Is the “curse of natural resources” really a curse?

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Abstract

This paper takes a new look at the long-run implications of resource abundance. Using a Schumpeterian growth model that yields an analyitical solution for the transition path, it derives conditions under which the “curse of natural resources” occurs and is in fact a curse, meaning that welfare falls, conditions under which it occurs but it is not a curse, meaning that growth slows down but welfare rises nevertheless, and conditions under which it does not occur at all. An effective way to summarize the results is to picture growth and welfare as hump-shaped functions of resource abundance. The property that the peak of growth occurs earlier than the peak of welfare captures the crucial role of initial consumption, which rises with resource abundance, and is an important reminder that the welfare effect of resource abundance depends on the whole path of consumption, not on a summary statistic of its slope.

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1 Introduction

The debate on the role that natural resources play in the long-run fortunes of the economy is as lively as ever. Currently it focuses on the influential work of Sachs and Warner (1995, 2001), who found a negative cross-country correlation between the share of natural resources exports in GDP and the growth rate of GDP per capita, interpreted the export share as a measure of natural resource abundance, and labeled this phenomenon the "curse of natural resources" (henceforth, the "curse" for short).

The finding strikes many as a paradox and poses tough questions for economic theory and policy.[1] Roughly speaking, there are two explanations. One is based on institutional deterioration, whereby natural resources create the opportunity for rent-seeking, corruption and conflict; the other is based on sectoral reallocation, whereby activities intensive in natural resources crowd out other, more technologically dynamic activities (e.g., manufacturing, knowledge-based services). Recently, a third point of view has emerged as some researchers find a positive correlation between growth and resource abundance and conclude that there is no "curse".[2]

In this paper I take a new look at the long-run implications of resource abundance through the lens of modern Schumpeterian growth theory. In particular, I use a model of the latest vintage that allows me to study welfare analytically. I derive conditions under which the "curse" occurs and is in fact a curse, meaning that welfare falls, conditions under which it occurs but it is not a curse, meaning that growth slows down but welfare rises nevertheless, and conditions under which it does not occur at all.

The model has two factors of production in exogenous supply, labor and a natural resource, and two sectors, primary production (or resource processing) and manufacturing. I define resource abundance as the endowment of the natural resource relative to labor. The primary sector uses labor to process the raw natural resource; the manufacturing sector uses labor and the processed natural resource to produce differentiated consumption goods. Because both sectors use labor, its reallocation from manufacturing to primary production drives the economy's adjustment to an increase in resource

[1] Not surprisingly, it has spurred a large literature. A comprehensive review is beyond the scope of the paper. The interested reader can consult Gylfason (2001a), Smulders (2005), Stevens (2003) or Lederman and Maloney (2007).

[2] The positive correlation is actually not new; see Sala-i-Martin (1997), Sala-i-Martin, Doppelhofer and Miller (2004). What is new is the explicit assertion that the "curse" does not exist; see Alexeev and Conrad (2008), Brunneschweiler and Bulte (2008), Lederman and Maloney (2007b).

abundance. The manufacturing sector is technologically dynamic: firms and entrepreneurs undertake R&D to learn how to use factors of production more efficiently and to design new products. Importantly, the process of product proliferation fragments the aggregate market into submarkets whose size does not increase with the size of the endowments and thereby sterilizes the scale effect. This means that the effect of resource abundance on growth is only *temporary*. The resulting structure is extremely tractable and yields a closed-form solution for the transition path.

The core mechanism is how the pattern of factor substitution in the two sectors determines the response of the natural resource price to an increase in the endowment ratio. The price response determines the income earned by the owners of the resource (the households) and thereby their expenditure on manufacturing goods. This mechanism links resource abundance to the size of the market for manufacturing goods. Since manufacturing is the economy's innovative sector, it drives how resource abundance affects incentives to undertake R&D.

Substitution matters because it determines the price elasticity of demand for the natural resource. Inelastic demand means that the price has to fall drastically to induce the market to absorb the additional quantity; elastic demand means that the adjustment requires a mild drop in the price. Importantly, the two cases are part of a continuum because I use technologies with factor substitution that changes with quantities. Now, if the economy exhibits substitution, demand is elastic, the price effect is mild, the quantity effect dominates, and resource income rises, spurring more spending on manufacturing and a temporary growth acceleration. If, instead, the economy exhibits complementarity, demand is inelastic, the price effect is strong, resource income falls, and we have a temporary growth deceleration, i.e., a "curse". Whether the economy experiences a growth acceleration or deceleration, however, is not sufficient to determine what happens to welfare, since, given technology, the lower resource price makes consumption goods cheaper. This means that to assess the welfare effect of the change in the endowment ratio I need to resolve the trade-off between short- and long-run effects if they differ in sign. As said, I can do this analytically since I have a closed-form solution for the transition path.

My main result is that I identify threshold values of the equilibrium price of the natural resource — and therefore threshold values of the endowment ratio — that yield the following sequence of scenarios as we gradually raise the endowment ratio from tiny to very large.

1. **There is no "curse" and thus no curse**. This happens when the

resource price is high because the endowment ratio is low. In this situation, demand is elastic, the quantity effect dominates over the price effect, and, consequently, resource income rises. In other words, starting from a situation of scarcity, the increase in resource abundance generates a growth acceleration associated to an initial jump up in consumption that yields higher welfare.

2. **There is a "curse" but it is not a curse**. That is, the rise in the endowment ratio causes a growth deceleration but the deceleration is offset by the fact that resource abundance raises the initial level of consumption. This happens when the endowment ratio is in the intermediate range in between the two thresholds and, correspondingly, the resource price is in its intermediate range. Relative to the previous case, demand becomes inelastic and the price effect dominates over the quantity effect, with the result that resource income falls.

3. **There is a "curse" and it is a curse**. This happens when the resource price is low because the endowment ratio is high. In this case, demand is inelastic, the price effect dominates over the quantity effect, and resource income falls. Differently from the previous case, the fall in resource income is now sufficiently large to cause the growth deceleration to dominate over the initial jump up in consumption.

One way to summarize this pattern is to picture growth and welfare as hump-shaped functions of resource abundance. The property that the peak of growth occurs earlier than the peak of welfare captures the crucial role of initial consumption, which rises with resource abundance. This is an important feature because the correlation between resource abundance and the initial level of GDP per capita is *positive* in the very data set used to document the negative correlation with the growth rate.[3] It is thus not obvious that resource abundant countries are worse off simply because they grow slower. Rather, one needs to resolve the standard intertemporal trade-off to sort out the welfare effect of resource abundance. The few papers that acknowledge the important role of this trade-off do not, however, resolve it; see Rodriguez and Sachs (1999), Bravo-Ortega and de Gregorio (2007), and in particular Eliasson and Turnovsky (2004, p. 1043).

Before getting into the details of the paper, it is worth discussing some features that help evaluate its strenghts and weaknesses in relation to the

[3] See Rodriguez and Sachs (1999), Bravo-Ortega and De Gregorio (2007) and, expecially, Alexeev and Conrad (2008) for cross-country evidence; see also Boyce and Emery (2007) for U.S. states

literature. First, I work with a closed economy and thus rule out trade-induced specialization (i.e., there is no Dutch disease) and dependence, in the traditional sense of primary exports being a crucial component of the national economy.[4] The main advantage of this approach is that there is no confusion about the definition of resource abundance. Moreover, it assigns a central role to endogenous price adjustments. This is important, in my judgment, because the assumption that prices are fixed (common in work that focuses on small open economies) removes drivers of income effects that should be part of the analysis. The main disadvantage is obvious: the paper's link to the empirical literature is not as direct. However, the recent emphasis of empirical researchers on relative endowments suggests that the problem might be more the literature's early focus on the primary exports share rather than this paper's focus on a closed economy.

The second feature is that I work with two sectors to capture the crucial aspect of specialization induced by resource abundance: the sectoral shares of activity and employment change and drive the economy's adjustment path and the associated welfare level. Importantly, I assume, in line with the rest of the literature, that one sector is technologically progressive and the other is not (or less so in a more general version of the model). This, of course, is necessary to obtain the reallocation and associated crowding-out mechanism that drives the "curse". Overall, then, the model emphasizes substitution and technological change as the driving forces of the economy's adjustment to changes in the relative endowment.

The paper's organization is as follows. Section 2 sets up the model. Section 3 constructs the general equilibrium of the market economy. Section 4 discusses the key properties of the equilibrium that drive the paper's main results. Section 5 derives the main results. It first studies the conditions under which the "curse" occurs and then studies the conditions under which it is, in fact, a curse. It also discusses interesting implications for our reading of the empirical literature. Section 6 concludes.

2 The model

2.1 Overview

The basic model that I build on is developed in Peretto and Connolly (2007), who build on Peretto (1998). A representative household supplies labor ser-

[4] I am working on an extension of the analysis of this paper to the open economy case. The preliminary results are in line with what I present here.

vices in a competitive market. It also borrows and lends in a competitive market for financial assets. The household values variety and buys as many differentiated consumption goods as possible. Manufacturing firms hire labor to produce differentiated consumption goods, undertake R&D, or, in the case of entrants, set up operations. Production of consumption goods also requires a processed resource, which is produced by competitive suppliers using labor and a raw natural resource. The introduction of this up-stream primary sector is the main innovation of this paper. The economy starts out with a given range of goods, each supplied by one firm. Entrepreneurs compare the present value of profits from introducing a new good to the entry cost. They only target new product lines because entering an existing product line in Bertrand competition with the existing supplier leads to losses. Once in the market, firms establish in-house R&D facilities to produce cost-reducing innovations. As each firm invests in R&D, it contributes to the pool of public knowledge and reduces the cost of future R&D. This allows the economy to grow at a constant rate in steady state.

2.2 Households

The representative household maximizes lifetime utility

$$U(t) = \int_t^\infty e^{-(\rho-\lambda)(s-t)} \log u(s) ds, \quad \rho > 0 \tag{1}$$

subject to the flow budget constraint

$$\dot{A} = rA + WL + p\Omega + \Pi_M - Y, \tag{2}$$

where ρ is the discount rate, A is assets holding, r is the rate of return on financial assets, W is the wage rate, $L = L_0 e^{\lambda t}$, $L_0 \equiv 1$, is population size, which equals labor supply since there is no preference for leisure, and Y is consumption expenditure. In addition to asset and labor income, the household receives rents from ownership of the endowment, Ω, of a natural resource whose market price is p and dividend income from resource-processing firms, Π_M. The household takes these terms as given.

The household has instantaneous preferences over a continuum of differentiated goods,

$$\log u = \log \left[\int_0^N \left(\frac{X_i}{L} \right)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}, \quad \epsilon > 1 \tag{3}$$

where ϵ is the elasticity of product substitution, X_i is the household's purchase of each differentiated good, and N is the mass of goods (the mass of firms) existing at time t.

The solution for the optimal expenditure plan is well known. The household saves according to

$$r = r_A \equiv \rho + \frac{\dot{Y}}{Y} - \lambda \tag{4}$$

and taking as given this time-path of expenditure maximizes (3) subject to $Y = \int_0^N P_i C_i di$. This yields the demand schedule for product i,

$$X_i = Y \frac{P_i^{-\epsilon}}{\int_0^N P_i^{1-\epsilon} di}. \tag{5}$$

With a continuum of goods, firms are atomistic and take the denominator of (5) as given; therefore, monopolistic competition prevails and firms face isoelastic demand curves.

2.3 Manufacturing: Production and Innovation

The typical firm produces one differentiated consumption good with the technology

$$X_i = Z_i^\theta \cdot F_X \left(L_{X_i} - \phi, M_i \right), \quad 0 < \theta < 1, \ \phi > 0 \tag{6}$$

where X_i is output, L_{X_i} is production employment, ϕ is a fixed labor cost, M_i is processed resource use (henceforth "materials" for short), and Z_i^θ is the firm's TFP, a function of the stock of firm-specific knowledge Z_i. The function $F_X(\cdot)$ is a standard neoclassical production function homogeneous of degree one in its arguments. Hence, the production technology exhibits constant returns to rival inputs, labor and materials, and overall increasing returns. (6) gives rise to total cost

$$W\phi + C_X(W, P_M) Z_i^{-\theta} X_i, \tag{7}$$

where the function $C_X(\cdot)$ is a standard unit-cost function homogeneous of degree one in its arguments.

The elasticity of unit cost reduction with respect to knowledge in (7) is the constant θ. The firm accumulates knowledge according to the R&D technology

$$\dot{Z}_i = \alpha K L_{Z_i}, \quad \alpha > 0 \tag{8}$$

where $\dot{Z}_i$ measures the flow of firm-specific knowledge generated by an R&D project employing L_{Z_i} units of labor for an interval of time dt and αK is the productivity of labor in R&D as determined by the exogenous parameter α and by the stock of public knowledge, K.

Public knowledge accumulates as a result of spillovers. When one firm generates a new idea to improve the production process, it also generates general-purpose knowledge which is not excludable and that other firms can exploit in their own research efforts. Firms appropriate the economic returns from firm-specific knowledge but cannot prevent others from using the general-purpose knowledge that spills over into the public domain. Formally, an R&D project that produces $\dot{Z}_i$ units of proprietary knowledge also generates $\dot{Z}_i$ units of public knowledge. The productivity of research is determined by some combination of all the different sources of knowledge. A simple way of capturing this notion is to write

$$K = \int_0^N \frac{1}{N} Z_i di,$$

which says that the technological frontier is determined by the average knowledge of all firms.[5]

The R&D technology (8), combined with public knowledge K, exhibits increasing returns to scale to knowledge and labor, and constant returns to scale to knowledge. This property makes constant, endogenous steady-state growth feasible.

2.4 The Primary or Resources Sector

In the primary sector competitive firms hire labor, L_R, to extract and process natural resources, R, into materials, M, according to the technology

$$M = F_M \left(L_M, R \right), \tag{9}$$

where the function $F_M (\cdot)$ is a standard neoclassical production function homogeneous of degree one in its arguments. The associated total cost is

$$C_M \left(W, p \right) M, \tag{10}$$

where C_M is a standard unit-cost function homogeneous of degree one in the wage W and the price of resources p.

[5] For a detailed discussion of the microfoundations of a spillovers function of this class, see Peretto and Smulders (2002).

This is the simplest way to model the primary sector for the purposes of this paper. Materials are produced with labor and a natural resource. The natural resource is in fixed endowment and earns rents. The primary sector competes for labor with the manufacturing sector. This captures the fundamental inter-sectoral allocation problem faced by this economy.

3 Equilibrium of the Market Economy

This section constructs the symmetric equilibrium of the manufacturing sector. It then characterizes the equilibrium of the primary sector. Finally, it imposes general equilibrium conditions to determine the aggregate dynamics of the economy. The wage rate is the numeraire, i.e., $W \equiv 1$.

3.1 Partial Equilibrium of the Manufacturing Sector

The typical manufacturing firm is subject to a death shock. Accordingly, it maximizes the present discounted value of net cash flow,

$$V_i(t) = \int_t^{\infty} e^{-\int_t^s [r(v)+\delta]dv} \Pi_i(s) ds, \quad \delta > 0$$

where $e^{-\delta t}$ is the instantaneous probability of death. Using the cost function (7), instantaneous profits are

$$\Pi_{Xi} = [P_i - C_X(1, P_M) Z_i^{-\theta}] X_i - \phi - L_{Z_i},$$

where L_{Z_i} is R&D expenditure. V_i is the value of the firm, the price of the ownership share of an equity holder. The firm maximizes V_i subject to the R&D technology (8), the demand schedule (5), $Z_i(t) > 0$ (the initial knowledge stock is given), $Z_j(t')$ for $t' \geq t$ and $j \neq i$ (the firm takes as given the rivals' innovation paths), and $\dot{Z}_j(t') \geq 0$ for $t' \geq t$ (innovation is irreversible). The solution of this problem yields the (maximized) value of the firm given the time path of the number of firms.

To characterize entry, I assume that upon payment of a sunk cost $\beta P_i X_i$, an entrepreneur can create a new firm that starts out its activity with productivity equal to the industry average.[6] Once in the market, the new firm implements price and R&D strategies that solve a problem identical to the one outlined above. Hence, entry yields value V_i and a free entry equilibrium requires $V_i = \beta P_i X_i$.

[6] See Etro (2004) and, in particular, Peretto and Connolly (2007) for a more detailed discussion of the microfoundations of this assumption.

The appendix shows that the equilibrium thus defined is symmetric and is characterized by the factor demands:

$$L_X = Y\frac{\epsilon - 1}{\epsilon}\left(1 - S_X^M\right) + \phi N; \tag{11}$$

$$M = Y\frac{\epsilon - 1}{\epsilon}\frac{S_X^M}{P_M}, \tag{12}$$

where

$$S_X^M \equiv \frac{P_M M_i}{C_X(W, P_M) Z_i^{-\theta} X_i} = \frac{\partial \log C_X(W, P_M)}{\partial \log P_M}.$$

Associated to these factor demands are the return to cost reduction and entry, respectively:

$$r = r_Z \equiv \alpha \left[\frac{Y\theta(\epsilon - 1)}{\epsilon N} - \frac{L_Z}{N}\right] - \delta; \tag{13}$$

$$r = r_N \equiv \frac{1}{\beta}\left[\frac{1}{\epsilon} - \frac{N}{Y}\left(\phi + \frac{L_Z}{N}\right)\right] + \hat{Y} - \hat{N} - \delta. \tag{14}$$

The dividend price ratio in (14) depends on the gross profit margin $\frac{1}{\epsilon}$. Anticipating one of the properties of the equilibria that I study below, note that in steady state the capital gain component of this rate of return, $\hat{Y} - \hat{N}$, is zero. Hence, the feasibility condition $\frac{1}{\epsilon} > (r + \delta)\beta$ must hold. This simply says that the firm expects to be able to repay the entry cost because it more than covers fixed operating and R&D costs.

3.2 General equilibrium

As mentioned, the resources sector is competitive. Hence, firms produce up to the point where $P_M = C_M(1, p)$ and demand factors according to:

$$R = S_M^R \frac{M P_M}{p} = Y\frac{\epsilon - 1}{\epsilon}\frac{S_X^M S_M^R}{p}, \tag{15}$$

$$L_M = \left(1 - S_M^R\right) M P_M = Y\frac{\epsilon - 1}{\epsilon} S_X^M \left(1 - S_M^R\right), \tag{16}$$

where

$$S_M^R \equiv \frac{\partial \log C_M(W, p)}{\partial \log p}$$

and I have used (11) and (12) to obtain the expressions after the second equality sign. These factor demands yield that the competitive resource firms make zero profits.

Equilibrium of the primary sector requires $R = \Omega$. One can thus think of (15) as the equation that determines the price of the natural resource, and therefore resource income for the household, given the level of economic activity, measured by expenditure on consumption goods Y.

The remainder of the model consists of the household's budget constraint (2), the labor demands (11) and (16), the returns to saving, cost reduction and entry in (4), (13) and (14). The household's budget constraint becomes the labor market clearing condition (see the appendix for the derivation):

$$L = L_N + L_X + L_Z + L_M,$$

where L_N is aggregate employment in entrepreneurial activity, $L_X + L_Z$ is aggregate employment in production and R&D operations of existing firms and L_M is aggregate employment of resources-processing firms. Assets market equilibrium requires equalization of all rates of return (no-arbitrage), $r = r_A = r_Z = r_N$, and that the value of the household's portfolio equal the value of the securities issued by firms, $A = NV = \beta Y$.

A useful feature of the model is that I can combine the equilibrium conditions for the labor and assets markets to obtain an equation that describes how the resource price affects household income and thus expenditure on consumption goods. Specifically, substituting $A = \beta Y$ into (2) and using the rate of return to saving (4), I obtain after rearranging terms

$$\frac{Y - L - p\Omega}{Y} = \beta\left(\rho - \lambda\right).$$

Notice how all dynamic terms dropped out. Recall that I am focussing on a closed economy where the fixed domestic resource supply Ω implies that the price of the resource reflects scarcity. Then, letting $y \equiv \frac{Y}{L}$ denote expenditure per capita and $\omega \equiv \frac{\Omega}{L}$ denote the endowment ratio, I can use (15) and the condition $R = \Omega$ to study the instantaneous equilibrium (p^*, y^*) as the intersection in (p, y) space of the following two curves:

$$y = \frac{1 + p\omega}{1 - \beta\left(\rho - \lambda\right)}; \tag{17}$$

$$y = \frac{1}{1 - \beta\left(\rho - \lambda\right) - \frac{\epsilon - 1}{\epsilon} S_M^R\left(p\right) S_X^M\left(p\right)}. \tag{18}$$

The first describes how resource income determines expenditure on consumption goods; the second how expenditure drives demand for the factors of production and thereby determines resource income.

Before proceeding, notice that population growth implies that a solution with constant p^* and y^* fails to exist if the endowment Ω is constant since the ratio ω then shrinks. There are three solutions to this problem. The first is to assume zero population growth. I have posited the death shock precisely for this purpose: negative *net* entry, i.e., $\dot{N} < 0$, is feasible and one does not need to deal with asymmetric dynamics due to sunk entry costs in the presence of constant population. The second solution is to allow for population growth and assume that Ω grows at the same rate so that ω stays constant. (Clearly, the first solution is a special case of this one.) The third is to posit Cobb-Douglas technologies so that the shares S_X^M and S_M^R are exogenous constants that do not change as ω changes.

Each solution has advantages and disadvantages. All three retain the desirable feature that y^* is constant and that consequently the interest rate is $r^* = \rho$ at all times; see the Euler equation (4). The Cobb-Douglas solution, popular in the literature, allows for explosive growth of p^* but has the drawback that while y^* remains constant it is independent of the endowment ratio and thereby shuts down crucial feedbacks. It is thus not appropriate for my purposes. To keep things as simple as possible, and retain the feature that the endowment ratio affects y^* through prices, I assume that ω is constant because Ω and L grow at the same rate, *possibly zero.*[7] Allowing for population growth and exponential decay of ω might be more realistic but requires one to work with time-varying y^* and r^*, which complicates things a great deal. (The model thus specified is not totally intractable but solving it would distract from the main focus of the paper.)

For the reminder of this section and the next, I posit that the equilibrium (p^*, y^*) exists and study its implications for the economy's dynamics. I discuss in detail existence and comparative statics properties with respect to ω in Section 5.

3.3 Dynamics[8]

It is useful to work with the variable $n \equiv \frac{N}{L}$. Taking into account the non-negativity constraint on R&D, the fact that y^* is constant and implies

[7] This assumption is actually quite common in empirical work, mainly because the best available data on natural capital yields only one observation per country for the year 1994 (World Bank 1997). See Gylfason (2001a,b) and Brunneschweiler and Bulte (2008) for further elaboration of the arguments that justify thinking of the empircal analog of ω as constant.

[8] This section draws on Peretto and Connolly (2007) where the model that I build on in this paper is developed in full.

$r^* = \rho$ allows me to solve (8) and (13) for

$$\hat{Z} = \alpha \frac{L_Z}{N} = \begin{cases} \frac{y^*}{n} \frac{\alpha\theta(\epsilon-1)}{\epsilon} - \rho - \delta & n < \bar{n} \\ 0 & n \geq \bar{n} \end{cases}, \tag{19}$$

where

$$\bar{n} \equiv y^* \frac{\alpha\theta(\epsilon - 1)}{(\rho + \delta)\epsilon}.$$

Substituting into (14) yields

$$\hat{n} = \begin{cases} \frac{1}{\beta}\left[\frac{1-\theta(\epsilon-1)}{\epsilon} - \left(\phi - \frac{\rho+\delta}{\alpha}\right)\frac{n}{y^*}\right] - (\rho+\delta) & n < \bar{n} \\ \frac{1}{\beta}\left[\frac{1}{\epsilon} - \phi\frac{n}{y^*}\right] - (\rho+\delta) & n \geq \bar{n} \end{cases}.$$

The general equilibrium of the model thus reduces to a single differential equation in the mass of firms per capita. The economy converges to

$$n^* = \begin{cases} \frac{\frac{1-\theta(\epsilon-1)}{\epsilon} - (\rho+\delta)\beta}{\phi - \frac{\rho+\delta}{\alpha}} y^* & \frac{\frac{1-\theta(\epsilon-1)}{\epsilon} - (\rho+\delta)\beta}{\phi\alpha - (\rho+\delta)} < \frac{\theta(\epsilon-1)}{(\rho+\delta)\epsilon} \\ \frac{\frac{1}{\epsilon} - (\rho+\delta)\beta}{\phi} y^* & \frac{\frac{1-\theta(\epsilon-1)}{\epsilon} - (\rho+\delta)\beta}{\phi\alpha - (\rho+\delta)} \geq \frac{\theta(\epsilon-1)}{(\rho+\delta)\epsilon} \end{cases}. \tag{20}$$

These solutions exist only if the feasibility condition $\frac{1}{\epsilon} > (\rho + \delta)\beta$ holds. The interior steady state with both vertical and horizontal R&D requires the more stringent conditions $\alpha\phi > \rho + \delta$ and

$$(\rho + \delta)\beta + \frac{\theta(\epsilon - 1)}{\epsilon} < \frac{1}{\epsilon} < (\rho + \delta)\beta + \frac{\alpha\phi}{\rho + \delta}\frac{\theta(\epsilon - 1)}{\epsilon}.$$

It then yields

$$\frac{y^*}{n^*} = \frac{\phi - \frac{\rho+\delta}{\alpha}}{\frac{1-\theta(\epsilon-1)}{\epsilon} - (\rho + \delta)\beta} \tag{21}$$

so that

$$\hat{Z}^* = \frac{\phi\alpha - (\rho + \delta)}{\frac{1-\theta(\epsilon-1)}{\epsilon} - (\rho + \delta)\beta} \frac{\theta(\epsilon - 1)}{\epsilon} - (\rho + \delta). \tag{22}$$

This steady-state growth rate is independent of the endowments L and Ω because there is no scale effect.

To perform experiments, I shall focus on this region of parameter space and work with the equation

$$\hat{n} = \nu - \left(\phi - \frac{\rho + \delta}{\alpha}\right)\frac{n}{\beta y^*}, \quad \nu \equiv \frac{1 - \theta(\epsilon - 1)}{\beta\epsilon} - (\rho + \delta).$$

This is a logistic equation (see, e.g., Banks 1994) with growth coefficient ν and crowding coefficient $\left(\phi - \frac{\rho+\delta}{\alpha}\right)\frac{1}{\beta y^*}$. Using the value n^* in (20), also called carrying capacity, I can rewrite it as

$$\hat{n} = \nu\left(1 - \frac{n}{n^*}\right), \tag{23}$$

which has solution

$$n(t) = \frac{n^*}{1 + e^{-\nu t}\left(\frac{n^*}{n_0} - 1\right)}, \tag{24}$$

where n_0 is the initial condition.

4 Properties of the equilibrium: technology, the path of consumption and welfare

The main advantage of this model is that one can solve explicitly for the level of welfare associated to the economy's transition to the steady state starting from any initial condition. In this section I bring this feature to the forefront as it provides the building blocs that I use in the derivation of the paper's main results.

In symmetric equilibrium (3), (5) and the fact that manufacturing firms set prices at a markup $\frac{\epsilon}{\epsilon-1}$ over marginal cost (see the Appendix) yield

$$\log u^* = \log\left(\frac{\epsilon - 1}{\epsilon}\frac{y^*}{c^*}Z^\theta N^{\frac{1}{\epsilon-1}}\right),$$

where

$$c^* \equiv C_X\left(1, C_M\left(1, p^*\right)\right)$$

One can reinterpret the utility function (3) as a production function for a final homogenous good assembled from intermediate goods, so that u is indeed a measure of output, and define aggregate TFP for this economy as

$$T \equiv Z^\theta N^{\frac{1}{\epsilon-1}}. \tag{25}$$

Taking logs and time derivatives this yields

$$\hat{T}(t) = \theta\hat{Z}(t) + \frac{1}{\epsilon - 1}\hat{N}(t) = \theta\hat{Z}(t) + \frac{1}{\epsilon - 1}\lambda + \frac{1}{\epsilon - 1}\hat{n}(t),$$

where $\hat{Z}(t)$ is given by (19), $\hat{n}(t)$ by (23) and $n(t)$ by (24). In steady state this gives

$$\hat{T}^* = \theta\hat{Z}^* + \frac{1}{\epsilon - 1}\lambda \equiv g^*,$$

which is independent of the endowments L and Ω, and thus of ω.

Observe now that according to (21) in steady state we have

$$\frac{y^*}{n^*} = \frac{y_0}{n_0} \Rightarrow \frac{y^*}{y_0} = \frac{n^*}{n_0},$$

and define

$$\Delta^* \equiv \frac{n^*}{n_0} - 1 = \frac{y^*}{y_0} - 1.$$

This is the percentage change in expenditure that the economy experiences in response to changes in fundamentals and/or policy parameters. It fully summarizes the effects of such changes on the scale of economic activity. Using this information, I obtain the following result.

Proposition 1 *Let $\log u^*(t)$ and U^* be, respectively, the instantaneous consumption index (3) and the welfare function (1) evaluated at y^*. Then, a path starting at time $t = 0$ with initial condition n_0 and converging to the steady state n^* is characterized by:*

$$\log T(t) = \log T_0 + g^* t + \left(\frac{\gamma}{\nu} + \frac{1}{\epsilon - 1}\right) \Delta^* \left(1 - e^{-\nu t}\right), \tag{26}$$

where

$$\gamma \equiv \theta \frac{\alpha\theta(\epsilon - 1)}{\epsilon} \frac{y^*}{n^*} = \theta \frac{\alpha\theta(\epsilon - 1)}{\epsilon} \frac{\phi - \frac{\rho+\delta}{\alpha}}{\frac{1-\theta(\epsilon-1)}{\epsilon} - (\rho + \delta)\beta}.$$

Therefore,

$$\log u^*(t) = \log \frac{y^*}{c^*} + g^* t + \left(\frac{\gamma}{\nu} + \frac{1}{\epsilon - 1}\right) \Delta^* \left(1 - e^{-\nu t}\right), \tag{27}$$

where without loss of generality $\frac{\epsilon-1}{\epsilon} T_0 = 1$. This yields

$$U^* = \frac{1}{\rho - \lambda} \left[\log\left(\frac{y^*}{c^*}\right) + \frac{g^*}{\rho - \lambda} + \mu\Delta^*\right], \tag{28}$$

where

$$\mu \equiv \frac{\gamma + \frac{\nu}{\epsilon-1}}{\rho - \lambda + \nu}.$$

Proof. See the Appendix. ■

The transitional component of the TFP operator in (26) summarizes the cumulated gain/loss due to above/below steady-state cost reduction and product variety expansion. The expression for flow utility (27) allows one to see separately the initial jump due to y^*/c^* and the gradual evolution due to T. Accordingly, the first term in (28) captures the role of steady-state real expenditure calculated holding technology constant; the second captures the role of steady-state growth; the third is the contribution from the gain/loss due to the transitional acceleration/deceleration of TFP relative to the steady state path. Following Peretto and Connolly (2007) we define the term μ as a *welfare multiplier* summarizing the transitional effects of the change in market size Δ^*.

5 Resource abundance, growth and welfare: Is there a "curse", is it a curse?

This section analyzes the effects of a change in the endowment ratio. It begins with a discussion of the conditions under which it raises or lowers consumption expenditure so that the market for manufacturing goods expands or contracts. It then shows how the interaction of the initial change in consumption and the transition dynamics after the shock produce a change in welfare whose sign can be assessed analytically.

5.1 Expenditure and prices

The first step in the assessment of the effects of resource abundance is to use (17) and (18) to characterize expenditure and prices. The following property of the demand functions (12) and (15) is useful.

Lemma 2 *Let:*

$$\epsilon_X^M \equiv -\frac{\partial \log M}{\partial \log P_M} = 1 - \frac{\partial \log S_X^M}{\partial \log P_M} = 1 - \frac{\partial S_X^M}{\partial P_M}\frac{P_M}{S_X^M};$$

$$\epsilon_M^R \equiv -\frac{\partial \log R}{\partial \log p} = 1 - \frac{\partial \log S_M^R}{\partial \log p} = 1 - \frac{\partial S_M^R}{\partial p}\frac{p}{S_M^R}.$$

Then,

$$\frac{\partial \left(S_M^R\left(p\right) S_X^M\left(p\right)\right)}{\partial p} = \Gamma\left(p\right)\frac{S_M^R\left(p\right) S_X^M\left(p\right)}{p}, \tag{29}$$

where

$$\Gamma\left(p\right) \equiv \left(1 - \epsilon_X^M\left(p\right)\right) S_M^R\left(p\right) + 1 - \epsilon_M^R\left(p\right).$$

Proof. See the Appendix. ■

$\Gamma(p)$ is the elasticity of $S_M^R(p) S_X^M(p)$ with respect to p. According to (12), therefore, it is the elasticity of the demand for the resource R with respect to its price p, holding constant expenditure per capita y. It thus captures the partial equilibrium effects of price changes in the resource and materials markets for given market size and regulates the shape of the income relation (18). To see how, differentiate (18), rearrange terms and use (15) to obtain:

$$\frac{d \log y(p)}{dp} = \frac{\frac{\epsilon-1}{\epsilon} \frac{d\left(S_M^R(p) S_X^M(p)\right)}{dp}}{1-\beta(\rho-\lambda)-\frac{\epsilon-1}{\epsilon} S_M^R(p) S_X^M(p)} = \omega \Gamma(p).$$

This says that the effect of changes in the resource price on expenditure on manufacturing goods depends on the overall pattern of substitution that is reflected in the price elasticities of materials and resource demand and in the resource share of materials production costs. To see the pattern most clearly, I pretend for the time being that $\Gamma(p)$ does not change sign with p. I comment later on how allowing $\Gamma(p)$ to change sign for some p makes the model even more interesting. The following proposition states the results formally, Figure 1 illustrates the mechanism.

Proposition 3 *Suppose that $\Gamma(p)$ is positive, zero or negative for all p. Then, there are three cases.*

1. ***Complementarity.*** *This occurs when $\Gamma(p) > 0$ and the income relation (18) is a monotonically increasing function of p with domain $p \in [0, \infty)$ and codomain $y \in [y^*(0), y^*(\infty))$, where*

$$\begin{aligned} y^*(0) &= \frac{1}{1-\beta(\rho-\lambda)-\frac{\epsilon-1}{\epsilon} S_M^R(0) S_X^M(0)}, \\ y^*(\infty) &= \frac{1}{1-\beta(\rho-\lambda)-\frac{\epsilon-1}{\epsilon} S_M^R(\infty) S_X^M(\infty)}. \end{aligned}$$

Then there exists a unique equilibrium $(p^*(\omega), y^*(\omega))$ with the property:

$$\begin{aligned} p^*(\omega) &: (0,\infty) \to (\infty, 0), \quad \frac{dp^*(\omega)}{d\omega} < 0 \quad \forall \omega; \\ y^*(\omega) &: (0,\infty) \to [y^*(\infty), y^*(0)), \quad \frac{dy^*(\omega)}{d\omega} < 0 \quad \forall \omega. \end{aligned}$$

2. ***Cobb-Douglas-like economy.*** *This occurs when S_M^R and S_X^M are exogenous constants, $\Gamma(p) = 0$ and the income relation (18) is the flat line*

$$y = \frac{1}{1 - \beta(\rho - \lambda) - \frac{\epsilon - 1}{\epsilon} S_M^R S_X^M} \equiv y_{CD}^*.$$

Then there exists a unique equilibrium $(p_{CD}^*(\omega), y_{CD}^*)$ with the property:

$$p_{CD}^*(\omega) : (0, \infty) \to (\infty, 0), \quad \frac{dp^*(\omega)}{d\omega} < 0 \quad \forall \omega.$$

3. ***Substitution.*** *This occurs when $\Gamma(p) < 0$ and the income relation (18) is a monotonically decreasing function of p with domain $p \in [0, \infty)$ and codomain $y \in (y^*(\infty), y^*(0)]$, where*

$$\begin{aligned} y^*(\infty) &= \frac{1}{1 - \beta(\rho - \lambda) - \frac{\epsilon - 1}{\epsilon} S_M^R(\infty) S_X^M(\infty)}, \\ y^*(0) &= \frac{1}{1 - \beta(\rho - \lambda) - \frac{\epsilon - 1}{\epsilon} S_M^R(0) S_X^M(0)}. \end{aligned}$$

Then there exists a unique equilibrium $(p^*(\omega), y^*(\omega))$ with the property:

$$\begin{aligned} p^*(\omega) &: (0, \infty) \to (\infty, 0), \quad \frac{dp^*(\omega)}{d\omega} < 0 \quad \forall \omega; \\ y^*(\omega) &: (0, \infty) \to [y^*(\infty), y^*(0)), \quad \frac{dy^*(\omega)}{d\omega} > 0 \quad \forall \omega. \end{aligned}$$

Proof. See the Appendix. ■

I refer to the second case as the Cobb-Douglas-like economy because this seemingly special configuration, requiring $\epsilon_M^R = 1 + \left(1 - \epsilon_X^M\right) S_M^R$, is in fact quite common in the literature as it occurs when both technologies are Cobb-Douglas and $\epsilon_X^M = \epsilon_M^R = 1$.

The effect of resource abundance on the resource price is negative in all cases, while the effect on expenditure changes sign according to the substitution possibilities between labor and materials in manufacturing and between labor and the natural resource in materials production. This brings us to the observation that if $\Gamma(p)$ changes sign for some p, the model generates endogenously a switch from substitution to complementarity. I illustrate this case in Figure 2 where the income relation (18) is a hump-shaped function of p. The pattern is best captured by looking at the properties of the function $\Gamma(p)$. Using the definitions in Lemma 2,

$$\begin{aligned}\frac{d\Gamma(p)}{dp} &= -\frac{d\epsilon_X^M(P_M)}{dP_M}\frac{dP_M}{dp}S_M^R(p) \\ &\quad + \left(1-\epsilon_X^M(P_M)\right)\left(1-\epsilon_M^R(p)\right) - \frac{d\epsilon_M^R(p)}{dp}.\end{aligned}$$

This derivative is negative if the following two conditions hold:

- the elasticities ϵ_X^M and ϵ_M^R are increasing in P_M and p, respectively;
- the terms $1-\epsilon_X^M$ and $1-\epsilon_M^R$ have opposite sign.

The hump-shaped income relation in Figure 2 then obtains if $\Gamma(0) > 0$ and $\Gamma(\infty) < 0$. The second condition says that if demand in one sector is elastic, say $1 < \epsilon_X^M$, then demand in the other sector is inelastic, $1 > \epsilon_M^R$. Below I provide an example of how one can use CES technologies to construct an economy where these conditions hold. Here I discuss the general property.

Proposition 4 *Suppose that there exists a price $\bar{p}$ where $\Gamma(p)$ changes sign, from positive to negative, so that the income relation (18) is a hump-shaped function of p with domain $p \in [0,\infty)$ and codomain $y \in [y^*(0), y^*(\infty))$ or $y \in [y^*(\infty), y^*(0))$, where*

$$\begin{aligned}y^*(0) &= \frac{1}{1-\beta(\rho-\lambda)-\frac{\epsilon-1}{\epsilon}S_M^R(0)S_X^M(0)}, \\ y^*(\infty) &= \frac{1}{1-\beta(\rho-\lambda)-\frac{\epsilon-1}{\epsilon}S_M^R(\infty)S_X^M(\infty)}.\end{aligned}$$

Then there exists a unique equilibrium $(p^*(\omega), y^*(\omega))$ with the property:

$$\begin{aligned}p^*(\omega) &: (0,\infty) \to (\infty, 0), \quad \frac{dp^*(\omega)}{d\omega} < 0 \quad \forall\omega; \\ y^*(\omega) &: (0,\infty) \to [y^*(\infty), y^*(0)), \quad \frac{dy^*(\omega)}{d\omega} \gtreqless 0 \quad \omega \lesseqgtr \bar{\omega},\end{aligned}$$

where $\bar{\omega}$ is the value of ω such that $p^*(\bar{\omega}) = \bar{p}$.

Proof. See the Appendix. ■

The main message of this analysis is that resource abundance raises expenditure, and thereby results into a larger market for manufacturing goods, when the economy exhibits overall substitution between labor and resources (processed and raw) in the manufacturing of consumption goods and the processing of the natural resource into materials. Conversely, when the economy exhibits overall complementarity resource abundance results into a smaller market for manufacturing goods. More importantly, whether the economy exhibits substitution or complementarity depends on equilibrium prices and thus on the endowment ratio itself. In other words, there are solid reasons to expect that the effect of the endowment ratio on the path of consumption is non-monotonic.

5.2 The path of consumption and welfare

For concreteness, consider an economy in steady state (p^*, y^*) and imagine an increment $d\omega$ in its endowment ratio. Then, by construction we have

$$\Delta^* = \frac{y^*(\omega) + \frac{dy^*(\omega)}{d\omega}}{y^*(\omega)} - 1 = \frac{1}{y^*(\omega)} \frac{dy^*(\omega)}{d\omega}$$

and we can use the results of the previous section in a straightforward manner. Let us look first at the initial effect on consumption.

Proposition 5 *The impact effect of the change in the endowment ratio is*

$$\frac{d \log\left(\frac{y^*(\omega)}{c^*(\omega)}\right)}{d\omega} = \left[\Gamma\left(p^*(\omega)\right) - \Psi\left(p^*(\omega)\right)\right] \omega \frac{dp^*(\omega)}{d\omega}, \tag{30}$$

where

$$\Psi\left(p^*(\omega)\right) \equiv \frac{\epsilon}{(\epsilon - 1)\, y^*(\omega)} = \kappa - S_M^R\left(p^*(\omega)\right) S_X^M\left(p^*(\omega)\right),$$

$$\kappa \equiv \frac{\epsilon}{\epsilon - 1}\left[1 - \beta(\rho - \lambda)\right].$$

Proof. See the Appendix. ■

Equation (27), Proposition 5 and the fact that $\frac{dp^*(\omega)}{d\omega} < 0\ \forall\omega$ then yield three possible configurations for the path of consumption:

1. $\Gamma\left(p^*(\omega)\right) < 0 < \Psi\left(p^*(\omega)\right)$. The initial jump is up and then we have a growth acceleration. There is no "curse" and welfare rises.

2. $0 < \Gamma(p^*(\omega)) < \Psi(p^*(\omega))$. The initial jump is up and then we have a growth deceleration. The welfare change is ambiguous since we have an intertemporal trade-off. Is the "curse" a curse?

3. $0 < \Psi(p^*(\omega)) < \Gamma(p^*(\omega))$. The initial jump is down and then we have a growth deceleration. Welfare falls. The "curse" is a curse.

Figure 3 illustrates these possibilities. Case 2 highlights the need to look at welfare directly to resolve the ambiguity.

Proposition 6 *The welfare effect of a change in the endowment ratio is*

$$\frac{dU^*(\omega)}{d\omega} = \frac{1}{\rho - \lambda}\left[(1+\mu)\Gamma(p^*(\omega)) - \Psi(p^*(\omega))\right]\omega\frac{dp^*(\omega)}{d\omega}. \tag{31}$$

Proof. See the Appendix. ■

We can then characterize four scenarios for welfare.

1. There is no curse, with or without quotation marks, when

$$(1+\mu)\Gamma(p^*(\omega)) < \Gamma(p^*(\omega)) \leq 0 < \Psi(p^*(\omega))$$

and the rise of the endowment ratio generates a growth acceleration associated to an initial jump up in consumption.[9]

2. There is a "curse", in that the rise of the endowment ratio causes a growth deceleration, but the deceleration is offset by the fact that resource abundance reduces prices and raises the level of consumption. This happens when

$$0 < \Gamma(p^*(\omega)) < (1+\mu)\Gamma(p^*(\omega)) < \Psi(p^*(\omega)).$$

In this case the "curse" is not a curse.

3. The "curse" is a curse when

$$0 < \Gamma(p^*(\omega)) < \Psi(p^*(\omega)) < (1+\mu)\Gamma(p^*(\omega))$$

and the rise of the endowment ratio generates a growth deceleration that dominates over the initial jump up in consumption.

[9] Notice that this includes the Cobb-Douglas economy since in that case $1 = \epsilon_X^M = \epsilon_M^R$ and growth does not respond to ω at all while lower prices yield higher utility.

4. The "curse" truly is a curse. This worst-case scenario happens when

$$0 < \Psi\left(p^*\left(\omega\right)\right) < \Gamma\left(p^*\left(\omega\right)\right) < \left(1+\mu\right)\Gamma\left(p^*\left(\omega\right)\right)$$

and the rise of the endowment ratio generates a growth deceleration associated to an initial fall of consumption.

One of course is interested in mapping these scenarios into values of the endowment ratio itself. To do this, it is useful (albeit not necessary) to impose more structure.

5.3 The role of ω: A CES economy

Consider the following technologies:

$$X_i = Z_i^\theta \left[\psi_X \left(L_{X_i} - \phi\right)^{\sigma_X} + \left(1-\psi_X\right) M_i^{\sigma_X}\right]^{\frac{1}{\sigma_X}}, \quad \sigma_X \leq 1;$$

$$M = \left[\psi_M L_M^{\sigma_M} + \left(1-\psi_M\right) R^{\sigma_M}\right]^{\frac{1}{\sigma_M}}, \quad \sigma_M \leq 1.$$

The associated variable unit-cost functions are:

$$C_{X_i} = Z_i^{-\theta}\left[\psi_X^{-\frac{1}{\sigma_X - 1}} W^{\frac{\sigma_X}{\sigma_X - 1}} + \left(1-\psi_X\right)^{-\frac{1}{\sigma_X - 1}} P_M^{\frac{\sigma_X}{\sigma_X - 1}}\right]^{\frac{\sigma_X - 1}{\sigma_X}};$$

$$C_M = \left[\psi_M^{-\frac{1}{\sigma_M - 1}} W^{\frac{\sigma_M}{\sigma_M - 1}} + \left(1-\psi_M\right)^{-\frac{1}{\sigma_M - 1}} p^{\frac{\sigma_M}{\sigma_M - 1}}\right]^{\frac{\sigma_M - 1}{\sigma_M}};$$

From these one derives (recall that $W \equiv 1$):

$$S_X^M = \frac{1}{1 + \left(\frac{\psi_X}{1-\psi_X}\right)^{\frac{1}{1-\sigma_X}} P_M^{\frac{\sigma_X}{1-\sigma_X}}}; \quad \epsilon_X^M = 1 + \frac{\sigma_X}{1-\sigma_X}\left(1 - S_X^M\right);$$

$$S_M^R = \frac{1}{1 + \left(\frac{\psi_M}{1-\psi_M}\right)^{\frac{1}{1-\sigma_M}} p^{\frac{\sigma_M}{1-\sigma_M}}}; \quad \epsilon_M^R = 1 + \frac{\sigma_M}{1-\sigma_M}\left(1 - S_M^R\right).$$

As is well known, the CES contains as special cases the linear production function ($\sigma = 1$) wherein inputs are perfect substitutes, the Cobb-Douglas ($\sigma = 0$) wherein the elasticity of substitution between inputs is equal to 1, and the Leontief ($\sigma = -\infty$) wherein inputs are perfect complements.

Let:

$$\Gamma\left(\omega\right) \equiv \Gamma\left(p^*\left(\omega\right)\right) = \left(1 - \epsilon_X^M\left(p^*\left(\omega\right)\right)\right) S_M^R\left(p^*\left(\omega\right)\right) + 1 - \epsilon_M^R\left(p^*\left(\omega\right)\right);$$

$$\Psi(\omega) \equiv \Psi(p^*(\omega)) = \kappa - S_M^R(p^*(\omega))\, S_X^M(p^*(\omega)).$$

Observe that if $\sigma_X > 0$ and $\sigma_M > 0$, then $\Gamma(p) < 0$ for all p and the "curse" never occurs. If $\sigma_X < 0$ and $\sigma_M < 0$, instead, $\Gamma(p) > 0$ for all p and the "curse" always occurs. Interestingly, if σ_X and σ_M have opposite signs we can capture how the economy moves from one case to the other as the endowment ratio changes. Specifically, let $\sigma_X > 0$ and $\sigma_M < 0$ so that the manufacturing sector exhibits gross substitution between labor and materials while the resource sector exhibits gross complementarity between labor and raw resources. Recall that this means that demand for processed resources, i.e., materials, in the manufacturing sector is elastic, while demand for raw resources in the primary sector is inelastic.

It is then straightforward to obtain the following three cases.

1. There is no "curse" and hence no curse when

$$\Gamma(\omega) \leq 0 \Leftrightarrow 0 < \omega < \bar{\omega}.$$

2. There is a "curse" that is not a curse when

$$0 < (1+\mu)\,\Gamma(\omega) < \Psi(\omega) \Leftrightarrow \bar{\omega} < \omega < \tilde{\omega}.$$

3. There is a "curse" that is a curse when

$$0 < \Psi(\omega) < (1+\mu)\,\Gamma(\omega) \Leftrightarrow \tilde{\omega} < \omega < \infty.$$

Figure 4 illustrates this analysis. The appendix establishes formally the properties of the curves $\Gamma(\omega)$ and $\Psi(\omega)$ used in the figure to find the threshold values of the endowment ratio. Here I focus on the economics.

First, and most important: Why does the overall pattern of substitution/complementarity matter so much for growth and welfare? Because it regulates the reduction in the price p required for the economy to absorb the extra endowment of R relative to L. Moreover, the primary and manufacturing sectors are vertically related so the adjustment involves the interdependent responses of P_M to the increase in supply of processed resources and of p to the increase in supply of raw resources. Now, inelastic demand means that the price has to fall drastically to induce the market to absorb the additional quantity. Hence, if demand is inelastic in both sectors the overall adjustment requires drastic drops in both P_M and p. What matters is that the drastic fall of p results in a fall of resources income $p\omega$ which depresses expenditure y. This is the case $\Gamma > 0$. In contrast, if demand is elastic in both sectors the overall adjustment requires mild drops in prices. The crucial difference is that in this case $\Gamma < 0$ so that resources income $p\omega$ rises because the quantity effect dominates the price effect.

With this intuition in hand, we can now interpret the analytical results of the model. The function $\Gamma(\omega)$ starts out negative and increases monotonically, changing sign at $\bar{\omega}$ and converging to its positive upper bound as $\omega \to \infty$. To see this, write

$$\Gamma'(\omega) = \frac{d\Gamma(p^*)}{dp^*}\frac{dp^*}{d\omega} > 0 \ \forall\omega$$

and observe that this economy satisfies the conditions for $d\Gamma(p)/dp < 0 \ \forall p$, namely: $\epsilon_X^M > 1$ and increasing in P_M; $\epsilon_M^R < 1$ and increasing in p.

The function $\Psi(\omega)$ is hump-shaped with a peak exactly at $\bar{\omega}$, where $\Gamma(\omega)$ changes sign. The reason is that this is the value where the derivative of $S_M^R(p)\,S_X^M(p)$ with respect to p equals zero. Notice also that $\Gamma(\omega) < \Psi(\omega)$ for all ω (see the appendix for the proof) so that Case 3 from figure 3 and Scenario 4 from the analysis of welfare no longer apply because initial consumption cannot fall.

The resulting pattern is the following: The endowment ratio is initially very low and prices are very high, that is, $\omega \to 0 \Rightarrow p \to \infty \Rightarrow P_M = C_M(1,p) \to \infty$. Under these conditions, $\Gamma(0) < 0 < \Psi(0)$ so that an increase in ω produces a growth acceleration associated to a jump up in initial consumption. Intuitively, this says that in a situation of extreme scarcity and extremely high price a helicopter drop of natural resource is good. As the relative endowment grows, the overall pattern of substitution changes. In particular, $\Gamma(\omega)$ changes sign at $\omega = \bar{\omega}$ and we enter the region where we get a "curse of natural resources" because an increase in resource abundance yields a decrease in expenditure on manufacturing goods that triggers a slowdown of TFP growth. This, "curse", however, is not really a curse since we are to the left of $\tilde{\omega}$ and the slowdown is associated to an initial jump up in consumption that dominates in the intertemporal tradeoff. As we keep moving to the right and enter the next region, $\tilde{\omega} < \omega < \infty$, the growth deceleration is again associated to an initial jump up in consumption but now the deceleration dominates in the intertemporal tradeoff and welfare falls. It is here that we have a curse.

An effective summary of this pattern is to picture growth and welfare as hump-shaped functions of resource abundance. To do this, we first must translate the path of the log of TFP into a measure of average growth in line with what is considered in the empirical literature. Specifically, let

$$g(0,t) \equiv \frac{1}{t}\left[\log T(t) - \log T_0\right]$$

be the average growth rate of TFP between time 0 and time t. Then, using (26) we have:

$$g(0,t) = g^* + \left(\frac{\gamma}{\nu} + \frac{1}{\epsilon - 1}\right)\Delta^* \frac{1 - e^{-\nu t}}{t}. \tag{32}$$

This says that the an increase in the endowment ratio is associated to a transitory period of above (below) trend average growth if it generates an increase (decrease) in per capita expenditure on consumption goods. Notice that as $t \to \infty$ this measure converges to g^*, which does *not* depend on ω because there is no scale effect. Notice also, that since it depends on ω only through Δ^* it is hump-shaped in ω with the maximum at $\omega = \bar{\omega}$.

For welfare we have two options. We can use the measure U^* calculated in (28), which is clearly hump-shaped in ω with the maximum at $\omega = \tilde{\omega}$. We then conclude directly that the peak of growth occurs earlier than the peak of welfare. This property captures the crucial role of initial consumption. As equation (28) makes clear, welfare depends on the whole path of consumption, not on a summary statistic of its slope.

One could argue that using (32) and (28) is inappropriate as they are summary statistics computed over different time intervals. The alternative is to compute welfare as the discounted integral of the same part of the consumption path that we use to compute (32), that is,

$$U^*(0,t) = \int_0^t e^{-(\rho-\lambda)s} \log u^*(s)\, ds.$$

Using (26), (27) and (32), we can write

$$U^*(0,t) = (a_1 - a_3)\, g^* + a_2 \cdot \log\left(\frac{y^*}{c^*}\right) + a_3 \cdot g(0,t),$$

where

$$a_1 \equiv \int_0^t e^{-(\rho-\lambda)s} s ds;\ a_2 \equiv \int_0^t e^{-(\rho-\lambda)s} ds;\ a_3 \equiv \frac{t\int_0^t e^{-(\rho-\lambda)s}\left(1 - e^{-\nu s}\right) ds}{1 - e^{-\nu t}}.$$

This more algebra-intensive exercise does not change the conclusion: the two summary statistics differ because the former excludes the initial jump in consumption. It is then obvious that at $\omega = \bar{\omega}$ the derivative of $U^* (0, t)$ with respect to ω is still positive and welfare is still rising. Interestingly, the expression above contains only items that in principle are observable so that one can do quick, back-of-the-envelope welfare calcuations.

It is useful to close this section with some remarks on the generality of the analysis. CES technologies yield a highly tractable structure. However, they are restrictive in that they yield that within each sector demand is either always inelastic or always elastic. To obtain the economy-wide cross over from complementarity to substitution one then needs to exploit the sectoral composition effect and posit that σ_X and σ_M have opposite sign. Nevertheless, the results above do not require this assumption. Inspection of the function $\Gamma(\omega)$ suggests that to obtain $\Gamma(0) < 0$ and $\Gamma(\infty) > 0$, and therefore the threshold $\bar{\omega}$, all that is needed is technologies that deliver price elasticities of demand that start out below one and turn larger than one as the price of the good rises.

5.4 The reallocation

One is interested in knowing how the outcomes above relate to the economy's reallocation of labor across sectors. Using (16) we have

$$\frac{L_M}{L} = y \frac{\epsilon - 1}{\epsilon} S_X^M \left(1 - S_M^R\right).$$

It is then easy to show that (see the Appendix)

$$\frac{d}{d\omega}\left(\frac{L_M}{L}\right) > 0 \quad \forall \omega,$$

so that, intuitively, resource abundance yields a reallocation of labor from manufacturing to primary production. This result implies that

$$\frac{L_X}{L} + \frac{L_Z + L_N}{L} = 1 - \frac{L_M}{L}$$

falls to its new steady state value when ω increases.

This is an interesting property as it says that there is a reallocation of labor from production of manufacturing goods to production of materials, but that this reallocation is *not necessarily* associated to a TFP slowdown. This is a fundamental difference between this model and models that generate the curse of natural resources by tying productivity growth to manufacturing employment through learning by doing mechanisms.

Since the inter-sectoral reallocation is instantaneous, the dynamics that drive the time path of TFP take place within manufacturing. Equations (4), (14) and the formulation of the entry cost yield that the R&D share of employment is

$$\begin{aligned}\frac{L_Z + L_N}{L} &= \frac{L_Z}{L} + \frac{\dot{N}}{L} \cdot \beta \frac{Y}{N} \\ &= \left[\frac{1}{\epsilon} - \beta\left(\rho - \lambda + \delta\right)\right] y - \phi n.\end{aligned}$$

Recall that y jumps on impact to its new steady state value y^* while n is predetermined and does not jump. Then, using (24) at any time t we have

$$\frac{L_Z + L_N}{L} = \left[\frac{1}{\epsilon} - \beta\left(\rho - \lambda + \delta\right)\right] y^* - \frac{\phi n^*}{1 + e^{-\nu t}\Delta^*}.$$

The new steady state value is

$$\left(\frac{L_Z + L_N}{L}\right)^* = y^* \left[\left[\frac{1}{\epsilon} - \beta\left(\rho - \lambda + \delta\right)\right] - \phi \frac{n^*}{y^*}\right],$$

where we know from the analysis of section 3 that $\frac{n^*}{y^*}$ is independent of ω. Thus, when the economy experiences a growth acceleration because y^* rises, the R&D share of employment jumps up and converges from above to a permanently higher value. Obviously, then, the ratio $\frac{L_X}{L}$ jumps down and converges from below to a permanently lower value. The reverse happens when y^* falls and the economy experiences a growth deceleration.

5.5 Other interesting considerations

It is also interesting to study in some detail the role of measures of abundance and specialization. I consider the following two measures of resource abundance that are close analogs to measures that have been studied in the empirical literature: the value of natural wealth as a share of GDP and the value of natural wealth per capita. Notice that since $A = \beta Y$ and y is constant, in this economy (nominal) GDP is

$$GDP = Y + \dot{A} = Y + \beta \dot{Y} = Y\left(1 + \beta\lambda\right).$$

Then, my two measures of abundance are:

$$\frac{p\Omega}{L} = p\omega;$$

$$\frac{p\Omega}{GDP} = \frac{p\omega}{(1+\beta\lambda)\, y}.$$

Differentiation yields:

$$\frac{d\,(p\omega)}{d\omega} = (\omega+1)\, p\omega\Gamma\frac{dp}{d\omega};$$

$$\frac{d}{d\omega}\left(\frac{p\Omega}{GDP}\right) = \frac{1}{(1+\beta\lambda)\, y}\left[\frac{d\,(p\omega)}{d\omega} - \frac{p\omega}{y}\frac{dy}{d\omega}\right] = \frac{p\omega}{(1+\beta\lambda)\, y}\Gamma\frac{dp}{d\omega}.$$

Notice how for both measures the effect of a change in ω *necessarily* has the same sign as the effect on expenditure,

$$\Delta^* = \frac{1}{y^*(\omega)}\frac{dy^*(\omega)}{d\omega} = \omega\Gamma\frac{dp}{d\omega}.$$

The model, in other words, predicts a positive correlation between natural wealth (per unit of GDP or per capita) and growth whether the "curse" occurs or not. It thus suggests that the positive correlation reported in the papers mentioned in the introduction does not provide sufficient information to assess the underlying economic mechanism — which here is driven by an income effect in which the adjustment of the resource price plays a crucial role — without specifying further what restrictions one imposes on the endogenous variables that show up in these measures.

For example, Brunneschewiler and Bulte (2008) note that there is a crucial difference between the two measures of abundance in that the latter has an endogenous variable at the denominator while the former does not. They argue that institutional factors drive down both the level and the growth rate of GDP per capita, so that it is quite possible that the ratio $\frac{p\Omega}{GDP}$ rises and regressions that do not control for its endogeneity reveal a negative correlation between growth and this measure of abundance. They thus suggest that one should use the former measure because it is not subject to this problem, and they show that doing so produces a positive correlation with growth. From this result they infer that the "curse" does not exist. The premise of this line of reasoning — that the endogeneity of the level of GDP at the denominator of the second measure of abundance must be taken into account — is absolutely correct. My analysis, moreover, shows that it runs deeper than that. The value of the resource endowment at the numerator of the ratio $\frac{p\Omega}{GDP}$ contains the price of the natural resource. The inference that the "curse" does not exist, then, is correct if we know that the price effect is zero so that an increase in ω surely yields higher resource income. To my knowledge, however, the question of whether the data on natural capital used in this type of regressions supports such an assumption has not been investigated.[10]

6 Conclusion

The debate on whether natural resource abundance is good or bad for the long-run fortunes of the economy is almost as old as economics itself. The modern incarnation, spurred by the work of Sachs and Warner (1995, 2001), hinges on the sign of the coefficient of measures of resource abundance in growth regressions. Differently from the past, thus, it is much more driven by solid and systematic econometric work. This is the clearest benefit of the comprehensive data sets now available. As this evidence accumulates and adds to the already existing large body of historical information and analyses, a definitive answer might finally emerge. We are not there yet, and the debate rages on.

A striking feature of this debate — and of the associated theoretical literature — is that it rarely takes into account that the correlation between resource abundance and the initial level of GDP per capita is positive in the very data set used to document the negative correlation with the growth rate, and that therefore it is not obvious that resource abundant countries are worse off simply because they grow slower. In this paper, I developed a model of endogenous growth that allowed me to focus on the role of the intertemporal trade-off in determining whether the effects of natural resource abundance on initial income and growth yield an increase or a decrease in welfare. It is only in the latter case that the "curse" is indeed a curse. I found that resource abundance has non-monotonic effects on growth and welfare. More precisely, both growth and welfare are hump-shaped functions of resource abundance. The property that the peak of growth occurs earlier than the peak of welfare captures the crucial role of initial consumption, which rises with resource abundance, and is an important reminder that the welfare effect of resource abundance depends on the whole path of consumption, not on a summary statistic of its slope.

If resource abundance means that society sacrifices growth to boost current consumption, it stands to reason that to figure out whether it better or worse off requires us to resolve the intertemporal trade-off. Consequently,

[10] This strikes me as a first-order question since much of the debate hinges on the performance of oil, gas or mineral exporting countries. Is it a fact that the discovery of new reserves of some resource in some country does *not* affect the world price of the resource? Even under price-taking behavior by the country's suppliers, the country's overall effect on the world price can be non-negligible.

growth regressions that ignore the effect of resource abundance on initial income provide only one piece of the puzzle, not the solution. This is not a simple restatement of the fact that correlation is not causation. We cannot infer from the negative correlation between resource abundance and growth that resource abundant economies are worse off even if we can establish beyond reasonable doubt that the relation is indeed negative and causal.

7 Appendix

7.1 The typical firm's behavior

To characterize the typical firm's behavior, consider the Current Value Hamiltonian

$$CVH_i = [P_i - C_X(1, P_M)Z_i^{-\theta}]X_i - \phi - L_{Z_i} + z_i \alpha K L_{Z_i},$$

where the costate variable, z_i, is the value of the marginal unit of knowledge. The firm's knowledge stock, Z_i, is the state variable; R&D investment, L_{Z_i}, and the product's price, P_i, are the control variables. Firms take the public knowledge stock, K, as given.

Since the Hamiltonian is linear, one has three cases. The case $1 > z_i \alpha K$ implies that the value of the marginal unit of knowledge is lower than its cost. The firm, then, does not invest. The case $1 < z_i \alpha K$ implies that the value of the marginal unit of knowledge is higher than its cost. Since the firm demands an infinite amount of labor to employ in R&D, this case violates the general equilibrium conditions and is ruled out. The first order conditions for the interior solution are given by equality between marginal revenue and marginal cost of knowledge, $1 = z_i \alpha K$, the constraint on the state variable, (8), the terminal condition,

$$\lim_{s \to \infty} e^{-\int_t^s [r(v)+\delta]dv} z_i(s) Z_i(s) = 0,$$

and a differential equation in the costate variable,

$$r + \delta = \frac{\dot{z}_i}{z_i} + \theta C_X(1, P_M) Z_i^{-\theta-1} \frac{X_i}{z_i},$$

that defines the rate of return to R&D as the ratio between revenues from the knowledge stock and its shadow price plus (minus) the appreciation (depreciation) in the value of knowledge. The revenue from the marginal unit of knowledge is given by the cost reduction it yields times the scale of production to which it applies. The price strategy is

$$P_i = C_X(1, P_M) Z_i^{-\theta} \frac{\epsilon}{\epsilon - 1}. \tag{33}$$

Peretto (1998, Proposition 1) shows that under the restriction $1 > \theta(\epsilon - 1)$ the firm is always at the interior solution, where $1 = z_i \alpha K$ holds, and equilibrium is symmetric.

The cost function (7) gives rise to the conditional factor demands:

$$L_{X_i} = \frac{\partial C_X(W, P_M)}{\partial W} Z_i^{-\theta} X_i + \phi;$$

$$M_i = \frac{\partial C_X(W, P_M)}{\partial P_M} Z_i^{-\theta} X_i.$$

Then, the price strategy (33), symmetry and aggregation across firms yield (11) and (12).

Also, in symmetric equilibrium $K = Z = Z_i$ yields $\dot{K}/K = \alpha L_Z / N$, where L_Z is aggregate R&D. Taking logs and time derivatives of $1 = z_i \alpha K$ and using the demand curve (5), the R&D technology (8) and the price strategy (33), one reduces the first-order conditions to (13).

Taking logs and time-derivatives of V_i yields

$$r + \delta = \frac{\Pi_{Xi}}{V_i} + \frac{\dot{V}_i}{V_i},$$

which is a perfect-foresight, no-arbitrage condition for the equilibrium of the capital market. It requires that the rate of return to firm ownership equal the rate of return to a loan of size V_i. The rate of return to firm ownership is the ratio between profits and the firm's stock market value plus the capital gain (loss) from the stock appreciation (depreciation).

In symmetric equilibrium the demand curve (5) yields that the cost of entry is $\beta \frac{Y}{N}$. The corresponding demand for labor in entry is

$$L_N = \left(\dot{N} + \delta N\right) \beta \frac{Y}{N}.$$

The case $V > \beta \frac{Y}{N}$ yields an unbounded demand for labor in entry, $L_N = +\infty$, and is ruled out since it violates the general equilibrium conditions. The case $V < \beta \frac{Y}{N}$ yields $L_N = -\infty$, which means that the non-negativity constraint on L_N binds and $\dot{N} = -\delta N$, which implies negative net entry due to the death shock. Free-entry requires $V = \beta \frac{Y}{N}$. Using the price strategy (33), the rate of return to entry becomes (14).

7.2 The economy's resources constraint

I now show that the household's budget constraint reduces to the economy's labor market clearing condition. Starting from (2), recall that $A = NV$ and $(r+\delta)V = \Pi_X + \dot{V}$. Substituting into (2) yields

$$\dot{N}V = N\Pi_X + L + p\Omega + \Pi_M - Y.$$

Observing that $N\Pi_X = NPX - L_X - L_Z - P_M M$, $NPX = Y$, $\Pi_M = P_M M - L_M - pR$, $R = \Omega$, and that the free entry condition yields that total employment in entrepreneurial activity is $L_N = \dot{N}V$, this becomes

$$L = L_N + L_X + L_Z + L_M.$$

7.3 Proof of Proposition 1

Taking logs of (25) yields

$$\log T(t) = \theta \log Z_0 + \theta \int_0^t \hat{Z}(s)\, ds + \frac{1}{\epsilon - 1} \log N(t).$$

Using the definition $n \equiv Ne^{-\lambda t}$ and adding and subtracting $\hat{Z}^*$ from $\hat{Z}(t)$ yields

$$\log T(t) = \theta \log Z_0 + gt + \theta \int_0^t \left[\hat{Z}(s) - \hat{Z}^*\right] ds + \frac{1}{\epsilon - 1} \log n(t).$$

Using (19) the integral becomes

$$\begin{aligned} \theta \int_0^t \left(\hat{Z}(s) - \hat{Z}^*\right) ds &= \theta \frac{\alpha\theta(\epsilon - 1)}{\epsilon} \int_0^t \left(\frac{y^*}{n(s)} - \frac{y^*}{n^*}\right) ds \\ &= \gamma \int_0^t \left(\frac{n^*}{n(s)} - 1\right) ds, \end{aligned}$$

where

$$\gamma \equiv \theta \frac{\alpha\theta(\epsilon - 1)}{\epsilon} \frac{y^*}{n^*} = \theta \frac{\alpha\theta(\epsilon - 1)}{\epsilon} \frac{\phi - \frac{\rho + \delta}{\alpha}}{\frac{1 - \theta(\epsilon - 1)}{\epsilon} - \rho\beta}.$$

Finally, (24) and the definition of Δ^* yield

$$\begin{aligned} \theta \int_0^t \left(\hat{Z}(s) - \hat{Z}^*\right) ds &= \gamma \int_0^t e^{-\nu s} \left(\frac{n^*}{n_0} - 1\right) ds \\ &= \frac{\gamma}{\nu} \Delta^* \left(1 - e^{-\nu t}\right). \end{aligned}$$

Using this result and (24) again, and noting that by construction $T_0 = Z_0^{\theta} n_0^{\frac{1}{\epsilon-1}}$ since $n_0 = N_0 e^{-\lambda \cdot 0} = N_0$, yields

$$\log T(t) = \log T_0 + g^* t + \frac{\gamma \Delta^*}{\nu}\left(1 - e^{-\nu t}\right) + \frac{1}{\epsilon - 1} \log \frac{1 + \Delta^*}{1 + e^{-\nu t}\Delta^*}.$$

Next consider

$$\log u^*(t) = \log \frac{\epsilon - 1}{\epsilon} + \log\left(\frac{y^*}{c^*}\right) + \log T(t)$$

and use the expression just derived to write

$$\log u^*(t) = \log \frac{y^*}{c^*} + g^* t + \frac{\gamma \Delta^*}{\nu}\left(1 - e^{-\nu t}\right) + \frac{1}{\epsilon - 1} \log \frac{1 + \Delta^*}{1 + e^{-\nu t}\Delta^*}.$$

Substituting this expression into (1) yields

$$\begin{aligned} U^* &= \int_0^{\infty} e^{-(\rho-\lambda)t} \log u^*(t)\, dt \\ &= \int_0^{\infty} e^{-(\rho-\lambda)t} \left[\log\left(\frac{y^*}{c^*}\right) + g^* t\right] dt \\ &\quad + \frac{\gamma}{\nu}\Delta^* \int_0^{\infty} e^{-(\rho-\lambda)t}\left(1 - e^{-\nu t}\right) dt \\ &\quad + \frac{1}{\epsilon - 1}\int_0^{\infty} e^{-(\rho-\lambda)t} \log \frac{1 + \Delta^*}{1 + e^{-\nu t}\Delta^*} dt. \end{aligned}$$

The first and second integrals have straightforward closed form solutions; the third has a complicated solution involving the hypergeometric function. For my purposes, it is useful to introduce the following approximation that allows me to simplify the expression for welfare. Since in general $\log(1 + x) \simeq x$, I can rewrite

$$\log \frac{1 + \Delta^*}{1 + e^{-\nu t}\Delta^*} = \log(1 + \Delta^*) - \log\left(1 + e^{-\nu t}\Delta^*\right) = \Delta^*\left(1 - e^{-\nu t}\right),$$

which yields (26), (27) and

$$\begin{aligned} U^* &= \int_0^{\infty} e^{-(\rho-\lambda)t} \left[\log\left(\frac{y^*}{c^*}\right) + g^* t\right] dt \\ &\quad + \left(\frac{\gamma}{\nu} + \frac{1}{\epsilon - 1}\right)\Delta^* \int_0^{\infty} e^{-(\rho-\lambda)t}\left(1 - e^{-\nu t}\right) dt, \end{aligned}$$

which upon integration yields (28).

7.4 Proof of Lemma 2

Differentiating and manipulating terms yields:

$$\begin{aligned}
\frac{\partial\left(S_X^M S_M^R\right)}{\partial p} &= \frac{\partial S_X^M}{\partial p} S_M^R + \frac{\partial S_M^R}{\partial p} S_X^M \\
&= \frac{\partial S_X^M}{\partial P_M}\frac{P_M}{S_X^M} \cdot \frac{\partial P_M}{\partial p}\frac{p}{P_M} \cdot \frac{S_X^M S_M^R}{p} + \frac{\partial S_M^R}{\partial p}\frac{p}{S_M^R} \cdot \frac{S_M^R S_X^M}{p} \\
&= \left[\frac{\partial S_X^M}{\partial P_M}\frac{P_M}{S_X^M} \cdot \frac{\partial P_M}{\partial p}\frac{p}{P_M} + \frac{\partial S_M^R}{\partial p}\frac{p}{S_M^R}\right]\frac{S_M^R S_X^M}{p} \\
&= \left[\frac{\partial S_X^M}{\partial P_M}\frac{P_M}{S_X^M} \cdot \frac{\partial C_M}{\partial p}\frac{p}{C_M} + \frac{\partial S_M^R}{\partial p}\frac{p}{S_M^R}\right]\frac{S_M^R S_X^M}{p}.
\end{aligned}$$

Recalling that

$$\begin{aligned}
\frac{\partial S_X^M}{\partial P_M}\frac{P_M}{S_X^M} &= 1 - \epsilon_X^M, \\
\frac{\partial S_M^R}{\partial p}\frac{p}{S_M^R} &= 1 - \epsilon_M^R, \\
\frac{\partial C_M}{\partial p}\frac{p}{C_M} &= S_M^R
\end{aligned}$$

and substituting into the expression above yields (29).

7.5 Proof of Proposition 3

Refer to Figure 1. In the case of complementarity, depicted in the upper panel, we have the following pattern. For $\omega \to 0$ the expenditure line (17) is almost, but not quite, flat and intersects the income relation (18) for $p \to \infty$ and $y \to y^*(\infty)$. As ω grows, the expenditure line rotates counterclockwise and the intersection shifts left, tracing the income relation. We thus obtain that both p^* and y^* fall. As $\omega \to \infty$, the expenditure line becomes vertical and the intersection occurs at $p \to 0$ and $y \to y^*(0)$.

In the case of substitution in the lower panel, we have a similar pattern with the difference that the income relation has negative slope so that y^* increases as the expenditure line rotates counterclockwise. Specifically, for $\omega \to 0$ the expenditure line is almost, but not quite, flat and intersects the income relation for $p \to \infty$ and $y \to y^*(\infty)$. As ω grows, the expenditure line rotates counterclockwise and the intersection shifts left yielding that

p^* falls while y^* rises. As $\omega \to \infty$, the expenditure line becomes vertical and the intersection occurs at $p \to 0$ and $y \to y^*(0)$.[11]

7.6 Proof of Proposition 4

Refer to Figure 2. The proof is essentially the same as above. The only difference is that as the expenditure line rotates it traces the hump-shaped income relation yielding that p^* always falls while y^* rises for $0 < \omega < \bar{\omega}$ and falls for $\bar{\omega} < \omega < \infty$.

7.7 Proof of Proposition 5

Observe first that

$$\frac{d\log\left(\frac{y}{c}\right)}{d\omega} = \frac{d\log y}{d\omega} - \frac{d\log c}{d\omega}.$$

Now,

$$\frac{d\log y}{d\omega} = \frac{d\log y}{dp}\frac{dp}{d\omega} = \omega\Gamma(p)\frac{dp}{d\omega}$$

and

$$\begin{aligned}\frac{d\log c}{d\omega} &= \frac{1}{C_X}\frac{dC_X}{d\omega} = \frac{1}{C_X}\frac{dC_X}{dP_M}\frac{dP_M}{dp}\frac{dp}{d\omega}\\ &= \frac{P_M}{C_X}\frac{dC_X}{dP_M}\frac{dP_M}{dp}\frac{p}{P_M}\frac{1}{p}\frac{dp}{d\omega}\\ &= \frac{S_X^M S_M^R}{p}\frac{dp}{d\omega}.\end{aligned}$$

[11] An interesting question is whether the model admits an equilibrium for $\omega = 0$. To investigate whether it does, it is useful to focus on the CES case considered in the text. The answer depends on wheteher resources are essential. Under $\sigma_M < 0$, $\sigma_X < 0$ they are and thus an equlibrium with $\omega = 0$ (due to $\Omega = 0$) cannot exist. To see this, notice that in this case $S_X^M(0)\,S_M^R(0) = 0$ and $S_X^M(\infty)\,S_M^R(\infty) = 1$ so that the representation in the upper panel of Figure 1 applies with the difference that the expenditure and income relations have the intercept $y^*(0)$ in common. (This point, obviously, is not an equilibrium.) We then have that for $\omega = 0$ the expenditure line is exactly flat and there is no intersection with the income relation. If instead we posit $\sigma_M > 0$, $\sigma_X > 0$ we have $S_X^M(0)\,S_M^R(0) > 0$ and $S_X^M(\infty)\,S_M^R(\infty) = 0$ and the representation in the lower panel of Figure 1 applies with the crucial difference that the expenditure line starts out at $y^*(\infty)$. Hence, for $\omega = 0$ the line is exactly tangent to the horizontal asymptote of the income relation implying that $p = \infty$ and $y^*(\infty)$ *is* a feasible equilibrium. This is because we now allow for substitution so that processed natural resources are *not* essential in manufacturing and the economy *can* exist with $\Omega = 0$.

Hence,

$$\frac{d\log\left(\frac{y}{c}\right)}{d\omega} = \left(\Gamma - \frac{S_X^M S_M^R}{p\omega}\right)\omega\frac{dp}{d\omega}.$$

which using (12) becomes (30).

7.8 Proof of Proposition 6

Observe that by construction

$$\frac{d\Delta^*}{d\omega} = \frac{1}{y^*}\frac{dy^*}{d\omega}$$

Differentiation of (28) then yields

$$\begin{aligned}\frac{dU^*}{d\omega} &= \frac{1}{\rho-\lambda}\left[\frac{d\log\left(\frac{y^*}{c^*}\right)}{d\omega} + \mu\frac{1}{y^*}\frac{dy^*}{d\omega}\right] \\ &= \frac{1}{\rho-\lambda}\left[(1+\mu)\frac{d\log y^*}{d\omega} - \frac{d\log c^*}{d\omega}\right].\end{aligned}$$

Using the expressions calculated above and rearranging terms yields (31).

7.9 The $\Gamma(\omega)$ and $\Psi(\omega)$ curves used in Figure 4

The $\Gamma(\omega)$ curve. Dropping stars to simplify notation, the equation becomes

$$\Gamma(\omega) = \left[-\frac{\sigma_X}{1-\sigma_X}\left(1 - S_X^M(p(\omega))\right) + \frac{\sigma_M}{1-\sigma_M}\right]S_M^R(p(\omega)) - \frac{\sigma_M}{1-\sigma_M}.$$

Proposition 4 yields that $p(0) = \infty$ and $p(\infty) = 0$. Therefore, for $\omega \to 0$:

$$S_M^R(0) = \lim_{p\to\infty}\frac{1}{1+\left(\frac{\psi_M}{1-\psi_M}\right)^{\frac{1}{1-\sigma_M}} p^{\frac{\sigma_M}{1-\sigma_M}}} = 1.$$

Also,

$$P_M(0) = \lim_{p\to\infty}\left[\psi_M^{\frac{1}{1-\sigma_M}} + (1-\psi_M)^{\frac{1}{1-\sigma_M}} p^{\frac{\sigma_M}{\sigma_M-1}}\right]^{\frac{\sigma_M-1}{\sigma_M}} = \infty,$$

so that

$$S_X^M(0) = \lim_{P_M\to\infty}\frac{1}{1+\left(\frac{\psi_X}{1-\psi_X}\right)^{\frac{1}{1-\sigma_X}} P_M^{\frac{\sigma_X}{1-\sigma_X}}} = 0.$$

Consequently,

$$\Gamma(0) = \frac{-\sigma_X}{1-\sigma_X} < 0.$$

In contrast, for $\omega \to \infty$:

$$S_M^R(\infty) = \lim_{p\to 0} \frac{1}{1 + \left(\frac{\psi_M}{1-\psi_M}\right)^{\frac{1}{1-\sigma_M}} p^{\frac{\sigma_M}{1-\sigma_M}}} = 0.$$

Also,

$$P_M(\infty) = \lim_{p\to 0} \left[\psi_M^{\frac{1}{1-\sigma_M}} + (1-\psi_M)^{\frac{1}{1-\sigma_M}} p^{\frac{\sigma_M}{\sigma_M - 1}}\right]^{\frac{\sigma_M - 1}{\sigma_M}} = \psi_M^{-\frac{1}{\sigma_M}},$$

so that

$$S_X^M(\infty) = \frac{1}{1 + \left(\frac{\psi_X}{1-\psi_X}\right)^{\frac{1}{1-\sigma_X}} \psi_M^{-\frac{1}{\sigma_M}\frac{\sigma_X}{1-\sigma_X}}}.$$

Consequently,

$$\Gamma(\infty) = \frac{-\sigma_M}{1-\sigma_M} > 0.$$

The next step is to show that the curve is monotonically increasing:

$$\begin{aligned}
\Gamma'(\omega) &= \underbrace{\frac{\sigma_X}{1-\sigma_X}}_{+} \cdot \underbrace{\frac{dS_X^M}{dP_M}}_{-} \cdot \underbrace{\frac{dP_M}{dp}}_{+} \cdot \underbrace{\frac{dp}{d\omega}}_{-} \cdot S_M^R \\
&\quad + \underbrace{\left[-\frac{\sigma_X}{1-\sigma_X}\left(1 - S_X^M\right) + \frac{\sigma_M}{1-\sigma_M}\right]}_{-} \cdot \underbrace{\frac{dS_M^R}{dp}}_{+} \cdot \underbrace{\frac{dp}{d\omega}}_{-} \\
&> 0.
\end{aligned}$$

Finally, by continuity there exists a value $\bar{\omega}$ where $\Gamma(\bar{\omega}) = 0$.

The $\Psi(\omega)$ curve. Again dropping stars, the equation is:

$$\Psi(\omega) = \kappa - S_M^R(p(\omega))\, S_X^M(p(\omega)).$$

The limiting behavior at 0 and ∞ is straightforward. The previous calculations yield:

$$\Psi(0) = \kappa - S_M^R(0)\, S_X^M(0) = \kappa;$$

$$\Psi(\infty) = \kappa - S_M^R(\infty)\, S_X^M(\infty) = \kappa.$$

Next observe that

$$\begin{aligned}\Psi'(\omega) &= \frac{d\left(S_M^R(p(\omega))\,S_X^M(p(\omega))\right)}{dp(\omega)}\frac{dp(\omega)}{d\omega}\\ &= \Gamma(\omega)\,\frac{S_M^R(p(\omega))\,S_X^M(p(\omega))}{p(\omega)}\frac{dp(\omega)}{d\omega},\end{aligned}$$

so that the curve is hump-shaped with its maximum exactly at the value $\bar{\omega}$ where $\Gamma(\omega)$ changes sign.

The threshold values. Observe that $\kappa > 1 > \frac{-\sigma_M}{1-\sigma_M}$. It is evident from Figure 2 then that $\Gamma(\omega) < \Psi(\omega)\ \forall\omega$. It follows that there is only one relevant intersection, of $\Psi(\omega)$ with $(1+\mu)\Gamma(\omega)$, that yields the threshold value $\tilde{\omega}$ such that $\bar{\omega} < \tilde{\omega}$.

7.10 The reallocation

The expression for the share of employment in the primary sector and the expressions derived in the proofs above yield

$$\begin{aligned}\frac{d}{d\omega}\left(\frac{L_M}{L}\right) &= \frac{\epsilon-1}{\epsilon}\left[\frac{dy}{d\omega}S_X^M\left(1-S_M^R\right)+y\frac{dS_X^M}{d\omega}-\frac{d\left(S_X^M S_M^R\right)}{d\omega}\right]\\ &= \frac{\epsilon-1}{\epsilon}\left[\frac{\Gamma}{\Psi}S_X^M\left(1-S_M^R\right)+1-\epsilon_X^M-\Gamma\right]\omega\frac{dp}{d\omega}.\end{aligned}$$

Recall that $\frac{dp}{d\omega} < 0$. Then

$$\frac{d}{d\omega}\left(\frac{L_M}{L}\right) > 0 \quad \forall\omega$$

because the term in brackets is

$$\frac{\Gamma}{\Psi}\left(S_X^M-\kappa\right)+1-\epsilon_X^M < 0$$

since $\kappa > 1$ and $1-\epsilon_X^M < 0$ under the assumption that manufacturing exhibits substitution between labor and materials.

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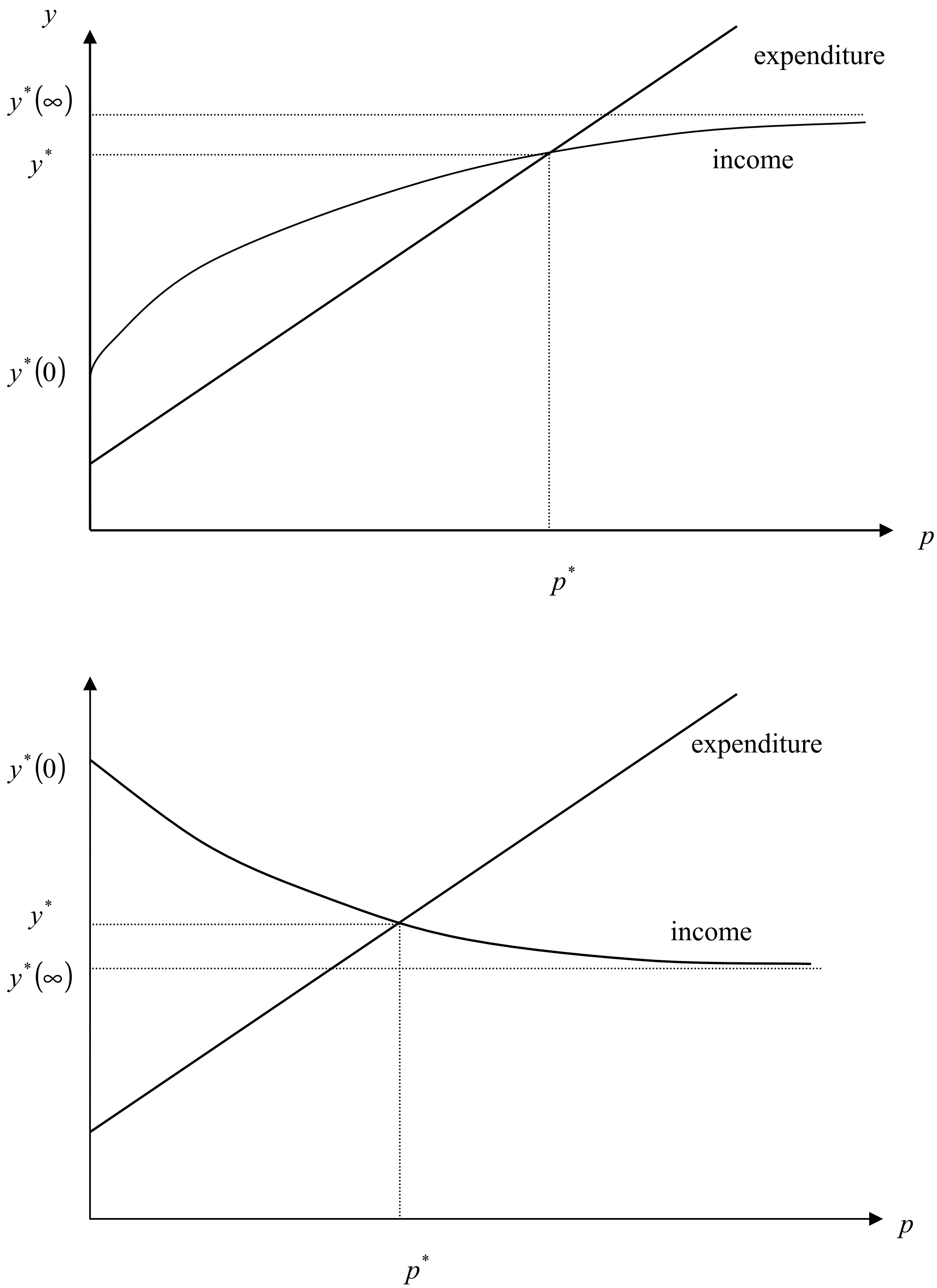

Figure 1: General Equilibrium: Monotonic income relation

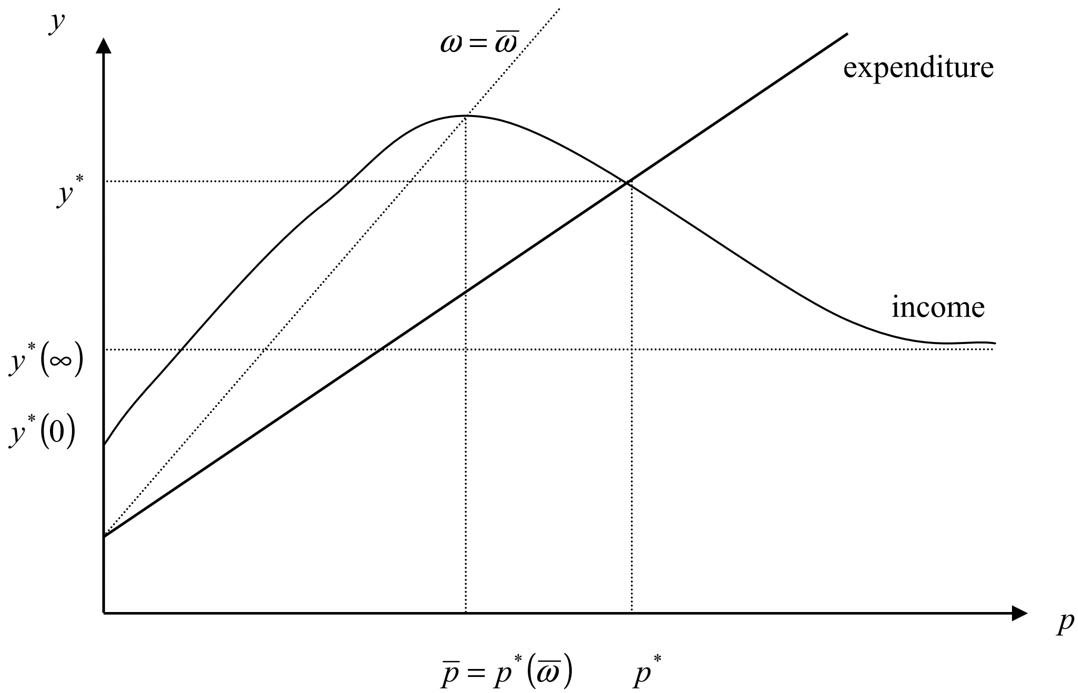

Figure 2: General Equilibrium: Hump-shaped income relation

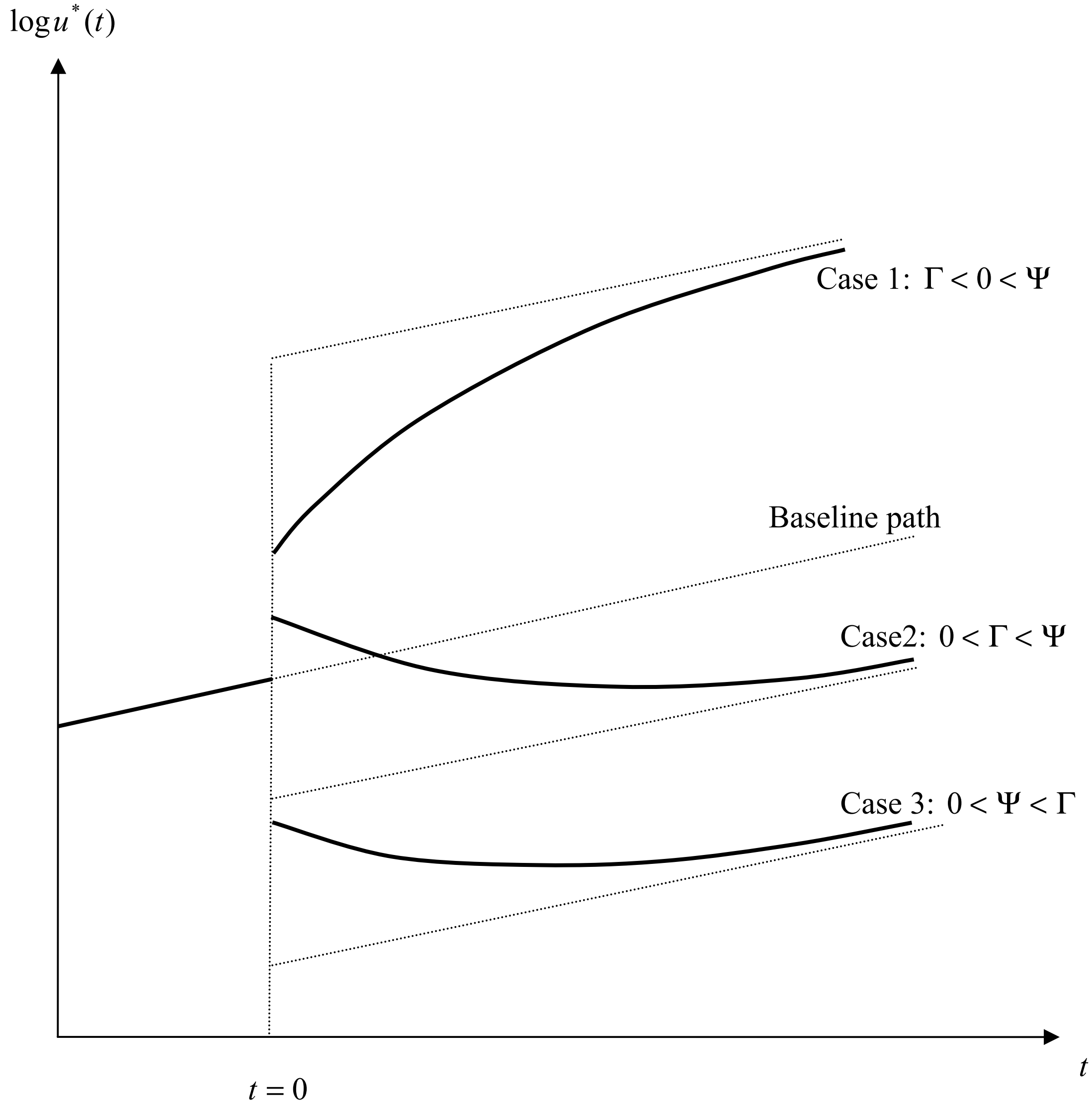

Figure 3: The path of consumption

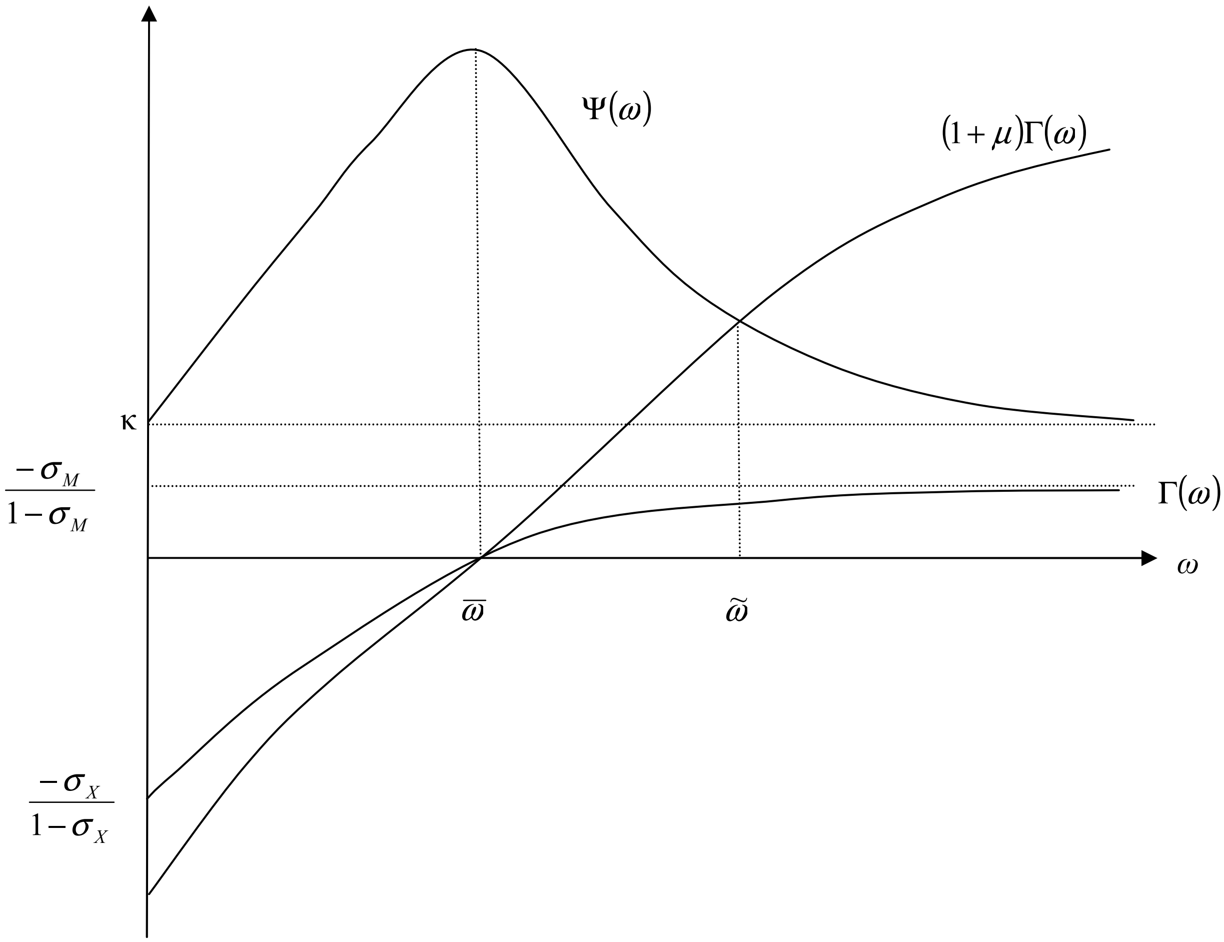

Figure 4: The endowment ratio and equilibrium outcomes