Growth with Deadly Spillovers[*]

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Abstract

Pollution is one of the world's primary causes of premature death, but macroeconomic analysis largely neglects the existence of such negative externality. We build a tractable growth model where vertical and horizontal innovations raise productivity, a polluting primary sector exploits natural resources, emissions increase mortality, and fertility is endogenous. The response of the mortality rate to changes in population size is generally non-monotonic and reflects a precise equilibrium relationship that combines emission intensity, dilution effects and labor reallocation effects. Deadly spillovers affect welfare and create steady states, including mortality traps, that do not exist in models without pollution-induced mortality. Exogenous shocks like environmental taxes increase population size, accelerate horizontal innovation in the transition and may even raise long-run growth if they reduce the long-run mortality rate. Subsidies to primary production have opposite consequences for the steady state and, especially if combined with new discoveries of primary resources, may push less populated resource-rich economies into mortality traps leading to population implosion.

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1 Introduction

Pollution kills. According to the Lancet Commission on pollution and health,

> "Diseases caused by pollution were responsible for an estimated 9 million premature deaths in 2015 – 16% of all deaths worldwide – three times more deaths than from AIDS, tuberculosis, and malaria combined and 15 times more than from all wars and other forms of violence." (The Lancet, 2017, p. 5).

These figures revise upwards previous estimates that had prompted the World Health Organization to consider pollution as one of the world's most significant causes of premature death (WHO, 2016). In fact, among all the risk factors held to explain world deaths in 2017, air pollution alone ranks fourth.[1]

The economics literature on the subject is mostly empirical and confirms the scale and pervasiveness of the problem (e.g., Ebenstein et al. 2015, Arceo et al. 2016). Despite this evidence, however, macroeconomic analysis neglects the role of *deadly spillovers*: there are no models that account for the simultaneous endogeneity of economic growth, environmental degradation, mortality and fertility. This type of models are however necessary to address fundamental questions, first and foremost: how does pollution affect macroeconomic performance through excess deaths?

Unlike the conventional pollution externalities studied in environmental economics, i.e., emissions that reduce the utility of individuals and/or the efficiency of firms, deadly spillovers work through excess deaths that reduce labor supply and household expenditure, activate reallocation of resources across sectors – including the polluting primary sector and R&D activities that drive productivity growth – and prompt households to revise saving and fertility decisions. Understanding how these propagation channels determine macroeconomic performance is a necessary first step to study a number of questions of direct interest to empirical research and policy making. What are the effects of deadly spillovers on income dynamics when we account for demographic change, in particular when both fertility and mortality are endogenous? What are the short- and long-run effects of pollution taxes and/or subsidies to polluting sectors? What are the consequence of pollution-caused mortality in less populated resource-rich countries that typically display high emissions per capita? If deadly spillovers generate mortality traps, is population implosion a possibility in countries with large, polluting primary sectors?

[1] Ritchie and Roser (2020a; 2020b) show that all four main causes of death – heart disease, cancer, respiratory diseases and infections – exhibit a strong relationship with air pollution. See also Schlenker and Walker (2016) and the literature cited therein. This evidence provides a lower bound on the importance of pollution for mortality since other forms of pollution also matter.

The answers to such questions hinge on how the mortality rate responds to changes in population size. We study this mechanism in a model where a polluting primary sector exploits a natural resource, horizontal and vertical innovations raise the productivity of intermediate producers, emissions increase mortality, and household choices determine fertility. A distinctive property of our framework is that it produces equilibrium paths where population converges to a finite size while income per capita grows via endogenous innovation. This property extends the results derived in Peretto and Valente (2015), which builds a theory of finite population on a finite planet. In contrast to many balanced-growth models that predict exponential population growth, the framework replicates the fertility decline experienced in most industrialized countries and is consistent with the view in demography and ecology that in the long run population must converge to a finite size because the earth's carrying capacity of people is finite.[2] In Peretto and Valente (2015) there is no pollution, mortality is exogenous, and population growth eventually stops because of the fertility response to income per capita. We extend the framework to introduce pollution externalities and endogenous mortality, obtaining a model where the *mortality response to pollution* affects economic growth and welfare, and becomes an independent force stabilizing the population level in the long run. Our main results are as follows.

First, we show that the equilibrium relation between the mortality rate and population size reflects the complex interactions between two main forces: the *primary-employment* effect and the *damage-dilution* effect. The primary-employment effect summarizes the causal chain linking labor supply to pollution generation: higher population increases total labor, which induces higher employment in the primary sector and higher emissions caused by commodity production. The damage-dilution effect summarizes the relation between population size and individual exposure to harmful pollutants, and incorporates two distinct mechanisms: dose dilution – i.e., the reduction in individual absorption of pollutants when population increases at given total emissions – and emission reduction – i.e., the reduction in individual exposure when a larger population causes emissions per capita to fall. Since the primary-employment and damage-dilution effects typically affect mortality in opposite directions, the general equilibrium relation between mortality rates and population size is either L-shaped (monotonically decreasing) or U-shaped.

Second, under broad conditions the economy converges to a *regular* steady state where the population is constant and income per capita grows due to endogenous R&D-based innovations. If a regular steady state already exists in the absence of pollution, including deadly spillovers modifies

[2] Demographers forecast a levelling off of world population within the next century, century and a half, and offer arguments based on first principles for why this must happen due to the feedback mechanisms that operate in a closed system, i.e., a finite planet.

its position and the path that leads to it: equilibrium dynamics are quantitatively different, and pollution-related deaths affect growth and welfare through channels that the existing literature neglects. This result provides new answers to traditional policy questions, like the effects of emission taxes or sectoral subsidies, as we discuss below. Moreover, deadly spillovers create under broad conditions mortality traps, that is, unstable steady states that split the state space in two basins of attraction: one for the regular steady state and one for the *extinction* steady state. We show that if the initial population-resource ratio is below a critical threshold, the population implodes due to increasing mortality despite *growing* fertility rates. In this scenario, deadly spillovers change the qualitative properties of the dynamical system and open the door to new policy questions. We also show that the existence of deadly spillovers can create regular steady states that would not exist otherwise, that is, endogenous pollution-caused mortality stabilizes the population even in the absence of other well-understood mechanisms.

Third, the demographic development of the economy has first-order effects on its economic development. This follows from the model's Schumpeterian block, which features endogenous R&D-driven innovation that responds to the dynamics of market size. The causal link is that population size is the key driver of market size. Along regular equilibrium paths, population growth expands market size and feeds transitional productivity growth through horizontal innovations that raise the number of firms. In the regular steady state, productivity growth is exclusively driven by the rate of vertical innovations, which is stronger the lower the mass of firms relative to population. Deadly spillovers affect both mechanisms because horizontal and vertical innovations depend on population dynamics that, in turn, respond to endogenous mortality. In particular, we show that exogenous shocks that increase the long-run population level *and* reduce the long-run mortality rate will typically yield a 'double growth dividend': population growth accelerates productivity growth via firms' entry during the transition, while a lower mortality rate increases long-run productivity growth via higher investment in vertical innovations. The fact that deadly spillovers create a channel through which the deep parameters regulating mortality have steady-state growth effects is a novel result in itself since our Schumpeterian framework belongs to a class of models known for the scale-invariance of the steady-state growth rate. This is not a manifestation of the traditional scale effect – a causal relationship running from the exogenous supply of labor to growth – but rather a distinctive outcome of our model where the (endogenous) dynamics of the population-resource ratio affect the (endogenous) dynamics of the mortality rate.

Fourth, deadly spillovers matter for environmental policy and the assessment of the effects of resource booms (discoveries of new natural resource endowments). We show that taxing polluting primary sectors yields a demographic double dividend: it increases the economy's carrying capacity of people, meaning that a given resource base supports a larger population in steady state, and it also reduces the size of the mortality trap. Also, taxing the polluting sector accelerates transitional productivity growth via horizontal innovations that expand the mass of firms along with population. In the long run, the tax may even yield an economic growth dividend by increasing the steady-state rate of vertical innovations: this will happen if the damage-dilution effect is sufficiently strong to guarantee a lower equilibrium mortality rate in the new steady state. Importantly, subsidies to the primary sector yield opposite effects and may be a recipe for disaster if they are implemented after resource booms.

Our analysis contributes to several literatures. It contributes to growth economics by providing a full account of the interactions between economic growth and demography when all the underlying determinants – fertility, mortality, innovation – are fully endogenous and produce a finite population. The view that demography matters for macroeconomic performance is well established but rarely implemented in models that produce finite population. The few growth models that do typically focus on Malthusian mechanisms (Eckstein et al., 1988; Galor and Weil, 2000; Brander and Taylor, 1998) or similar market-based mechanisms where resource scarcity causes relative-price dynamics that eventually bring population growth to a halt (Strulik and Weisdorf, 2008; Peretto and Valente, 2015).[3] While they provide useful insights, none of these works study pollution-caused mortality, which in contrast is central to our analysis.

The few existing theories explicitly linking emissions to mortality assume that pollution reduces life expectancy and analyze equilibrium paths in models of capital accumulation; see Mariani et al. (2010), Varvarigos (2014), and Goenka et al. (2020). In this framework, the average death rate grows with emissions but people can undertake defensive expenditures that mitigate the effect. This interaction can create multiple steady states with different income levels.[4] These conclusions relate to Nelson's (1956) notion of under-development traps: non-linearities in the returns to investment generate regular high-*income* steady states and low-*income* steady states that constitute poverty traps.[5] Our analysis differs from these contributions in two key dimensions. First, the economic

[3] A different approcah is in Brunnschweiler et al. (2020), where population is stabilized by the dilution of financial wealth in an OLG economy populated by disconnected generations of agents with finite expected lifetime.

[4] In models with fixed saving and investment rates, a high rate of pollution-reducing technical change is a general pre-condition for sustainable long-term growth (Brock and Taylor, 2005). Models of optimal pollution control study whether the sustainability condition is satisfied ex-post once savings and investment in clean technologies are endogenous. The rise of poverty traps induced by pollution with state-dependent abatement efficiency is formally demonstrated in Smulders and Gradus (1996) and Xepapadeas (1997).

[5] An interesting application of this reasoning can be found in models with endogenous lifetime (Blackburn and Cipriani, 2002; Chakraborty, 2004) where households optimize over finite horizons and longevity rises with income, e.g., via better nutrition and health care. The interaction produces a stable steady state with high income but also

block of our economy is a scale-invariant Schumpeterian model of endogenous R&D-based innovation where demography and productivity dynamics eventually decouple: with or without pollution, the regular steady state features constant, finite, endogenous population size while income per capita grows at a constant rate. This property yields distinctive predictions, like the impact of demography on different engines of growth and the dividends generated by environmental taxes, that cannot be replicated in one-sector models of the neoclassical type. Second, the demographic block of our economy hinges on a mortality function satisfying the properties of the relative-risk functions used in the medical literature on pollution-attributed deaths (e.g., IHME, 2018) and includes explicit population-exposure interactions in the form of dose dilution, which occurs at the point of contact between humans and pollutants, and emission reduction, which occurs at the point of origin of pollutants as a by-product of household activity. This characterization of the interaction between humans and pollutants yields a rich and yet tractable model where the response of the mortality rate to population is generally non-monotonic due to the combination of primary-employment and damage-dilution effects. Accordingly, our mortality traps are conceptually distinct from the poverty traps discussed in development economics: demographic implosion is triggered by a low population-resource ratio, not by low income levels.

Our result that the emission tax can generate a demographic double dividend and an economic growth dividend is a novel contribution to the literature on environmental macroeconomics and policy. The traditional notion of a double dividend is that emission taxes can reduce aggregate efficiency losses by shifting distortionary taxes from clean production factors to dirty ones (Bovenberg and Goulder, 2002). A complementary notion is that emission taxes can encourage innovation (Porter and van der Linde, 1995). Neither mechanism works through demography. Our result, in contrast, follows entirely from the endogenous demographic response to deadly spillovers. The related result that a subsidy to the primary sector has the opposite effects of the emission tax is relevant from a policy perspective because resource-rich developing countries often implement such subsidies by invoking the need to boost income via resource rents (Bretschger and Valente, 2018). In our analysis, the subsidy reduces the population and pushes the economy closer to the mortality trap. Similarly, our result on the effects of the discovery of a new endowment of the natural resource contributes to the literature on the Resource Curse hypothesis, which studies the mechanisms through which natural resource abundance undermines economic performance (e.g., Mehlum et al. 2006) but typically neglects demography.

Our theoretical analysis also offers a potential contribution to the empirical literature that inspired it. Our characterization of excess deaths builds on the established concept of pollution

a poverty trap with low income and short lifetime.

attributed fraction (PAF) of total deaths formalized and estimated by a growing body of the medical literature, including the Global Burden of Disease Study (IHME, 2018). The key difference is that we deviate from the standard practice that equates dose absorption to concentrations as we allow dose absorption to depend separately on concentrations (as the measure of emissions) and population (as the measure of the number of people potentially exposed to the emissions). The econometric models currently used to estimate PAFs retrospectively ignore damage-dilution effects, but our model suggests that including such effects would have relevant implications for projections about future mortality rates. In particular, neglecting damage dilution can introduce undesirable biases in applied studies, like OECD (2016), that aim at constructing good forecasts of long-run economic and demographic trends. Hopefully, our framework can offer some guidance on how to avoid such biases.

2 The model

We study a decentralized economy where the competitive primary sector produces a commodity using labor and a raw natural resource (henceforth, resource). The monopolistically competitive intermediate sector uses the commodity to produce differentiated goods that the competitive final sector uses to produce a homogeneous consumption good. Endogenous economic growth results from horizontal and vertical innovations in the intermediate sector. The decisions of households facing child-rearing costs drive endogenous fertility. Commodity production and household activities generate harmful pollution that increases mortality. We begin our analysis with a discussion of the interactions between demography and pollution.

2.1 Demography with deadly spillovers

Central to our model is the relation between population and mortality that arises from deadly spillovers. The relation has two components. The first incorporates insights developed in medicine, epidemiology, public health and health economics to characterize how pollution causes mortality. The second characterizes how primary production and household activity generate pollution. The combination of the two components produces a rich and yet tractable representation of mortality as a function of population size. The properties of this function drive our novel results on the demographic evolution of our model economy. In this subsection we concentrate on the first component, the pollution-mortality causal channel. We present the second component, the population-pollution channel, in subsection 2.2.

2.1.1 Pollution kills

Time is continuous and indexed by $t \in [0, \infty)$. The dynamics of population, L, are

$$\dot{L}(t) = B(t) - M(t) = [b(t) - m(t)] \cdot L(t), \tag{1}$$

where B is births and M is deaths. For future use, we also specify the dynamics in terms of the birth rate, $b = B/L$, and the death rate, $m = M/L$. The novel ingredient is the function

$$M(t) = \underbrace{\bar{m}L(t)}_{\text{baseline deaths}} + \underbrace{(1 - \bar{m})\, L(t) \cdot D(t)}_{\text{excess deaths caused by pollution, } M_p} \tag{2}$$

that decomposes total deaths in baseline deaths unrelated to pollution, $\bar{m}L$, and excess deaths caused by pollution, M_p. The exogenous constant $\bar{m} > 0$ is the baseline mortality rate that prevails in the absence of deadly spillovers. Deaths from pollution are a fraction D of $(1 - \bar{m})\, L$, the mass of people that survive the baseline causes of death. Drawing on the insights developed in several literatures, we model D as the ratio between the flow of excess deaths due to pollution and the population. The former is the output of a matching process, $f(E, L)$, with two inputs: the population, L, as the measure of the mass of individuals that can potentially absorb harmful pollutants, and aggregate emissions, E.[6] The fraction D, therefore, is the outcome of a process in which individuals and pollutants collide at random. Each collision results in an individual's exposure to and possible absorption of the pollutants, an event that the literature calls *dose absorption*, which can result in the death of the individual. Given this interpretation and its construction, in the language of matching models we call D the *dose-absorption rate*.

Formally, we write the matching process as a differentiable function increasing in each one of its inputs, i.e., $f_E(\cdot) > 0$ and $f_L(\cdot) > 0$. We also reasonably assume that each input is essential, i.e., $f(0, L) = f(E, 0) = 0$. To maximize tractability we write

$$D(t) = \frac{f(E(t), L(t))}{L(t)} = \mu_0 \cdot E(t)^{\chi} \cdot L(t)^{-\chi\zeta}, \tag{3}$$

where $\chi > 0$ and $0 \leq \zeta < 1/\chi$. With this representation, excess deaths caused by pollution are

$$M_p(t) = (1 - \bar{m})\, L(t) \cdot D(t) = \mu E(t)^{\chi} L(t)^{1-\chi\zeta}, \tag{4}$$

where $\mu = \mu_0 (1 - \bar{m}) > 0$ collects all the constant terms and $0 < 1 - \chi\zeta \leq 1$. For future use, we define the *pollution-caused excess mortality rate*

$$m_p(t) \equiv \frac{M_p(t)}{L(t)} = \mu \cdot \left(\frac{E(t)}{L(t)^{\zeta}} \right)^{\chi}, \tag{5}$$

[6] We treat pollution as a flow because it makes things clearer and we noted that, aside from requiring a lot of extra algebra, nothing of substance in our analysis changes if we treat pollution as a stock.

where the denominator L^ζ captures the *dose-dilution effect*, that is, the property that for given emissions E a larger population reduces individual dose absorption. Letting ζ vary between zero and one we obtain three cases of particular interest:

$$\begin{array}{llll} \zeta = 1 & \to M_p = \mu E^\chi L^{1-\chi}, & m_p = \mu \cdot (E/L)^\chi\,; & \text{balanced dose dilution} \\ 0 < \zeta < 1 & \to M_p = \mu E^\chi L^{1-\zeta\chi}, & m_p = \mu \cdot \left(E/L^\zeta\right)^\chi\,; & \text{weak dose dilution} \\ \zeta = 0 & \to M_p = \mu E^\chi L, & m_p = \mu \cdot E^\chi. & \text{no dose dilution} \end{array}$$

In the first polar case, $\zeta = 1$, the excess mortality rate depends on emissions per capita. The opposite polar case, $\zeta = 0$, yields no dilution: population size does not affect individual dose absorption and thus the excess mortality rate depends on aggregate emissions. In the intermediate case, $0 < \zeta < 1$, the excess mortality rate depends on aggregate emissions and population size with different elasticities.[7]

In writing the excess mortality rate (5) as a function of population size we deviate from the practice in the literature that typically considers the no dilution case. The reason is that we want to allow for a wide range of processes caused by different types of pollutants. This feature of our analysis is sufficiently important to warrant a detailed discussion.

2.1.2 Pollution absorption and dose-dilution effects

In this subsection we show that our specification of pollution-caused excess mortality is firmly grounded in the literature while it allows for more generality.

Dose, exposure, and concentration. According to conventional definitions in the scientific literature, *dose* is the amount of the pollutant that crosses one of the body's boundaries and reaches the target tissue, *exposure* refers to any (outer or inner) contact between a contaminant and the human body, *concentration* is the amount of pollutant per unit volume, e.g., micrograms of PM per cubic meter of air.[8] The empirical literature typically uses concentration as a measure of individual doses (see, e.g., Burnett and Cohen, 2020). This is an understandable way to circumvent the huge practical problem of measuring actual individual doses, but it is at best a crude approximation that neglects the distinction between pollution – a physical characteristic of the environment at a certain place and time – and individual dose – the result of the interaction between the environment and a specific individual. Our model captures this distinction: concentration is aggregate emissions E, whereas the individual dose absorption is D, the outcome of a matching process with well-defined

[7] We omit the case $\zeta < 0$ because we are interested in dose dilution, not its opposite. Allowing for $\zeta < 0$ makes m_p increasing in population size, which yields general equilibrium results that are qualitatively the same as in the no dose dilution case.

[8] See Watson et al. (1988) for a detailed discussion of concepts and conventional definitions.

characteristics. Assuming *concentration=exposure=dose*, as the empirical literature does, rules out dose-dilution effects a priori. The empirical literature seems to be well aware of this problem but to our knowledge there have been no attempts to estimate the effect of population size on individual dose absorption at given emission levels.[9]

Empirical methodologies. The most cited estimates of pollution-caused mortality rates (e.g., the Global Burden Disease Study: GBD, 2018) are based on the empirical model

$$\mathrm{PAF} = 1 - \left(\mathrm{RR}_{(d,d_{\min})}\right)^{-1} \tag{6}$$

where the left-hand side is the *pollution attributed fraction* (PAF) of total deaths and $\mathrm{RR}_{(d,d_{\min})}$ is a relative risk function built on the concept of odds ratio, namely, the probability of death if the average person absorbs d units divided by the probability of death if the person absorbs $d_{\min}$ units, where $d_{\min}$ is the counterfactual with zero deaths from pollution. Relative risk functions can take many different shapes, but in all cases they must be increasing in d and satisfy $\mathrm{RR}_{(d,d_{\min})} \geqslant 1$, with equality when $d = d_{\min}$ (see, e.g., Burnett and Cohen, 2020). We now derive the explicit relation between these objects and our model. First, we solve equation (6) for RR and note that in our model $d_{\min} = 0$. This gives us $\mathrm{RR}_{(d,0)} = (1-\mathrm{PAF})^{-1}$. Next, we note that PAF is the empirical counterpart of the ratio M_p/M in our model. We thus use equation (4) to write

$$\mathrm{RR}_{(d,0)} = 1 + \frac{1-\bar{m}}{\bar{m}} \cdot D\left(E, L\right). \tag{7}$$

Our representation of excess deaths is thus directly related to the empirical literature. In particular, with our characterization of the matching process for $D\left(E, L\right)$ the right-hand side of (7) satisfies the necessary properties for relative risk functions identified in the empirical literature. As noted, the difference between our theory and the existing literature is that the latter assumes that concentration is a good proxy for individual dose absorption. As we argue below, however, this assumption presumes that pollutants are non-rival. Some studies convert the scale of E from aggregate to individual levels, but they do so without adjusting for population size. For example, Pope et al. (2011) convert the standard scale of mean concentration, $\bar{E} = \mu\mathrm{g/m}^3$ micrograms per cubic metre per day, to the individual scale of inhaled mass, $\bar{E}_i = \bar{E} \cdot 18\mathrm{m}^3$ micrograms per day, since the average individual inhales $18\mathrm{m}^3$ of air daily. The unstated assumption of this methodology is that

[9] Concentration is a valid proxy if it approximates well the dose actually absorbed by the average individual (Watson et al., 1988, p.208). Case studies report substantial gaps between mean concentrations of air pollutants and individual exposure; see, e.g., Shaddick et al. (2008) for estimates of personal exposures in Greater London compared with the traditional approach of assuming that a single pollution affects simultaneously and equally the entire population.

if population doubles at given concentration $\bar{E}$, the individual dose $\bar{E}_i$ is unchanged, but this would mean total human absorption of the pollutant $L\bar{E}_i$ doubles despite unchanged total pollution. This procedure thus neglects the fact that changes in population size modify actual individual doses at given concentration – an interaction that we must include in our general-equilibrium analysis in order to study the mortality-population relationship. More precisely, rescaling procedures like Pope et al. (2011) do not recognize that as long as the harmful pollutant is rival a larger population at given concentration dilutes the individual dose that is relevant for mortality. We now elaborate further on this point.[10]

Pollutants and humans. Suppose a group of $L(t)$ identical individuals located in a particular volume of space at time t. Inject in that space a quantity $E(t)$ of a pollutant. If $E_A(t)$ units of the pollutant make contact with the bodies of the individuals (exposure), the residual amount $E(t) - E_A(t)$ fully dissipates within the time interval dt. This flow-pollution hypothesis greatly simplifies our exposition of the key concepts without affecting the conclusions. Henceforth, we thus drop the time argument unless necessary to avoid confusion. Next, we follow the literature and write the dose absorption rate as an increasing and concave function of average exposure, i.e.,

$$D = h\left(\frac{E_A}{L}\right). \tag{8}$$

The precise mapping between the literature and our model hinges on whether the pollutant is non-rival or rival across individuals.

Non-rival pollutants. A pollutant is non-rival if multiple individuals can absorb it simultaneously with no reduction of its effect on each individual's health. An example of a process that meets this definition is climate change: rising temperature is viewed as affecting every individual simultaneously and equally. The empirical literature on climate change models individual damage as an increasing function of temperature, and temperature is an increasing function of aggregate measures of greenhouse gasses (see, e.g., Bressler, 2021). In the formal structure proposed above, this case yields $E_A = E \cdot L$ and thus $D = h(E)$. Our model captures this type of process in the no dose dilution case, $\zeta = 0$, which yields $m_p = \mu E^{\chi}$. However, our focus in this paper is neither climate change nor rising temperature. Moreover, the definition above suggests that, despite its popularity, the representation of pollutants as non-rival across individuals is suspect in light of the first-principles of physics and chemistry (Bolin, 2003). Indeed, the literature itself suggests that the

[10] These considerations fully apply to empirical methodologies that identify exposure with *population-adjusted mean concentrations* (e.g., Mackie et al., 2016). In those studies, the role of population weights is to provide a more accurate measure of local concentration $\bar{E}$ faced by local residents, but individual doses $\bar{E}_i$ are still identified with concentration levels that are not divided (nor 'partially diluted') by the local population level.

most relevant risks for individual health come from particulate matter (PM) and water pollution: these pollutants cause non-communicable diseases through absorption of individual doses that are rival: contaminated water consumed (or PM units inhaled) by an individual cannot be consumed (inhaled) by other individuals. We now turn to this concept.

Rival pollutants and dose dilution. A pollutant is rival when its absorption by one individual prevents its absorption by other individuals. In this case, we have $E_A = E$ and thus $D = h\left(E/L\right)$. Our model describes this scenario in the balanced-dilution specification where $\zeta = 1$ yields $m_p = \mu\left(E/L\right)^{\chi}$. As noted, however, our model is more general because it recognizes that the absorption process exhibits neither pure non-rivalry nor pure rivalry. One way to see this is to suppose $E > E_A$ and assume an increase dL of the mass of individuals in a given unit of space holding E in that space constant. Since exposure of each of the dL newcomers includes previously unabsorbed units of the pollutant, the final outcome ranges between two polar cases. At one extreme is the case where aggregate exposure E_A does not change because the dL newcomers exclusively make contact with pollution units that would have made contact with the pre-existing population L. In this scenario, we can think of aggregate exposure as a fixed fraction of aggregate emissions, which yields that the larger population dilutes linearly individual exposure. At the opposite end is the case where individual exposure remains the same because the dL newcomers exclusively make contact with pollution units that would otherwise dissipate. In this scenario, we can think of individual exposure as a fixed fraction of aggregate emissions regardless of population size. In the intermediate scenario, contact between one of the dL newcomers and a specific pollution unit prevents to some degree contact of that unit with the pre-existing population. The reason why this is a matter of degree is that chemical and physiological processes exhibit threshold, congestion and saturation mechanisms that are difficult to capture and cannot be described by simple proportionality relations. This suggests thinking of individual exposure as some general function of both aggregate emissions and population size. Inserting such function in (8) yields $D = h\left(E, L\right)$. Setting $h(E, L) = \mu_0 E^{\chi} L^{-\chi\zeta}$ yields our model, which one can thus interpret as a convenient reduced form that captures the considerations above. For example, more precise estimates of our parameter ζ would be especially relevant for studies, like OECD (2016), that use current estimates of pollution-caused mortality to make long-term projections of future mortality. We emphasize that dose dilution follows from the fact that individuals and pollutants must be rival to some degree because they are physical objects. The reason why perfect rivalry might not apply is that not all pollution is absorbed by individuals and nature is complex. Our structure accommodates such nuance. Nevertheless, none of the observations above support viewing pollutants as non-rival except in exceptional cases like temperature.

2.2 Pollution generation and emission-reducing effects

Pollution has two sources: commodity production (e.g., mining or generation of energy from fossil fuels) and household or, equivalently, personal activities (e.g., transport services, waste disposal, residential use of environmental amenities). Pollution generation thus takes the form

$$E(t) = \Gamma(E_\omega(t), E_h(t)), \quad \frac{\partial \Gamma(E_\omega, E_h)}{\partial E_\omega} > 0, \quad \frac{\partial \Gamma(E_\omega, E_h)}{\partial E_h} > 0, \tag{9}$$

where E_ω is emissions from commodity production and E_h is emissions from household activities.

Commodity production is $Q = \mathcal{F}(\Omega, L_Q)$ where Q is output, L_Q is the labor input and $\mathcal{F}$ is a linearly homogeneous function with the standard property of positive and diminishing marginal product of each input. For simplicity, we model the resource input as the constant flow, Ω, of productive services from a fixed endowment.[11] Resource processing generates one unit of emissions per unit of output, i.e.,

$$E_\omega(t) = Q(t) = \mathcal{F}(\Omega, L_Q(t)). \tag{10}$$

The representation $E_\omega = Q$ is not restrictive since further elasticities come into play when we consider the other source of pollution.

Household emissions per person is $E_h/L = \Psi(L)$. In line with the literature, we interpret the argument of this function as population density, that is, population L in a reference geographical area that we normalize to unity (e.g., people per square mile). It would be tempting to assume that a larger population L produces more emissions, $E_h = L \cdot \Psi(L)$, simply because $\Psi'(L) \geq 0$. However, a growing body of literature on urbanization documents density effects that yield $\Psi'(L) < 0$. Empirically, less populated areas tend to exhibit higher emissions per capita and, in some cases, also higher aggregate emissions (Stone, 2008). One explanation is that high density allows people to pursue personal activities in less polluting ways (e.g., public transport instead of individual transport) by providing stronger incentives (e.g., congestion effects) or better access to pollution-saving technologies (e.g., infrastructure with strong economies of scale). A second explanation is that pollution abatement activities, private and public, are more likely to take place in high-density areas due to stronger individual awareness and public support for tighter regulations. These and similar arguments suggest that population density reduces the pollution intensity of household activities (see Bork and Schrauth 2021 and the literature cited therein). If this effect is sufficiently strong, population size has a generally ambiguous effect on personal emissions, that is, at least over some range we cannot rule out $dE_h/dL < 0$.

[11] We assume that one unit of the endowment provides one unit of services. Thus, Ω denotes the endowment as well. We considered extensions where the natural resource is either renewable or exhaustible (Ω is an endogenous state variable) and noted that they complicate the analysis substantially without adding insight. We thus decided to focus on the simpler case of a fixed endowment.

These considerations suggest that the effect of population size on aggregate emissions is the outcome of complex interactions. We identify three channels:

$$\frac{dE}{dL} = \underbrace{\frac{\partial \Gamma\left(E_{\omega}, E_{h}\right)}{\partial E_{\omega}} \cdot \frac{dE_{\omega}}{dL}}_{\text{labor supply} > 0} + \underbrace{\frac{\partial \Gamma\left(E_{\omega}, E_{h}\right)}{\partial E_{h}} \cdot \Psi\left(L\right)}_{\text{scale} > 0} + \underbrace{\frac{\partial \Gamma\left(E_{\omega}, E_{h}\right)}{\partial E_{a}} \cdot L \cdot \Psi'\left(L\right)}_{\text{density} < 0}.$$

This decomposition identifies two empirically relevant cases. Weak emission reduction occurs when a larger population raises aggregate emissions but reduces emissions per capita. This requires $0 < (dE/dL)\,(L/E) < 1$. Strong emission reduction occurs when the larger population reduces emissions in both aggregate and per-capita terms. This requires $(dE/dL)\,(L/E) < 0$. The recent empirical literature estimates the elasticity $(dE/dL)\,(L/E)$ using population density and ground-level concentrations as the measures of L and E. These estimates suggest that strong emission reduction holds for ozone while weak emission reduction appears to hold for air pollution. For example, Bork and Schrauth (2021) obtain -0.14 for O3 ground-level concentrations and 0.08 for PM in Germany, Carozzi and Roth (2023) estimate an elasticity for PM of 0.14 in the US, and Chen et al. (2020) find a negative elasticity for PM of -0.26 in Chinese cities after controlling for city fixed effects in a panel specification. According to Ahlfeldt Pietrostefani (2019), the elasticity for PM ranges from 0.08 to 0.15 in advanced western economies.

We work with the functional form $E = E_{\omega}^{\upsilon} E_{h}^{1-\upsilon} = Q^{\upsilon}\left(L\Psi\left(L\right)\right)^{1-\upsilon}$, with $0 < \upsilon < 1$ and $\Psi\left(L\right) = L^{-(1+\xi)}$, where $\xi \geqslant -1$. If $\xi < 0$, density effects are weaker than scale effects in household emissions and E_h increases with the population. The reverse occurs if $\xi > 0$. Next, we define the elasticity of commodity output with respect to population size

$$\varepsilon_{Q,L} \equiv \frac{dQ}{dL_Q} \cdot \frac{L_Q}{Q} = \left(\frac{\partial Q}{\partial L_Q} \cdot \frac{L_Q}{\mathcal{F}}\right) \cdot \left(\frac{dL_Q}{dL} \cdot \frac{L}{L_Q}\right). \tag{11}$$

Note that this definition allows for the general-equilibrium dependence of primary employment, L_Q, on labor supply, L, and is thus a macroeconomic object. This structure yields the elasticity

$$\frac{dE}{dL} \cdot \frac{L}{E} = \underbrace{\upsilon \cdot \varepsilon_{Q,L}}_{\text{labor supply}} - \underbrace{\xi\left(1-\upsilon\right)}_{\text{emission reduction}}, \tag{12}$$

which identifies our two main channels for the dependence of aggregate emissions on population size: labor supply as the driver of primary employment; the balance between scale and density effects in household emissions. The expression identifies parametric conditions for weak or strong emission reduction that depend on the properties of the production technology of the primary sector.

2.3 Mortality, employment and damage dilution

From (2), (3) and (9), the crude mortality rate equals

$$\underbrace{m\left(t\right)}_{\text{crude}}=\underbrace{\bar{m}}_{\text{baseline}}+\underbrace{\mu\cdot Q\left(t\right)^{\chi\upsilon}\cdot L\left(t\right)^{-\chi\left[\zeta+\xi\left(1-\upsilon\right)\right]}}_{\text{excess death rate } m_p(t)}. \tag{13}$$

Given primary production, Q, population size, L, reduces the excess death rate via dose dilution at given total emissions and via emission reduction from population density. We can thus define the overall *damage-dilution effect* of larger population at given primary production as

$$\chi\left[\zeta+\xi\left(1-\upsilon\right)\right]=\underbrace{\text{damage intensity}\cdot\left[\text{dose dilution}+\text{emission reduction}\right]}_{\text{per capita damage reduction (given } Q\text{)}} \tag{14}$$

The sign of the damage-dilution effect is a priori ambiguous but it can be positive under a variety of plausible circumstances because dose dilution and emission reduction operate independently and can substitute for each other. Considering pollutants for which dose dilution is substantial, like water-contaminating elements, damage dilution may be positive even if population density does not yield substantial emissions reduction, that is, $\zeta>0$ with $\xi\left(1-\upsilon\right)$ negligible. Symmetrically, pollutants for which population density induces substantial emissions reduction, like ozone, exhibits positive damage dilution even if we treat O3 doses as non-rival, that is, $\zeta=0$ with $\xi\left(1-\upsilon\right)$ positive. For intermediate cases where dose dilution and emissions reduction can be positive but moderate, like particulate matter, damage dilution can still be positive. Total differentiation of (13) yields

$$\frac{dm_p}{dL}\cdot\frac{L}{m_p}=\underbrace{\chi\upsilon\varepsilon_{Q,L}}_{\text{primary-employment effect}}-\underbrace{\chi\left[\zeta+\xi\left(1-\upsilon\right)\right]}_{\text{damage-dilution effect}}. \tag{15}$$

This expression shows that larger population increases or decreases the mortality rate depending on the difference of two effects that fully summarize the several channels identified above. This property drives our analysis of the equilibrium path of the economy.

2.4 Consumption and reproduction choices

We use the Peretto-Valente (2015) extension of the textbook formulation of fertility theory (see, e.g., Barro and Sala-i-Martin, 2004, Ch. 9). The extension gives full control over expenditure per child to the household and allows for a "quantity-quality" trade-off with no additional complexity. Specifically, a representative household maximizes the dynastic utility function

$$U_0=\int_0^\infty e^{-\rho t}\ln u\left(C_L\left(t\right),C_B\left(t\right),L\left(t\right),B\left(t\right)\right)dt,\quad \rho>0 \tag{16}$$

where ρ is the individual discount rate, C_L is consumption of the adults, C_B is consumption of the children, L is the stock of adults and B is the instantaneous flow of newly born children per unit of time. Instantaneous utility is

$$u\left(C_L, C_B, L, B\right) = \left(\frac{C_L}{L}\right)^{\alpha}\left(\frac{C_B}{B}\right)^{1-\alpha}\left(L^{\alpha}B^{1-\alpha}\right)^{\psi}, \quad 0 < \alpha < 1, \quad 0 < \psi < 1. \tag{17}$$

In this structure, agents obtain utility from the consumption and presence of adults and from the consumption and presence of children with weights, respectively, α and $1-\alpha$. The parameter ψ regulates the trade-off between the individual consumption of the members of each group (adults and children) and the size of each group.[12]

Household expenditure is $Y = p_c C_L + p_c C_B$, where p_c is the price of the final good. The fertility choice is thus characterized by a trade-off between the utility benefit from reproduction and expenditure on the children's consumption. The price-taking household supplies the services of labor and of the natural resource inelastically. The household's budget is

$$\dot{A}\left(t\right) = r\left(t\right)A\left(t\right) + w\left(t\right)L\left(t\right) + p_{\omega}\left(t\right)\Omega + S\left(t\right) - Y\left(t\right), \tag{18}$$

where r is the rate of return on financial assets, A is asset holdings, w is the wage, p_{ω} is the per-unit resource royalty and Ω is the natural resource endowment over which the household has full property rights. The household chooses the time paths of C_L, C_B and B to maximize (16) subject to (18) and (1). The household takes the path of the mortality rate as given because private agents are unable to internalize the effects of emissions on mortality. Nonetheless, the household internalizes the intertemporal trade-off caused by population growth: a larger mass of adults expands the dynasty's consumption possibilities via additional labor income but, at the same time, reduces individual consumption possibilities via dilution effects.

The solution to the household problem is described in the Appendix. The conditions for utility maximization are the familiar Euler equation for consumption growth

$$\frac{\dot{Y}\left(t\right)}{Y\left(t\right)} = r\left(t\right) - \rho \tag{19}$$

and the associated equation for the birth rate,

$$\frac{\dot{b}\left(t\right)}{b\left(t\right)} = \frac{b\left(t\right)}{\left(1-\alpha\right)\left(1-\psi\right)}\left[\psi + \frac{w\left(t\right)L\left(t\right) - Y\left(t\right)}{Y\left(t\right)}\right] - \rho. \tag{20}$$

Equation (19) determines the growth rate of household consumption expenditure according to the traditional trade-off: the marginal benefit of asset accumulation versus the marginal cost of

[12] The restriction $0 < \psi < 1$ implies that for each group the elasticity of utility with respect to individual consumption exceeds the elasticity of utility with respect to the size of the group. Moreover, as we show in the Appendix, the maximization probem of the household is well defined only if the condition $\psi\left(1-\alpha\right) < 1-\alpha$ holds.

sacrificing current consumption. Equation (20) says that the birth rate increases over time when the anticipated rate of return from generating future adults exceeds the utility discount rate, ρ. The term in square brackets shows the components of this rate of return: the gross elasticity of utility to the mass of adults, ψ, plus their contribution to asset accumulation, given by the difference between labor income and consumption expenditure.

2.5 Producers: Final and Intermediate Sectors

Final sector. The final sector is competitive and produces with the technology

$$C(t) = \left(\int_0^{N(t)} x_i(t)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}, \quad \epsilon > 1 \tag{21}$$

where C is output, N is the mass of intermediate goods, x_i is the quantity of good i and ϵ is the elasticity of substitution between pairs of intermediate goods. Final producers maximize profits taking as given the mass of intermediate goods and the price, p_{x_i}, of each intermediate good. The solution to this problem yields the demand schedule

$$p_{x_i}(t) = \frac{Y(t)}{\int_0^{N(t)} x_i(t)^{\frac{\epsilon-1}{\epsilon}} di} \cdot x_i(t)^{-\frac{1}{\epsilon}} \tag{22}$$

for each intermediate good.

Intermediate sector: incumbents. Each intermediate good is supplied by a monopolist that operates the production technology

$$x_i(t) = z_i(t)^{\theta} \cdot Q_{x_i}(t)^{\gamma} (L_{x_i}(t) - \phi)^{1-\gamma}, \quad 0 < \theta < 1, \quad 0 < \gamma < 1, \tag{23}$$

where x_i is output, Q_{x_i} is the commodity input, L_{x_i} is production labor and $\phi > 0$ is overhead labor. The productivity term z_i^{θ} is Hicks-neutral with respect to the rival inputs, labor and the commodity, and depends on the stock of firm-specific knowledge z_i. The firm's cost minimization problem yields the total cost function

$$TC_i(x_i(t); w(t), p_q(t)) = w(t)\phi + \gamma^{-\gamma}(1-\gamma)^{-1+\gamma} p_q(t)^{\gamma} w^{1-\gamma} z_i(t)^{-\theta} x_i(t) \tag{24}$$

and the associated conditional factor demands:

$$L_{x_i}(t) = (1-\gamma)\frac{\epsilon-1}{\epsilon} \cdot \frac{p_{x_i}(t) x_i(t)}{w(t)} + \phi; \tag{25}$$

$$Q_{x_i}(t) = \gamma \frac{\epsilon-1}{\epsilon} \cdot \frac{p_{x_i}(t) x_i(t)}{p_q(t)}. \tag{26}$$

The firm accumulates firm-specific knowledge according to the technology

$$\dot{z}_i(t) = \kappa \cdot \left[\int_0^{N(t)} \frac{1}{N(t)} z_j(t)\, dj \right] \cdot L_{z_i}(t), \quad \kappa > 0 \tag{27}$$

where L_{z_i} is R&D labor, κ is an exogenous parameter and the term in bracket is the stock of public knowledge that accumulates as a result of spillovers across firms: when one firm develops a new idea, it also generates non-excludable knowledge that benefits the R&D of other firms. The firm's instantaneous profit is

$$\pi_i(t) = \left[p_{x_i}(t) - \gamma^{-\gamma}(1-\gamma)^{-1+\gamma} p_q(t)^\gamma w^{1-\gamma} z_i(t)^{-\theta} \right] x_i(t) - w(t)\phi - w(t) L_{z_i}(t), \tag{28}$$

The value of the firm is

$$V_i(t) = \int_t^\infty \pi_i(v) \exp\left(-\int_t^v (r(s) + \delta)\, ds \right) dv, \quad \delta > 0 \tag{29}$$

where δ is an exit shock. (To avoid confusion with the death rate of people, m, we refer to δ as the exit rate.) At time t the firm chooses the paths $\{p_{x_i}, L_{z_i}\}$ that maximize (29) subject to the demand schedule (22) and the R&D technology (27). The solution to this problem (see the Appendix) yields the maximized value of the firm given the time path of the mass of firms, $N(t)$. Under the assumption $z_i(t) = z(t)$, i.e., all incumbent firms start with the same stock of knowledge, the equilibrium is symmetric. That is, at each instant t each monopolist charges the same price $p_{x_i} = p_x$ and produces the same quantity $x_i = x$. Combining this result with the final producer's behavior, we obtain

$$p_x(t)\, x(t) = \frac{Y(t)}{N(t)}. \tag{30}$$

This equation says that aggregate intermediate sales equal consumption expenditure, Y, and that each monopolist captures a share, $1/N$, of the market.

Intermediate sector: entrants. Entrepreneurs hire labor to develop new intermediate goods and set up firms to serve the market. Denoting the typical entrant i without loss of generality, and denoting L_{N_i} the amount of labor required to start the new firm that enters the market with knowledge $z_i(t)$ equal to the industry average, the cost of entry is $wL_{N_i} = w\beta L/N$, where $\beta > 0$ is a parameter representing technological opportunity.[13] The entrant anticipates that once in the market the new firm solves an intertemporal problem identical to that of the generic incumbent and therefore that the value of the new firm is the maximized value $V_i(t)$ defined in (29). Free entry then requires

$$V_i(t) = w(t) L_{N_i}(t) = w(t)\beta L(t)/N(t) \tag{31}$$

for each entrant.

[13] See Peretto and Connolly (2007) for the microfoundations of this assumption and a discussion of alternatives that deliver the same qualitative results.

2.6 Primary sector

Since the intermediate sector is symmetric, we write the quantity of the commodity demanded by intermediate producers as $Q = NQ_{x_i}$, with Q_{x_i} given by (26). A representative competitive firm produces the commodity by combining the resource with labor under constant returns to scale. The firm maximizes profit

$$\Pi_q = p_q(t) Q(t)(1-\tau) - p_\omega(t)\Omega - w(t) L_Q(t) \tag{32}$$

subject to the technology (10) taking all prices and the tax rate as given. To simplify the exposition, we work with the CES specification of (10)

$$Q(t) = \mathcal{F}(\Omega, L_Q(t)) = \left[\eta \cdot \Omega^{\frac{\sigma-1}{\sigma}} + (1-\eta) \cdot L_Q(t)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \quad \sigma \geq 0,\ \eta \in (0,1), \tag{33}$$

where σ is the elasticity of input substitution and η governs the input shares. The resource and labor are complements if $\sigma < 1$ and substitutes if $\sigma > 1$. Letting $\sigma \to 1$ we obtain the Cobb-Douglas case $Q = \Omega^\eta L_Q^{1-\eta}$. Let $\Theta(w, p_\omega) \equiv \eta^\sigma p_\omega^{1-\sigma} + (1-\eta)^\sigma w^{1-\sigma}$ denote the unit cost function associated to the technology (33). The profit-maximizing decisions of the commodity producer yield

$$p_q = \frac{\Theta(w, p_\omega)}{1-\tau} \tag{34}$$

and the resource *cost-share* function (see the Appendix)

$$\Upsilon(t) \equiv \frac{\mathrm{d}\ln\Theta(w(t), p_\omega(t))}{\mathrm{d}\ln p_\omega(t)} = \frac{p_\omega(t)\Omega}{p_\omega(t)\Omega + w(t) L_Q(t)} = \frac{\eta^\sigma p_\omega(t)^{1-\sigma}}{\eta^\sigma p_\omega(t)^{1-\sigma} + (1-\eta)^\sigma w(t)^{1-\sigma}}. \tag{35}$$

The resource cost share Υ is the ratio between royalties paid by firms to resource owners and the firm's total expenditures on inputs. In the Cobb-Douglas case, $\sigma \to 1$, the cost-share is constant, $\Upsilon \to \eta$. In the other cases, a higher resource price reduces (increases) the resource cost-share when primary inputs are substitutes (complements). These *cost-share effects* determine the equilibrium response of household income and consumption expenditure to changes in the relative scarcity of the resource, as we show below.

3 Equilibrium and the mortality rate

This section summarizes the key interactions taking place in equilibrium between demographic and economic variables. Expenditures per capita reflect the response of income to changes in resource scarcity, while mortality responds to changes in the population-resource ratio according to precise relationship among the equilibrium mortality rate, per capita emission damages, and the allocation of labor allocation commodity production.

3.1 Output and input markets

To determine the general equilibrium of the economy, we impose several market clearing conditions. The resource market clears when supply by the representative household equals demand by the representative commodity producer, i.e., when

$$p_\omega(t)\,\Omega = \Upsilon(t) \cdot p_q(t)\,Q(t)\,(1-\tau)\,. \tag{36}$$

This equation says that the commodity producer spends on the resource a fraction Υ of the after-tax value of its sales, where Υ is the cost-share function defined in (35).

The commodity market clears when supply by the commodity producer equals demand by intermediate firms. Using (26) and (30) we obtain

$$p_q(t)\,Q(t) = \gamma\frac{\epsilon-1}{\epsilon} \cdot Y(t)\,. \tag{37}$$

The labor market clears when $L = L_X + L_Z + L_N + L_Q$, where L is labor supply, $L_X + L_Z = N\left(L_{x_i} + L_{z_i}\right)$ is labor demand by intermediate producers (for production and in-house R&D), $L_N = \dot{N} L_{N_i}$ is labor demand by entrants and L_Q is labor demand by the primary sector.

Finally, the financial market clears when the value of the household's portfolio equals the value of the securities issued by firms, $A = NV_i$. The free-entry condition (31) then yields

$$A(t) = \beta w(t)\,L(t)\,. \tag{38}$$

In the remainder of the analysis we normalize the wage, $w(t) \equiv 1$. This choice of numeraire implies that expenditure on final output, Y, is an index of the value added of labor services.[14] Also, we let $y \equiv Y/L$ denote *consumption expenditure per capita* and $\ell \equiv L/\Omega$ denote the ratio of labor supply (population) to resource supply, henceforth *population-resource ratio* for short. High ℓ represents relative abundance of labor or, equivalently, relative scarcity of the resource.

3.2 Expenditure and resource use

Two relationships between consumption expenditure and resource income characterize the intratemporal equilibrium of the economy (see the Appendix for the derivation). The first follows from combining the household's budget constraint (18) and the Euler equation (19) with the equilibrium

[14] With the wage set at $w = 1$, p_c is the price of the final good in units of labor. Therefore, the real wage, w/p_c, grows when $\dot{p}_c/p_c < 0$ and a long-run equilibrium featuring constant expenditure Y and growth of the physical variables $c_L \equiv C_L/L$ and $c_B \equiv C_B/B$ is characterized by $\dot{c}_L/c_L = \dot{c}_B/c_B = -\dot{p}_c/p_c$, that is, growth comes from the rate of decline of the relative price of the final good.

condition of the assets market (38). It reads

$$y\left(t\right)=\frac{1+\beta\rho+\frac{p_{\omega}(t)}{\ell(t)}}{1-\tau\gamma\frac{\epsilon-1}{\epsilon}} \tag{39}$$

and says that consumption expenditure per capita, y, is a constant fraction of income per capita, the sum of the wage, $w=1$, asset income per capita, $\rho A/L=\beta\rho w=\beta\rho$, and resource income per capita, $p_{\omega}\Omega/L=p_{\omega}/\ell$. The presence of the commodity tax at the denominator is due to the balanced-budget assumption and captures the positive effect of public transfers on household expenditure. The second relationship follows from (36) and (37). It reads

$$\frac{p_{\omega}\left(t\right)}{\ell\left(t\right)}=\left[\left(1-\tau\right)\cdot\Upsilon\left(p_{\omega}\left(t\right)\right)\cdot\gamma\frac{\epsilon-1}{\epsilon}\right]\cdot y\left(t\right) \tag{40}$$

and says that resource income per capita is a fraction (in brackets) of consumption expenditure per capita. We call this fraction the *royalty share.*

The royalty share depends on the technological parameters of all production sectors and on the commodity tax. The tax reduces the royalty share despite the lump-sum rebate because it distorts the use of the commodity in primary production and thus generates a traditional deadweight loss. With $w=1$, the resource cost-share defined in (35) is a function $\Upsilon\equiv\Upsilon\left(p_{\omega}\right)$ of the resource price only. Therefore, equations (39) and (40) form a system of two equations in three variables (y,p_{ω},ℓ). To characterize the interaction of the resource market equilibrium with household consumption-saving decisions, we solve for the resource price p_{ω} and expenditure per capita y as functions of the population-resource ratio ℓ.

Proposition 1 *Given population-resource ratio $\ell\left(t\right)>0$, at each instant $t\in[0,\infty)$ the solution of equations (39)-(40) yields a unique equilibrium pair*

$$\left\{p_{\omega}^{*}\left(\ell\left(t\right)\right),y^{*}\left(\ell\left(t\right)\right)\right\}$$

with the following properties. The resource price is monotonically increasing in the population-resource ratio, i.e., $dp_{\omega}^{*}\left(\ell\right)/d\ell>0$ for all $\ell>0$. The effect of the population-resource ratio on expenditure per capita, instead, depends on the elasticity of substitution between inputs in commodity production. In terms of elasticity,

$$\frac{d\ln y^{*}\left(\ell\right)}{d\ln\ell}=\left(1-\tau\right)\gamma\frac{\epsilon-1}{\epsilon}\ell y^{*}\left(\ell\right)\cdot\frac{d\Upsilon\left(p_{\omega}\left(\ell\right)\right)}{d\ell},$$

where

$$\frac{d\Upsilon\left(p_{\omega}\left(\ell\right)\right)}{d\ell}=\begin{cases}<0 & \textit{if } \sigma>1\\ =0 & \textit{if } \sigma=1\\ >0 & \textit{if } \sigma<1\end{cases}.$$

Using equation (34) the equilibrium commodity price is

$$p_q^*(\ell) \equiv \frac{1}{1-\tau}\Theta\left(1, p_\omega^*(\ell)\right) \quad \textit{with} \quad \frac{dp_q^*(\ell)}{d\ell} = \begin{cases} < 0 & \textit{if } \sigma > 1 \\ = 0 & \textit{if } \sigma = 1 \\ > 0 & \textit{if } \sigma < 1 \end{cases}.$$

Proof: *see the Appendix.*

The effects of the population-resource ratio, ℓ, on expenditure per capita, y, are a direct consequence of the *cost-share effects* discussed earlier. When ℓ rises, the resource becomes relatively more scarce and its price, p_ω, rises. When labor and the resource are substitutes (complements), an increase in the resource price reduces (increases) the resource cost share in primary production and thereby reduces (increases) resource royalties per capita.[15] The important insight of Proposition 1 is thus that the cost-share effects push expenditure per capita in the same direction as resource income per capita. Under substitutability, $\sigma > 1$, we have $\partial y^*(\ell)/\partial \ell < 0$ because the quantity channel at the denominator of p_ω/ℓ dominates as the resource price falls less than one-for-one with ℓ. With $\sigma < 1$, instead, we have $\partial y^*(\ell)/\partial \ell > 0$ because the price channel at the numerator of p_ω/ℓ dominates as the resource price falls more than one-for-one with ℓ. In the Cobb-Douglas case, changes in ℓ leave resource income per capita and expenditure per capita unchanged.

3.3 The equilibrium mortality rate

Expressions (13) and (15) yield the relationship between mortality and the population-resource ratio. We stress that our definition $\ell = L/\Omega$ implies that comparative-statics statement concerning ℓ qualitatively apply to L as well. Therefore, in the following, one can use "population-resource ratio" and "population" interchangeably. The next Proposition provides a full characterization of the response of the equilibrium mortality rate to population and emphasizes the crucial role played by the primary sector's technology.[16]

Proposition 2 *The equilibrium mortality rate is a function of the population-resource ratio, i.e., $m = m^*(\ell)$. In the Cobb-Douglas case, $\sigma \to 1$, we have*

$$m = m^*(\ell) \equiv \bar{m} + \tilde{\mu} \cdot \ell^{\chi\{\upsilon(1-\eta) - [\zeta + \xi(1-\upsilon)]\}}, \tag{41}$$

where

$$\tilde{\mu} \equiv \mu \left(\frac{1-\eta}{\eta} \cdot \frac{(1+\beta\rho)(1-\tau)\eta\gamma^{\frac{\epsilon-1}{\epsilon}}}{1-\tau\gamma(1-\eta)^{\frac{\epsilon-1}{\epsilon}} - \eta\gamma^{\frac{\epsilon-1}{\epsilon}}} \right)^{\chi\upsilon(1-\eta)} \Omega^{\chi\upsilon - \chi[\zeta+\xi(1-\upsilon)]}$$

[15] Expression (35) yields $\partial\Upsilon(p_\omega)/\partial p_\omega < 0$ if $\sigma > 1$, $\partial\Upsilon(p_\omega)/\partial p_\omega = 0$ if $\sigma = 1$, and $\partial\Upsilon(p_\omega)/\partial p_\omega > 0$ if $\sigma < 1$.

[16] Proposition 2 characterizes the equilibrium relations among endogenous variables: to avoid confusion, we drop the time argument unless necessary.

is constant over time. Under substitutability or complementarity, $\sigma \lesseqgtr 1$, we have

$$m = m^*(\ell) \equiv \bar{m} + \bar{\mu} \cdot \Upsilon(\ell)^{\frac{\sigma}{1-\sigma}\chi\upsilon} \cdot \ell^{-\chi[\zeta+\xi(1-\upsilon)]}, \tag{42}$$

where $\bar{\mu} \equiv \mu\eta^{\chi\upsilon\frac{\sigma}{\sigma-1}}\Omega^{\chi\{\upsilon-[\zeta+\xi(1-\upsilon)]\}} > 0$ is constant over time and $\Upsilon(\ell) \equiv \Upsilon(p^*_\omega(\ell))$ is the equilibrium cost share of resource use with the property:

$$\begin{array}{llll} \sigma > 1 \;\rightarrow & \frac{d\Upsilon(\ell)}{d\ell} < 0, & \lim_{\ell\to 0^+} \Upsilon(\ell) = 1, & \lim_{\ell\to\infty} \Upsilon(\ell) = 0; \\ \sigma < 1 \;\rightarrow & \frac{d\Upsilon(\ell)}{d\ell} > 0, & \lim_{\ell\to 0^+} \Upsilon(\ell) = 0, & \lim_{\ell\to\infty} \Upsilon(\ell) = 1. \end{array} \tag{43}$$

***Proof**: see the Appendix.*

Figure 1 illustrates the equilibrium mortality rates defined in Proposition 2. The mortality response to larger population-resource ratio is ambiguous and often non-monotonic. In the Cobb-Douglas case, the mortality rate responds to ℓ monotonically, but in different directions depending on the underlying parameters. Under substitutability and complementarity, $m^*(\ell)$ can be non-monotonic because it depends on the resource cost-share, $\Upsilon(\ell) \equiv \Upsilon(p^*_\omega(\ell))$, which affects the strength of the labor-supply channel. We prove in the Appendix all the subcases appearing in Figure 1. In this subsection, we emphasize the intuition behind the results for the Cobb-Douglas and substitutability cases, which are particularly relevant for our results.

Cobb-Douglas. For $\sigma \to 1$ the employment share of the primary sector is an exogenous constant and we obtain $\varepsilon_{Q,L} = 1 - \eta$. Therefore, the response of the mortality rate to ℓ obeys a simple knife-edge condition. When $\upsilon(1-\eta) < \zeta + \xi(1-\upsilon)$, the damage-dilution effect dominates the primary-employment effect and the mortality rate is decreasing in ℓ. When $\upsilon(1-\eta) > \zeta+\xi(1-\upsilon)$, instead, the primary-employment effect dominates: as ℓ grows, the damage-dilution effect does not compensate for higher emissions and the mortality rate increases. The special case $\upsilon(1-\eta) = \zeta + \xi(1-\upsilon)$ yields $m = \bar{m} + \tilde{\mu}$, that is, the mortality rate is invariant to ℓ.

Substitutability. When $\sigma > 1$, the labor-supply effect is weak for small ℓ and strong for large ℓ. In particular (see Appendix),

$$\lim_{\ell\to 0^+} \varepsilon_{Q,L} = 0 \text{ and } \lim_{\ell\to\infty} \varepsilon_{Q,L} = 1. \tag{44}$$

To grasp the intuition for (44), note that a decline in population reduces ℓ because it reduces labor supply. Given $\sigma > 1$, as labor becomes relatively scarce, its relative price rises and the primary sector substitutes labor with the primary resource. As ℓ keeps falling, this process continues until the labor cost share in the primary sector, and thus the elasticity $\varepsilon_{Q,L}$, converges to zero. The same mechanism in reverse explains why the primary-employment effect becomes stronger when the population-resource ratio increases. As a result of these forces, the mortality response to

ℓ is generally ambiguous and possibly non-monotonic. If we rule out damage dilution setting $\zeta + \xi(1-\upsilon) = 0$, the mortality rate increases with ℓ via the primary-employment effect. Allowing for damage dilution, $\zeta + \xi(1-\upsilon) > 0$, makes the mortality response non-monotonic in ℓ. Moreover, the mortality rate explodes as ℓ becomes very small.

Lemma 3 *With substitutability, $\sigma > 1$, and damage dilution, $\zeta + \xi(1-\upsilon) > 0$, the mortality rate approaches infinity as the resource-population ratio approaches zero:*

$$\sigma > 1 \ \rightarrow \lim_{\ell \to 0^+} m^*(\ell) = \lim_{\ell \to 0^+} \bar{m} + \bar{\mu} \cdot \ell^{-\chi[\zeta + \xi(1-\upsilon)]} = +\infty. \tag{45}$$

Proof: *see the Appendix.*

The intuition for this result follows from the fact that the elasticity of commodity output with respect to employment, $\varepsilon_{Q,L}$, approaches zero as $\ell \to 0$. When ℓ decreases because population declines, primary producers substitute labor with resource use at increasing rates. This implies that while primary production declines, emissions per worker increase and the resulting excess deaths caused by deadly spillovers eventually explode. In this scenario, a small population is bad for mortality because emissions per capita become very high and the resulting damage cannot be relieved by dose dilution and/or emissions reduction from population density.

Lemma 3 implies that countries with small population and/or abundant primary resources may exhibit very high mortality rates. What happens for large ℓ, instead, depends on parameter values. Since $\varepsilon_{Q,L}$ approaches one as $\ell \to \infty$, we have the cases in Figure 1. If $\zeta + \xi(1-\upsilon) \geqslant \upsilon$, the mortality rate is *L-shaped*, that is, $m^*(\ell)$ is monotonically decreasing in ℓ because the labor-supply effect is weaker than the damage-dilution effect for all ℓ. If $0 < \zeta + \xi(1-\upsilon) < \upsilon$, the mortality rate is *U-shaped*, that is, $m^*(\ell)$ reaches a minimum and then increases with ℓ, because a large ℓ combined with a high elasticity $\varepsilon_{Q,L}$ makes the primary-employment effect strong enough to dominate the damage-dilution effect.[17]

4 Population dynamics

This section characterizes the equilibrium dynamics of fertility, mortality and population in a self-contained sub-system describing the demography block of our economy. The property that makes our model this tractable is the scale-invariance of the Schumpeterian model of endogenous innovation that provides the industry block of our economy.

[17] Figure 1 shows that for any value of σ, there are cases in which $m^*(\ell)$ is decreasing at least locally. Decreasing mortality occurs less under complementarity, $\sigma < 1$, because the cost-share effects underlying result (44) are reversed. This makes the primary-employment effect stronger for small population.

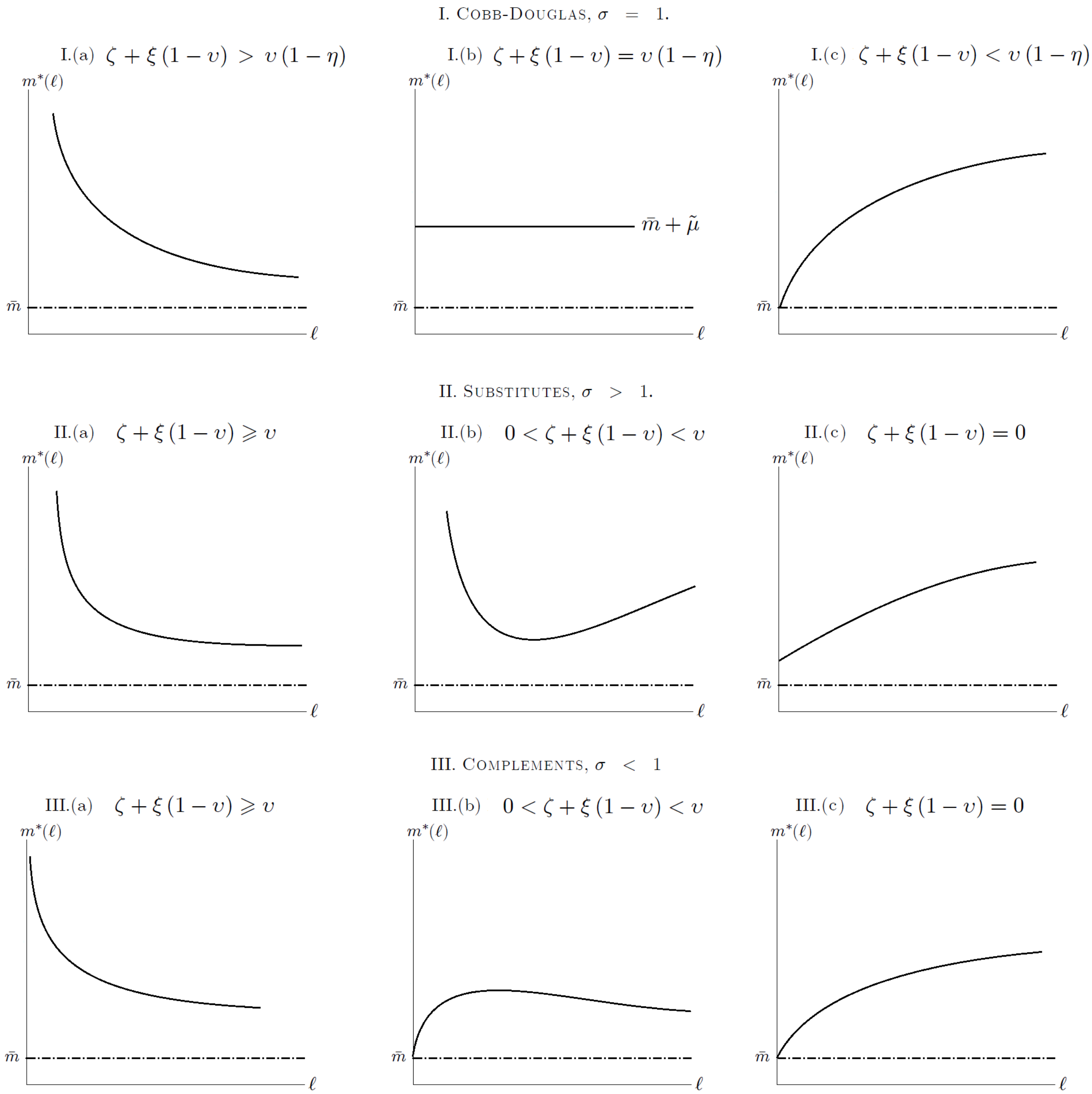

Figure 1: Equilibrium mortality rates as functions of the population-resource ratio, $m = m^*(\ell)$. Without deadly spillovers – i.e., setting $\mu = 0$ – the mortality rate would coincide with the baseline rate $m = \bar{m}$ in all cases.

4.1 Demography-scarcity interactions

Since the resource endowment Ω is fixed, the population-resource ratio, $\ell = L/\Omega$, grows at the same rate as population, i.e.,

$$\frac{\dot{\ell}(t)}{\ell(t)} = b(t) - m^*(\ell(t)), \tag{46}$$

where $m^*(\ell)$ is the equilibrium mortality rate characterized in Proposition 2. The Euler equation for the birth rate (20) yields

$$\frac{\dot{b}(t)}{b(t)} = \frac{b(t)}{(1-\alpha)(1-\psi)}\left[\frac{1-(1-\psi)\cdot y^*(\ell(t))}{y^*(\ell(t))}\right] - \rho, \tag{47}$$

where $y^*(\ell)$ is the equilibrium expenditure per capita characterized in Proposition 1. Equations (46) and (47) form a 2D dynamic system that fully determines the equilibrium interactions between fertility, resource scarcity and mortality. Since the system can generate multiple steady states, we distinguish between stable and unstable cases with the following definition.

Definition 4 *A regular steady state is a point (ℓ_{ss}, b_{ss}) in (ℓ, b) space such that the values (ℓ_{ss}, b_{ss}) are positive and finite and satisfy $\dot{b} = \dot{\ell} = 0$. Moreover, the point exhibits (at least local) stability, i.e., there is a thick set of initial conditions $\ell(0) > 0$ starting from which the equilibrium trajectory $(\ell(t), b(t))$ converges to (ℓ_{ss}, b_{ss}) and population converges to the finite value $L_{ss} = \ell_{ss}\Omega > 0$.*

Our notion of regular steady state is conventional in the sense that, being a stable rest point, it represents the long-run attractor of the dynamics when certain initial conditions hold. The distinctive property is that (ℓ_{ss}, b_{ss}) features constant population size in the long run, L_{ss}, while per capita income grows via innovation. In this light, it is worth noting that our approach makes two distinct contributions to the existing analytical framework. First, if a regular steady state exists independently of pollution, deadly spillovers modify its position and the path that leads to it. While the qualitative properties of the dynamics in the two models are similar, their *quantitative* properties are obviously different and potentially very much so. Second, deadly spillovers can create steady states that would not otherwise exist, and such steady states may be regular or not. In other words, deadly spillovers change the *qualitative* propertied of the dynamics rather drastically. To highlight this feature, we first summarize the predictions of the model with no deadly spillovers (subsection 4.2) and then analyze the model with deadly spillovers (subsection 4.3).

4.2 Special case with exogenous mortality

We set $\mu = 0$ in (13) to obtain the special case with exogenous mortality nested in our model. The steady-state loci are, respectively, $\dot{\ell} = 0 \to b = \bar{m}$ and

$$\dot{b} = 0 \to b = \frac{(1-\alpha)(1-\psi)\rho}{y^*(\ell)^{-1} - (1-\psi)}. \tag{48}$$

This special case delivers the following results (see the Appendix for details and proofs).

First, combining the two steady-state equations yields

$$\bar{m} = \frac{(1-\alpha)(1-\psi)\rho}{y^{-1} - (1-\psi)} \to y_{ss} = \frac{(1-\psi)\bar{m}}{(1-\alpha)\rho - \bar{m}}.$$

This result says that steady-state expenditure depends only on preference parameters and demography via the exogenous mortality rate. It thus has a strong Malthusian flavor. It differs from the standard Malthusian result, however, because y_{ss} is consumption expenditure per capita, not real consumption per capita, and therefore it is not a measure of living standards. As stated, in our model constant expenditure per capita is associated to constant growth of consumption per capita via innovations that reduce the price of consumption.[18]

Second, when the primary sector's technology is Cobb-Douglas, $\sigma = 1$, population grows or declines at a constant rate because the stationary loci are horizontal straight lines that in general do not coincide. In Figure 2, phase diagram (a) shows the case in which the equilibrium birth rate exceeds $\bar{m}$, implying a constant and positive population growth rate.

Third, under substitutability, $\sigma > 1$, there exists a regular steady state (ℓ_{ss}, b_{ss}) that is saddle-point stable; see phase diagram (e) in Figure 2. If the economy starts with $\ell(0) < \ell_{ss}$, the equilibrium path features *positive population growth* with a *declining fertility rate* until b reaches $\bar{m}$ and stabilizes the population. The reason for these dynamics is that, with $\sigma > 1$, expenditure per capita declines with ℓ because the rising resource scarcity yields lower resource income per capita. This mechanism produces the negative slope of the $\dot{b} = 0$ locus, which is the key to the stability of the process. In fact, in the opposite case of complementarity, the income response to ℓ is reversed and the steady state (ℓ_{ss}, b_{ss}) becomes unstable: with $\sigma < 1$, the economy follows diverging paths, leading to either population explosion or human extinction depending on the initial level of the population-resource ratio (see the Appendix for details).

The main takeaway of this analysis is that $\sigma \geqslant 1$ deserves special emphasis. The Cobb-Douglas case is interesting because the prediction of exponential population growth rests on a knife-edge hypothesis about technology: only for $\sigma = 1$ no steady state exists unless the two stationary loci are on top of each other. Substitutability, $\sigma > 1$, is even more relevant because it generates a plausible path of demographic development: assuming $\ell(0) < \ell_{ss}$, population converges to a finite size because resources per worker and births per adult shrink over time. This is consistent with the well-known fertility decline observed throughout the industrialized world and with the widely shared idea that population growth cannot outstrip the finite natural resource base. Introducing deadly spillovers in this context identifies how pollution changes at the margin the steady state and thus the equilibrium path of the economy. We thus focus on $\sigma \geqslant 1$ in the remainder of the analysis.[19]

[18] A version of the model with a time cost of reproduction delivers the same qualitative results as the one presenrted here. It follows that this property of our approach does not depend on the cost of reproduction being in units of the consumption good.

[19] The analysis of the dynamic system with deadly spillovers and strict complementarity, $\sigma < 1$, is in the Appendix

for completeness.

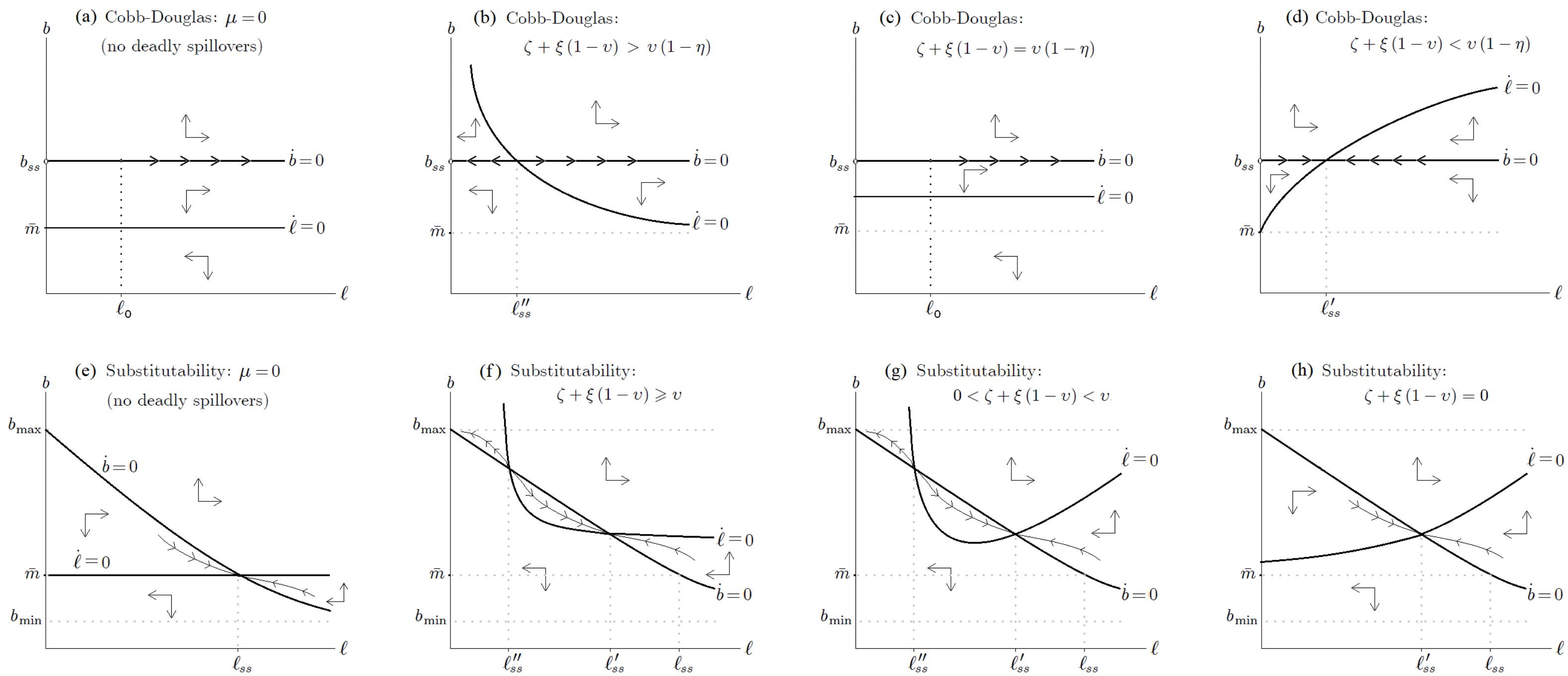

Figure 2. Phase diagrams of system (37)-(38) under exogenous versus endogenous mortality under $\sigma \geqslant 1$. Diagrams (a) and (e) rule out deadly spillovers by setting $\mu = 0$ in the model. Diagrams (b)-(d) and (f)-(g) assume deadly spillovers under different combinations of parameters. See Appendix (Figure A2) for the case of complementarity.

4.3 Dynamics with endogenous mortality

The analysis of the previous subsection allows us to study the dynamical system with endogenous mortality (46)-(47) in a straightforward manner. The $\dot{b} = 0$ locus is still given by expression (48). Equation (46), instead, yields

$$\dot{\ell} = 0 \qquad \rightarrow \qquad b = m^*(\ell) \equiv \begin{cases} \bar{m} + \tilde{\mu} \cdot \ell^{\chi\{\upsilon(1-\eta)-[\zeta+\xi(1-\upsilon)]\}} & \text{if } \sigma = 1, \\ \bar{m} + \bar{\mu} \cdot \Upsilon(\ell)^{\frac{\sigma}{1-\sigma}\chi\upsilon} \cdot \ell^{-\chi[\zeta+\xi(1-\upsilon)]} & \text{if } \sigma \gtrless 1. \end{cases} \tag{49}$$

This expression shows that the shape of the $\dot{\ell} = 0$ locus matches the shape of the equilibrium mortality rate defined in Proposition 2. A property of note is that we no longer obtain a simple analytical solution for expenditure per capita because the mortality rate is endogenous. Combining the $\dot{b} = 0$ and $\dot{\ell} = 0$ equations yields that ℓ is the solution of an implicit equation, i.e.,

$$\ell_{ss} = \arg\text{solve}\left\{ m^*(\ell) = \frac{(1-\alpha)(1-\psi)\rho}{y^*(\ell)^{-1} - (1-\psi)} \right\}. \tag{50}$$

We then obtain

$$y_{ss} = y^*(\ell_{ss}) = \frac{(1-\psi)\, m^*(\ell_{ss})}{(1-\alpha)\rho - m^*(\ell_{ss})}. \tag{51}$$

Figure 2 shows the resulting phase diagrams for the Cobb-Douglas case and for substitutability. Both cases deliver novel results.

Cobb-Douglas. With $\sigma = 1$, the gross fertility rate determined by (48) is constant but deadly spillovers generally affect population growth via the mortality rate: whenever the primary-employment effect does not match exactly the damage-dilution effect, $\upsilon(1-\eta) \neq \zeta + \xi(1-\upsilon)$, deadly spillovers create a steady state that would not exist otherwise. The steady state (ℓ_{ss}, b_{ss}) can be stable or unstable depending on the relative strength of the primary-employment and damage-dilution effects.

Proposition 5 *(Cobb-Douglas) For $\sigma = 1$ and $\upsilon(1-\eta) \neq \zeta + \xi(1-\upsilon)$, deadly spillovers create a steady state (ℓ_{ss}, b_{ss}), which may be stable or unstable: it is a regular steady state for $\upsilon(1-\eta) > \zeta + \xi(1-\upsilon)$; it creates a mortality trap for $\upsilon(1-\eta) < \zeta + \xi(1-\upsilon)$. For $\sigma = 1$ and $\upsilon(1-\eta) = \zeta + \xi(1-\upsilon)$, there is no steady state and deadly spillovers permanently reduce the constant population growth rate.* ***Proof****: see the Appendix.*

The top panel of Figure 2 shows the phase diagrams for the Cobb-Douglas commodity production technology. It is worth stressing that, except for specific knife-edge cases, the main message of the Cobb-Douglas technology is that pollution-caused mortality creates steady states that would not exist otherwise and thus delivers novel qualitative results. In diagram (a), $\upsilon(1-\eta) < \zeta + \xi(1-\upsilon)$

yields the cases where the damage-dilution effect dominates the primary-employment effect. In this configuration, $m^*(\ell)$ is decreasing in ℓ, the $\dot{\ell} = 0$ locus is decreasing in ℓ, and the steady state (ℓ''_{ss}, b_{ss}) is unstable. The population-resource ratio ℓ''_{ss} is thus an *extinction threshold*: if labor is initially abundant relative to the resource, $\ell_0 > \ell''_{ss}$, the economy experiences sustained population growth whereas in the opposite situation, $\ell_0 < \ell''_{ss}$, the economy is in a mortality trap characterized by a vicious circle of ever-declining population and ever-increasing mortality. In this scenario, population must be initially large enough, relative to the resource endowment, to generate positive population growth at time zero and thereafter. In diagram (b), $\upsilon(1-\eta) > \zeta + \xi(1-\upsilon)$ yields the case in which the primary-employment effect dominates the damage-dilution effect. In this configuration, $m^*(\ell)$ is decreasing in ℓ and deadly spillovers create a stable steady state (ℓ'_{ss}, b_{ss}). Starting from $\ell_0 > \ell'_{ss}$, population increases at a declining rate due to pollution-induced mortality until population growth becomes zero. We thus have the insight that introducing pollution-caused mortality in a model that would otherwise feature exploding population is sufficient to produce a finite population. In other words, deadly spillovers are the only force that stabilizes the population in the long run. Diagram (c) considers the knife-edge case $\upsilon(1-\eta) = \zeta + \xi(1-\upsilon)$ that does not feature steady states and may predict opposite dynamics depending on the strength of pollution-caused mortality. In this scenario, equation (48) yields $b_{ss} > \bar{m}$ and the equilibrium mortality rate determined by (49) is $m^* = \bar{m} + \tilde{\mu}$. If $\tilde{\mu}$ is relatively small, that is, if pollution induces moderate excess mortality, we obtain positive constant population growth, $b_{ss} - \bar{m} - \tilde{\mu} > 0$, like in diagram (c). If, instead, $\tilde{\mu}$ is sufficiently large, we obtain $b_{ss} - \bar{m} - \tilde{\mu} < 0$ and deadly spillovers reverse the sign of the constant population growth rate from positive to negative.[20]

The bottom panel of Figure 2 shows the phase diagrams under substitutability, $\sigma > 1$, which delivers further interesting results. First, even if damage dilution is positive, deadly spillovers reduce the steady-state size of the population: while the model without pollution exhibits a regular steady state (ℓ_{ss}, b_{ss}), the model with deadly spillovers generates a regular steady state (ℓ'_{ss}, b'_{ss}) with $\ell'_{ss} < \ell_{ss}$. Second, recalling Lemma 3, substitutability makes the mortality rate explode for small ℓ when damage dilution is positive, $\zeta + \xi(1-\upsilon) > 0$. Phase diagrams (f) and (g) in Figure 2 illustrate this mechanism: deadly spillovers shift the $\dot{\ell} = 0$ locus up and bend it upwards as ℓ approaches zero. If deadly spillovers are extremely strong, the regular steady state disappears.[21] More generally, when the regular steady state exists, the mortality effect of pollution at low population-resource ratio creates an additional, unstable steady state that yields a mortality

[20] The case $b_{ss} - \bar{m} - \tilde{\mu} < 0$ is like diagram (c) but with the $\dot{\ell} = 0$ locus lying above b_{ss}. This reverses the direction of the arrows along the $\dot{b} = 0$ locus and delivers persistent population decline.

[21] The case with no steady states looks like Figure 2, graphs (f)-(g), but with the $\dot{\ell} = 0$ locus so high that there is no intersection with the $\dot{b} = 0$ locus.

trap.

Proposition 6 *(Substitutability) Assume* $\sigma > 1$*. With deadly spillovers, the regular steady state* (ℓ'_{ss}, b'_{ss}) *has smaller population-resource ratio,* $\ell'_{ss} < \ell_{ss}$*, than the regular steady state* (ℓ_{ss}, b_{ss}) *of the model without pollution. In addition, if* $\zeta + \xi(1-\upsilon) > 0$*, deadly spillovers create a second, unstable steady state* (ℓ''_{ss}, b''_{ss}) *with* $b''_{ss} > b'_{ss}$ *and* $\ell''_{ss} < \ell'_{ss}$*. The interval* $(0, \ell''_{ss})$ *is the mortality trap caused by deadly spillovers. If* $\ell(0) > \ell''_{ss}$*, the economy converges to the regular steady state. If* $\ell(0) < \ell''_{ss}$*, the equilibrium path exhibits* $\lim_{t\to\infty} \ell(t) = 0$*.* ***Proof****: see the Appendix.*

Figure 2 illustrates the two main results delivered by Proposition 6. First, deadly spillovers *reduce the population* by modifying the position of the regular steady state: as the economy converges to (ℓ'_{ss}, b'_{ss}), the long-run population-resource ratio is lower because of higher mortality. This conclusion, which holds regardless of the damage-dilution effect, is self-evident in Figure 2: with respect to the case with no pollution, diagram (e), deadly spillovers reduce ℓ'_{ss} in all cases, even when no mortality trap arises like in diagram (h). More generally, endogenous mortality due to pollution affects the whole equilibrium path of the economy and, as we shall see, has substantial consequences for welfare through multiple channels, including firms' incentives to innovate since these depend on the anticipated dynamics of the size of the market.

The second result is that deadly spillovers can create the mortality trap, the region $(0, \ell''_{ss})$ of the state space where implosive population dynamics prevail. The unstable steady state (ℓ''_{ss}, b''_{ss}) is an extinction threshold: if population is initially too small relative to the resource endowment, $\ell(0) < \ell''_{ss}$, the economy does not converge to the regular steady state (ℓ'_{ss}, b'_{ss}) and follows, instead, an equilibrium path leading to zero population. Such population implosion *does not* result from falling fertility. Rather, starting from $\ell(0) < \ell''_{ss}$, the transition exhibits increasing fertility as well as increasing mortality. The reason is that the fertility rate is constrained by household income, whereas the mortality rate is unbounded: as population shrinks, growing deadly spillovers lead to exploding mortality while households may only raise the fertility rate up to $b_{\max}$, the highest birth rate consistent with their budget constraint. The economy escapes the mortality trap and converges to the regular steady state only if the initial population-resource ratio is sufficiently high, $\ell(0) > \ell''_{ss}$. This result delivers specific insights for less populated, resource-rich economies. Diagrams (f)-(g) in Figure 2 show that economies that are closer to the mortality trap feature a *low* population-resource ratio and a *high* birth rate. Given resource abundance, economies with a small population tend to be ceteris paribus closer to the mortality trap even though they may exhibit higher birth rates. By the same token, exogenous shocks that reduce population push the economy toward the trap. A similar though not identical mechanism applies to resource abundance and exogenous shocks expanding the endowment (e.g., discoveries of new stocks of natural resources): given population, a larger resource base can push the economy toward the trap not only by reducing the current population-resource ratio, but also by expanding the mortality trap itself by pushing ℓ''_{ss} to the right. We discuss these and related points in the next section.

5 Growth, emission taxes and resource booms

In this section we derive the equilibrium paths of consumption, innovation rates, income growth and utility. We then study the effects of emission taxes, subsidies to the primary sector, resource booms, and discuss the framework's implications for empirical analysis and policy making.

5.1 Consumption, growth and utility

The model's measure of gross domestic product is final output, C. Since household expenditure on consumption is Y, we have (see the Appendix)

$$\frac{C(t)}{L(t)} = \frac{y(t)}{p_c(t)} = y(t) \cdot \frac{z(t)^{\theta} N(t)^{\frac{1}{\epsilon-1}}}{(1-\gamma)^{-(1-\gamma)} \gamma^{-\gamma} \frac{\epsilon}{\epsilon-1} p_q(t)^{\gamma}}. \tag{52}$$

This expression says that GDP per capita equals consumption expenditure per capita divided by the price index of intermediate goods. The price index, in turn, depends on the endogenous components of technology, product variety and firm-specific knowledge, and on the relative price of the commodity. For clarity, we separate the role of endogenous technology from that of the vertical production structure. In the last term of (52), the numerator is a reduced-form representation of *total factor productivity* (TFP), which we henceforth denote as $T \equiv z^{\theta} N^{\frac{1}{\epsilon-1}}$. The denominator is an index of how markup-pricing and the cost of inputs drive the price of intermediates.

Differentiating (52) with respect to time, we obtain

$$g(t) \equiv \frac{\dot{C}(t)}{C(t)} - \frac{\dot{L}}{L} = \frac{\dot{T}(t)}{T(t)} + \frac{\dot{y}(t)}{y(t)} - \gamma \frac{\dot{p}_q(t)}{p_q(t)}. \tag{53}$$

The first term is the growth rate of TFP, which in turn equals a weighted sum of the rates of vertical innovation, $\dot{z}/z$, and horizontal innovation, $\dot{N}/N$. The second term is expenditure per capita growth. The third term is the standard *scarcity drag* of models with finite natural resources. Recalling Proposition 1, the equilibrium commodity price is $p_q^*(\ell) = \frac{1}{1-\tau}\Theta(1, p_\omega^*(\ell))$ and its growth rate over time thus reads

$$\frac{\dot{p}_q(t)}{p_q(t)} = \frac{\mathrm{d}\ln\Theta(w, p_\omega^*(\ell(t)))}{\mathrm{d}\ln p_\omega^*(\ell(t))} \frac{\dot{\ell}(t)}{\ell(t)} = \Upsilon(t) \frac{\dot{\ell}(t)}{\ell(t)}, \tag{54}$$

where Υ is the resource-cost share defined in (35). Therefore, using (54) and the results in Proposition 1, we can write the growth rate of income per capita as

$$g(t) = \frac{\dot{T}(t)}{T(t)} + \gamma \left[(1-\tau) \frac{\epsilon - 1}{\epsilon} \ell y^* (\ell) \underbrace{\frac{\mathrm{d}\Upsilon (p_\omega (\ell))}{\mathrm{d}\ell}}_{- \text{ for } \sigma > 1} - \Upsilon (t) \right] \frac{\dot{\ell}(t)}{\ell(t)}. \tag{55}$$

The second term represents transitional effects that operate only when ℓ changes over time.

Equation (55) describes the equilibrium dynamic relation between income per capita and fertility. Under substitutability (our preferred case), the bracket is negative and thus, holding constant TFP growth, there is a negative relation between income per capita growth and fertility since $\dot{\ell}/\ell = b - m$. Moreover, an economy that approaches the steady state with rising population (from the left) exhibits falling fertility and rising income per capita as long as TFP growth is faster than the scarcity drag, i.e., as long as $g > 0$. In this case, the model produces the negative comovement between income per capita and fertility that characterizes advanced economies. We stress that in this description we hold TFP growth constant, whereas TFP growth is endogenous and jointly determined with population growth. Nevertheless, the argument provides a sufficient condition, i.e., $g > 0$, for a negative income-fertility relation.

When the population-resource ratio becomes constant, $\dot{\ell} = 0$, the only source of economic growth is innovation. More precisely, if the economy converges to a regular steady state (ℓ_{ss}, b_{ss}), the only source of economic growth is *vertical* innovation: firm-specific knowledge grows at a constant rate while the mass of firms is constant, $N(t) = N_{ss}$. The mechanism driving this property is that vertical and horizontal innovation exhibit a negative comovement during the transition: entry of new firms reduces the profitability of firm-specific knowledge investment through market fragmentation while investment in firm-specific knowledge slows down entry by diverting labor away from horizontal R&D. As we show in the Appendix, these comovements eventually bring the economy to a steady state where the mass of firms is constant and the engine of growth is firm-specific knowledge accumulation.

Proposition 7 *Assume*

$$\frac{\frac{\epsilon-1}{\epsilon} \kappa \theta \left(\phi - \frac{\rho+\delta}{\kappa} \right) y_{ss}}{\frac{1-\theta(\epsilon-1)}{\epsilon} y_{ss} - \beta (\rho + \delta)} > \rho + \delta$$

and let the economy converge to the steady state (ℓ_{ss}, b_{ss})*. Then, the mass of firms is*

$$N_{ss} = \frac{\frac{1-\theta(\epsilon-1)}{\epsilon} y_{ss} - \beta (\rho + \delta)}{\phi - \frac{\rho+\delta}{\kappa}} \cdot L_{ss} > 0, \tag{56}$$

firm-specific knowledge grows at rate

$$\left(\frac{\dot{z}}{z}\right)_{ss}=\frac{\frac{\epsilon-1}{\epsilon}\kappa\theta\left(\phi-\frac{\rho+\delta}{\kappa}\right)y_{ss}}{\frac{1-\theta(\epsilon-1)}{\epsilon}y_{ss}-\beta\left(\rho+\delta\right)}-\rho-\delta>0, \tag{57}$$

and final output grows at rate

$$g_{ss}=\theta\left(\frac{\dot{z}}{z}\right)_{ss}.$$

Proof*: see the Appendix.*

This proposition highlights an important property of our model. While the model belongs to a class known for the scale-invariance of the steady-state growth rate, deadly spillovers create a novel channel through which the deep parameters regulating pollution-induced mortality have steady-state growth effects. To see this, note that equation (57) contains steady-state expenditure per capita, y_{ss}, which according to equation (51) is a function of the steady-state resource-population ratio, ℓ_{ss}. The economic intuition and the direction of the key relationships is the following. First, a higher steady-state mortality rate $m^*\left(\ell_{ss}\right)$ implies a higher steady-state expenditure per capita $y^*\left(\ell_{ss}\right)$ via the presudo-Malthusian relationship (51) – an effect that we label as the *mortality-expenditure channel*. Second, a higher steady-state expenditure per capita $y^*\left(\ell_{ss}\right)$ reduces steady-state growth g_{ss} because higher expenditure per capita expands the size of the market and this attracts entry: each firm captures a smaller market share and thus reduces in-house R&D efforts[22] – an effect that we label as the *expenditure-innovation channel*. These two mechanisms imply that the steady-state growth rate is a function of the steady-state population-resource ratio ℓ_{ss} as a result of endogenous mortality. Importantly, this is not a scale effect linking population size to economic growth because population L_{ss} is endogenous, and the relationship between L_{ss} and $m^*\left(\ell_{ss}\right)$ has a generally ambiguous sign. In fact, the relation that we obtain is between growth and the model's deep parameters characterizing the generation and propagation of pollution through the population, with the resulting effect on mortality, and not a relation between growth and the size of a particular endowment. The sign of steady-state growth effects induced by an exogenous shock ultimately depends on the origin of the shock, which may affect the shape of the mortality function $m^*\left(\cdot\right)$ and the steady-state input ratio ℓ_{ss} at the same time – a case in point is the analysis of tax changes in the next subsection.

[22] This mechanism is analyzed in detail in Peretto and Connolly (2007). Since the entry cost is proportional to the population-firms ratio, L/N, the long-run mass of firms relative to population N_{ss}/L_{ss} responds positively to expenditure per capita $y^*\left(\ell_{ss}\right)$ because as expenditure per capita rises, the size of the market expands more than proportionally to the entry cost and thus attracts entry. Entry, in turn, dilutes firms' market shares inducing each firm to reduce their in-house R&D efforts.

The model's key measure of living standards is individual utility in equation (17), which evaluated at the equilibrium reads (see the Appendix)

$$\ln u = \underbrace{\bar{\alpha} + \ln T + \ln y^* (\ell) - \gamma \ln p_q^* (\ell)}_{\text{Economic channel}} + \underbrace{\ln L^{\psi} b^{-(1-\psi)(1-\alpha)}}_{\text{Demographic channel}}, \tag{58}$$

where $\bar{\alpha} \equiv \ln \alpha^{\alpha} \gamma^{\gamma} (1-\alpha)^{1-\alpha} (1-\gamma)^{1-\gamma} \frac{\epsilon-1}{\epsilon}$. Equation (58) allows us to distinguish the different components of instantaneous utility. The economic channel shows how the components of economic activity affect utility at each point in time. The demographic channel summarizes the effects of population level and birth rate on utility: it combines direct effects, i.e., the household's preference for adults and children, and the indirect effects of family composition on the allocation of consumption among adults and children. Differentiating (58) with respect to time yields

$$\frac{\dot{u}(t)}{u(t)} = g(t) + (\psi + 1) \frac{\dot{L}(t)}{L(t)} - (1-\psi)(1-\alpha) \frac{\dot{b}(t)}{b(t)}, \tag{59}$$

where g is the growth rate computed in (55). Equation (59) shows the distinct contribution of economic and demographic channels to the dynamics of utility. The model's dynamics, worked out in detail in the Appendix and briefly discussed above, show that in response to a permanent expansion of the market for intermediate goods both firm-specific knowledge growth and net entry accelerate until they revert to $(\dot{z}/z)_{ss}$ and N_{ss}. Changes in fundamentals therefore modify the dynamics of (ℓ, b) and affect welfare through the underlying components of utility, namely, the consumption expenditure channel, $\ln y^* (\ell)$, the commodity price channel, $-\gamma \ln \left(p_q^* (\ell)\right)$, and the demographic channel, $\ln L^{\psi} b^{-(1-\psi)(1-\alpha)}$. We next provide concrete examples by studying the effects of the commodity tax and of a resource boom.

5.2 Commodity tax

We consider the scenario in which substitutability and deadly spillovers create a regular steady state and a mortality trap (Proposition 6). The following Proposition provides the comparative statics effects of τ on both the regular steady state and the size of the mortality trap.

Proposition 8 *(Commodity Tax) Assume $\sigma > 1$ and $\zeta + \xi (1-\upsilon) > 0$. The increase in the commodity tax, τ, shifts the $\dot{\ell} = 0$ locus down and the $\dot{b} = 0$ locus up. Therefore, it yields a higher regular-steady-state population-resource ratio, $d\ell'_{ss}/d\tau > 0$, as well as a smaller mortality-trap threshold, $d\ell''_{ss}/d\tau < 0$.* ***Proof:*** *see the Appendix.*

To understand the mechanism driving these comparative-statics effects, start holding the resource-population ratio constant at the initial steady-state. The increase in τ reduces the demand for the resource, triggering a reduction in the resource price and an increase in expenditure per capita. Given substitutability, the lower resource price raises the resource cost-share and thus drives down the mortality rate via the primary-employment effect. In graphical terms, there is an upward shift of the expenditure schedule $y^*(\ell)$ that yields an upward shift of the $\dot{b}=0$ locus and a downward shift of mortality schedule, $m^*(\ell)$, that yields a downward shift of the $\dot{\ell}=0$ locus; see Figure 3, diagrams (a)-(c). The consequence of these shifts is a widening gap between the two steady states, with a higher regular-steady-state population-resource ratio, ℓ'_{ss}, and a lower mortality-trap threshold, ℓ''_{ss}.

As an example of the forces at play, assume that the tax change is relatively small so that the initial steady state remains in the basin of attraction of the regular steady state. The phase diagram shows that the population-resource ratio, ℓ, increases monotonically over time as the economy converges to the new regular steady state. The commodity tax increase, therefore, triggers a permanent, monotonic expansion of the population. The birth rate, b, in contrast, exhibits *overshooting*: it jumps up on the new saddle path and then declines gradually and monotonically during the transition, converging from above to the new steady state. The position of the new steady state on the vertical axis, however, can be either above or below the old one. Before discussing this property, we note that the permanent expansion of the population causes a permanent expansion of the market for intermediate goods. This means that initially TFP growth accelerates because of both more net entry and more investment by incumbent firms. Along the transition, the expansion of the mass of firms weakens the incentive of incumbents to invest, causing a slowdown of firm-level productivity growth that counteracts the expansion of product variety. This negative comovoment beween mass of firms and firm growth is at the heart of the Schumpeterian model that we use. As discussed, absent deadly spillovers, this mechanism would produce scale-invariance, which in this context would yield that steady-state TFP growth is invariant to the commodity tax. However, with deadly spillovers, steady-state TFP growth responds to the steady-state population-resource ratio. This means that the commodity tax has an effect on steady-state TFP growth that has the opposite sign of the effect on the mortality rate. We stress once again that this channel for growth effects is solely and entirely due to the endogeneity of the mortality rate.

As mentioned, the birth rate overshoots: it jumps up on the new saddle path and then declines along the transition toward a new steady-state value that can be larger or smaller than the old one because in steady state the birth rate must equal the mortality rate. We have two cases.

- If $\zeta+\xi(1-\upsilon)\geqslant\upsilon$, damage dilution is suffficiently strong to guarantee that the mortality rate does not increase despite the larger ℓ and higher aggregate emissions at the new steady state.[23] This implies the the new steady-state birth rate is not higher than the old one. This case is in Figure 3, diagram (a). Note that because the new steady state has a lower mortality rate, it has a higher growth rate.

- If $\zeta + \xi (1 - \upsilon) < \upsilon$, damage dilution is weaker and thus in the new steady state the mortality rate may be higher or lower than the initial one because it is subject to opposing forces: the downward shift of the schedule, $m^* (\ell)$, that reduces mortality for given ℓ; the larger ℓ that can result in higher aggregate emissions that dominate damage dilution.[24] If the net effect is a lower or unchanged mortality rate, the conclusions are the same as for the previous case; see Figure 3, diagram (b). If, instead, the net effect is a higher mortality rate, like in diagram (c) of Figure 3, the new steady state has a lower growth rate.

One takeaway of this analysis is that the commodity tax yields a double *demographic* dividend: it expands the size of the population *and* it reduces the size of the mortality trap by pushing the mortality threshold ℓ''_{ss} to the left. Associated to these gains there is an *economic growth* dividend because the lower mortality rate yields a higher TFP growth rate. The same mechanism in reverse, i.e., reducing the commodity tax, yields a double loss, namely, a lower population and a larger mortality trap. A large enough cut of the commodity tax can actually put the economy in the mortality trap, as shown in Figure 3, diagram (d). If the initial population-resource ratio is ℓ_2, the economy converges to the regular steady state under the old tax rate but falls in the mortality trap with the new tax rate, following a path that eventually leads to extinction. This scenario offers a sobering lesson for less populated resource-rich countries that implement low commodity and/or emission taxes and/or subsidize their primary sectors. In an economy with population-resource ratio close to the mortality trap, subsidizing the primary sector is functionally equivalent to introducing a negative emission tax. Empirical evidence suggests that many real-world economies face such a situation, in particular oil-exporting countries where subsidies to the extractive industry are pervasive and high (Gupta et al., 2002; Metschies, 2005). Below, we pursue this argument further by showing that the combination of subsidies to the primary sector and new discoveries of the resource can be a recipe for disaster.

[23] In graphical terms, $\zeta + \xi (1 - \upsilon) \geqslant \upsilon$ implies that the $\dot{\ell} = 0$ is L-shaped, so that the combined shifts of the $\dot{b} = 0$ locus (upward) and of the $\dot{\ell} = 0$ locus (downward) imply a steady state with lower mortality rate $m^* (\ell_{ss}) = b_{ss}$.

[24] In graphical terms, $\zeta + \xi (1 - \upsilon) < \upsilon$ implies that the $\dot{\ell} = 0$ is U-shaped, and the regular steady state is likely to be found in the part of the curve with positive slope. Hence, the combined shifts of the $\dot{b} = 0$ locus (upward) and of the $\dot{\ell} = 0$ locus (downward) may imply either a lower or higher mortality rate $m^* (\ell_{ss}) = b_{ss}$ in the regular steady state, depending on the curvature and on the extent of the shift of the $\dot{\ell} = 0$ locus.

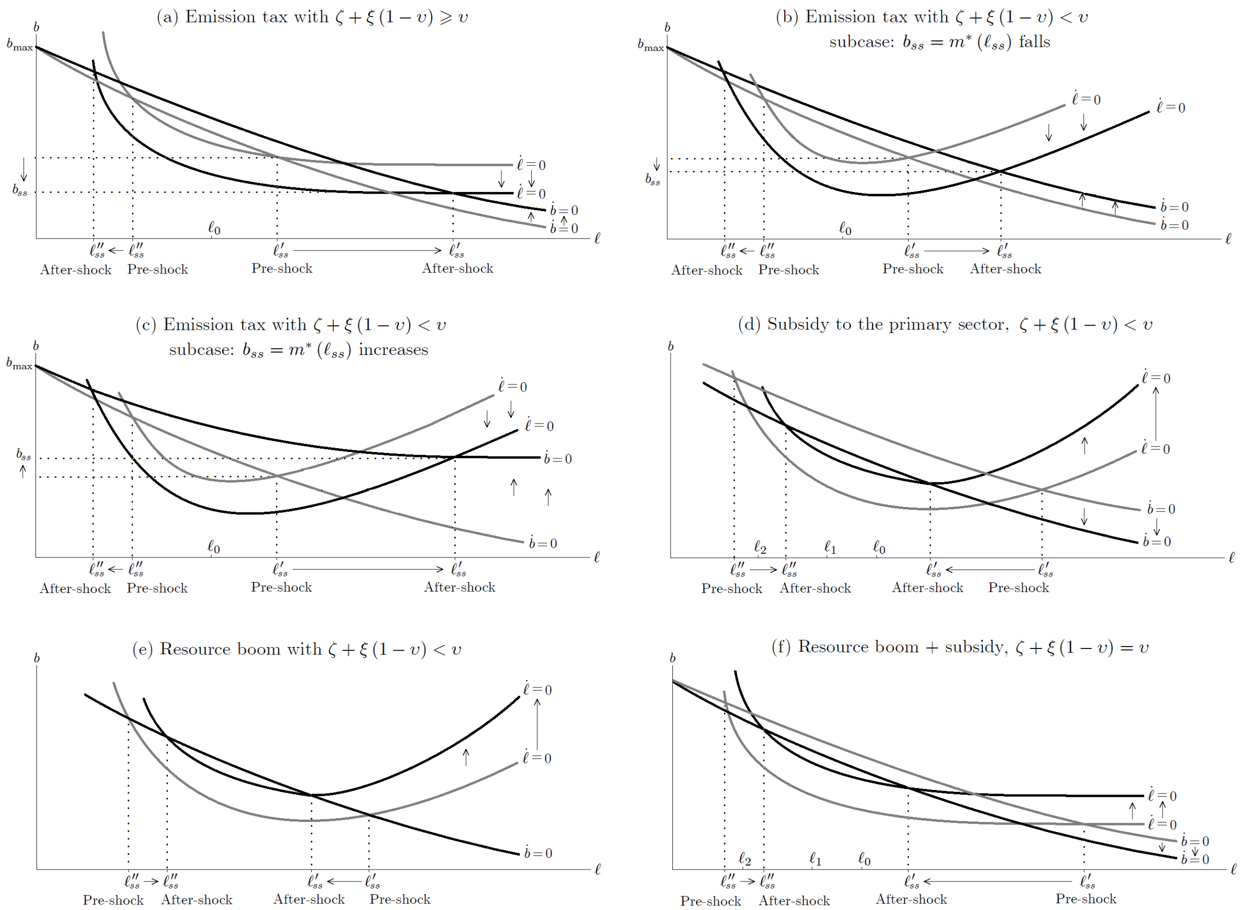

Figure 3. Exogenous shocks under substitutability, $\sigma > 1$, and $\zeta+\xi(1-\upsilon) > 0$. See main text for detailed descriptions.

5.3 Resource booms

A *resource boom* is an exogenous increase at time $t = 0$ of the resource endowment, Ω. By definition, therefore, it reduces the resource-population ratio, $\ell(0) = L(0)/\Omega$. All else equal, this immediate effect brings the economy closer to the mortality trap. But the shock may further increase the threat of population implosion by expanding the mortality trap depending on the value of the damage elasticity. Note, moreover, that, as in the case of the commodity tax, the resource boom has an effect on steady-state TFP growth of the opposite sign of its effect on the mortality rate. The following proposition summarizes the demographic effects of the boom.

Proposition 9 *(Resource boom) Assume $\sigma > 1$ and $\zeta + \xi(1-\upsilon) > 0$. An increase in the resource endowment, Ω, affects the equilibrium mortality function $m^*(\ell)$ as follows*

$$\frac{dm^*(\ell)}{d\Omega} \left\{ \begin{array}{ll} > 0 & \text{if } \upsilon > \zeta + \xi(1-\upsilon) \\ = 0 & \text{if } \upsilon = \zeta + \xi(1-\upsilon) \\ < 0 & \text{if } \upsilon < \zeta + \xi(1-\upsilon) \end{array} \right\} \text{ for any } \ell > 0. \tag{60}$$

When $\upsilon > \zeta + \xi(1-\upsilon)$, a resource boom enlarges the mortality trap, $(0, \ell''_{ss})$. ***Proof:*** *see the Appendix.*

The mechanism driving this result is that the emission damage incorporated in the mortality function (42) depends on the resource endowment Ω with elasticity $\upsilon - [\zeta + \xi(1-\upsilon)]$. If this elasticity is positive, the increase in Ω raises the mortality rate associated with the regular steady state. This phenomenon is a type of *resource curse* seldom recognized in the literature. Diagram (e) in Figure 3 describes the effect of the resource boom assuming $\upsilon > \zeta + \xi(1-\upsilon) > 0$. As the endowment increases from Ω_0 to Ω_1, the $\dot{\ell} = 0$ locus shifts up and yields a lower population-resource ratio in the regular steady state, ℓ'_{ss}, and a higher mortality-trap threshold, ℓ''_{ss}. At the same time, the population-resource ratio at time zero moves from the pre-shock level $\ell_0 = L_0/\Omega_0$ to the lower after-shock level $\ell_1 \equiv \ell_1(0) = L_0/\Omega_1$. The welfare effects of these shocks are generally ambiguous. Moreover, the shock itself may drive the economy into the mortality trap, yielding drastically opposite results: if $\ell_1 < \ell''_{ss}$, the population decline deletes and eventually overturns the consumption gains, while both the demographic components of utility – adult population and flow of children – yield net losses both in the transition and in the long run as the mortality rate grows.

In the case $\upsilon \leqslant \zeta + \xi(1-\upsilon)$, the resource boom does not expand the mortality trap but this does not mean that the trap is less threatening: even when the mortality trap, $(0, \ell''_{ss})$, shrinks or remains the same, the increase in Ω reduces the population-resource ratio. With $\upsilon < \zeta + \xi(1-\upsilon)$, the $\dot{\ell} = 0$ locus shifts down but the initial reource-population ratio can fall more than the mortality threshold ℓ''_{ss}. With $\upsilon = \zeta + \xi(1-\upsilon)$, the resource boom moves the economy closer to population implosion because ℓ falls instantaneously: even though the steady-state levels of ℓ and b do not change, the endowment shock may exhert substantial effects on the population level and on the mass of firms in the long run – and, hence, on transitional economic growth – as we show below.

5.4 Resource booms and subsidies: numerical illustration

Consider the polar case $\upsilon = \zeta + \xi(1-\upsilon)$ under substitutability, $\sigma > 1$, with a linear damage function, $\chi = 1$, and assume the following scenario: the economy experiences a resource boom and the government decides to subsidize the primary sector by reducing the commodity tax rate, τ, below zero. Policies of this kind are frequently implemented in resource-rich countries, with various justifications. The resource boom has a straightforward graphical representation, namely, a displacement to the left of the current population-resource ratio with no change in the steady-state loci and the associated points (ℓ'_{ss}, b'_{ss}) and (ℓ''_{ss}, b''_{ss}). The subsidy, instead, modifies the positions of both the regular steady state and the extinction threshold by reducing ℓ'_{ss} and increasing ℓ''_{ss}. Figure 3, diagram (f), illustrates the simultaneous shift in the stationary loci caused by the subsidy shock. Table 1 distinguishes the effects of resource booms from those of subsidies by considering each shock in turn, and clarifies the implications of both shocks occurring at the same time for population size L'_{ss}, mortality rates m'_{ss}, mass of firms N'_{ss}, and long-term growth g'_{ss} when the economy converges to the regular steady state (central columns). Table 1 reports numerical results for the unstable steady state (right-end columns).[25]

Consider the combined whereby Ω grows by 2% relative to the baseline, while the tax rate falls from $\tau = 0$ to $\tau = -5\%$. If the economy still converges to a regular steady state (ℓ'_{ss}, b'_{ss}) after the shock, the effect on long-run population L'_{ss} is negative because subsidies to the primary sector increase the mortality rate m'_{ss}, and the effect on economic growth is twofold. Out of the steady state, convergence towards a smaller population level induces a smaller number of firms in the long run, N'_{ss}, which reduces TFP growth by depressing horizontal innovations. In the long run, increased expenditure per capita, y'_{ss}, depresses vertical innovations, although the decline in g'_{ss} is quantitatively negligible for the vast majority of plausible parametrizations.

Besides the effects on the regular steady state, a resource boom combined with subsidies to the primary sector moves the economy closer to the mortality trap for two independent reasons: while the larger resource endowment reduces the current population-resource ratio, the lower tax rate τ shifts the mortality-trap threshold, ℓ''_{ss}, to the right. Both these effects push the economy away from the pre-shock regular steady state and, if they are strong enough, may even derail the economy from the regular path and push it into the mortality trap. Figure 3, diagram (f), allows us to describe the two possible outcomes. Suppose the pre-shock level of the population-resource ratio is ℓ_0, that is, the economy is initially converging to the pre-shock regular steady state. If the resource boom is relatively small, the post-shock population-resource ratio may fall to a moderately lower level like $\ell_1 > \ell''_{ss}$, which still guarantees convergence to the (new, post-shock) regular steady state. If the increase in Ω is substantial, instead, the post-shock population-resource ratio may fall down to $\ell_2 < \ell''_{ss}$ and trigger population implosion: the fertility rate jumps up and keeps growing but never reaches the exploding mortality rate.

We stress that the numerical exercise reported in Table 1 and in Figure 3(f) assumes parameter values yielding no effects of the resource boom on the steady state loci, $\upsilon = \zeta + \xi(1-\upsilon)$. With weaker damage dilution, $\upsilon > \zeta + \xi(1-\upsilon)$, we obtain an even larger increase in the mortality-trap threshold for scenarios (iv)-(v) because, as established in Proposition 9, the increase in Ω further reduces the gap between the two steady states.[26] These and the previous considerations make

[25] The parameter values assumed in calculating the numbers reported in Table 1 are: $\chi = 1$, $\beta = 1.587$, $\rho = 0.015$, $\epsilon = 4.3$, $\gamma = 0.3$, $\eta = 0.5$, $\sigma = 2$, $\alpha = 0.8$, $\psi = 0.3$, $\bar{m} = 0.016$, $\mu = 0.2$, $\upsilon = 0.2$, $\zeta = 0.1$.

[26] In graphical terms, scenario (v) with $\upsilon > \zeta + \xi(1-\upsilon)$ would feature one downward shift of the $\dot{b} = 0$ locus and *two* upward shifts of the $\dot{\ell} = 0$ locus, one due to the subsidy and one due to the resource boom.

Scenario	τ	Ω	ℓ'_{ss}	L'_{ss}	m'_{ss}	N'_{ss}	g'_{ss}	ℓ''_{ss}	L''_{ss}	m''_{ss}
(*i*) Baseline	0.00	25.0 mln	1.241	31,022,161	2.081%	234,553	1.501%	0.014	353,243	3.98%
(*ii*) Positive tax	0.05	25.0 mln	1.473	36,831,505	2.057%	277,871	1.502%	0.014	350,262	3.98%
(*iii*) Subsidy	-0.05	25.0 mln	1.055	26,387,205	2.105%	199,940	1.500%	0.014	356,448	3.97%
(*iv*) Resource boom	0.00	25.5 mln	1.241	31,642,605	2.081%	239,244	1.501%	0.014	360,307	3.98%
(*v*) Boom & Subsidy	-0.05	25.5 mln	1.055	26,914,949	2.105%	203,939	1.500%	0.014	363,577	3.97%

Table 1. Taxes, subsidies, resource booms: quantitative effects on regular steady state and extinction trap.

our general conclusion evident: labor-poor countries with abundant polluting resources face larger mortality traps. If the governments of these countries respond to new resource discoveries with higher subsidies to the primary sector – a policy often justified with the need to escape under-development traps – the possibility of falling into a different trap characterized by ever-growing mortality should be taken seriously.

6 Conclusion

In stark contrast to the magnitude of pollution-induced mortality reported in the empirical literature, there is little to no recognition of such an important phenomenon in macroeconomic models of growth and development. Filling this gap requires tractable models in which economic growth, fertility and mortality are simultaneously endogenous and interconnected via equilibrium relationships. We have shown that unlike conventional pollution externalities, deadly spillovers affect welfare through multiple channels – labor-supply effects, consumption-saving decisions, reproduction choices, changes in market size that affect incentives to innovate and thereby productivity growth – and that the response of the equilibrium mortality rate to population size is generally ambiguous and often non-monotonic. This relationship between mortality and population reflects not only the emission intensity of primary production but also damage dilution effects induced by population size/density and labor reallocation effects caused by technology. Under parametrizations that yield empirically plausible paths – prominently, a transitional fertility decline leading to a finite endogenous population level – deadly spillovers modify potential population in the long run, productivity growth in both the short and the long run, and may even create mortality traps that, unlike the typical poverty traps studied in development economics, threaten less populated economies with abundant natural resources. From a growth perspective, our framework shows that exogenous shocks that (a) increase long-run population capacity accelerate TFP growth during the transition via net entry of firms, while shocks that (b) reduce lower long-run mortality rates increase TFP growth in the long run via faster rates of vertical innovations. Under certain conditions, policy-induced shocks – like an increase in environmental taxes in the presence of substantial damage dilution – can simultaneously produce both outcomes, (a) and (b). More generally, our model suggests that emission taxes may yield double dividends by increasing long-run population capacity. To the contrary, subsidies to primary production reduce long-run population capacity and may increase the risk of population implosion. Consequently, subsidizing commodity production during a resource boom can have disastrous consequences if the primary sector's technology does not change. These considerations suggest that some novel thinking is called for in the debate on the prospects of many developing countries where discoveries of natural resources are accompanied by (implicit or explicit) subsidies designed to foster their exploitation.

Our framework also delivers novel insights for applied research. Our analysis shows that the pollution-attributed fraction (PAF) of total deaths will respond to changes in population size according to a combination of (i) labour-supply, (ii) dose-dilution, and (iii) emission-reducing effects that bear substantial quantitative and qualitative implications for the equilibrium mortality rate. While channels (ii)-(iii) are crucial determinants of long-run economic and demographic outcomes in our model, applied studies that forecast future PAFs should do not typically include them in their reference models nor in their pre-estimated parameters. On the one hand, the medical literature is generally aware of the existence of (ii) dose-dilution effects but the actual PAF estimates do not take these into account because the conventional estimation methods use the 'mean concentration of pollutants' as a proxy for individual doses – a procedure that neglects feedback effects of population size on individual doses. On the other hand, the literature on environmental and urban economics clearly documents the importance of (iii) emission-reducing effects induced by population density in the real world. However, applied studies that use PAFs estimated by the medical literature to make projections about future mortality rates – e.g., OECD (2016) – tend to incorporate exclusively (i) labour-supply effects. In principle, these studies can incorporate both (ii)-(iii) by introducing, ex post, damage-dilution effects in the economic model used to obtain projections of future exposure to pollution: this has not been done so far, but it would bridge the gap, at least in part, between applied work on pollution-induced mortality and the notion of equilibrium mortality rate that we define and explore in our analysis.

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Growth with Deadly Spillovers

(Online Appendix)

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A Appendix: The model

A.1 Consumption and Reproduction Choices

Utility maximization and derivation of equations (19)-(20). The maximization problem can be specified as (omitting time arguments when no ambiguity arises)

$$\max_{\{c_L(t),c_B(t),B(t)\}} \int_0^\infty e^{-\rho t} \ln\left[\left(C_L L^{\psi-1}\right)^\alpha \left(C_B B^{\psi-1}\right)^{1-\alpha}\right] dt$$

subject to

$$\dot{A} = rA + wL + p_\omega \Omega + S - p_c C_L - p_c C_B, \tag{A.1}$$

$$\dot{L} = B - mL, \tag{A.2}$$

where (A.1) is the asset accumulation law (18) and (A.2) is the demographic law (1) where the path of the mortality rate m is taken as given. The current value Hamiltonian for this problem reads

$$\begin{aligned}\mathcal{L} \equiv\ & \ln\left[\left(C_L L^{\psi-1}\right)^\alpha \left(C_B B^{\psi-1}\right)^{1-\alpha}\right] + \\ & +\vartheta_A \left(rA + wL + p_\omega \Omega + S - p_c C_L - p_c C_B\right) + \\ & +\vartheta_L \left(B - mL\right),\end{aligned} \tag{A.3}$$

where ϑ_A and ϑ_L are the dynamic multipliers associated with asset accumulation and with population growth, respectively.

The necessary conditions for utility maximization read

$$\begin{array}{lcll}
\mathcal{L}_{C_L} = 0 & \rightarrow & \frac{\alpha}{C_L} = \vartheta_A p_c & \text{(i)} \\
\mathcal{L}_{C_B} = 0 & \rightarrow & \frac{1-\alpha}{C_B} = \vartheta_A p_c & \text{(ii)} \\
\mathcal{L}_{B} = 0 & \rightarrow & \frac{\psi(1-\alpha)}{B} + \vartheta_L = \vartheta_A p_c \frac{C_B}{B} & \text{(iii)} \\
\mathcal{L}_{A} = \rho\vartheta_A - \dot{\vartheta}_A & \rightarrow & \vartheta_A r = \rho\vartheta_A - \dot{\vartheta}_A & \text{(iv)} \\
\mathcal{L}_{L} = \rho\vartheta_L - \dot{\vartheta}_L & \rightarrow & \frac{\psi\alpha}{L} + \vartheta_A\left(w - p_c\frac{C_L}{L}\right) - \vartheta_L m = \rho\vartheta_L - \dot{\vartheta}_L & \text{(v)} \\
\text{TVC assets} & \rightarrow & \lim_{t\to\infty} e^{-\rho t}\vartheta_A(t)\, A(t) = 0 & \text{(vi)} \\
\text{TVC population} & \rightarrow & \lim_{t\to\infty} e^{-\rho t}\vartheta_L(t)\, L(t) = 0 & \text{(vii)}
\end{array} \tag{A.4}$$

Conditions (A.4.i)-(A.4.ii) yield constant consumption expenditure shares for adults and children:

$$p_c C_L = \alpha Y \quad \text{and} \quad p_c C_B = (1-\alpha) Y. \tag{A.5}$$

From (A.5), total household consumption expenditure equals

$$Y = p_c C_L + p_c C_B = \frac{1}{\vartheta_A}, \tag{A.6}$$

so that (A.4.iv) yields the Euler equation

$$\frac{\dot{Y}(t)}{Y(t)} = -\frac{\dot{\vartheta}_A(t)}{\vartheta_A(t)} = r(t) - \rho, \tag{A.7}$$

which is (19) in the main text.

From the fertility condition (A.4.iii) we have

$$\psi(1-\alpha) + \vartheta_L(t) B(t) = \vartheta_A p_c(t) C_B(t) \tag{A.8}$$

where we can substitute $\vartheta_A p_c C_B = 1-\alpha$ from (A.4.ii), to obtain

$$\vartheta_L(t) B(t) = (1-\alpha)(1-\psi). \tag{A.9}$$

This expression shows that positive fertility $B > 0$ is consistent with a positive marginal shadow value of the population $\vartheta_L > 0$ if and only if $\psi < 1$. The expression also shows that the shadow value of children $\vartheta_L(t) B(t)$ is constant. Time-differentiating (A.9) yields

$$\frac{\dot{\vartheta}_L(t)}{\vartheta_L(t)} = -\frac{\dot{B}(t)}{B(t)}. \tag{A.10}$$

From (A.6) and (A.9), the ratio ϑ_A/ϑ_L is

$$\frac{\vartheta_A(t)}{\vartheta_L(t)} = \frac{1}{(1-\alpha)(1-\psi)} \cdot \frac{B(t)}{Y(t)}. \tag{A.11}$$

Now consider the co-state equation for population (v), which we can write

$$-\frac{\dot{\vartheta}_L}{\vartheta_L} = \frac{\psi\alpha}{\vartheta_L L} + \frac{\vartheta_A}{\vartheta_L}\left(w - p_c\frac{C_L}{L}\right) - m - \rho \tag{A.12}$$

Substituting (A.9), (A.11), and (A.10) in (A.12), we have

$$\frac{\dot{B}}{B} = \frac{b}{(1-\alpha)(1-\psi)} \frac{\psi\alpha Y + wL - \alpha Y}{Y} - m - \rho. \tag{A.13}$$

Recalling that the left hand side of (A.13) equals $\frac{\dot{B}}{B} = \frac{\dot{b}}{b} + \frac{\dot{L}}{L}$, (1) yields

$$\frac{\dot{b}}{b} = \frac{b}{(1-\alpha)(1-\psi)} \frac{\psi\alpha Y + wL - \alpha Y}{Y} - b - \rho$$

and therefore

$$\frac{\dot{b}(t)}{b(t)} = \frac{b(t)}{(1-\alpha)(1-\psi)} \left[\psi + \frac{w(t) L(t) - Y(t)}{Y(t)}\right] - \rho, \tag{A.14}$$

which is equation (20) in the main text.

A.2 Producers: Final and Intermediate sectors

Final producers. The representative competitive firm maximizes aggregate profits, $Y-\int_0^N p_{x_i}x_i di$, subject to (21) taking prices as given. The first order condition for the quantity x_i of each intermediate variety i yields the demand schedule

$$p_{x_i}(t)=\frac{Y(t)}{\int_0^{N(t)} x_i(t)^{\frac{\epsilon-1}{\epsilon}}di}\cdot x_i(t)^{-\frac{1}{\epsilon}}. \tag{A.15}$$

Incumbents: profit maximization. The maximization problem is

$$\begin{aligned} \max_{\{p_{x_i},L_{z_i}\}} V_i(t) &= \int_t^{\infty}\pi_i(t)\exp\left(-\int_t^v (r(s)+\delta)\,ds\right)dv \\ &\quad \text{subject to} \\ \pi_i &= p_{x_i}x_i - TC_i(x_i;w,p_q)-wL_{z_i}, & \text{(A.16)}\\ \dot{z}_i &= \kappa\cdot\bar{z}\cdot L_{z_i}, & \text{(A.17)}\\ p_{x_i} &= \Psi\cdot x_i^{-\frac{1}{\epsilon}}. & \text{(A.18)} \end{aligned}$$

The monopolist takes as given input prices, the term $\Psi=Y/\int_0^N x_i^{\frac{\epsilon-1}{\epsilon}}di$ in (A.18) and the path of public knowledge $\bar{z}$. Substituting the constraints (A.16), (**??**) and (A.18) in the objective function, the Hamiltonian for this problem can be written as

$$\bar{\mathcal{L}}=p_{x_i}x_i-TC_i(x_i;w,p_q)-wL_{z_i}-wL_{z_i}+\vartheta_z\cdot\kappa\cdot\bar{z}\cdot L_{z_i}, \tag{A.19}$$

where ϑ_z is the dynamic multiplier associated to z_i. The necessary conditions for maximization read

$$\begin{array}{llll} \bar{\mathcal{L}}_{p_{x_i}}=0 & \rightarrow & p_{x_i}=\frac{\epsilon}{\epsilon-1}\frac{\partial TC_i(x_i;w,p_q)}{\partial x_i} & \text{(i)} \\ \bar{\mathcal{L}}_{L_{z_i}}=0 & \rightarrow & \vartheta_z\kappa\bar{z}=w & \text{(ii)} \\ \bar{\mathcal{L}}_{z_i}=(r+\delta)\vartheta_z-\dot{\vartheta}_z & \rightarrow & \frac{\epsilon-1}{\epsilon}\cdot\theta\frac{p_{x_i}x_i}{z_i}=(r+\delta)\vartheta_z-\dot{\vartheta}_z & \text{(iii)} \\ \text{TVC knowledge} & \rightarrow & lim_{t\rightarrow\infty}\exp\left(-\int_t^v(r(s)+\delta)ds\right)\vartheta_z z_i=0 & \text{(iv)} \end{array} \tag{A.20}$$

For future reference, note that (i) yields

$$TC_i=p_qQ_i+wL_{x_i}=\frac{\epsilon-1}{\epsilon}p_{x_i}x_i+w\phi \tag{A.21}$$

and (ii)-(iii) yield

$$\frac{\dot{\vartheta}_z}{\vartheta_z}=r+\delta-\frac{\epsilon-1}{\epsilon}\cdot\theta\frac{p_{x_i}x_i}{\vartheta_z z_i}=r+\delta-\frac{\epsilon-1}{\epsilon}\cdot\theta p_{x_i}x_i\cdot\frac{\kappa\bar{z}}{wz_i}, \tag{A.22}$$

where the last term follows from using (ii) to substitute ϑ_z.

A.3 Primary sector

Derivation of the cost share of resource use (35). The profit maximizing conditions with respect to resource use and labor respectively yield

$$p_q(1-\tau)\cdot\left[\eta\cdot\Omega^{\frac{\sigma-1}{\sigma}}+(1-\eta)\cdot L_Q^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}-1}\eta\cdot\Omega^{\frac{\sigma-1}{\sigma}} = p_\omega\Omega, \tag{A.23}$$

$$p_q(1-\tau)\cdot\left[\eta\cdot\Omega^{\frac{\sigma-1}{\sigma}}+(1-\eta)\cdot L_Q^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}-1}(1-\eta)\cdot L_Q^{\frac{\sigma-1}{\sigma}} = wL_Q. \tag{A.24}$$

From (A.23) and (A.24), the cost share of resource use is

$$\Upsilon\equiv\frac{p_\omega\Omega}{p_\omega\Omega+wL_Q}=\frac{\eta\cdot\Omega^{\frac{\sigma-1}{\sigma}}}{\eta\cdot\Omega^{\frac{\sigma-1}{\sigma}}+(1-\eta)\cdot L_Q^{\frac{\sigma-1}{\sigma}}}. \tag{A.25}$$

In order to rewrite (A.25) in terms of factor prices, note that (A.23) and (A.24) imply

$$\frac{\eta\cdot\Omega^{\frac{\sigma-1}{\sigma}}}{(1-\eta)\cdot L_Q^{\frac{\sigma-1}{\sigma}}}=\frac{\eta^\sigma p_\omega^{1-\sigma}}{(1-\eta)^\sigma w^{1-\sigma}}. \tag{A.26}$$

Substituting (A.26) in the right hand side of (A.25) yields

$$\Upsilon\equiv\frac{p_\omega\Omega}{p_\omega\Omega+wL_Q}=\frac{\eta^\sigma p_\omega^{1-\sigma}}{\eta^\sigma p_\omega^{1-\sigma}+(1-\eta)^\sigma w^{1-\sigma}}, \tag{A.27}$$

which is expression (35) in the main text.

B Appendix: Equilibrium and Mortality Rates

B.1 Output and Input Markets

Derivation of (30). Since each solves the maximization problem given the same initial level of firm-specific knowledge, the equilibrium of the intermediate sector is symmetric. Given $p_{x_i}=p_x$ for each $i\in[0,N]$, the demand curve (A.15) yields (30).

Derivation of (??). From the intermediate producers' problem, the profit maximization condition (A.20.i) can be aggregated as

$$\gamma\frac{\epsilon-1}{\epsilon}N(t)\,p_{x_i}(t)\,x_i(t)=p_q(t)\,Q(t). \tag{B.1}$$

Substituting the equilibrium condition (30) in (B.1) we obtain

$$\gamma\frac{\epsilon-1}{\epsilon}Y(t)=p_q(t)\,Q(t), \tag{B.2}$$

which is equation (??) in the main text.

Derivation of (??). The zero-profit condition in the primary sector implies

$$p_q(t)\,Q(t)\,(1-\tau)=p_\omega(t)\,\Omega+w(t)\,L_Q(t). \tag{B.3}$$

Combining (B.3) with (35) yields (??) in the main text.

B.2 Expenditure and Resource Use

Derivation of (39). Substituting the government budget constraint (18) into the wealth constraint (18) yields

$$\frac{\dot{A}(t)}{A(t)} = r(t) + \frac{w(t)L(t)}{A(t)} + \frac{p_\omega(t)\Omega + \tau p_q(t)Q(t)}{A(t)} - \frac{Y(t)}{A(t)}. \tag{B.4}$$

Substituting $A = \beta wL$ from (38) and $p_q Q = \gamma\frac{\epsilon-1}{\epsilon}Y$ from (37), we have

$$\frac{\dot{Y}(t)}{Y(t)} = r(t) + \frac{w(t)L(t)}{\beta w(t)L(t)} + \frac{p_\omega(t)}{\beta w(t)}\frac{\Omega}{L(t)} - \frac{1-\tau\gamma\frac{\epsilon-1}{\epsilon}}{\beta w(t)}\frac{Y(t)}{L(t)}. \tag{B.5}$$

Using the Euler equation (19) yields

$$0 = \rho + \frac{w(t)L(t)}{\beta w(t)L(t)} + \frac{p_\omega(t)}{\beta w(t)}\frac{\Omega}{L(t)} - \frac{1-\tau\gamma\frac{\epsilon-1}{\epsilon}}{\beta w(t)}\frac{Y(t)}{L(t)}. \tag{B.6}$$

Multiplying through by $\beta w(t)$ and rearranging terms, we obtain

$$\frac{Y(t)}{L(t)} = \frac{w(t) + \rho\beta w(t) + p_\omega(t)\frac{\Omega}{L(t)}}{1-\tau\gamma\frac{\epsilon-1}{\epsilon}}. \tag{B.7}$$

Using the definitions $y = Y/L$ and $\ell = L/\Omega$, and the choice of numeraire $w = 1$, we obtain equation (39) in the main text.

Proof of Proposition 1. From (35), the cost share of resource use with normalized wage $w = 1$ reads

$$\Upsilon(p_\omega) \equiv \frac{p_\omega\Omega}{p_\omega\Omega + L_Q} = \frac{\eta^\sigma p_\omega^{1-\sigma}}{\eta^\sigma p_\omega^{1-\sigma} + (1-\eta)^\sigma} = \frac{1}{1+\frac{(1-\eta)^\sigma}{\eta^\sigma p_\omega^{1-\sigma}}} \tag{B.8}$$

and thus exhibits the following properties:

$$\left\{\begin{array}{lll} (\sigma<1) & \to & \lim_{p_\omega\to 0}\Upsilon(p_\omega)=0, \quad \lim_{p_\omega\to\infty}\Upsilon(p_\omega)=1, \\ (\sigma>1) & \to & \lim_{p_\omega\to 0}\Upsilon(p_\omega)=1, \quad \lim_{p_\omega\to\infty}\Upsilon(p_\omega)=0, \\ (\sigma=1) & \to & \Upsilon(p_\omega)=\eta. \end{array}\right\} \tag{B.9}$$

Recalling the definition $\ell = L/\Omega$, rewrite (39) and (40) as

$$y_1(p_\omega;\ell) = \frac{1+\beta\rho+(p_\omega/\ell)}{1-\tau\gamma\frac{\epsilon-1}{\epsilon}} \tag{B.10}$$

$$y_2(p_\omega) = \frac{1+\beta\rho}{1-\gamma\frac{\epsilon-1}{\epsilon}\left[\tau+(1-\tau)\cdot\Upsilon(p_\omega)\right]} \tag{B.11}$$

where (B.11) is obtained by substituting (40) in (39) to eliminate p/ℓ and solving for y. In (B.10) and (B.11), we have defined $y_1(p_\omega;\ell)$ and $y_2(p_\omega)$ as functions that treat p_ω as the explicit argument and ℓ as a parameter. The fixed point

$$\left(y^*(\ell), p_\omega^*(\ell)\right) = \arg\text{solve}\left\{y_1(p_\omega;\ell) = y_2(p_\omega)\right\} \tag{B.12}$$

characterizes the intratemporal equilibrium of the economy. The proof of Proposition 1 involves two steps. First, we prove existence and uniqueness of the equilibrium. Second, we assess the marginal effects of variations in ℓ.

Step #1. System (B.10)-(B.11) can be represented graphically in the (p_ω, y) plane: given ℓ, function $y_1(p_\omega;\ell)$ is a linear increasing function of p_ω, whereas the behavior of $y_2(p_\omega)$ depends on the value of σ. From (B.8), $y_2(p_\omega)$ is decreasing and convex for $\sigma > 1$; a flat horizontal line for $\sigma = 1$; increasing and concave for $\sigma < 1$. The three cases are described in Figure A1. The vertical intercepts and horizontal asymptotes of $y_2(p_\omega)$ are defined in (B.9) and (B.11). In all cases, the intersection $y_1(p_\omega;\ell) = y_2(p_\omega)$ is unique and determines the conditional values $y^*(\ell)$ and $p^*_\omega(\ell)$. In particular, $y^*(\ell)$ exhibits the property

$$y_{\min} \equiv \frac{1+\beta\rho}{1-\tau\gamma\frac{\epsilon-1}{\epsilon}} < y^*(\ell) < \frac{1+\beta\rho}{1-\gamma\frac{\epsilon-1}{\epsilon}} \equiv y_{\max}. \tag{B.13}$$

Step #2. The marginal effects of ℓ can be studied by means of Figure A1. In all cases, an increase in ℓ reduces the slope of $y_1(p_\omega;\ell)$ leaving $y_2(p_\omega)$ unchanged, so that the results

$$\frac{\mathrm{d}p^*_\omega(\ell)}{\mathrm{d}\ell} > 0, \quad \lim_{\ell\to 0^+} p^*_\omega(\ell) = 0, \quad \lim_{\ell\to\infty} p^*_\omega(\ell) = \infty, \tag{B.14}$$

hold independently of the elasticity of substitution. With respect to $y^*(\ell)$, we have

$$(\sigma < 1) \rightarrow \frac{\mathrm{d}y^*(\ell)}{\mathrm{d}\ell} > 0, \quad \lim_{\ell\to 0^+} y^*(\ell) = y_{\min}, \quad \lim_{\ell\to\infty} y^*(\ell) = y_{\max}, \tag{B.15}$$

$$(\sigma > 1) \rightarrow \frac{\mathrm{d}y^*(\ell)}{\mathrm{d}\ell} < 0, \quad \lim_{\ell\to 0^+} y^*(\ell) = y_{\max}, \quad \lim_{\ell\to\infty} y^*(\ell) = y_{\min}, \tag{B.16}$$

$$(\sigma = 1) \rightarrow \frac{\mathrm{d}y^*(\ell)}{\mathrm{d}\ell} = 0, \quad y = \frac{1+\beta\rho}{1-\gamma\frac{\epsilon-1}{\epsilon}\left[\tau + (1-\tau)\eta\right]}. \tag{B.17}$$

Finally, the equilibrium commodity price is a function of ℓ via the zero-profit condition in the primary sector: from $\Theta(w, p_\omega) \equiv \eta^\sigma p_\omega^{1-\sigma} + (1-\eta)^\sigma w^{1-\sigma}$ and $p_q = \frac{1}{1-\tau}\Theta(w, p_\omega)$ with $w = 1$, we have

$$p^*_q(\ell) \equiv \frac{1}{1-\tau} \cdot \left[\frac{\eta^\sigma}{p^*_\omega(\ell)^{\sigma-1}} + (1-\eta)^\sigma\right]. \tag{B.18}$$

Since $p^*_\omega(\ell)/\mathrm{d}\ell > 0$, substitutability $\sigma > 1$ implies $p^*_q(\ell)/\mathrm{d}\ell < 0$; complementarity $\sigma < 1$ implies $p^*_q(\ell)/\mathrm{d}\ell > 0$; the Cobb-Douglas case $\sigma = 1$ implies $p^*_q(\ell)/\mathrm{d}\ell = 0$. ■

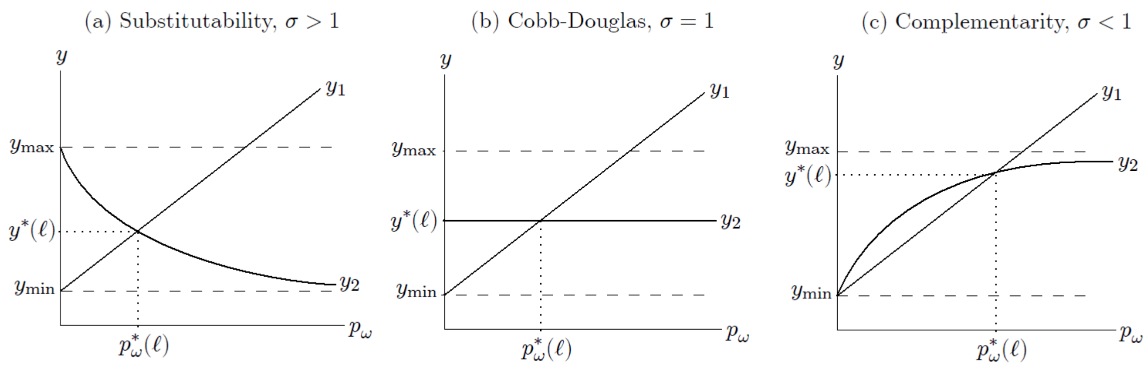

Figure A1. Determination of the equilibrium couple $(y^*(\ell), p^*_\omega(\ell))$ in the proof of Proposition 1.

B.3 The Equilibrium Mortality Rate

Derivation of (13). From (9) and the subsequent hypotheses on emission generation in subsection 2.3, the emission generation function reads

$$E = (E_f)^{\upsilon}(E_a)^{1-\upsilon} = (E_f)^{\upsilon}(\varsigma(L)L)^{1-\upsilon} = (E_f)^{\upsilon}\left(\varsigma_0 \frac{L}{(L/\varsigma_1)^{1+\xi}}\right)^{1-\upsilon}$$

where ς_0 is a proportionality index and ς_1 is a fixed measure of area/surface determining density L/ς_1. Normalizing $\varsigma_0 = \varsigma_1 = 1$ without loss of generality for our results, we obtain $E = E_f^{\upsilon}L^{-\xi(1-\upsilon)}$, which we can substitute in (5) to obtain (13).

Proof of Proposition 2. First, consider the general case $\sigma \gtrless 1$. Denoting commodity output per adult by $q \equiv Q/L$, and recalling that $E_f = Q$, we can rewrite (13) as

$$m(t) = \bar{m} + \mu \cdot q(t)^{\chi\upsilon} \cdot L(t)^{\chi\upsilon - \chi[\zeta + \xi(1-\upsilon)]}. \tag{B.19}$$

Considering the primary sector, the equilibrium zero-profit condition (A.23) and the resource demand schedule of commodity producers (**??**) can be respectively rewritten as

$$\frac{p_\omega(t)}{p_q(t)} = \Upsilon(t) \cdot q(t)\,\ell(t)(1-\tau), \tag{B.20}$$

$$\frac{p_\omega(t)}{p_q(t)} = \eta(1-\tau) \cdot (q(t)\,\ell(t))^{\frac{1}{\sigma}}. \tag{B.21}$$

Combining the above equations to eliminate p_ω/p_q and solving for $q\ell$, we obtain

$$q(t)\,\ell(t) = \left(\frac{\eta}{\Upsilon(t)}\right)^{\frac{\sigma}{\sigma-1}}. \tag{B.22}$$

Substituting (B.22) into (B.19), the mortality rate becomes

$$m(t) = \bar{m} + \mu \cdot \eta^{\chi\upsilon\frac{\sigma}{\sigma-1}} \cdot \ell(t)^{-\chi\upsilon} \cdot L(t)^{\chi\upsilon - \chi[\zeta+\xi(1-\upsilon)]} \cdot \Upsilon(\ell(t))^{\frac{\sigma}{1-\sigma}\chi\upsilon} \tag{B.23}$$

where the we have substituted $\Upsilon(\ell)$ with the cost share of resource use evaluated in equilibrium, obtained from substituting the equilibrium resource price $p_\omega = p_\omega^*(\ell)$ defined in Proposition 1 inside the expression for the cost share $\Upsilon(p_\omega)$ defined by the last term in equation (35) with normalized wage $w = 1$. Substituting $L(t) = \ell(t)\Omega$ to eliminate population in (B.23), we get

$$m(t) = \bar{m} + \mu \cdot \eta^{\chi\upsilon\frac{\sigma}{\sigma-1}} \cdot \Omega^{\chi\upsilon-\chi[\zeta+\xi(1-\upsilon)]} \cdot \ell(t)^{-\chi[\zeta+\xi(1-\upsilon)]} \cdot \Upsilon(\ell(t))^{\frac{\sigma}{1-\sigma}\chi\upsilon}, \tag{B.24}$$

which, after defining the convenient constant $\bar{\mu} \equiv \mu\eta^{\chi\upsilon\frac{\sigma}{\sigma-1}}\Omega^{\chi\upsilon-\chi[\zeta+\xi(1-\upsilon)]} > 0$, reduces to equation (42). Considering the asymptotic behavior of $\Upsilon(\ell)$, we combine results (B.9) with results (B.14) to obtain

$$\begin{cases} \sigma > 1 \;\rightarrow\; \frac{\mathrm{d}\Upsilon(\ell)}{\mathrm{d}\ell} < 0, \quad \lim_{\ell\to\infty}\Upsilon(\ell) = 0, \quad \lim_{\ell\to 0^+}\Upsilon(\ell) = 1, \\ \sigma < 1 \;\rightarrow\; \frac{\mathrm{d}\Upsilon(\ell)}{\mathrm{d}\ell} > 0, \quad \lim_{\ell\to\infty}\Upsilon(\ell) = 1, \quad \lim_{\ell\to 0^+}\Upsilon(\ell) = 0, \end{cases} \tag{B.25}$$

which proves expression (43). Next, consider the Cobb-Douglas case. Profit maximization in the primary sector implies the factor income shares $p_\omega\Omega = \eta \cdot p_q(1-\tau)Q$ and $wL_Q = (1-\eta) \cdot p_q(1-\tau)Q$. Normalizing the wage rate $w = 1$, we can combine these conditions to write

$$p_\omega(t)/\ell(t) = \frac{\eta}{1-\eta} \cdot \frac{L_Q(t)}{L(t)}. \tag{B.26}$$

From Proposition 1, rents per adult p_ω/ℓ are independent of ℓ in the Cobb-Douglas case. Therefore, the employment share of the primary sector, L_Q/L, is independent of ℓ and, given the constant tax rate τ, we can define the convenient constant

$$\left(\frac{L_Q}{L}\right)^{1-\eta} = \left(\frac{1-\eta}{\eta} \cdot \frac{p_\omega}{\ell}\right)^{1-\eta} \equiv \bar{\vartheta}. \tag{B.27}$$

Note that $\sigma = 1$ implies $\Upsilon(p_\omega) \to \eta$ so that, from (39)-(40), we have

$$\frac{p_\omega}{\ell} = \frac{(1+\beta\rho)(1-\tau)\eta\gamma^{\frac{\epsilon-1}{\epsilon}}}{1-\tau\gamma(1-\eta)\frac{\epsilon-1}{\epsilon} - \eta\gamma^{\frac{\epsilon-1}{\epsilon}}} \tag{B.28}$$

so that

$$\bar{\vartheta} = \left(\frac{1-\eta}{\eta} \cdot \frac{(1+\beta\rho)(1-\tau)\eta\gamma^{\frac{\epsilon-1}{\epsilon}}}{1-\tau\gamma(1-\eta)\frac{\epsilon-1}{\epsilon} - \eta\gamma^{\frac{\epsilon-1}{\epsilon}}}\right)^{1-\eta}. \tag{B.29}$$

From the primary sector's technology $Q = \Omega^\eta L_Q^{1-\eta}$, we can use (B.27) to write $q = Q/L = (\Omega/L)^\eta (L_Q/L)^{1-\eta}$ as

$$q(t) = \bar{\vartheta}\ell(t)^{-\eta}. \tag{B.30}$$

Substituting this result into (**??**), the equilibrium mortality rate (B.19) can be written as

$$m(t) = \bar{m} + \mu\bar{\vartheta}^{\chi\upsilon}\Omega^{\chi\upsilon-\chi[\zeta+\xi(1-\upsilon)]} \cdot \ell(t)^{\chi\upsilon(1-\eta)-\chi[\zeta+\xi(1-\upsilon)]}, \tag{B.31}$$

which, after defining the convenient constant $\tilde{\mu} \equiv \mu \bar{\vartheta}^{\chi \upsilon} \Omega^{\chi \upsilon - \chi[\zeta + \xi(1-\upsilon)]} > 0$, reduces to equation (41). ■

Proof of Lemma 3. Under substitutability, result $\lim_{\ell \to 0^+} \Upsilon(\ell) = 1$ in (43) implies that the equilibrium mortality rate (42) exhibits

$$\ell(t)^{-\chi[\zeta+\xi(1-\upsilon)]} \cdot \Upsilon(\ell(t))^{\frac{\sigma}{1-\sigma}\chi\upsilon}$$

$$\sigma > 1 \;\; \rightarrow \lim_{\ell \to 0^+} m^*(\ell) = \bar{m} + \bar{\mu} \cdot \lim_{\ell \to 0^+} \Upsilon(\ell)^{\frac{\sigma}{1-\sigma}\chi\upsilon} \cdot \ell^{-\chi[\zeta+\xi(1-\upsilon)]} = \bar{m} + \bar{\mu} \cdot \lim_{\ell \to 0^+} \frac{1}{\ell^{\chi[\zeta+\xi(1-\upsilon)]}}, \quad \text{(B.32)}$$

so that, for any $\zeta + \xi(1-\upsilon) > 0$, substitutability implies $\lim_{\ell \to 0^+} m(\ell) = +\infty$. ■

Derivation of result (44) and extension to the case of complementarity. For future reference, rewrite equation (11) as

$$\varepsilon_{Q,L} \equiv \left(\frac{\partial \mathcal{F}}{\partial L_Q} \cdot \frac{L_Q}{\mathcal{F}} \right) \cdot \left(\frac{\mathrm{d}L_Q}{\mathrm{d}L} \cdot \frac{L}{L_Q} \right) = \varepsilon_{Q,L_Q} \cdot \varepsilon_{L_Q,L} \quad \text{(B.33)}$$

where both the sub-elasticities ε_{Q,L_Q} and $\varepsilon_{L_Q,L}$ are evaluated in equilibrium. Note that, from (A.26), the ratio between primary inputs can be written as

$$\left(\frac{\Omega}{L_Q} \right)^{\frac{\sigma-1}{\sigma}} = \frac{1-\eta}{\eta} \cdot \frac{\eta^\sigma p_\omega^{1-\sigma}}{(1-\eta)^\sigma w^{1-\sigma}},$$

which can be solved for L_Q with $w = 1$ as

$$L_Q = \Omega \cdot \left(\frac{1-\eta}{\eta} \right)^\sigma \cdot p_\omega^\sigma. \quad \text{(B.34)}$$

From the technology (33), the elasticity of commodity output to primary employment reads

$$\varepsilon_{Q,L_Q} \equiv \frac{\partial \mathcal{F}}{\partial L_Q} \cdot \frac{L_Q}{\mathcal{F}} = \frac{(1-\eta) L_Q^{\frac{\sigma-1}{\sigma}}}{\eta \Omega^{\frac{\sigma-1}{\sigma}} + (1-\eta) L_Q^{\frac{\sigma-1}{\sigma}}},$$

where we can substitute L_Q by (B.34) and the equilibrium price $p_\omega = p_\omega^*(\ell)$, obtaining

$$\varepsilon_{Q,L_Q} = \frac{(1-\eta)^\sigma \eta^{1-\sigma} \cdot p_\omega^{\sigma-1}}{\eta + (1-\eta)^\sigma \eta^{1-\sigma} \cdot p_\omega^{\sigma-1}} = \frac{1}{\frac{\eta}{(1-\eta)^\sigma \eta^{1-\sigma} \cdot (p_\omega^*)^{\sigma-1}} + 1}. \quad \text{(B.35)}$$

From (B.35), and recalling the asymptotic properties of $p_\omega^*(\ell)$ established in (B.14), the equilibrium elasticity ε_{Q,L_Q} exhibits

$$\begin{array}{lll} \sigma > 1 \;\; \rightarrow & \frac{\mathrm{d}\varepsilon_{Q,L_Q}}{\mathrm{d}\ell} > 0, \quad \lim_{\ell \to 0^+} \varepsilon_{Q,L_Q} = 0, \quad \lim_{\ell \to \infty} \varepsilon_{Q,L_Q} = 1, \\ \sigma < 1 \;\; \rightarrow & \frac{\mathrm{d}\varepsilon_{Q,L_Q}}{\mathrm{d}\ell} < 0, \quad \lim_{\ell \to 0^+} \varepsilon_{Q,L_Q} = 1, \quad \lim_{\ell \to \infty} \varepsilon_{Q,L_Q} = 0. \end{array} \quad \text{(B.36)}$$

Next, consider the following definitions

$$\varepsilon_{\Upsilon,p_\omega^*} \equiv \frac{\partial \Upsilon(p_\omega^*)}{\partial p_\omega^*} \cdot \frac{p_\omega^*}{\Upsilon}, \quad \varepsilon_{p_\omega^*,\ell} \equiv \frac{\partial p_\omega^*(\ell)}{\partial \ell} \cdot \frac{\ell}{p_\omega^*}, \quad \text{(B.37)}$$

where $p_\omega^*(\ell)$ is the equilibrium resource price defined in Proposition 1 and $\Upsilon(p_\omega^*)$ is the resource cost share (35) evaluated in the equilibrium with $w = 1$. Focusing on $\Upsilon(p_\omega^*)$, from (35) we can calculate

$$\varepsilon_{\Upsilon,p_\omega^*} = \frac{(1-\sigma)(1-\eta)^\sigma}{\eta^\sigma (p_\omega^*)^{1-\sigma} + (1-\eta)^\sigma}. \tag{B.38}$$

Next, log-differentiating the static equilibrium conditions (39) and (40) evaluated in equilibrium, we have

$$\varepsilon_{y^*,\ell} = \left(\varepsilon_{p_\omega^*,\ell} - 1\right) \frac{p_\omega^*/\ell}{1 + p_\omega^*/\ell} \quad \text{and} \quad \varepsilon_{p_\omega^*,\ell} - 1 = \varepsilon_{\Upsilon,p_\omega^*} \cdot \varepsilon_{p_\omega^*,\ell} + \varepsilon_{y^*,\ell}, \tag{B.39}$$

where $\varepsilon_{y^*,\ell} \equiv \frac{\partial y^*(\ell)}{\partial \ell} \cdot \frac{\ell}{y^*}$. Combining the two conditions in (B.39) to eliminate $\varepsilon_{y^*,\ell}$ and solving for $\varepsilon_{p_\omega^*,\ell}$, we obtain

$$\varepsilon_{p_\omega^*,\ell} = \frac{1}{1 - \varepsilon_{\Upsilon,p_\omega^*}(1 + p_\omega^*/\ell)}. \tag{B.40}$$

Using (B.38) to substitute $\varepsilon_{\Upsilon,p_\omega^*}$ in (B.40), we have

$$\varepsilon_{p_\omega^*,\ell} = \frac{1}{1 - \frac{(1-\sigma)(1-\eta)^\sigma(1+p_\omega^*/\ell)}{\eta^\sigma(p_\omega^*)^{1-\sigma}+(1-\eta)^\sigma}} > 0. \tag{B.41}$$

We now have all the elements to characterize the response of primary employment to variations in total labor supply. Time-differentiation of (B.34) yields

$$\frac{\dot{L}_Q(t)}{L_Q(t)} = \sigma \cdot \frac{\dot{p}_\omega(t)}{p_\omega(t)} = \sigma \cdot \varepsilon_{p_\omega^*,\ell} \cdot \frac{\dot{\ell}(t)}{\ell(t)}, \tag{B.42}$$

where the last term follows from substituting the equilibrium price $p_\omega = p_\omega^*(\ell)$. Since $\ell = L/\Omega$, we can rewrite (B.42) as

$$\frac{\dot{L}_Q(t)}{L_Q(t)} = \underbrace{\sigma\varepsilon_{p_\omega^*,\ell}}_{\varepsilon_{L_Q,L}} \cdot \frac{\dot{L}(t)}{L(t)} = \varepsilon_{L_Q,L} \cdot \frac{\dot{L}(t)}{L(t)} \tag{B.43}$$

where $\varepsilon_{L_Q,L}$ is the elasticity of primary employment to total labor supply in equilibrium. Using result (B.41) to substitute $\varepsilon_{p_\omega^*,\ell}$, we obtain

$$\varepsilon_{L_Q,L} = \frac{\sigma}{1 - \frac{(1-\sigma)(1-\eta)^\sigma(1+p_\omega^*/\ell)}{\eta^\sigma(p_\omega^*)^{1-\sigma}+(1-\eta)^\sigma}} > 0. \tag{B.44}$$

The asymptotic behavior of $\varepsilon_{L_Q,L}$ when ℓ ranges from 0 to $+\infty$ is determined by the asymptotic behavior of the resource price p_ω^* and rents per adult p_ω^*/ℓ. The behavior of p_ω^* is already described in expression (B.14). The behavior of p_ω^*/ℓ can be tracked as follows. Evaluating condition (40) in equilibrium, rents per adult equal

$$p_\omega^*(\ell)/\ell = (1-\tau)\gamma\frac{\epsilon - 1}{\epsilon} \cdot \Upsilon(\ell) \cdot y^*(\ell). \tag{B.45}$$

The asymptotic properties of equilibrium expenditure $y^*(\ell)$ are already described in (B.15)-(B.16), and the asymptotic properties of the equilibrium resource cost share $\Upsilon(\ell)$ are already described in (B.25). Therefore, (B.45) implies

$$\begin{array}{llll} \sigma > 1 \ \rightarrow & \frac{\mathrm{d}p^*_\omega/\ell}{\mathrm{d}\ell} < 0, & \lim_{\ell\to 0^+} p^*_\omega/\ell = (1-\tau)\,\gamma\frac{\epsilon-1}{\epsilon}\cdot y_{\max}, & \lim_{\ell\to\infty} p^*_\omega/\ell = 0. \\ \sigma < 1 \ \rightarrow & \frac{\mathrm{d}p^*_\omega/\ell}{\mathrm{d}\ell} > 0, & \lim_{\ell\to 0^+} p^*_\omega/\ell = 0, & \lim_{\ell\to\infty} p^*_\omega/\ell = (1-\tau)\,\gamma\frac{\epsilon-1}{\epsilon}\cdot y_{\max}. \end{array} \tag{B.46}$$

Using (B.46), we can establish the following results about the elasticity $\varepsilon_{L_Q,L}$ derived in (B.44).

$$\begin{array}{lll} \sigma > 1 \ \rightarrow & \lim_{\ell\to 0^+} \varepsilon_{L_Q,L} = \sigma, & \lim_{\ell\to\infty} \varepsilon_{L_Q,L} = 1, \\ \sigma < 1 \ \rightarrow & \lim_{\ell\to 0^+} \varepsilon_{L_Q,L} = 1 & \lim_{\ell\to\infty} \varepsilon_{L_Q,L} = \sigma, \end{array} \tag{B.47}$$

Going back to definition of ε_E in (B.33), we can thus substitute results (B.35) and (B.44) to obtain

$$\varepsilon_{Q,L} = \varepsilon_{Q,L_Q}\cdot\varepsilon_{L_Q,L} = \frac{(1-\eta)^\sigma\,\eta^{1-\sigma}\cdot p_\omega^{\sigma-1}}{\eta+(1-\eta)^\sigma\,\eta^{1-\sigma}\cdot p_\omega^{\sigma-1}}\cdot\frac{\sigma}{1-\frac{(1-\sigma)(1-\eta)^\sigma(1+p^*_\omega/\ell)}{\eta^\sigma(p^*_\omega)^{1-\sigma}+(1-\eta)^\sigma}}, \tag{B.48}$$

which, from (B.36) and (B.47), exhibits the properties

$$\begin{array}{lll} \sigma > 1 \ \rightarrow & \lim_{\ell\to 0^+} \varepsilon_{Q,L} = 0, & \lim_{\ell\to\infty} \varepsilon_{Q,L} = 1, \\ \sigma < 1 \ \rightarrow & \lim_{\ell\to 0^+} \varepsilon_{Q,L} = 1, & \lim_{\ell\to\infty} \varepsilon_{Q,L} = 0. \end{array} \tag{B.49}$$

Expression (B.49) proves result (44) in the main text for the case of substitutability.

Behavior of the equilibrium mortality rate (comprehensive proof of the diagrams in Figure 1). From (15), we can define the elasticity of the excess mortality rate to population size as

$$\varepsilon_{m,L} \equiv \frac{dm_p}{dL}\cdot\frac{L}{m_p} = \frac{\mathrm{d}\,(m-\bar{m})}{\mathrm{d}L}\cdot\frac{L}{(m-\bar{m})} = \chi\upsilon\varepsilon_{Q,L} - \chi\left[\zeta+\xi\,(1-\upsilon)\right], \tag{B.50}$$

Combining results (B.49) with expression (B.50), we have

$$\begin{array}{lll} \sigma > 1 \ \rightarrow & \lim_{\ell\to 0^+} \varepsilon_{m,L} = -\chi\left[\zeta+\xi\,(1-\upsilon)\right], & \lim_{\ell\to\infty} \varepsilon_{m,L} = \chi\left\{\upsilon-\left[\zeta+\xi\,(1-\upsilon)\right]\right\}, \\ \sigma < 1 \ \rightarrow & \lim_{\ell\to 0^+} \varepsilon_{m,L} = \chi\left\{\upsilon-\left[\zeta+\xi\,(1-\upsilon)\right]\right\}, & \lim_{\ell\to\infty} \varepsilon_{m,L} = -\chi\left[\zeta+\xi\,(1-\upsilon)\right]. \end{array} \tag{B.51}$$

Result (B.51) provide a comprehensive proof of the behavior of the equilibrium mortality rate in all the subcases depicted in Figure 1 for $\sigma \neq 1$ (the Cobb-Douglas case is already discussed in the main text). First, consider all the sub-cases with $\sigma > 1$. Under substitutability, the limit for $\ell \to 0^+$ is strictly negative for any $\chi\left[\zeta+\xi\,(1-\upsilon)\right] > 0$, so that $m^*(\ell)$ is surely decreasing in ℓ for low values of ℓ. The limit for $\ell\to\infty$ shows that $m^*(\ell)$ remains declining for $\upsilon \leqslant \zeta+\xi\,(1-\upsilon)$ – that is, a monotonically declining 'L-shaped' function – whereas it bends upward for $\upsilon > \zeta+\xi\,(1-\upsilon)$ – that is, a non-monotonic 'U-shaped' function. In the special case $\zeta+\xi\,(1-\upsilon) = 0$, we have a monotonically increasing function satisfying $\lim_{\ell\to 0^+}\varepsilon_{m,L} = 0$ and $\lim_{\ell\to\infty}\varepsilon_{m,L} = \chi\upsilon > 0$. Second, consider all the sub-cases with $\sigma < 1$. Under complementarity, the limit for $\ell \to \infty$ is strictly negative for any $\chi [\zeta + \xi (1 - \upsilon)] > 0$, so that $m^* (\ell)$ is surely decreasing in ℓ for high values of ℓ. The limit for $\ell \to 0^+$ shows that $m^* (\ell)$ is declining for $\upsilon \leqslant \zeta + \xi (1 - \upsilon)$ – that is, a monotonically declining 'L-shaped' function – whereas it is initially increasing for $\upsilon > \zeta + \xi (1 - \upsilon)$ – that is, a non-monotonic 'hump-shaped' function. In the special case $\zeta + \xi (1 - \upsilon) = 0$, we have an increasing concave function since $\lim_{\ell \to 0^+} \varepsilon_{m,L} = \chi > 0$ and $\lim_{\ell \to \infty} \varepsilon_{m,L} = 0$.

C Appendix: Population Dynamics

C.1 Special Case with Exogenous Mortality

Dynamics with exogenous mortality (including strict complementarity). Setting $m(t) = \bar{m}$ in each $t \in [0, \infty)$, the dynamic system (46)-(47) becomes

$$\frac{\dot{\ell}(t)}{\ell(t)} = b(t) - \bar{m} \tag{C.1}$$

$$\frac{\dot{b}(t)}{b(t)} = \frac{b(t)}{(1-\alpha)(1-\psi)} \left[\frac{1 - (1-\psi) y^* (\ell(t))}{y^* (\ell(t))} \right] - \rho \tag{C.2}$$

and the stationary loci read

$$\dot{\ell} = 0 \to \Lambda^{\ell} \equiv b = \bar{m} \tag{C.3}$$

$$\dot{b} = 0 \to \Lambda^{b} (\ell) \equiv b = \frac{\rho (1-\alpha)(1-\psi) y^* (\ell)}{1 - (1-\psi) y^* (\ell)}. \tag{C.4}$$

From the definition of $\Lambda^b (\ell)$ in (C.4) we can rewrite (C.2) as $\dot{b} = \rho \frac{b^2}{\Lambda^b(\ell)} - \rho b$. Therefore, system (C.1)-(C.2) exhibits the coefficient matrix

$$\Xi \equiv \begin{pmatrix} \frac{\partial \dot{\ell}}{\partial \ell} = b - \bar{m} & \frac{\partial \dot{\ell}}{\partial b} = \ell \\ & \\ \frac{\partial \dot{b}}{\partial \ell} = -\frac{\rho b^2}{\Lambda^b(\ell)^2} \frac{\partial \Lambda^b(\ell)}{\partial \ell} & \frac{\partial \dot{b}}{\partial b} = 2\rho \frac{b}{\Lambda^b(\ell)} - \rho \end{pmatrix} \tag{C.5}$$

which can be evaluated in any generic simultaneous steady state $\dot{\ell} = \dot{b} = 0$ as

$$\Xi_{ss} \equiv \begin{pmatrix} \frac{\partial \dot{\ell}}{\partial \ell} = 0 & \frac{\partial \dot{\ell}}{\partial b} = \ell_{ss} \\ & \\ \frac{\partial \dot{b}}{\partial \ell} = -\rho \frac{\partial \Lambda^b(\ell)}{\partial \ell} & \frac{\partial \dot{b}}{\partial b} = \rho \end{pmatrix} \tag{C.6}$$

The determinant of (C.6) is given by

$$|\Xi_{ss}| = \rho \ell_{ss} \frac{\partial \Lambda^b (\ell)}{\partial \ell} \tag{C.7}$$

and the eigenvalues $(\varkappa_1, \varkappa_2)$ of (C.6) are determined by the second-order equation

$$\varkappa^2 - \rho\varkappa + \rho\ell_{ss}\frac{\partial\Lambda^b(\ell)}{\partial\ell} = 0 \tag{C.8}$$

The three possible cases (Cobb-Douglas, substitutability, complementarity) are discussed below.

Cobb-Douglas case. From (B.17), setting $\sigma = 1$ implies a constant expenditure level

$$y(t) = \frac{1+\beta\rho}{1-\gamma^{\frac{\epsilon-1}{\epsilon}}[\tau + (1-\tau)\eta]} \equiv \bar{y}. \tag{C.9}$$

From (C.9), the stationary locus $\dot{b} = 0$ in (C.4) becomes

$$\dot{b} = 0 \rightarrow \Lambda^b(\ell) \equiv b = \frac{\rho(1-\alpha)(1-\psi)\bar{y}}{1-(1-\psi)\bar{y}} \tag{C.10}$$

which is independent of the population-resource ratio ℓ. Therefore, as shown in phase diagram (a) of Figure 2, the two loci are horizontal straight lines. In order to satisfy all the utility-maximizing conditions, the fertility rate must jump onto the $\dot{b} = 0$ at time zero, which implies a constant gross fertility rate forever. When the parameters satisfy

$$\frac{\rho(1-\alpha)(1-\psi)\bar{y}}{1-(1-\psi)\bar{y}} > \bar{m} \tag{C.11}$$

the gross fertility rate exceeds the mortality rate, $\Lambda^b(\ell) \equiv b > \bar{m}$, in which case the economy displays positive population growth. This is the case depicted in Figure 2, graph (a). In the long run, the economy converges asymptotically to zero resources per capita and an infinite population.

Substitutability. By Proposition 1, setting $\sigma > 1$ implies that $y^*(\ell)$ is strictly decreasing in ℓ. Therefore, the stationary locus $\dot{b} = 0$ in (C.4) is also decreasing in ℓ. In particular, combining (C.4) with results (B.16), we have

$$\frac{\partial\Lambda^b(\ell)}{\partial\ell} < 0, \quad \lim_{\ell\to 0}\Lambda^b(\ell) = \frac{\rho(1-\alpha)(1-\psi)y_{\max}}{1-(1-\psi)y_{\max}}, \quad \lim_{\ell\to\infty}\Lambda^b(\ell) = \frac{\rho(1-\alpha)(1-\psi)y_{\min}}{1-(1-\psi)y_{\min}}, \tag{C.12}$$

where $y_{\min} \equiv (1+\beta\rho)/\left(1-\tau\gamma^{\frac{\epsilon-1}{\epsilon}}\right)$ and $y_{\max} \equiv (1+\beta\rho)/\left(1-\gamma^{\frac{\epsilon-1}{\epsilon}}\right)$ from (B.13). Since the $\dot{\ell} = 0$ locus is a horizontal straight line, $\Lambda^\ell \equiv b = \bar{m}$, result (C.12) allows us to define suitable parameter restrictions such that there exists a simultaneous steady state (b_{ss}, ℓ_{ss}) in which $b = \bar{m}$ and $\ell_{ss} > 0$. The fact that such steady state (b_{ss}, ℓ_{ss}) is saddle-point stable is proved as follows. From $\partial\Lambda^b(\ell)/\partial\ell < 0$ in (C.12), we have $\rho^2 - 4\rho\ell_{ss}\frac{\partial\Lambda^b(\ell)}{\partial\ell} > 0$ and this implies that both the eigenvalues $(\varkappa_1, \varkappa_2)$ solving (C.8) are real. Moreover, the fact that $\sqrt{\rho^2 - 4\rho\ell_{ss}\frac{\partial\Lambda^b(\ell)}{\partial\ell}} > \rho$ guarantees that $(\varkappa_1, \varkappa_2)$ have opposite sign. The direction of the arrows shown in phase diagram (e) of Figure 2 is determined by the signs of the coefficients in matrix (C.6). Therefore, under substitutability, the steady state (b_{ss}, ℓ_{ss}) is a global attractor of the dynamics. If the economy has initial endowments such that $\ell(0) > \ell_{ss}$, the economy jumps on the branch of the stable arm featuring negative population growth. Instead, if the economy has initial endowments such that $\ell(0) < \ell_{ss}$, the economy jumps on the opposite branch of the stable arm featuring positive population growth. In either case, the economy approaches asymptotically a finite endogenous level of population $L_{ss} = \ell_{ss}\Omega$ and constant resources per capita in the long run.

Complementarity, $\sigma < 1$. By Proposition 1, setting $\sigma < 1$ implies that $y^*(\ell)$ is strictly increasing in ℓ. Therefore, the stationary locus $\dot{b} = 0$ in (C.4) is also increasing in ℓ. In particular, combining (C.4) with results (B.15), we have

$$\frac{\partial \Lambda^b(\ell)}{\partial \ell} > 0, \quad \lim_{\ell \to 0} \Lambda^b(\ell) = \frac{\rho(1-\alpha)(1-\psi)\,y_{\min}}{1-(1-\psi)\,y_{\min}}, \quad \lim_{\ell \to \infty} \Lambda^b(\ell) = \frac{\rho(1-\alpha)(1-\psi)\,y_{\max}}{1-(1-\psi)\,y_{\max}}, \tag{C.13}$$

where $y_{\min} \equiv (1+\beta\rho)/\left(1-\tau\gamma\frac{\epsilon-1}{\epsilon}\right)$ and $y_{\max} \equiv (1+\beta\rho)/\left(1-\gamma\frac{\epsilon-1}{\epsilon}\right)$ from (B.13). Since the $\dot{\ell} = 0$ locus is a horizontal straight line, $\Lambda^{\ell} \equiv b = \bar{m}$, result (C.13) allows us to define suitable parameter restrictions such that there exists a unique simultaneous steady state (b_{ss}, ℓ_{ss}) in which $b = \bar{m}$ and $\ell_{ss} > 0$. The fact that such steady state (b_{ss}, ℓ_{ss}) is globally unstable is proved as follows. From $\partial \Lambda^b(\ell)/\partial \ell > 0$ in (C.13), the determinant (C.7) is strictly positive, and $|\Xi_{ss}| > 0$ implies that the eigenvalues $(\varkappa_1, \varkappa_2)$ are both real and have the same sign. From (C.8), the polynomial exhibits the signs $(+,-,+)$, which implies by Descartes rule that no root can be negative. Hence, both $(\varkappa_1, \varkappa_2)$ must be strictly positive. Therefore the simultaneous steady state (b_{ss}, ℓ_{ss}) under complementarity is globally unstable. Since this case is not discussed in the main text, we report the associated phase diagram in Figure A2, graph (a). The direction of the arrows is determined by the signs of the coefficients in matrix (C.6). If the economy has initial endowments such that $\ell(0) > \ell_{ss}$, the economy jumps on the diverging path featuring positive population growth and increasing population-resource ratio, and approaches asymptotically an infinite population and zero resources per capita. Instead, if the economy has initial endowments such that $\ell(0) < \ell_{ss}$, the economy jumps on the diverging path featuring declining population, and ultimately implosion in the long run. The cause of the instability is that expenditure per capita increases with population because the rising resource scarcity yields higher resource income per capita. When labor is initially relatively scarce, the rising resource *abundance* drives down resource income per capita inducing further reductions in fertility and, eventually, population implosion. When labor is initially abundant relative to the resource base, a positive initial net fertility rate triggers a self-reinforcing circle of rising incomes and rising population via sustained fertility rates.

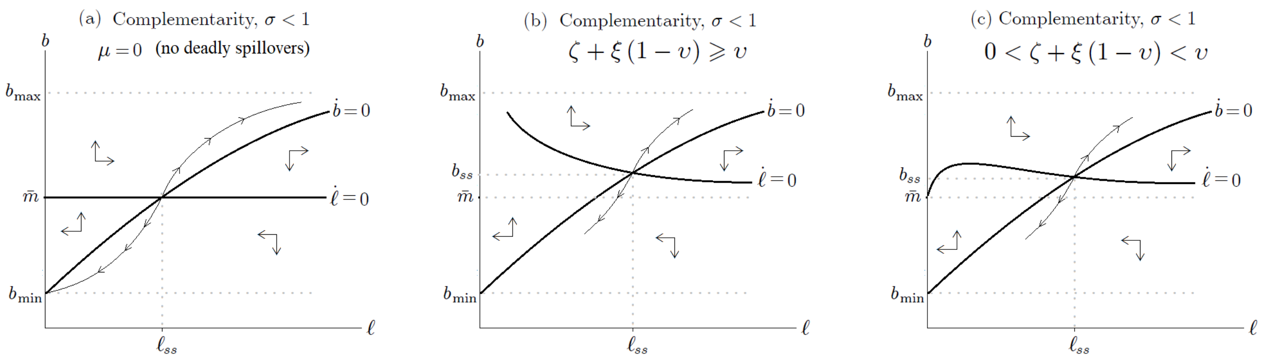

Figure A2. Demographic dynamics with and without deadly spillovers under complementarity.

C.2 Dynamics with Endogenous Mortality

Dynamics with endogenous mortality (including strict complementarity). The dynamic system (46)-(47) with endogenous mortality reads

$$\begin{aligned} \frac{\dot{\ell}(t)}{\ell(t)} &= b(t) - m^*(\ell(t)) \\ \frac{\dot{b}(t)}{b(t)} &= \frac{b(t)}{(1-\alpha)(1-\psi)}\left[\frac{1-(1-\psi)\,y^*(\ell(t))}{y^*(\ell(t))}\right] - \rho \end{aligned}$$

and the stationary loci read

$$\dot{\ell} = 0 \rightarrow \Lambda^{\ell}(\ell) \equiv b = m^*(\ell) \tag{C.14}$$

$$\dot{b} = 0 \rightarrow \Lambda^{b}(\ell) \equiv b = \frac{\rho(1-\alpha)(1-\psi)\,y^*(\ell)}{1-(1-\psi)\,y^*(\ell)}. \tag{C.15}$$

For future reference, note that the elasticity of the stationary locus (C.4) with respect to ℓ is

$$\frac{\partial \Lambda^b(\ell)}{\partial \ell}\frac{\ell}{\Lambda^b(\ell)} = \frac{1}{1-(1-\psi)\,y^*(\ell)} \cdot \frac{\partial y^*(\ell)}{\partial \ell}\frac{\ell}{y^*(\ell)} \tag{C.16}$$

From the definition of $\Lambda^b(\ell)$ in (C.15) we can rewrite (47) as $\dot{b} = \rho\frac{b^2}{\Lambda^b(\ell)} - \rho b$. Therefore, system (46)-(47) exhibits the coefficient matrix

$$\Xi \equiv \begin{pmatrix} \frac{\partial \dot{\ell}}{\partial \ell} = b - m^*(\ell) - \ell \cdot \frac{\partial m^*(\ell)}{\partial \ell} & \frac{\partial \dot{\ell}}{\partial b} = \ell \\ & \\ \frac{\partial \dot{b}}{\partial \ell} = -\frac{\rho b^2}{\Lambda^b(\ell)^2}\frac{\partial \Lambda^b(\ell)}{\partial \ell} & \frac{\partial \dot{b}}{\partial b} = 2\rho\frac{b}{\Lambda^b(\ell)} - \rho \end{pmatrix} \tag{C.17}$$

which can be evaluated in any generic simultaneous steady state $\dot{\ell} = \dot{b} = 0$ as

$$\Xi_{ss} \equiv \begin{pmatrix} \frac{\partial \dot{\ell}}{\partial \ell} = -\ell_{ss} \cdot \frac{\partial m^*(\ell)}{\partial \ell} & \frac{\partial \dot{\ell}}{\partial b} = \ell_{ss} \\ & \\ \frac{\partial \dot{b}}{\partial \ell} = -\rho\frac{\partial \Lambda^b(\ell)}{\partial \ell} & \frac{\partial \dot{b}}{\partial b} = \rho \end{pmatrix} \tag{C.18}$$

The determinant of (C.6) is given by

$$|\Xi_{ss}| = \rho \ell_{ss} \left[\frac{\partial \Lambda^b(\ell)}{\partial \ell} - \frac{\partial m^*(\ell)}{\partial \ell} \right] \tag{C.19}$$

and the eigenvalues $(\varkappa_1, \varkappa_2)$ of (C.6) are determined by the second-order equation

$$\varkappa^2 - \varkappa \left(\rho - \ell_{ss} \cdot \frac{\partial m^*(\ell)}{\partial \ell} \right) + \rho \ell_{ss} \left[\frac{\partial \Lambda^b(\ell)}{\partial \ell} - \frac{\partial m^*(\ell)}{\partial \ell} \right] = 0 \tag{C.20}$$

Note that expression (C.14) implies that in the (ℓ, b) plane, the $\dot{\ell} = 0$ locus exhibits the same shape as that of the equilbrium mortality rate $m^*(\ell)$ characterized in Proposition 2. Expression (C.15) is the same as that for the case of exogenous mortality (see (C.4) above) so that it satisfies all the properties previously derived. The cases with strict substitutability $\sigma > 1$ and Cobb-Douglas $\sigma = 1$ are discussed in the proofs of Propositions 6 and 5 below. The cases with strict substitutability $\sigma < 1$ are discussed further below.

Substitutability: dynamics with $\sigma > 1$ and proof of Proposition 6. Throughout this proof we assume $\zeta + \xi(1 - \upsilon) > 0$. By Proposition 1, setting $\sigma > 1$ implies that $y^*(\ell)$ is strictly decreasing in ℓ. Therefore, the stationary locus $\dot{b} = 0$ in (C.15) is also decreasing in ℓ. In particular, combining (C.15) with results (B.16), we have

$$\frac{\partial \Lambda^b(\ell)}{\partial \ell} < 0, \quad \lim_{\ell \to 0} \Lambda^b(\ell) = \frac{\rho(1-\alpha)(1-\psi)\, y_{\max}}{1-(1-\psi)\, y_{\max}}, \quad \lim_{\ell \to \infty} \Lambda^b(\ell) = \frac{\rho(1-\alpha)(1-\psi)\, y_{\min}}{1-(1-\psi)\, y_{\min}}, \tag{C.21}$$

where $y_{\min} \equiv (1+\beta\rho) / \left(1 - \tau\gamma^{\frac{\epsilon-1}{\epsilon}}\right)$ and $y_{\max} \equiv (1+\beta\rho) / \left(1 - \gamma^{\frac{\epsilon-1}{\epsilon}}\right)$ from (B.13). Consider now the stationary locus $\dot{\ell} = 0$ in (C.14). Recalling Proposition 2 and results (45) and (44), substitutability implies

$$\lim_{\ell \to 0^+} \Lambda^\ell(\ell) = \bar{m} + \frac{\bar{\mu}}{\ell \chi[\zeta + \xi(1-\upsilon)]} = +\infty \quad \text{with} \lim_{\ell \to 0^+} \varepsilon_{m,L} = -\chi[\zeta + \xi(1-\upsilon)] \text{ and } \lim_{\ell \to \infty} \tilde{\varepsilon} = \varepsilon_{m,L} = \chi\{\upsilon - [\zeta + \xi(1 - \tag{C.22}$$

and the exact shape of the $\dot{\ell} = 0$ locus for high values of ℓ depends on the sign of $\upsilon - [\zeta + \xi(1-\upsilon)]$. For $\upsilon \leqslant \zeta + \xi(1-\upsilon)$, the $\dot{\ell} = 0$ locus is monotonously decreasing and asymptotically horizontal,

$$\upsilon \leqslant \zeta + \xi(1-\upsilon) \rightarrow \frac{\mathrm{d}\Lambda^\ell(\ell)}{\mathrm{d}\ell} < 0 \text{ with } \lim_{\ell \to +\infty} \Lambda^\ell(\ell) = \bar{m} \tag{C.23}$$

whereas for $\upsilon > \zeta + \xi(1-\upsilon)$, the $\dot{\ell} = 0$ locus is *U-shaped*,

$$\upsilon > \zeta + \xi(1-\upsilon) \rightarrow \exists\, \hat{\ell} > 0 : \frac{\mathrm{d}\Lambda^\ell(\ell)}{\mathrm{d}\ell} \begin{cases} < 0 & \text{for} \quad 0 < \ell < \hat{\ell} \\ > 0 & \text{for} \quad \hat{\ell} < \ell < \infty \end{cases} < 0 \text{ and } \lim_{\ell \to +\infty} \Lambda^\ell(\ell) = \infty \tag{C.24}$$

Recalling the properties of the $\dot{b} = 0$ locus derived in (C.21), it follows that the existence of simultaneous steady states satisfying $\dot{b} = \dot{\ell} = 0$ falls into the following cases and subcases:

$\upsilon \leqslant \zeta + \xi(1-\upsilon)$ In this case, the combination of (C.21) and (C.23) implies that, provided the general existence condition $b_{\max} < \bar{m} < b_{\min}$ is satisfied, there certainly exist *two* simultaneous steady states $\dot{b} = \dot{\ell} = 0$ respectively charcterized by the labor-resource ratios ℓ'_{ss} and ℓ''_{ss} with $\ell'_{ss} > \ell''_{ss}$, as shown in Figure 2, phase diagram (f).

$\upsilon > \zeta + \xi(1-\upsilon)$ In this case, the combination of (C.21) and (C.24) implies that, provided the general existence condition $b_{\max} < \bar{m} < b_{\min}$ is satisfied, we can either have no steady state (that is, the $\dot{\ell} = 0$ locus is always strictly above the $\dot{b} = 0$ locus) or *two* simultaneous steady states charcterized by the labor-resource ratios ℓ'_{ss} and ℓ''_{ss} with $\ell'_{ss} > \ell''_{ss}$, as shown in Figure 2, phase diagram (g).[27] The case with no steady state arises when the spillover is extremely strong: in graphical terms, the $\dot{\ell} = 0$ locus shifts upwards so much that no intersection with the $\dot{b} = 0$ locus exists. But when spillovers are not that strong, the intersections between the two loci are two, as shown in Figure 2, diagram (g).

Assuming that two steady states (b'_{ss}, ℓ'_{ss}) and (b''_{ss}, ℓ''_{ss}) exist, their stability properties can be derived as follows. First, consider the steady state (b''_{ss}, ℓ''_{ss}) characterized by low labor-resource ratio. As shown in Figure 2, this is an intersection in which the $\dot{\ell} = 0$ locus cuts the $\dot{b} = 0$ locus from above while both loci are strictly declinling, that is,

$$\left.\left(\frac{\partial \Lambda^b(\ell)}{\partial \ell} - \frac{\partial m^*(\ell)}{\partial \ell}\right)\right|_{\ell=\ell''_{ss}} > 0. \tag{C.25}$$

Result (C.25) implies that the determinant (C.19) evaluated in the steady state (b''_{ss}, ℓ''_{ss}) is strictly positive, $|\Xi''_{ss}| > 0$, and this implies that the eigenvalues $(\varkappa_1, \varkappa_2)$ are both real and have the same sign. Since $\left.\frac{\partial m^*(\ell)}{\partial \ell}\right|_{\ell=\ell''_{ss}} < 0$, the polynomial in (C.20) exhibits the signs $(+,-,+)$, which implies by Descartes rule that no root can be negative. Hence, both $(\varkappa_1, \varkappa_2)$ must be strictly positive. Therefore the steady state (b''_{ss}, ℓ''_{ss}) is an *unstable node* and acts as a "mortality trap": if the initial labor-resource ratio $\ell(0)$ is strictly below ℓ''_{ss}, the equilibrium path diverges to $\lim_{t\to\infty} \ell(t) = 0$ and thereby population implosion, $\lim_{t\to\infty} L(t) = 0$.

Next, consider the steady state (b'_{ss}, ℓ'_{ss}) characterized by low-fertility and high labor-resource ratio. As shown in Figure 2, this is an intersection in which the $\dot{\ell} = 0$ locus cuts the $\dot{b} = 0$ locus from below, that is,

$$\left.\left(\frac{\partial \Lambda^b(\ell)}{\partial \ell} - \frac{\partial m^*(\ell)}{\partial \ell}\right)\right|_{\ell=\ell'_{ss}} < 0. \tag{C.26}$$

Result (C.26) implies real roots because the solution to (C.20) includes the positive term

$$\left(\rho - \ell_{ss} \cdot \frac{\partial m^*(\ell)}{\partial \ell}\right)^2 - 4\rho \ell_{ss}\left[\frac{\partial \Lambda^b(\ell)}{\partial \ell} - \frac{\partial m^*(\ell)}{\partial \ell}\right] > 0.$$

[27] The only potential exception would be the rather implausible case in which the $\dot{\ell} = 0$ locus is tangent to the $\dot{b} = 0$ locus from above, which we do not discuss for simplicity.

Also, result (C.26) implies that the determinant (C.19) evaluated in the steady state (b'_{ss}, ℓ'_{ss}) is strictly negative, $|\Xi'_{ss}| > 0$, so that the real roots have opposite sign. Therefore the steady state (b'_{ss}, ℓ'_{ss}) is *saddle-point stable* and acts as an attractor: if the initial labor-resource ratio $\ell(0)$ is strictly above ℓ''_{ss}, the equilibrium path converges to $\lim_{t\to\infty} \ell(t) = \ell'_{ss}$ along the stable arm of the saddle.

Cobb-Douglas: dynamics with $\sigma = 1$ **and proof of Proposition 6**. The properties of the stationary locus (C.14) under $\sigma = 1$ directly follow from result (41) in Proposition 2. On the one hand, From (C.10), the stationary locus $\dot{b} = 0$ is the horizontal straight line

$$\dot{b} = 0 \rightarrow \Lambda^b \equiv b = \frac{\rho (1-\alpha)(1-\psi)\bar{y}}{1-(1-\psi)\bar{y}}.$$

On the other hand, from (C.14) and (41), the stationary locus $\dot{\ell} = 0$ becomes

$$\dot{\ell} = 0 \rightarrow \Lambda^{\ell}(\ell) \equiv b = m^*(\ell) = \bar{m} + \tilde{\mu} \cdot \ell(t)^{\chi\{\upsilon(1-\eta)-[\zeta+\xi(1-\upsilon)]\}}.$$

Assuming that the general existence condition $\Lambda^b > \bar{m}$ is satisfied, we obtain the general cases described in diagrams (b)-(c)-(d) of Figure 2. If $\upsilon(1-\eta) < \zeta + \xi(1-\upsilon)$, the $\dot{\ell} = 0$ locus is decreasing: since

$$\frac{\partial \Lambda^b}{\partial \ell} - \frac{\partial m^*(\ell)}{\partial \ell} = -\frac{\partial m^*(\ell)}{\partial \ell} > 0, \tag{C.27}$$

the eigenvalues are both real and positive, the steady state is an *unstable node* and thus acts as a mortality trap generated by deadly spillovers. Instead, if $\upsilon(1-\eta) > \zeta + \xi(1-\upsilon)$, the $\dot{\ell} = 0$ locus is increasing: since

$$\frac{\partial \Lambda^b}{\partial \ell} - \frac{\partial m^*(\ell)}{\partial \ell} = -\frac{\partial m^*(\ell)}{\partial \ell} < 0 \tag{C.28}$$

the eigenvalues are both real and have opposite signs, the steady state is *saddle-point stable* and thus acts as a regular steady state generated by deadly spillovers. The knife-edge case $\upsilon(1-\eta) = \zeta + \xi(1-\upsilon)$ predicts exponential population growth or decline (including the possibility that population implodes exponentially with deadly spillovers though it would explode exponentially without spillovers).

Complementarity: dynamics with $\sigma < 1$. Under complementarity, deadly spillovers bear quantitative effects by modifying the position of the unstable steady state but do not yield qualitative effects: contrary to the case with substitutability, pollution externalities do not create additional steady states. The intuition for this result can be easily verified in Figure A2, where phase diagram (a) refers to the model without deadly spillovers (see the Appendix section on "Dynamics with exogenous mortality" above) and phase diagrams (b) and (c) refer to the model with deadly spillovers under different parametrizations. Phase diagrams (b) and (c) can be straightforwardly obtained by superimposing the equilibrium mortality rates derived in Figure 1 in the phase diagram without spillovers 2A.(a). Without deadly spillovers, there only exists one steady state, which is unstable. With deadly spillovers, the unstable steady state still exists and is pushed north-east, but there no additional steady states created by endogenous mortality.

D Appendix: Growth, Emission Taxes and Resource Booms

D.1 Consumption, growth and welfare

Equilibrium utility: derivation of (58). The price index of the consumption good is

$$p_c(t) = \left(\int_0^{N(t)} p_{xi}^{1-\epsilon}(t)\, di \right)^{\frac{1}{1-\epsilon}}.$$

In symmetric equilibrium, it reduces to

$$p_c(t) = N(t)^{-\frac{1}{\epsilon-1}} p_{xi}(t) = N(t)^{-\frac{1}{\epsilon-1}} \cdot \frac{\epsilon}{\epsilon-1} \cdot \gamma^{-\gamma} (1-\gamma)^{-1+\gamma} z(t)^{-\theta} p_q(t)^{\gamma}, \tag{D.1}$$

where $\gamma^{-\gamma}(1-\gamma)^{-1+\gamma} z^{-\theta} p_q^{\gamma}$ is the marginal cost of production of intermediate firms. The household's expenditiure allocation rule is $p_c c_L L = \alpha Y$ and $p_c c_B B = (1-\alpha) Y$. Using these expressions, we write directly

$$u = c_L^{\alpha} c_B^{1-\alpha} b^{\psi(1-\alpha)} L^{\psi} = \alpha^{\alpha} (1-\alpha)^{1-\alpha} \cdot \frac{y}{p_c} \cdot b^{\psi(1-\alpha)} L^{\psi},$$

which using (D.1) becomes

$$u = \alpha^{\alpha}(1-\alpha)^{1-\alpha} \gamma^{\gamma} (1-\gamma)^{1-\gamma} \frac{\epsilon-1}{\epsilon} \cdot y \cdot T \cdot p_q^{-\gamma} \cdot b^{(\psi-1)(1-\alpha)} L^{\psi}.$$

and therefore

$$\ln u = \bar{\alpha} + \ln y + \ln T - \gamma \ln p_q - (1-\psi)(1-\alpha) \ln b + \psi \ln L$$

where we have defined $\bar{\alpha} \equiv \ln \alpha^{\alpha} (1-\alpha)^{1-\alpha} \gamma^{\gamma} (1-\gamma)^{1-\gamma} \frac{\epsilon-1}{\epsilon}$.

Proof of Proposition 7. As a first step, we derive the equilibrium growth rate of firms knowledge. From (A.22) with $w = 1$ and $\bar{z} = z_i$ by symmetry, we have

$$\frac{\dot{\vartheta}_z}{\vartheta_z} = r + \delta - \frac{\epsilon-1}{\epsilon} \cdot \kappa \theta p_{x_i} x_i. \tag{D.2}$$

From foc (iii) in expression (A.20), symmetry implies $\vartheta_z \kappa z_i = 1$ where κ is constant, so that $\dot{\vartheta}_z / \vartheta_z = -\dot{z}_i / z_i$. Equation (D.2) thus yields

$$\frac{\dot{z}_i}{z_i} = \frac{\epsilon-1}{\epsilon} \cdot \kappa \theta p_{x_i} x_i - r - \delta. \tag{D.3}$$

Exploiting the definitions $y = Y/L$ and $\ell = L/\Omega$, we can rewrite the equilibrium output condition (30) and the Keynes-Ramsey rule (19), respectively, as

$$p_{x_i} x_i = \frac{yL}{N}, \tag{D.4}$$

$$r = \frac{\dot{y}}{y} + \frac{\dot{\ell}}{\ell} + \rho. \tag{D.5}$$

Substituting both (D.4) and (D.5) in (D.3), the equilibrium growth rate of firms knowledge reads

$$\frac{\dot{z}_i(t)}{z_i(t)} = \frac{\epsilon - 1}{\epsilon} \cdot \kappa\theta \frac{y(t) L(t)}{N(t)} - \frac{\dot{y}(t)}{y(t)} - \frac{\dot{\ell}(t)}{\ell(t)} - \rho - \delta. \tag{D.6}$$

Next consider horizontal innovations. Time-differentiating the free-entry condition (31) we have

$$\frac{\dot{V}_i}{V_i} = \frac{\dot{L}}{L} - \frac{\dot{N}}{N}. \tag{D.7}$$

From the definition of present-value profits (29), the growth rate of V_i must obey the dynamic no-arbitrage condition $\dot{V}_i/V_i = r + \delta - (\pi_i/V_i)$. Substituting this condition in (D.7) and solving the resulting expression for $\dot{N}/N$, we obtain

$$\frac{\dot{N}}{N} = \frac{\pi_i}{V_i} + \frac{\dot{L}}{L} - r - \delta = \frac{\pi_i}{V_i} - \rho - \delta - \frac{\dot{y}}{y}, \tag{D.8}$$

where the last term follows from substituing r with (D.5). From (28) and (A.21), the profit rate with $w = 1$ can be written as

$$\frac{\pi_i}{V_i} = \frac{\frac{1}{\epsilon} p_{x_i} x_i - \phi - L_{z_i}}{V_i} = \frac{\frac{1}{\epsilon} p_{x_i} x_i - \phi - L_{z_i}}{\beta \frac{L}{N}} \tag{D.9}$$

where the last term follows from the free-entry condition (31). Substituting (D.9) in (D.8), and using (D.4) to substitute $p_{x_i} x_i$, yields

$$\frac{\dot{N}(t)}{N(t)} = \frac{1}{\beta L(t)} \left[\frac{Y(t)}{\epsilon} - (\phi + L_{z_i}(t)) N(t) \right] - \rho - \delta - \frac{\dot{y}(t)}{y(t)}. \tag{D.10}$$

From the knowledge accumulation equation (27) under symmetry, we can substitute $L_{z_i} = \frac{1}{\kappa} \frac{\dot{z}_i}{z_i}$ in (D.10) to obtain the equilibrium growth rate of the mass of firms

$$\frac{\dot{N}(t)}{N(t)} = \frac{1}{\beta L(t)} \left[\frac{Y(t)}{\epsilon} - \left(\phi + \frac{1}{\kappa} \frac{\dot{z}_i(t)}{z_i(t)} \right) N(t) \right] - \rho - \delta - \frac{\dot{y}(t)}{y(t)}. \tag{D.11}$$

Equations (D.6) and (D.11) determine the joint dynamics of vertical and horizontal innovation rates. The growth rate of knowledge may be either strictly positive – i.e., the case in which vertical R&D activities are operative – or zero – i.e., the case in which parameters are such that no labor is invested in knowledge accumulation. In either case, it is already apparent from (D.11) that the mass of firms $N(t)$ follows a logistic process with time-varying coefficients. For the sake of generality, we hereby focus on the case in which vertical R&D is operative. Using (D.6) to substitute the growth rate of knowledge in (D.11), we obtain

$$\frac{\dot{N}(t)}{N(t)} = \frac{1}{\beta}\left[\frac{1-\theta(\epsilon-1)}{\epsilon}\frac{Y(t)}{L(t)} - \left(\phi - \frac{\rho+\delta+\frac{\dot{y}(t)}{y(t)}+\frac{\dot{\ell}(t)}{\ell(t)}}{\kappa}\right)\frac{N(t)}{L(t)}\right] - \rho - \delta - \frac{\dot{y}(t)}{y(t)}. \tag{D.12}$$

Clearly, if the economy converges to a regular steady state (b_{ss}, ℓ_{ss}), population $L = L_{ss}$ and expenditure per adult $y^*(\ell_{ss}) = y_{ss}$ are both constant and the growth rate of the mass of firms reduces to

$$\frac{\dot{N}(t)}{N(t)} = \frac{1}{\beta}\left[\frac{1-\theta(\epsilon-1)}{\epsilon}y_{ss} - \left(\phi - \frac{\rho+\delta}{\kappa}\right)\frac{N(t)}{L_{ss}}\right] - \rho - \delta, \tag{D.13}$$

which converges to zero with a constant mass of firms given by

$$N_{ss} \equiv \lim_{t\to\infty} N(t) = \frac{\frac{1-\theta(\epsilon-1)}{\epsilon}y_{ss} - \beta(\rho+\delta)}{\phi - \frac{\rho+\delta}{\kappa}} \cdot L_{ss}, \tag{D.14}$$

which proves expression (56) in Proposition 7. Note that, from the equilibrium condition (39) evaluated in the steady state, the product $y_{ss} \cdot L_{ss}$ equals

$$y_{ss} \cdot L_{ss} = \frac{(1+\beta\rho)L_{ss} + p_{\omega,ss}\Omega}{1 - \tau\gamma\frac{\epsilon-1}{\epsilon}}. \tag{D.15}$$

Based on result (D.14), we can calculate the long-run growth rate of firms knowledge from (D.6) as

$$\lim_{t\to\infty}\frac{\dot{z}_i(t)}{z_i(t)} = \frac{\epsilon-1}{\epsilon}\cdot\kappa\theta\frac{y_{ss}L_{ss}}{N_{ss}} - \rho - \delta = \frac{\frac{\epsilon-1}{\epsilon}\kappa\theta\left(\phi - \frac{\rho+\delta}{\kappa}\right)y_{ss}}{\frac{1-\theta(\epsilon-1)}{\epsilon}y_{ss} - \beta(\rho+\delta)} - \rho - \delta, \tag{D.16}$$

which proves expression (57) in Proposition 7.

D.2 Commodity Tax

Proof of Proposition 8. The proof comprises five steps, namely, (i) proving $\mathrm{d}y^*(\bar{\ell})/\mathrm{d}\tau > 0$, (ii) proving $\mathrm{d}p^*_\omega(\bar{\ell})/\mathrm{d}\tau < 0$, (iii) proving $\mathrm{d}m^*(\bar{\ell})/\mathrm{d}\tau > 0$, (iv) proving $\mathrm{d}\ell'_{ss}/\mathrm{d}\tau > 0$ and $\mathrm{d}\ell''_{ss}/\mathrm{d}\tau < 0$, and (v) proving that $\mathrm{d}p^*_\omega(\bar{\ell})/\mathrm{d}\tau > 0$.

Step 1: proof of $\mathrm{d}y^*(\bar{\ell})/\mathrm{d}\tau > 0$. This result can be easily proved graphically by means of Figure A1. From the static equilibrium conditions (B.10)-(B.11), recalling that $0 < \Upsilon(p_\omega) < 1$, it is easily established that changes in τ for given ℓ yield the following marginal effects

$$\frac{\partial y_1(p_\omega;\ell)}{\partial\tau} > 0, \quad \frac{\partial y_2(p_\omega)}{\partial\tau} > 0 \quad \text{and} \quad \frac{\partial y_{\min}}{\partial\tau} > 0. \tag{D.17}$$

Since both $y_1(p_\omega;\ell)$ and $y_2(p_\omega)$ shift upwards following an increase in τ, it follows that the equilibrium level of expenditure per adult is also increasing in the tax rate, $\mathrm{d}y^*(\bar{\ell})/\mathrm{d}\tau > 0$ for given $\bar{\ell}$.

Step 2: proof of $\mathrm{d}p_\omega^*\left(\bar{\ell}\right)/\mathrm{d}\tau<0$. To simplify notation, in the remainder of this proof we will denote $y^*\left(\bar{\ell}\right)$, $p_\omega^*\left(\bar{\ell}\right)$ and $\Upsilon\left(p_\omega^*\left(\bar{\ell}\right)\right)$ by y^*, p_ω^*, and $\Upsilon\left(p_\omega^*\right)$, respectively. Total differentiation of (39) and (40) with respect to τ in equilibrium gives, respectively,

$$\frac{\mathrm{d}p_\omega^*}{\mathrm{d}\tau}\cdot\frac{1}{\ell} = \frac{\mathrm{d}y^*}{\mathrm{d}\tau}\cdot\left(1-\tau\gamma\frac{\epsilon-1}{\epsilon}\right)-y^*\cdot\gamma\frac{\epsilon-1}{\epsilon}, \tag{D.18}$$

$$\frac{\mathrm{d}p_\omega^*}{\mathrm{d}\tau}\cdot\frac{1}{p_\omega^*} = -\frac{1}{1-\tau}+\frac{\mathrm{d}\Upsilon\left(p_\omega^*\right)}{\mathrm{d}\tau}\cdot\frac{1}{\Upsilon\left(p_\omega^*\right)}+\frac{\mathrm{d}y^*}{\mathrm{d}\tau}\cdot\frac{1}{y^*}. \tag{D.19}$$

Focusing on (D.19), note that by construction, $\mathrm{d}\Upsilon\left(p_\omega^*\right)/\mathrm{d}\tau=\left(\partial\Upsilon/\partial p_\omega^*\right)\cdot\left(\mathrm{d}p_\omega^*/\mathrm{d}\tau\right)$ so that, from the definition of $\varepsilon_{\Upsilon,p_\omega^*}$ in (B.38), we have

$$\frac{\mathrm{d}\Upsilon\left(p_\omega^*\right)}{\mathrm{d}\tau}\cdot\frac{1}{\Upsilon\left(p_\omega^*\right)}=\underbrace{\frac{\partial\Upsilon\left(p_\omega^*\right)}{\partial p_\omega^*}\cdot\frac{p_\omega^*}{\Upsilon\left(p_\omega^*\right)}}_{\varepsilon_{\Upsilon,p_\omega^*}}\cdot\frac{\mathrm{d}p_\omega^*}{\mathrm{d}\tau}\cdot\frac{1}{p_\omega^*}=\varepsilon_{\Upsilon,p_\omega^*}\cdot\frac{\mathrm{d}p_\omega^*}{\mathrm{d}\tau}\cdot\frac{1}{p_\omega^*}. \tag{D.20}$$

Substituting result (D.20) into (D.19), and solving for $\left(\mathrm{d}p_\omega^*/\mathrm{d}\tau\right)$, we obtain

$$\frac{\mathrm{d}p_\omega^*}{\mathrm{d}\tau}\cdot\frac{1}{p_\omega^*}\cdot\left(1-\varepsilon_{\Upsilon,p_\omega^*}\right)=\frac{\mathrm{d}y^*}{\mathrm{d}\tau}\cdot\frac{1}{y^*}-\frac{1}{1-\tau}. \tag{D.21}$$

Now consider equation (D.18): rearranging terms to solve for $\left(\mathrm{d}y^*/\mathrm{d}\tau\right)$, we have

$$\frac{\mathrm{d}y^*}{\mathrm{d}\tau}\cdot\frac{1}{y^*}\cdot\left(1-\tau\gamma\frac{\epsilon-1}{\epsilon}\right)=\frac{\mathrm{d}p_\omega^*}{\mathrm{d}\tau}\cdot\frac{1}{\ell}\cdot\frac{1}{y^*}+\gamma\frac{\epsilon-1}{\epsilon},$$

where we can substitute $\left(1-\tau\gamma\frac{\epsilon-1}{\epsilon}\right)=\frac{1+p_\omega^*/\ell}{y^*}$ from (39) as well as $\gamma\frac{\epsilon-1}{\epsilon}=\frac{p_\omega^*/\ell}{(1-\tau)\cdot\Upsilon\cdot y^*}$ from (40), to obtain

$$\frac{\mathrm{d}y^*}{\mathrm{d}\tau}\cdot\frac{1}{y^*}=\frac{p_\omega^*/\ell}{1+p_\omega^*/\ell}\cdot\left[\frac{\mathrm{d}p_\omega^*}{\mathrm{d}\tau}\cdot\frac{1}{p_\omega^*}+\frac{1}{(1-\tau)\cdot\Upsilon\left(p_\omega^*\right)}\right]. \tag{D.22}$$

Using (D.22) to substitute $\left(\mathrm{d}y^*/\mathrm{d}\tau\right)$ into the right hand side of (D.21), we have

$$\frac{\mathrm{d}p_\omega^*}{\mathrm{d}\tau}\cdot\frac{1}{p_\omega^*}\cdot\underbrace{\left(1-\varepsilon_{\Upsilon,p_\omega^*}-\frac{p_\omega^*/\ell}{1+p_\omega^*/\ell}\right)}_{\text{strictly positive}}=\frac{1}{1-\tau}\left[\frac{p_\omega^*/\ell}{1+p_\omega^*/\ell}\cdot\frac{1}{\Upsilon\left(p_\omega^*\right)}-1\right], \tag{D.23}$$

where the term in round brackets in the left hand side is strictly positive because, under strict substitutability, $\varepsilon_{\Upsilon,p_\omega^*}<0$ holds (see expression (B.38) above). Therefore, the sign of $\left(\mathrm{d}p_\omega^*/\mathrm{d}\tau\right)$ is determined by the last term in square brackets in the right hand side of (D.23). By definition (35), the cost share of resource use can be written as $\Upsilon\equiv\frac{p_\omega/\ell}{(p_\omega/\ell)+(L_Q/L)}$. Therefore, we have

$$\frac{p_\omega^*/\ell}{1+p_\omega^*/\ell}\cdot\frac{1}{\Upsilon\left(p_\omega^*\right)}=\frac{\left(L_Q/L\right)+\left(p_\omega^*/\ell\right)}{1+\left(p_\omega^*/\ell\right)}<1, \tag{D.24}$$

where the strict inequality must hold because $L_Q/L<1$ is necessary to have positive production in the intermediate sector. It follows from (D.24) that the last term in square brackets in the right hand side of (D.23) is strictly negative. Hence, the equilibrium resource price is strictly decreasing in the tax rate, $\mathrm{d}p_\omega^*/\mathrm{d}\tau < 0$ for given ℓ.

Step 3: proof of $\mathrm{d}m^*\left(\bar{\ell}\right)/\mathrm{d}\tau < 0$. To simplify notation, denote $m^*\left(\bar{\ell}\right)$ by m^*. Since $\sigma > 1$ implies $\partial\Upsilon/\partial p_\omega^* < 0$, it follows from the previous result $\mathrm{d}p_\omega^*/\mathrm{d}\tau < 0$ that, for given ℓ,

$$\frac{\mathrm{d}\Upsilon\left(p_\omega^*\right)}{\mathrm{d}\tau} = \frac{\partial\Upsilon\left(p_\omega^*\right)}{\partial p_\omega^*} \cdot \frac{\mathrm{d}p_\omega^*}{\mathrm{d}\tau} > 0. \tag{D.25}$$

From the equilibrium mortality rate (42) in Proposition 2 we thus have

$$\frac{\mathrm{d}m^*}{\mathrm{d}\tau} = \frac{\mathrm{d}}{\mathrm{d}\tau}\left(\bar{m} + \bar{\mu}\cdot\Upsilon^{\frac{\sigma}{1-\sigma}\chi}\cdot\ell^{-(1-\varphi)}\right) = \bar{\mu}\ell^{-(1-\varphi)}\cdot\frac{\sigma}{1-\sigma}\chi\cdot\frac{\mathrm{d}\Upsilon\left(p_\omega^*\right)}{\mathrm{d}\tau} < 0, \tag{D.26}$$

where the negative sign comes from $\sigma > 1$ combined with (D.25) above.

Step 4: proof of $\mathrm{d}\ell'_{ss}/\mathrm{d}\tau > 0$ *and* $\mathrm{d}\ell''_{ss}/\mathrm{d}\tau < 0$. This result hinges on two effects that correspond to two shifts in the steady-state loci of the dynamic system (46)-(47), as graphically shown in Figure 3, diagram (a). First, the $\dot{\ell} = 0$ locus reads $b = m^*(\ell)$ and shifts downwards in the phase plane in view of the result $\mathrm{d}m^*\left(\bar{\ell}\right)/\mathrm{d}\tau < 0$. Second, from (C.15), the $\dot{b} = 0$ locus is strictly increasing in expenditure per adult y^* and therefore shifts upwards in the phase plane in view of the result $\mathrm{d}y^*\left(\bar{\ell}\right)/\mathrm{d}\tau > 0$. Both these shifts imply that an increase in τ widens the distance between the two steady states, pushing the regular input ratio ℓ'_{ss} to the right and the extinction threshold ℓ''_{ss} to the left, as shown in Figure 3, diagram (a).

Step 5: proof of $\mathrm{d}p_q^*\left(\bar{\ell}\right)/\mathrm{d}\tau > 0$. From (B.18), we have

$$p_q^*\left(\bar{\ell}\right) = \frac{1}{1-\tau}\cdot\left[\frac{\eta^\sigma}{p_\omega^*\left(\bar{\ell}\right)^{\sigma-1}} + (1-\eta)^\sigma\right]$$

so that an increase in τ for given $\bar{\ell}$ has two effects: a direct one, which is strictly positive, and an indirect on working through $p_\omega^*\left(\bar{\ell}\right)$. Having proved that $\mathrm{d}p_\omega^*\left(\bar{\ell}\right)/\mathrm{d}\tau < 0$, the fact that $\sigma > 1$ implies a negative relationship between $p_\omega^*\left(\bar{\ell}\right)$ on $p_q^*\left(\bar{\ell}\right)$ yields a strictly positive indirect effect of τ on $p_q^*\left(\bar{\ell}\right)$ and thereby a strictly positive overall effect of τ on $p_q^*\left(\bar{\ell}\right)$.

D.3 Resource Booms

Proof of Proposition 9. From Proposition 2, the $\dot{\ell} = 0$ locus with $\sigma > 1$ reads

$$\dot{\ell} = 0 \rightarrow b = m^*(\ell) \equiv \bar{m} + \mu\eta^{\chi\upsilon\frac{\sigma}{\sigma-1}}\Omega^{\chi\{\upsilon-[\zeta+\xi(1-\upsilon)]\}}\cdot\Upsilon(\ell)^{\frac{\sigma}{1-\sigma}\chi\upsilon}\cdot\ell^{-\chi[\zeta+\xi(1-\upsilon)]}$$

where the term $\Omega^{\chi\{\upsilon-[\zeta+\xi(1-\upsilon)]\}}$ implies that

$$\frac{\mathrm{d}m^*(\ell)}{\mathrm{d}\Omega}\left\{\begin{array}{ll} > 0 & \text{if } \upsilon > \zeta + \xi(1-\upsilon) \\ = 0 & \text{if } \upsilon = \zeta + \xi(1-\upsilon) \\ < 0 & \text{if } \upsilon < \zeta + \xi(1-\upsilon) \end{array}\right\} \text{ for any } \ell > 0 \tag{D.27}$$

From (D.27), following an increase in the resource base Ω, the $\dot{\ell} = 0$ locus in the phase diagram shifts upwards when $\upsilon > \zeta + \xi(1-\upsilon)$, shifts downwards when $\upsilon < \zeta + \xi(1-\upsilon)$, and does not shift when $\upsilon = \zeta + \xi(1-\upsilon)$. Since the position $\dot{b} = 0$ locus is not affected by the resource base Ω, the input ratio levels associated to the mortality threshold and the regular steady state respectively react to the resource boom as follows

$$\frac{\mathrm{d}\ell''_{ss}}{\mathrm{d}\Omega} = \left\{ \begin{array}{ll} > 0 & \text{if } \upsilon > \zeta + \xi(1-\upsilon) \\ = 0 & \text{if } \upsilon = \zeta + \xi(1-\upsilon) \\ < 0 & \text{if } \upsilon < \zeta + \xi(1-\upsilon) \end{array} \right\} \quad \text{and} \quad \frac{\mathrm{d}\ell'_{ss}}{\mathrm{d}\Omega} = \left\{ \begin{array}{ll} < 0 & \text{if } \upsilon > \zeta + \xi(1-\upsilon) \\ = 0 & \text{if } \upsilon = \zeta + \xi(1-\upsilon) \\ > 0 & \text{if } \upsilon < \zeta + \xi(1-\upsilon) \end{array} \right\}$$

which completes the proof. ■