}$)	36,207,651	1.370%	0.619	1,181,975	1.403%	2.978%
Higher child cost	(1% increase in γ)	34,739,921	1.358%	0.594	1,124,670	1.416%	2.995%

Note: See Appendix for details on sources and identification.

Table 3
Transitional, long-run and welfare effects of permanent shocks: reduced immigration versus increase in reproduction cost.

A. Exogenous shocks: transitional and long run effects										
Shock	Migration shock (25% drop)					Child cost shock (6% increase)				
Parameter variation	$\bar{M}=84,009$ falls to $\bar{M}=63,006$ in year 2020					$\gamma=2.41$ rises to $\gamma=2.56$ in year 2020				
Endogenous variable	L(mln)	N(mln)	$w\tilde{L}/A$	$\dot{A}/A$	r	L(mln)	N(mln)	$w\tilde{L}/A$	$\dot{A}/A$	r
Year 2020 (pre-shock)	36.526	1.194	0.624	1.40%	2.98%	36.526	1.194	0.624	1.40%	2.98%
Year 2035	36.217	1.188	0.619	1.39%	2.97%	36.111	1.186	0.617	1.39%	2.96%
Year 2065	35.634	1.166	0.609	1.40%	2.97%	35.343	1.157	0.604	1.39%	2.97%
Year 2120	34.679	1.128	0.593	1.41%	2.98%	34.123	1.108	0.583	1.41%	2.99%
Steady state	28.587	0.884	0.488	1.49%	3.09%	28.199	0.869	0.482	1.50%	3.09%
B. Individual ex-post welfare										
Cohort-specific index	EPW_{2025}	EPW_{2035}	EPW_{2045}	EPW_{2055}	EPW_{2065}	EPW_{2075}	EPW_{2085}	EPW_{2095}	EPW_{2105}	EPW_{2115}
No shock ($\chi=1.05$)	27.91	33.59	39.26	44.94	50.62	56.30	61.98	67.66	73.33	79.01
Migration shock ($\chi=1.05$)	27.83	33.49	39.15	44.82	50.50	56.19	61.88	67.58	73.28	79.00
Child cost shock ($\chi=1.05$)	27.73	33.38	39.04	44.70	50.38	56.08	61.77	67.48	73.19	78.92
No shock ($\chi=1.025$)	13.72	19.40	25.07	30.75	36.43	42.11	47.79	53.47	59.14	64.82
Migration shock ($\chi=1.025$)	13.66	19.32	24.99	30.67	36.35	42.04	47.74	53.45	**59.16**	**64.88**
Child cost shock ($\chi=1.025$)	13.56	19.22	24.89	30.56	36.25	41.95	47.65	53.37	59.09	**64.82**
No shock ($\chi=1.10$)	56.28	61.96	67.64	73.32	79.00	84.68	90.35	96.03	101.71	107.39
Migration shock ($\chi=1.10$)	56.17	61.82	67.47	73.13	78.80	84.47	90.15	95.84	101.53	107.23
Child cost shock ($\chi=1.10$)	56.06	61.69	67.34	72.99	78.65	84.32	90.01	95.70	101.40	107.11

(Section 5.1) on the opposite effects of δ and ρ. The third and fourth scenarios emphasize, instead, the similar consequences of reductions in $\bar{M}$ and increases in γ. Reduced immigration and increased reproduction costs produce a qualitatively different response of the fertility rate. The reduction in $\bar{M}$ increases b_{ss} because a permanent fall in net inflows is ultimately compensated by increased domestic births in the steady state. The increase in γ, instead, reduces transitional fertility rates via higher private costs of reproduction leaving the long-run rate b_{ss} unchanged. Despite the asymmetric effects on fertility, the two shocks bear qualitatively identical consequences on the other endogenous variables. Reduced immigration and increased reproduction costs reduce the long-run population level and drive down labor incomes relative to assets; the long-run mass of firms shrinks, and the reallocation of workers to vertical R&D boosts interest rates in the long run. The welfare consequences of such shocks are neither clear-cut nor symmetric across generations: the next subsection tackles this point by studying both the transitional and the long-run effects of large unexpected permanent shocks.

6.3. Demographic shocks, transition and welfare

Consider two independent scenarios in which ‘reduced immigration’ or ‘increased reproduction cost’ bear similar quantitative effects on steady-state population. The first scenario assumes a *migration shock* whereby the net inflows $\bar{M}$ fall permanently by 25% of the baseline value, from 84,009 to 63,006, which may be interpreted as an ‘immigration barrier’ set by a policymaker. The second scenario assumes a *child-cost shock* whereby γ permanently increases by 6% of its baseline value. The reference time zero is the year 2015, and the shocks hit the economy from year 2020 onwards. Panel A in Table 3 provides a summary comparison in terms of initial, short-to-medium-run and steady-state effects on selected variables. Fig. 2 presents a detailed analysis of the transitional paths generated by the two shocks over a century-long horizon. Importantly, the impact of both shocks on growth-related variables in the short-to-medium run is *reversed* with respect to the steady state outcomes: although wealth creation and interest rates are higher in the very long run, the transition features several decades of slower growth and low rates of return. The reason is that the decline in population creates net exit of firms from the market during the whole transition, $\dot{N}/N<0$, which reduces the overall rate of wealth creation, $\dot{A}/A$, in the short-to-medium run. During the transition, labor is reallocated from entry to production activities, interest rates decline and aggregate consumption falls substantially even in the medium run.[26] These ‘reversed growth effects’ bear specific consequences for intergenerational welfare: Cohorts that happen to be alive when the shocks occur may experience net welfare losses. This is particularly relevant for newborn generations since they heavily rely on labor incomes and experience a productivity slowdown that reduces real wages. We can verify this conclusion by means of a cohort-specific utility index,

$$EPW_j \equiv \int_j^{j+(1/\delta)} \left[\ln c_j(t) + \psi \ln b_j(t)\right] \cdot e^{-\rho(t-j)} dt, \tag{51}$$

[26] The smaller drop and the subsequent recovery that we observe in consumption *per capita* during this phase is actually due to the population decline rather than to faster output growth. The different paths of aggregate and per capita consumption are shown in the bottom panel of Figure 2.

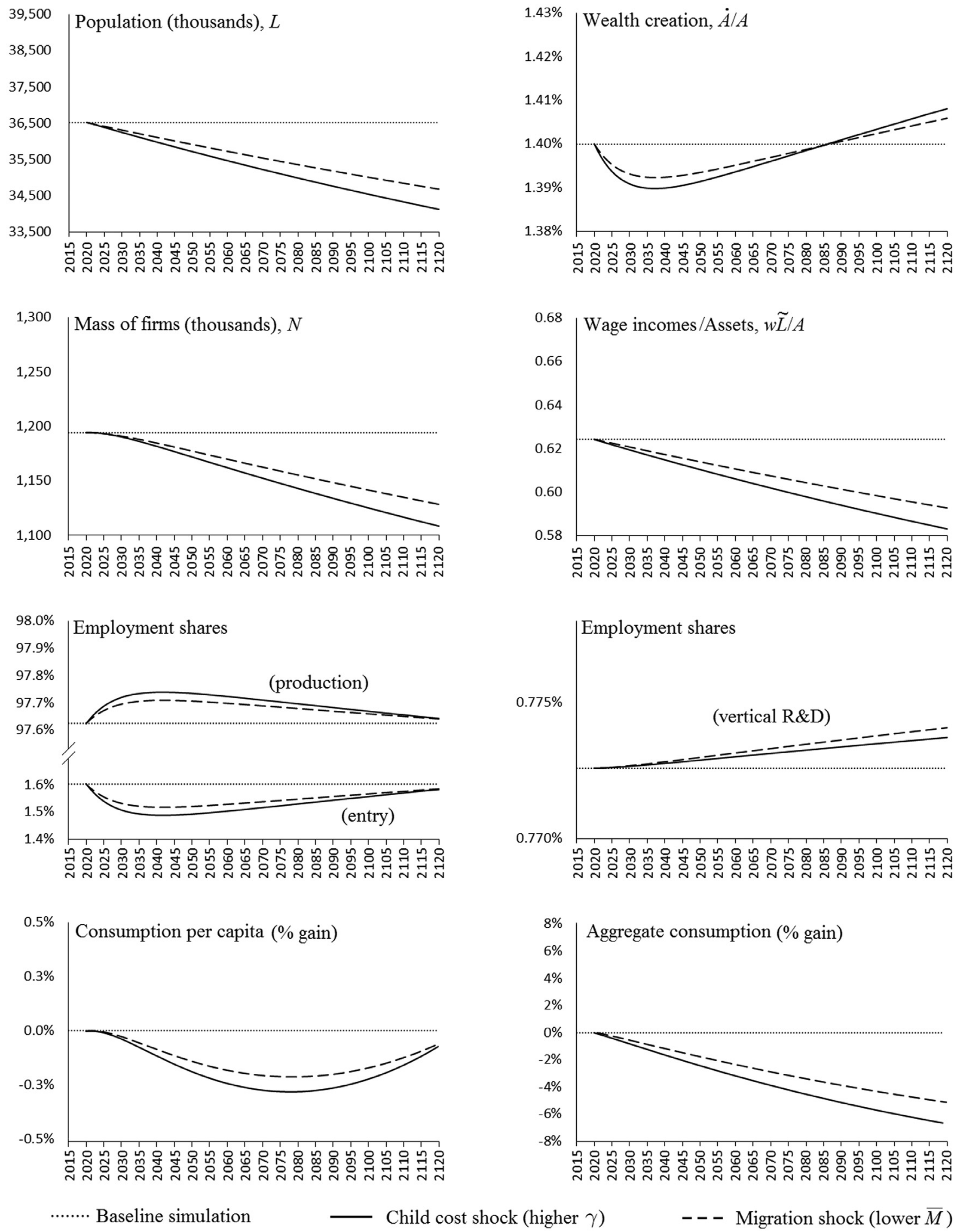

Fig. 2. Transitional dynamics generated by exogenous increases in reproduction costs and by reduced immigration.

which represents the *ex-post welfare* level enjoyed by a typical member of cohort j whose actual lifetime exactly coincides with life expectancy $1/\delta$. Table 3, panel B, reports the values of EPW_j for ten different cohorts born in the years $j = 2025, 2035, \ldots, 2115$, and compares their welfare levels in the three cases of interest: the 'no shock' scenario in which the economy remains in the baseline steady state forever, the migration shock, and the child-cost shock. To check sensitivity, we repeat this exercise under alternative values of parameter χ, which determines the relative contribution of the firms' net entry rate to the overall rate of wealth creation.[27] We set $\chi = (1.025, 1.05, 1.10)$, where 1.05 is our benchmark, assuming the same initial stock of knowledge in all scenarios. All the cohorts born within a century after the child-cost shock suffer net welfare losses in the cases $\chi = (1.05, 1.10)$. Assuming $\chi = 1.025$, we observe net welfare gains for the cohorts born after 2100. The general, robust conclusion is that while both these shocks may raise economic growth in the very long run, they also permanently reduce the mass of firms and the wage bill relative to assets, generating decades of stagnating growth, low interest rates and wages and, hence, net welfare losses for a large set of cohorts.

7. Conclusion

Endogenous interactions between fertility and productivity growth can explain why and how demography matters for macroeconomic performance even in the long run. In our model with disconnected generations, financial wealth dilution and the wage response to population size stabilize population despite positive output growth, and demographic shocks bear first-order effects on consumption, the functional income distribution and welfare. In particular, barriers to immigration or higher reproduction costs reduce the number of firms in steady state, raise output growth in the very long run but reduce the welfare of many generations by causing permanent reductions in labor income shares as well as prolonged stagnation during the transition. Our results suggest a number of critical questions for applied research. Measuring the impact of identifiable demographic shocks on productivity growth and on factor income shares is challenging but is clearly a central issue from the perspective of both positive analysis and policymaking. We are unaware of any studies testing, at the macro level, the quantitative relevance of wealth dilution effects on consumption and asset prices: this is a novel insight of our model that deserves empirical scrutiny. Also, our analysis suggest that the long-run effects of public policies related to demography – e.g., welfare systems, child-cost subsidies, immigration policies – depend on endogenous fertility responses that are typically overlooked in policy evaluation studies. Tackling these issues is our main suggestion for future research.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.jmoneco.2020.02.002.

CRediT authorship contribution statement

Christa N. Brunnschweiler: Methodology, Formal analysis, Investigation, Data curation, Writing - original draft. **Pietro F. Peretto:** Methodology, Formal analysis, Conceptualization, Writing - original draft. **Simone Valente:** Methodology, Formal analysis, Conceptualization, Software, Writing - original draft.

References

Attanasio, O., Low, H., Sanchez-Marcos, V., 2008. Explaining changes in female labor supply in a life-cycle model. Am. Econ. Rev. 98 (4), 1517–1552.
Backus, D., Cooley, T., Henriksen, E., 2014. Demography and low-frequency capital flows. J. Int. Econ. 92, 94–102.
Barro, R., Becker, G., 1989. Fertility choice in a model of economic growth. Econometrica 57 (2), 481–501.
Barro, R., Sala-i Martin, X., 2004. Economic Growth. MIT Press, Cambridge, MA..
Blanchard, O., 1985. Debt, deficits, and finite horizons. J. Political Econ. 93, 223–247.
Brander, J., Taylor, M.S., 1998. The simple economics of easter island: aRicardo-Malthus model of renewable resource use. Am. Econ. Rev. 88 (1), 119–138.
Buiter, W., 1988. Death, birth, productivity growth and debt neutrality. Econ. J. 98, 279–293.
Carvalho, C., Ferrero, A., Nechio, F., 2016. Demographics and real interest rates: inspecting the mechanism. Eur. Econ. Rev. 88, 208–226.
Chu, A., Cozzi, G., Liao, C.H., 2013. Endogenous fertility and human capital in a schumpeterian growth model. J. Popul. Econ. 26, 181–202.
Cooley, T., Henriksen, E., 2018. The demographic deficit. J. Monet. Econ. 93, 45–62.
Decker, R., Haltiwanger, J., Jarmin, R., Miranda, J., 2016. Declining business dynamism: what we know and the way forward. Am. Econ. Rev. 106 (5), 203–207.
Eckstein, Z., Stern, S., Wolpin, K., 1988. Fertility choice, land, and the malthusian hypothesis. Int. Econ. Rev. 29, 353–361.
Ehrlich, I., Lui, F., 1997. The problem of population and growth: a review of the literature from malthus to contemporary models of endogenous population and endogenous growth. J. Econ. Dyn. Control 21 (1), 205–242.
Galor, O., 2011. Unified Growth Theory. University Press, Princeton, NJ: Princeton.
Galor, O., Weil, D., 2000. Population, technology, and growth: from malthusian stagnation to the demographic transition and beyond. Am. Econ. Rev. 90 (4), 806–828.
Gordon, R., 2018. Why has economic growth slowed when innovation appears to be accelerating?NBER Working Paper No. 24554, April.
Jones, C., 1995. R&d-based models of economic growth. J. Political Econ. 103 (4), 759–784.
Jones, L. E., Tertilt, M., 2006. An economic history of fertility in the U.S.: 1826–1960. NBER Working Paper No. 12796.
Karahan, F., Pugsley, B., Sahin, A., 2019. Demographic origins of the startup deficit. Manuscript.
Madsen, J., 2010. The anatomy of growth in the OECD since 1870. J. Monet. Econ. 57, 753–767.
OECD, 2017. OECD Statistics, http://www.stats.oecd.org (accessed July 2017). Organisation for Economic Co-operation and Development.
Peretto, P., 1998. Technological change and population growth. J. Econ. Growth 3, 283–311.
Peretto, P., Connolly, M., 2007. The manhattan metaphor. J. Econ. Growth 12 (4), 329–350.
Peretto, P., Smulders, S., 2002. Technological distance growth and scale effects. Econ. J. 112, 603–624.

[27] The transition paths in Fig. 2 assume the benchmark value $\chi = 1.05$ and show that the reversed effect on wealth creation lasts 70 years: after the shock occurring in 2020, the growth rate of assets is below the baseline level until 2090. Assuming higher (lower) values of χ would further delay (anticipate) the switching date, i.e., the instant at which $\dot{A}/A$ crosses the pre-shock level from below.

Peretto, P., Valente, S., 2015. Growth on a finite planet: resources, technology and population in the long run. J. Econ. Growth 20, 305–331.
Romer, P., 1990. Endogenous technological change. J. Political Econ. 98 (5), 71–102.
Strulik, H., Weisdorf, J., 2008. Population, food, and knowledge: a simple unified growth theory. J. Econ. Growth 13 (3), 195–216.
Weil, P., 1989. Overlapping families of infinitely-lived agents. J. Public Econ. 38 (2), 183–198.
Yaari, M., 1965. Uncertain lifetime, life insurance, and the theory of the consumer. Rev. Econ. Stud. 32, 137–150.