Energy taxes and endogenous technological change
(Running Title: Energy taxes)

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Abstract

This paper studies the effects of a tax on energy use in a growth model where market structure is endogenous and jointly determined with the rate of technological change. Because this economy does not exhibit the scale effect (a positive relation between TFP growth and aggregate R&D), the tax has no effect on the steady-state growth rate. It has, however, important transitional effects that give rise to surprising results. Specifically, under the plausible assumption that energy demand is inelastic, there may exist a hump-shaped relation between the energy tax and welfare. This shape stems from the fact that the reallocation of resources from energy production to manufacturing triggers a *temporary acceleration* of TFP growth that generates a ✓-shaped time profile of consumption. If endogenous technological change raises consumption sufficiently fast and by a sufficient amount in the long run, and households are sufficiently patient, the tax raises welfare despite the fact that – in line with standard intuition – it lowers consumption in the short run.

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1 Introduction

This paper studies the effects of a tax on energy use in a growth model where technological change and market structure are endogenous. Of particular interest is the interaction between changes in the inter-industry allocation of resources across manufacturing and energy production and the intra-industry effects within manufacturing. The latter are important because the manufacturing sector is the engine of growth of the economy.

There are several reasons why such an analysis is worthwhile. The current spike in the price of oil stands out as it has once again focussed attention on how *energy prices* affect the economy in the short and the long run.

At business cycle frequency, the evidence on the macroeconomic effects of energy prices is mixed. Hamilton argues that exogenous shocks to the price of oil explain most of the fluctuations of the US economy (see, e.g., [9] and [10]); Barsky and Killian, in contrast, argue that they matter very little (see [5], [11] and [12]). It is fair to say, however, that the conventional wisdom emerging from time-series studies is in line with Hamilton's view – that is, the price of oil drives economic fluctuations and growth. A corollary to this view is the widespread belief (particularly in the US) that high standards of living require low energy prices.

An alternative approach is to look at cross sections of countries. The best, and most recent, example is [6], a very interesting study that covers some of the ground that I cover here. The main differences are that that paper treats energy as a primary input (i.e., not produced by means of other inputs) and thus does not allow for the intersectoral reallocation of resources that drives my results. (Also the model exhibits the scale effect and does not have transitional dynamics.) More importantly it does not study welfare, which instead is the main focus of my analysis. However, it includes an empirical section that provides results directly relevant to my own. First, it shows that energy demand is inelastic: in a sample of 37 developed countries with 5-year average panel data over the period 1975-2004, the estimated mean elasticity is -0.06. Also, it documents that energy use crowds out investment in physical, knowledge and human capital and therefore crowds out long-run growth: the estimated overall steady-state effect of energy use on growth is -0.3. This is a rather large effect. Moreover, it is an effect that begs further research. As Bretschger puts is: "That high energy prices can be good for growth is somewhat counterintuitive. However, intuition may have been relying too much on the business cycle in the 1970s, and not necessarily on long-run growth experience" (p. 3).

In summary, there is ample motivation for studying the role of energy prices and energy policy in a growth context. In light of many governments' stated goal of reducing energy intensity (the ratio of energy use to GDP) without inflicting undue harm, moreover, understanding the role of specific instruments like energy taxes becomes very important.

Over the last 10 years economists have placed more and more emphasis on the role of technological change in the analysis of energy, environmental, and climate policy.[1] The reason is that technology is now seen as a crucial factor in the assessment of the long-run costs and benefits of the proposed interventions. Perhaps surprisingly, however, the literature has not exploited to its full potential the modern theory of endogenous technological change to shed new light on these issues.[2] With this paper, I try to fill this gap.

I take a new look at the long-run implications of energy taxation (with lump-sum recycling of revenues) through the lens of modern Schumpeterian growth theory. In particular, I use a model of the latest vintage that sterilizes the scale effect through a process of product proliferation that fragments the aggregate market into submarkets whose size does not increase with the size of the workforce.[3] The model is extremely tractable and yields a closed-form solution for the economy's transition path. This in turn allows me to study analytically the welfare effects of the energy tax.

My main finding is that, under the assumption that energy demand is inelastic, in an economy with growth-favoring fundamentals and patient households there exists a hump-shaped relation between the energy tax and welfare. Interestingly, I obtain this shape abstracting from environmental externalities – a modeling choice that brings to the forefront how endogenous technological change alters dramatically the assessment of the short- and long-run *economic costs* of the energy tax.

The mechanism driving this result is the following. The tax on energy use changes relative after-tax input prices and induces manufacturing firms to substitute other inputs for energy in their production operations. As energy demand falls, the economy experiences a reallocation of resources from the energy sector to the manufacturing sector. If energy demand is inelastic, associated to this reallocation is an increase in expenditure on manufacturing goods that induces an increase of aggregate R&D, the sum of cost-reducing R&D internal to the firm and entrepreneurial R&D aimed at product variety expansion. Despite this increase, however, steady-state growth does not change because the dispersion effect due to entry offsets the increase in aggregate R&D. This follows from the fact that the increase in the size of the manufacturing sector attracts entry and, over time, the larger number of firms generates dispersion of R&D resources across firms and thus sterilizes the scale effect. Consequently, the growth rate of total factor productivity (TFP) in manufacturing is independent of the size of the manufacturing sector.

The core of this mechanism is the reallocation of resources from energy to manufacturing that generates a *temporary acceleration* of TFP growth. Under empirically plausible conditions there exists a range of tax rates such that this acceleration generates a ✓-shaped time profile of consumption whereby consumption drops on impact and then rises sufficiently fast and by a sufficient amount that welfare rises. In other words, the long-run gain due to endogenous technological change more than offsets the short-run pain – the fact that holding technology constant, the higher after-tax price of energy makes goods more expensive so that consumption falls.

It is worth stressing how in light of [6] discussed earlier, the model's main ingredients, especially the assumption of inelastic energy demand, and its emphasis on factor reallocation across sectors rest on solid empirical ground.

I mentioned that the main body of the analysis abstracts from environmental externalities so that the capability of the tax to enhance welfare stems solely from its effect on the intersectoral allocation of resources. The intuition is that this reallocation mitigates some of the distortions – monopolistic pricing, firms' failure to internalize technological spillovers and other pecuniary externalities related to the interaction between incumbents and entrants – that characterize models of endogenous innovation. Hence, my *positive* analysis suggests that as a second-best instrument the energy tax has desirable effects independently of its role in addressing environmental problems.[4] This feature of the analysis emphasizes how allowing for endogenous technological change alters drastically the assessment of the costs of policy interventions.

The paper is organized as follows. Section 2 discusses the setup of the model. Section 3 constructs the equilibrium of the market economy. Section 4 characterizes the dynamic effects of the energy tax. Section 5 concludes with a discussion of several other extensions that likewise yield qualitative results consistent with those shown but were not included in the main presentation because they complicate matters without adding insight.

2 The model

2.1 Overview

The basic model that I build on is developed in [16], who in turn build on [13]. The innovation of this paper is that I add an energy sector. To keep things as simple as possible, I assume that energy producers are competitive and face an infinitely elastic world supply of natural resources.[5] The structure of the model and the key economic mechanisms are as follows.

The economy is populated by a representative household that supplies labor services inelastically in a competitive market. The household can also freely borrow and lend in a competitive market for financial assets. Manufacturing firms hire labor to produce differentiated consumption goods, undertake R&D, or, in the case of entrants, set up operations. In addition to labor, production of consumption goods requires energy, which is supplied by the competitive energy producers mentioned above. The government taxes energy purchases and returns the proceeds in a lump-sum fashion to the household.[6]

The economy starts out with a given range of goods, each supplied by one firm. The household values variety and is willing to buy as many differentiated goods as possible. Entrepreneurs compare the present value of profits from introducing a new good to the entry cost. They only target new product lines because entering an existing product line in Bertrand competition with the existing supplier leads to losses.

Once in the market, firms establish in-house R&D facilities to produce a stable flow of cost-reducing innovations. As each firm invests in R&D, it contributes to the pool of public knowledge and reduces the cost of future R&D. This allows growth at a constant rate in steady state, which is reached when the economy settles into a stable industrial structure.

2.2 Households

The representative household maximizes lifetime utility

$$U(t) = \int_t^\infty e^{-(\rho-\lambda)(s-t)} \log u(s) ds, \quad \rho > \lambda > 0 \tag{1}$$

subject to the flow budget constraint

$$\dot{A} = rA + WL + \tau E + \Pi_E - Y, \quad \tau \geq 0 \tag{2}$$

where ρ is the discount rate, λ is population growth, A is assets holding, r is the rate of return on financial assets, W is the wage rate, $L = L_0 e^{\lambda t}$, $L_0 \equiv 1$, is population size, which equals labor supply since there is no preference for leisure, and Y is consumption expenditure. In addition to asset and labor income, the household receives the lump-sum rebate of the energy tax revenues, τE, where τ is a per-unit tax and E is aggregate energy use. It also receives dividends Π_E from the energy sector.

The household has instantaneous preferences over a continuum of differentiated goods,[7]

$$\log u = \log \left[\int_0^N \left(\frac{X_i}{L} \right)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}, \quad \epsilon > 1 \tag{3}$$

where ϵ is the elasticity of product substitution, X_i is the household's purchase of each differentiated good, and N is the mass of goods (the mass of firms) existing at time t.

The solution for the optimal expenditure plan is well known. The household saves if assets earn the reservation rate of return

$$r = r_A \equiv \rho + \hat{Y} - \lambda \tag{4}$$

(a hat on top of a variable denotes a proportional growth rate) and taking as given this time-path of expenditure maximizes (3) subject to $Y = \int_0^N P_i X_i di$. This yields the demand schedule for product i,

$$X_i = Y \frac{P_i^{-\epsilon}}{\int_0^N P_i^{1-\epsilon} di}. \tag{5}$$

With a continuum of goods, firms are atomistic and take the denominator of (5) as given; therefore, monopolistic competition prevails and firms face isoelastic demand curves.

2.3 Manufacturing: Production and Innovation

The typical firm produces one differentiated consumption good with the technology

$$X_i = Z_i^{\theta} \cdot F\left(L_{X_i} - \phi, E_i\right), \quad 0 < \theta < 1, \ \phi > 0 \tag{6}$$

where X_i is output, L_{X_i} is production employment, ϕ is a fixed labor cost, E_i is energy use, and Z_i^{θ} is the firm's TFP, a function of the stock of firm-specific knowledge Z_i. The function $F(\cdot)$ is a standard neoclassical production function homogeneous of degree one in its arguments. Hence, (6) exhibits constant returns to rival inputs, labor and energy, and overall increasing returns. The associated total cost is

$$W\phi + C_X(W, P_E + \tau)Z_i^{-\theta} \cdot X_i, \tag{7}$$

where $C_X(\cdot)$ is a standard unit-cost function homogeneous of degree one in its arguments. The elasticity of unit cost reduction with respect to knowledge is the constant θ.

The firm accumulates knowledge according to the R&D technology

$$\dot{Z}_i = \alpha K L_{Z_i}, \quad \alpha > 0 \tag{8}$$

where $\dot{Z}_i$ measures the flow of firm-specific knowledge generated by an R&D project employing L_{Z_i} units of labor for an interval of time dt and αK is the productivity of labor in R&D as determined by the exogenous parameter α and by the stock of public knowledge, K.

Public knowledge accumulates as a result of spillovers. When one firm generates a new idea to improve the production process, it also generates general-purpose knowledge which is not excludable and that other firms can exploit in their own research efforts. Firms appropriate the economic returns from firm-specific knowledge but cannot prevent others from using the general-purpose knowledge that spills over into the public domain. Formally, an R&D project that produces $\dot{Z}_i$ units of proprietary knowledge also generates $\dot{Z}_i$ units of public knowledge. The productivity of research is determined by some combination of all the different sources of knowledge. A simple way of capturing this notion is to write

$$K = \int_0^N \frac{1}{N} Z_i di,$$

which says that the technological frontier is determined by the average knowledge of all firms.[8]

The R&D technology (8), combined with public knowledge K, exhibits increasing returns to scale to knowledge and labor, and constant returns to scale to knowledge. This property makes constant, endogenous steady-state growth feasible.

2.4 The Energy Sector and the rest of the world

Energy firms hire labor, L_E, to extract energy from natural resources (e.g. carbon, oil, gas), O. The energy-generation technology is $E = G(L_E, O)$, where $G(\cdot)$ is a standard neoclassical production function homogeneous of degree one in its arguments. The associated total cost is

$$C_E(W, P_O)\,E, \tag{9}$$

where $C_E(\cdot)$ is a standard unit-cost function homogeneous of degree one in the wage W and the price of resources P_O.

To fix ideas, I refer to resources as "oil" and assume that domestic supply is zero. In other words, I think of this as a small open economy that faces an infinitely elastic world supply. Corresponding to oil purchases, then, there is a flow of payments to the rest of the world. I show below that this flow is in units of labor (my numeraire) so that the economy trades labor services for oil and the balanced trade conditions holds.

I ignore international assets flows and any other sort of interaction with the rest of the world, e.g., technological spillovers, import-export of goods other than the exchange of labor services for oil, and so on.[9]

This is the simplest way to model the energy sector for the purposes of this paper. Energy is produced with labor and oil purchased at a given price in the world market. The energy sector competes for labor with the manufacturing sector. This captures the fundamental intersectoral allocation problem faced by this economy. Energy purchases are taxed on a per-unit basis. This affects manufacturers' production costs, their demand for energy and labor, and the general equilibrium path of the economy.

3 Equilibrium of the Market Economy

This section constructs the symmetric Nash equilibrium of the manufacturing sector. It then characterizes the equilibrium of the energy sector. Finally, it imposes general equilibrium conditions to determine the aggregate dynamics of the economy. The wage rate is the numeraire, i.e., $W \equiv 1$.

3.1 Equilibrium of the Manufacturing Sector

The typical manufacturing firm maximizes the present discounted value of net cash flow,

$$V_i(t) = \int_t^{\infty} e^{-\int_t^s r(v)dv} \Pi_{Xi}(s) ds.$$

Using the cost function (7), instantaneous profits are

$$\Pi_{Xi} = [P_i - C_X(1, P_E + \tau) Z_i^{-\theta}] X_i - \phi - L_{Z_i},$$

where L_{Z_i} is R&D expenditure. V_i is the value of the firm, the price of the ownership share of an equity holder. The firm maximizes V_i subject to the R&D technology (8), the demand schedule (5), $Z_i(t) > 0$ (the initial knowledge stock is given), $Z_j(t')$ for $t' \geq t$ and $j \neq i$ (the firm takes as given the rivals' innovation paths), and $\dot{Z}_j(t') \geq 0$ for $t' \geq t$ (innovation is irreversible). The solution of this problem yields the (maximized) value of the firm given the time path of the number of firms.

To characterize entry, I assume that upon payment of a sunk cost $\beta P_i X_i$, an entrepreneur can create a new firm that starts out its activity with productivity equal to the industry average.[10] Once in the market, the new firm implements price and R&D strategies that solve a problem identical to the one outlined above. Hence, entry yields value V_i. A free entry equilibrium, therefore, requires $V_i = \beta P_i X_i$.

The appendix shows that the equilibrium thus defined is symmetric and is characterized by the factor demands:

$$L_X = Y\frac{\epsilon - 1}{\epsilon}\left(1 - S_X^E\right) + \phi N; \tag{10}$$

$$E = Y\frac{\epsilon - 1}{\epsilon}\frac{S_X^E}{P_E + \tau}, \tag{11}$$

where the share of energy in the firm's variable costs is

$$S_X^E \equiv \frac{(P_E + \tau)\, E_i}{C_X(1, P_E + \tau) Z_i^{-\theta} X_i} = \frac{\partial \log C_X(W, P_E + \tau)}{\partial \log (P_E + \tau)}.$$

Note that S_X^E depends only on the after-tax price of energy since $W \equiv 1$.

Associated to these factor demands are the return to cost reduction and entry, respectively:

$$r = r_Z \equiv \alpha \left[\frac{Y\theta(\epsilon - 1)}{\epsilon N} - \frac{L_Z}{N}\right]; \tag{12}$$

$$r = r_N \equiv \frac{1}{\beta}\left[\frac{1}{\epsilon} - \frac{N}{Y}\left(\phi + \frac{L_Z}{N}\right)\right] + \hat{Y} - \hat{N}. \tag{13}$$

The dividend price ratio in (13) depends on the gross profit margin $\frac{1}{\epsilon}$. Anticipating one of the properties of the equilibria that I study below, note that in steady state the capital gain component of this rate of return, $\hat{Y} - \hat{N}$, is zero. Hence, the feasibility condition $\frac{1}{\epsilon} > r\beta$ must hold. This simply says that the firm expects to be able to repay the entry cost because it more than covers fixed operating and R&D costs.

3.2 Equilibrium of the Energy Sector

Given the cost function (9), competitive energy producers operate along the infinitely elastic supply curve

$$P_E = C_E\left(1, P_O\right). \tag{14}$$

In equilibrium, then, energy production is given by (11) evaluated at this pre-tax price. Defining the share of oil in energy costs as

$$S_E^O \equiv \frac{P_O O}{C_E\left(1, P_O\right) E} = \frac{\partial \log C_E(W, P_O)}{\partial \log P_O},$$

I can write the associated demands for labor and oil as:

$$L_E = E\frac{\partial C_E(W, P_O)}{\partial W} = Y\frac{\epsilon - 1}{\epsilon}\frac{P_E S_X^E}{P_E + \tau}\left(1 - S_E^O\right); \tag{15}$$

$$P_O O = E\frac{\partial C_E(W, P_O)}{\partial P_O} = Y\frac{\epsilon - 1}{\epsilon}\frac{P_E S_X^E}{P_E + \tau}S_E^O. \tag{16}$$

The share S_E^O depends only on the exogenous price of oil. Not surprisingly, (15) and (16) yield that the competitive energy producers make zero profits and thus pay zero dividends to the household. Consequently, I can set $\Pi_E = 0$ in (2).

3.3 General equilibrium

The model consists of the returns to saving, cost reduction and entry in (4), (12) and (13), the labor demands (10), (15), and the household's budget constraint (2). The household's budget constraint becomes the labor market clearing condition (see the appendix for the derivation):

$$L = L_N + L_X + L_Z + L_E + L_O,$$

where L_N is aggregate employment in entrepreneurial activity, $L_X + L_Z$ is aggregate employment in production and R&D operations of existing firms, L_E is aggregate employment in generation activity of energy firms and $L_O = P_O O$ is the balanced trade condition that states that the country exchanges labor services for oil. Therefore, to construct the general equilibrium of the economy I now only need to look at the financial market.

Assets market equilibrium requires equalization of all rates of return (no-arbitrage), $r = r_A = r_Z = r_N$, and that the value of the household's portfolio equal the value of the securities issued by firms, $A = NV = \beta Y$. Thus, the economy features a constant wealth to expenditure ratio. This property and log utility deliver a result that simplifies dramatically the analysis of dynamics. Substituting $A = \beta Y$ into (2), using the rate of return to saving in (4), and recalling that $\Pi_E = 0$, I obtain

$$0 = \beta (\rho - \lambda) + \frac{L + \tau E - Y}{Y},$$

which I can rewrite

$$\frac{Y}{L} = \frac{1}{1 - \beta (\rho - \lambda) - \tau \frac{E}{Y}} \equiv y^*, \tag{17}$$

where

$$\frac{E}{Y} = \frac{\epsilon - 1}{\epsilon} \frac{S_X^E}{P_E + \tau}.$$

This term depends only on parameters and the exogenous price of oil P_O. Since y^* is constant, then, the interest rate is $r = \rho$ at all times.

3.4 Dynamics

Because population grows, it is useful to work with the variable $n \equiv \frac{N}{L}$. Taking into account the non-negativity constraint on R&D, the results just derived allow me to solve (8) and (12) for

$$\hat{Z} = \alpha \frac{L_Z}{N} = \begin{cases} \frac{y^*}{n} \frac{\alpha\theta(\epsilon-1)}{\epsilon} - \rho & n < \bar{n} \\ 0 & n \geq \bar{n} \end{cases}, \tag{18}$$

where

$$\bar{n} \equiv y^* \frac{\alpha\theta (\epsilon - 1)}{\rho\epsilon}.$$

Substituting into (13) yields

$$\hat{n} = \begin{cases} \frac{1}{\beta} \left[\frac{1-\theta(\epsilon-1)}{\epsilon} - \left(\phi - \frac{\rho}{\alpha} \right) \frac{n}{y^*} \right] - \rho & n < \bar{n} \\ \frac{1}{\beta} \left[\frac{1}{\epsilon} - \phi \frac{n}{y^*} \right] - \rho & n \geq \bar{n} \end{cases}.$$

The general equilibrium of the model thus reduces to a single differential equation in the mass of firms per capita. Figure 1 illustrates dynamics.[11] If $\alpha\phi > \rho$, the entry rate is always falling. In contrast, if $\alpha\phi \leq \rho$ the entry rate is initially rising or constant, until the economy crosses the threshold $\bar{n}$ when the entry rate starts falling. In all cases, the economy converges to

$$n^* = \begin{cases} \frac{\frac{1-\theta(\epsilon-1)}{\epsilon}-\rho\beta}{\phi-\frac{\rho}{\alpha}} y^* & \frac{\frac{1-\theta(\epsilon-1)}{\epsilon}-\rho\beta}{\phi\alpha-\rho} < \frac{\theta(\epsilon-1)}{\rho\epsilon} \\ \frac{\frac{1}{\epsilon}-\rho\beta}{\phi} y^* & \frac{\frac{1-\theta(\epsilon-1)}{\epsilon}-\rho\beta}{\phi\alpha-\rho} \geq \frac{\theta(\epsilon-1)}{\rho\epsilon} \end{cases} . \tag{19}$$

These dynamics make clear that $\phi > 0$ kills the possibility of endogenous growth through product proliferation because the term ϕN on the right hand side of the resources constraint implies that the equation cannot hold for a given labor endowment if N grows too large.[12]

The solutions in (19) exist only if the feasibility condition $\frac{1}{\epsilon} > \rho\beta$ holds. The interior steady state with both vertical and horizontal R&D requires the more stringent conditions $\alpha\phi > \rho$ and

$$\rho\beta + \frac{\theta(\epsilon-1)}{\epsilon} < \frac{1}{\epsilon} < \rho\beta + \frac{\alpha\phi}{\rho}\frac{\theta(\epsilon-1)}{\epsilon}.$$

It then yields

$$\frac{y^*}{n^*} = \frac{\phi - \frac{\rho}{\alpha}}{\frac{1-\theta(\epsilon-1)}{\epsilon} - \rho\beta} \tag{20}$$

so that

$$\hat{Z}^* = \frac{\phi\alpha - \rho}{\frac{1-\theta(\epsilon-1)}{\epsilon} - \rho\beta}\frac{\theta(\epsilon-1)}{\epsilon} - \rho. \tag{21}$$

Notice how the steady-state growth rate of productivity in manufacturing is independent of conditions in the energy market because the sterilization of the scale effect implies that it does not depend on the size of the manufacturing sector and therefore on the intersectoral allocation of labor.

To perform experiments, I shall focus on this region of parameter space and work with the equation

$$\hat{n} = \nu - \left(\phi - \frac{\rho}{\alpha}\right)\frac{n}{\beta y^*}, \quad \nu \equiv \frac{1-\theta(\epsilon-1)}{\beta\epsilon} - \rho.$$

This is a logistic equation (see, e.g., [3]) with growth coefficient ν and crowding coefficient $\left(\phi - \frac{\rho}{\alpha}\right) \frac{1}{\beta y^*}$. Using the value n^* in (19), also called carrying capacity, I can rewrite it as

$$\hat{n} = \nu \left(1 - \frac{n}{n^*}\right), \tag{22}$$

which has solution

$$n(t) = \frac{n^*}{1 + e^{-\nu t}\left(\frac{n^*}{n_0} - 1\right)}, \tag{23}$$

where n_0 is the initial condition.

4 The effects of the energy tax

This section analyzes the effects of the energy tax. It begins with a discussion of the conditions under which the tax raises consumption expenditure so that the market for manufacturing goods expands. Next, it shows how the reallocation of resources associated to that expansion affects the path of TFP, the CPI and therefore (real) consumption per capita. Finally, it shows that under plausible conditions welfare is hump-shaped in the tax.

4.1 Expenditure

The first step in the evaluation of the effects of the energy tax on growth and welfare is to assess its effect on expenditure. The following property of the energy demand function (11) is quite useful.

Lemma 1 *Let*

$$\epsilon_X^E \equiv -\frac{\partial \log E}{\partial \log (P_E + \tau)}.$$

Then, $\epsilon_X^E \leq 1$ *if*

$$\frac{\partial S_X^E}{\partial (P_E + \tau)} \geq 0,$$

which is true if

$$\frac{\partial L_X}{\partial \left(P_E+\tau\right)} \leq 0,$$

that is, if labor and energy are gross complements in (6).

Proof. See the Appendix. ■

In words, this says that energy demand is inelastic, i.e., $\epsilon_X^E \leq 1$, when the energy share of manufacturing cost, S_X^E, is non-decreasing in the after-tax price of energy. Energy demand, conversely, is elastic when the energy cost share in manufacturing is decreasing in $P_E + \tau$. The effect of the after-tax price of energy on the energy cost share, in turn, depends on whether labor and energy are gross complements or gross substitutes. I now use this result to derive one of the key ingredients for the analysis of the growth and welfare effects of the tax.

Proposition 2 *Assume that the production technology (6) exhibits gross complementarity between labor and energy so that energy demand is inelastic,* $\epsilon_X^E \leq 1$*. Then,* $y^*(\tau)$ *is a monotonically increasing function with domain* $\tau \in [0,\infty)$ *and codomain* $[y^*(0), y^*(\infty))$*, where:*

$$y^*(0) = \frac{1}{1-\beta\left(\rho-\lambda\right)};$$

$$y^*(\infty) = \frac{1}{1-\beta\left(\rho-\lambda\right)-\frac{\epsilon-1}{\epsilon}}.$$

Assume, in contrast, that labor and energy are gross substitutes so that $\epsilon_X^E > 1$*. Then,* $y^*(\tau)$ *is a hump-shaped function of* τ *with the same domain as before and codomain* $[y^*(0), y^*(\infty))$*, where:*

$$y^*(0) = y^*(\infty) = \frac{1}{1-\beta\left(\rho-\lambda\right)}.$$

Proof. See the Appendix. ■

The mechanism driving this result highlights the importance of the substitution possibilities between labor and energy in manufacturing. If labor and energy are gross complements, higher manufacturing employment requires higher energy use, and this dampens the negative effect of the increase in the after-tax price of energy on energy demand. If, in contrast, labor and energy are gross substitutes, higher manufacturing employment requires lower energy use and thereby amplifies the negative effect of the after-tax price increase on energy demand.

To see this property in sharper detail, it is useful to consider the following example.

Example 3 *Consider a CES production function of the form*

$$X_i = Z_i^{\theta}\left[(L_{X_i} - \phi)^{\sigma} + E_i^{\sigma}\right]^{\frac{1}{\sigma}}, \quad \sigma \leq 1$$

As is well known, this contains as special cases the linear production function ($\sigma = 1$) wherein inputs are perfect substitutes, the Cobb-Douglas ($\sigma = 0$) wherein the elasticity of substitution between inputs is equal to 1, and the Leontief ($\sigma = -\infty$) wherein inputs are perfect complements. The associated unit-cost function is

$$C_{X_i} = Z_i^{-\theta}\left[W^{\frac{\sigma}{\sigma-1}} + (P_E+\tau)^{\frac{\sigma}{\sigma-1}}\right]^{\frac{\sigma-1}{\sigma}}.$$

From this one derives (recall that $W \equiv 1$):

$$\begin{aligned} S_X^E &= \frac{1}{1+(P_E+\tau)^{\frac{\sigma}{1-\sigma}}}; \\ \epsilon_X^E &= 1 + \frac{\sigma}{1-\sigma}\frac{1}{1+(P_E+\tau)^{-\frac{\sigma}{1-\sigma}}} = 1 + \frac{\sigma}{1-\sigma}\left(1 - S_X^E\right). \end{aligned}$$

Hence, $\sigma \leq 0$ yields $\epsilon_X^E \leq 1$ and $\frac{dy^*}{d\tau} > 0$ for all τ. In contrast, $0 < \sigma \leq 1$ yields $\epsilon_X^E > 1$ and $\frac{dy^*}{d\tau} > 0$ for $\tau < \bar{\tau}$ where

$$\bar{\tau} \equiv \arg\text{solve}\left\{1 = \frac{\tau\epsilon_X^E}{P_E+\tau}\right\}.$$

The main message of Proposition 2 is that expenditure rises with the tax when energy demand is inelastic because the induced fall in energy use is not so dramatic that total tax revenues fall. In other words, the economy operates on the upward sloping part of the energy tax revenue curve.

4.2 The CPI: how the cost of energy and TFP affect consumption

Consider now the effects on consumption. The price index of a basket of consumption goods – the CPI of this economy – is

$$P_Y = \left[\int_0^N P_j^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}}.$$

Accordingly, the price strategy $P_j = C_X (1, P_E + \tau) Z_j^{-\theta} \frac{\epsilon}{\epsilon - 1}$ (see the appendix for the derivation) yields *real* expenditure per capita as

$$\frac{y^*}{P_Y} = \frac{\epsilon - 1}{\epsilon} \frac{y^*}{c^*} N^{\frac{1}{\epsilon - 1}} Z^{\theta},$$

where to simplify the notation I define $c^* \equiv C_X (1, C_E (1, P_O) + \tau)$. This component of the unit cost is pinned down by exogenous parameters.

Why look at real expenditure? Because it measures the flow of consumption in the utility function (3) and thus is relevant for welfare. Moreover, one can reinterpret (3) as a production function for a final homogenous good assembled from intermediate goods and define aggregate TFP as

$$T = N^{\frac{1}{\epsilon - 1}} Z^{\theta}. \tag{24}$$

Accordingly,

$$\hat{T}(t) = \frac{1}{\epsilon - 1} \hat{N}(t) + \theta \hat{Z}(t).$$

In steady state this gives

$$\hat{T}^* = \frac{\lambda}{\epsilon - 1} + \theta \hat{Z}^* \equiv g^*, \tag{25}$$

where $\hat{Z}^*$ is given by (21). Observe how g^* is independent of conditions in the energy market and of population size and growth.

A nice feature of this model is that I can compute TFP in closed form along the transition path. To bring this feature to the forefront, notice that according to (20) in steady state

$$\frac{y^*}{n^*} = \frac{y_0}{n_0} \Rightarrow \frac{y^*}{y_0} = \frac{n^*}{n_0}.$$

Now define

$$\Delta \equiv \frac{n^*}{n_0} - 1 = \frac{y^*}{y_0} - 1.$$

This is the percentage change in expenditure that the economy experiences in response to changes in fundamentals and/or policy parameters. It fully summarizes the effects of such changes on the scale of economic activity. The following proposition characterizes how changes in scale affect the manufacturing sector.

Proposition 4 *At any time $t > 0$ the log of TFP is*

$$\begin{aligned} \log T(t) &= \log\left(Z_0^\theta n_0^{\frac{1}{\epsilon-1}}\right) + g^* t \\ &\quad + \frac{\gamma\Delta}{\nu}\left(1 - e^{-\nu t}\right) + \frac{1}{\epsilon - 1}\log\frac{1+\Delta}{1+\Delta e^{-\nu t}}, \end{aligned} \tag{26}$$

where

$$\gamma \equiv \theta\frac{\alpha\theta(\epsilon-1)}{\epsilon}\frac{y^*}{n^*} = \theta\frac{\theta(\epsilon-1)}{\epsilon}\frac{\phi\alpha - \rho}{\frac{1-\theta(\epsilon-1)}{\epsilon} - \rho\beta}.$$

Moreover,

$$\frac{d\log T(t)}{d\tau} = \left[\frac{\gamma}{\nu}\left(1 - e^{-\nu t}\right) + \frac{1}{\epsilon-1}\frac{1}{1+\Delta}\frac{1-e^{-\nu t}}{1+\Delta e^{-\nu t}}\right]\frac{d\Delta}{d\tau}.$$

Proof. See the Appendix. ■

Recall that Proposition 2 provides a *sufficient* condition – inelastic energy demand – for $\Delta > 0$. It thus allows one to tell when the energy tax is potentially beneficial, that is, when $\frac{d \log T(t)}{d\tau} > 0$. In this case, the two transitional components of the TFP operator in (26) represent, respectively, the cumulated gain from cost reduction, $Z(t)$, and the cumulated gain from product variety per capita, $n(t)$. The mechanism driving these components is quite intuitive: a tax on energy use changes relative after-tax input prices and induces manufacturing firms to substitute labor for energy in their production operations. This standard effect is associated to an increase of aggregate R&D employment, the sum of cost-reducing R&D internal to the firm and entrepreneurial R&D aimed at product variety expansion. Despite this reallocation, however, steady-state growth does not change because the dispersion effect due to entry offsets the increase in aggregate R&D. Average R&D, in other words, does not increase. This follows from the fact that the increase in the size of the manufacturing sector, measured by the rise in aggregate expenditure on consumption goods, raises the returns to entry. Over time the larger number of firms generates dispersion of R&D resources and sterilizes the scale effect. Consequently, the growth rate is independent of the size of the manufacturing sector.

Summarizing, the energy tax reallocates labor from the energy sector to the manufacturing sector. If energy demand is inelastic, associated to this reallocation is an increase in expenditure on manufacturing goods, driven by the lump-sum recycling of tax revenues, that generates a *temporary acceleration* of TFP growth.

If one ignores these effects of endogenous technological change – if one posits that T grows at an exogenous rate – consumption depends on the tax only through y^*/c^*. The following proposition characterizes this case.

Proposition 5 *Assume that technology does not adjust in response to the energy tax. Then,*

$$\frac{d}{d\tau}\left(\log \frac{y^*}{c^*}\right) < 0 \text{ for all } \tau.$$

Proof. See the Appendix. ■

This is an important result. It says that endogenous technological change is *necessary* to obtain welfare gains from the energy tax. The reason is that, in line with intuition, the energy tax raises the cost of production of goods so that the CPI rises and consumption falls.

4.3 Welfare

I now investigate how the long-run effects of endogenous technological change offset the short-run cost (if any) of the energy tax. As I mentioned, to emphasize the importance of endogenous technology, I ignore for now environmental quality in the household's preferences so that the welfare gains of the tax stem solely from the fact that the reallocation of resources from energy to manufacturing accelerates temporarily the pace of endogenous technological change and yields a long-run TFP gain.

Proposition 6 *Let* $\log u^*(t)$ *and* U^* *be, respectively, the instantaneous utility index (3) and welfare function (1) evaluated at* y^**. Then, a path starting at time* $t = 0$ *is characterized by:*

$$\begin{aligned} \log u^*(t) &= \log \frac{y^*}{c^*} + g^* t \\ &\quad + \frac{\gamma \Delta}{\nu}\left(1 - e^{-\nu t}\right) + \frac{1}{\epsilon - 1} \log \frac{1 + \Delta}{1 + \Delta e^{-\nu t}}; \end{aligned} \tag{27}$$

$$\begin{aligned} U^* &= \frac{1}{\rho - \lambda}\left[\log \frac{y^*}{c^*} + \frac{g^*}{\rho - \lambda} + \frac{\gamma \Delta}{\rho - \lambda + \nu}\right] \\ &\quad + \frac{1}{\epsilon - 1} \int_0^\infty e^{-(\rho - \lambda)t} \log \frac{1 + \Delta}{1 + \Delta e^{-\nu t}} dt. \end{aligned} \tag{28}$$

Proof. See the Appendix. ■

The expression in (28) separates out the contribution to welfare of: steady-state real expenditure calculated holding technology constant, $\frac{y^*}{c^*}$; steady-state TFP growth, g^*; the change in firm-level productivity, Z, due to the transitional acceleration (for $\Delta > 0$) or deceleration (for $\Delta < 0$) of firm-level cost reduction; the change in product variety per capita, n, due to the transitional acceleration (for $\Delta > 0$) or deceleration (for $\Delta < 0$) of product variety expansion. Figure 2 illustrates the underlying path of flow utility in the case in which the tax has a long-run beneficial effect.[13] Since the CPI jumps up on impact, there is an instantaneous drop in consumption, followed by faster than trend growth, with eventual convergence to a higher steady-state growth path, parallel to the initial one. Thus, all the gains from the energy tax stem from *level effects* spread over time. This is where the model's property that the transition path has an analytical solution comes very handy since it allows one to see exactly how the intertemporal trade-off plays out.

I now show that under plausible conditions endogenous technological change yields that the level of welfare attained by the decentralized market equilibrium is a hump-shaped function of τ, as in Figure 3.

Proposition 7 *Assume that the production technology (6) exhibits gross complementarity between labor and energy so that energy demand is inelastic, $\epsilon_X^E \leq 1$. Consider an economy on the equilibrium path induced by the introduction of an energy tax $\tau > 0$, starting from the equilibrium with no energy tax, i.e., $\tau = 0$. Then, U^* is a hump-shaped function of τ iff*

$$\frac{\gamma + \frac{\nu}{\epsilon - 1}}{\rho - \lambda + \nu} > \frac{\epsilon}{\epsilon - 1}\left[\frac{1}{\epsilon} - \beta\left(\rho - \lambda\right)\right]. \tag{29}$$

Otherwise, U^* is decreasing in τ for all $\tau \geq 0$. A sufficient condition for inequality (29) to hold is

$$\frac{g^*}{\rho} > \frac{1}{\epsilon - 1} - \theta.$$

Proof. See the Appendix. ■

The second inequality in this proposition provides a *sufficient* condition for existence of the hump-shaped relation in Figure 3. In essence, it says that the hump shape occurs *if* the economy has growth-favoring fundamentals (since g^* is increasing in θ and ϵ) and patient households. In practice, this means that the economy exhibits a sufficiently high steady-state ratio of growth to the interest rate, $\frac{g^*}{r^*}$, since $\rho = r^*$. To see whether this condition is likely to hold in reality, one can compute from the data for the US economy (see, e.g., [4])

$$\frac{g^*}{r^*} = \frac{.02}{.04} = \frac{1}{2}.$$

The sufficient condition then holds if

$$\frac{1}{2} > \frac{1}{\epsilon - 1} - \theta.$$

There are two ways to think about this inequality. The first is to observe that ϵ is the elasticity of substitution among products and that, accordingly, $\frac{1}{\epsilon}$ is the typical firm profit margin. The inequality then requires observed profit margins to satisfy

$$\frac{1}{\epsilon} < \frac{\frac{1}{2} + \theta}{\frac{1}{2} + \theta + 1}.$$

Independent estimates of θ, the elasticity of cost reduction, unfortunately do not exist yet. However, one can check that for $\theta = 0$ the condition requires profit margins of less than 33%, for $\theta = \frac{1}{2}$ it requires profit margins of less than 50%, for $\theta = \frac{3}{4}$, it requires profit margins of less than 56%. In the US observed profit margins are typically less than 30-40%, suggesting that the sufficient condition likely holds.

The other way to think about the inequality above is to realize that the term $\frac{1}{\epsilon - 1}$ is the love-of-variety effect in the preferences in (3). As is well known, this effect can be disentangled from the elasticity of substitution so that data on profit margins are not relevant to the sufficient condition in the proposition. Rather, what is relevant is data on the weights assigned to product variety, N, and firm-level productivity, Z, in the TFP operator (24). Borrowing the argument developed in [16], for example, one could rewrite

(3) as

$$\log u = \log \left(N^{\omega - \frac{\epsilon}{\epsilon-1}} \left[\int_0^N \left(\frac{X_i}{L} \right)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}} \right), \quad \epsilon > 1,\, \omega \geq 0, \tag{30}$$

where ω measures love-of-variety. Because this is an external effect, it does not affect the behavior of agents. Nothing changes, then, in the characterization of the equilibrium, except that $\frac{1}{\epsilon-1}$ in the sufficient condition above is replaced by ω. Accordingly, the condition surely holds if the weight of the vertical dimension of innovation is larger than that of the horizontal dimension, i.e., $\omega - \theta < 0$. Is there any reason to believe that this is so? Yes. Empirically the correlation between steady-state income per capita growth, g^*, and population growth, λ, is approximately zero. This suggests that ω is small and thus that $\omega < \theta$ is quite plausible. Even more plausible, of course, is the less restrictive condition actually required that $\omega < \theta + \frac{g^*}{r^*}$.

5 Discussion and suggestions for further research

In this paper I studied the effects of a tax on energy use in a growth model where technological change and market structure are endogenous. I focussed in particular on the interaction between changes in the inter-industry allocation of resources across the manufacturing and energy sectors and the intra-industry effects within manufacturing. I found that, under the plausible assumption that energy demand is inelastic, in an economy with growth-favoring fundamentals and patient households there exists a hump-shaped relation between the tax and welfare.

The mechanism driving this result is the following. The tax induces manufacturing firms to substitute labor for energy in their production operations. As energy demand falls, the economy experiences a reallocation of labor from the energy sector to the manufacturing sector. This reallocation induces an increase of aggregate R&D employment. Despite this increase, however, steady-state growth does not change because the dispersion effect due to entry offsets the increase in aggregate R&D. Average R&D, in other words, does not increase.

The core of the mechanism, thus, is that the energy tax reallocates labor from energy to manufacturing and thereby generates a *temporary acceleration* of TFP growth that in the long run can offset the short-run pain of the tax due to the fact that, holding technology constant, the higher after-tax price of energy makes manufacturing goods more expensive. If the economy has growth-favoring fundamentals and patient households, there is a range of tax rates such that the lower prices at the end of the transition drive consumption up to the point where it dominates the intertemporal trade-off and welfare rises.

Skeptics of environmental taxes usually point out that results like these depend crucially on the willingness and capability of the government not to divert revenues to wasteful uses. This is a legitimate point. However, it is an argument concerning the proper functioning of the government in general, not an objection to environmental taxes per se. Waste is bad regardless of the particular tax instrument that funds it. Moreover, my analysis does not require the government to make a particularly enlightened use of tax revenues; it simply posits that it rebates them to the households in a lump-sum fashion. Whether this implies an unrealistic belief in the proper functioning of the political process is beyond the scope of this paper's analysis.

As a robustness check on my story, I have investigated several extensions of the model. I did not include them in the paper because they complicate the analysis without adding insight. Some of them, nevertheless, are worth exploring further in future research.

The most obvious extension is to allow for technological change in the energy sector (more on this below). How do things change in this case? If the issue is the robustness of my qualitative results, the interesting case is when the reallocation of labor due to the energy tax produces a slowdown of productivity growth in the energy sector that counteracts the acceleration in manufacturing. The conditions for my novel result then must include the requirement that the acceleration of productivity growth in manufacturing more than offsets the deceleration in energy. This is just a matter of suitably setting the model's parameters. Conceptually, it requires that innovation-related distortions are worse in manufacturing than in energy. If not, then the novel result – that there exists a range of values of the tax rate such that welfare rises – does not occur and welfare falls monotonically with the energy tax. The model thus built, however, is cumbersome and in my judgment the complexity adds no insight, since it simply (re)produces the qualitative results that I obtain with the simpler version that I used in the paper.[14]

Another obvious extension is endogenous labor supply. It is easy to show that with log utility defined over consumption and leisure the qualitative results are exactly the same. However, the interesting aspect of this extension is that the results can in fact be much stronger. If instead of rebating the energy tax revenues in a lump-sum fashion, the government uses them to pay down, say, a distortionary tax on labor income, then there is an additional *revenue-recycling effect* that expands labor supply, aggregate employment, and thereby boosts the growth acceleration driving my welfare result.

In future work, one item that deserves attention is population growth since it puts pressure on the economy's resource endowment and thus generates escalating resource prices. I bypassed this complication by using the small open economy assumption, which in practice turns a finite supply of oil into an infinitely elastic one. This convenient simplification might raise doubts about the robustness of my result. It turns out, however, that the result carries over to the case of scarcity due either to the fact that the economy is closed and has a finite endowment of oil or to the fact that the economy is not small and thus affects the world price of oil. This is important because it implies that the result is robust to plausible extensions that allow for a positive relation between (domestic) oil demand and the price at which (domestic) energy firms purchase oil.

It is worth emphasizing that at issue here is the behavior of resource prices, not whether the economy is capable of long-run growth. Escalating resource prices are consistent with long-run growth in the presence of scarcity. In fact, escalating prices is precisely how the economy copes with resource scarcity. My assumption that TFP in manufacturing is Hicks neutral implies that growth of income per capita is feasible at *unchanged physical inputs use*, and thereby makes quite starkly the point that the scarcity signaled by escalating resource prices is due to population growth, not to growth of income per capita.

An important aspect of scarcity is that it *requires* resource-augmenting technical change in the energy sector in order to offset the upward pressure on prices due to population growth. There is then an additional trade-off because resource-augmenting R&D in the energy sector competes for resources with R&D in manufacturing. On the other hand, this gives cumulative effects along the vertical production structure so that cost reductions in energy production ultimately show up in the prices of consumption goods. In this paper I abstracted from this issue to keep things as simple as possible. In the more general case, the basic mechanism does not change but the formal analysis becomes much more cumbersome.

6 Appendix

6.1 The typical firm's behavior

To characterize the typical firm's behavior, consider the Current Value Hamiltonian

$$CVH_i = [P_i - C_X(1, P_E + \tau) Z_i^{-\theta}] X_i - \phi - L_{Z_i} + z_i \alpha K L_{Z_i},$$

where the costate variable, z_i, is the value of the marginal unit of knowledge. The firm's knowledge stock, Z_i, is the state variable; R&D investment, L_{Z_i}, and the product's price, P_i, are the control variables. Firms take the public knowledge stock, K, as given.

Since the Hamiltonian is linear, one has three cases. The case $1 > z_i \alpha K$ implies that the value of the marginal unit of knowledge is lower than its cost. The firm, then, does not invest. The case $1 < z_i \alpha K$ implies that the value of the marginal unit of knowledge is higher than its cost. Since the firm demands an infinite amount of labor to employ in R&D, this case violates the general equilibrium conditions and is ruled out. The first order conditions for the interior solution are given by equality between marginal revenue and marginal cost of knowledge, $1 = z_i \alpha K$, the constraint on the state variable, (8), the terminal condition,

$$\lim_{s \to \infty} e^{-\int_t^s r(v)dv} z_i(s) Z_i(s) = 0,$$

and a differential equation in the costate variable,

$$r = \frac{\dot{z}_i}{z_i} + \theta C_X(1, P_E + \tau) Z_i^{-\theta-1} \frac{X_i}{z_i},$$

that defines the rate of return to R&D as the ratio between revenues from the knowledge stock and its shadow price plus (minus) the appreciation (depreciation) in the value of knowledge. The revenue from the marginal unit of knowledge is given by the cost reduction it yields times the scale of production to which it applies. The price strategy is

$$P_i = C_X(1, P_E + \tau) Z_i^{-\theta} \frac{\epsilon}{\epsilon - 1}. \tag{31}$$

[13] (Proposition 1) shows that under the restriction $1 > \theta(\epsilon - 1)$ the firm is always at the interior solution, where $1 = z_i \alpha K$ holds, and equilibrium is symmetric.

The cost function (7) gives rise to the conditional factor demands:

$$L_{X_i} = \frac{\partial C_X(W, P_E + \tau)}{\partial W} Z_i^{-\theta} X_i + \phi;$$

$$E_i = \frac{\partial C_X(W, P_E + \tau)}{\partial (P_E + \tau)} Z_i^{-\theta} X_i.$$

Then, the price strategy (31), symmetry and aggregation across firms yield (10) and (11).

Also, in symmetric equilibrium $K = Z = Z_i$ yields $\dot{K}/K = \alpha L_Z/N$, where L_Z is aggregate R&D. Taking logs and time derivatives of $1 = z_i \alpha K$ and using the demand curve (5), the R&D technology (8) and the price strategy (31), one reduces the first-order conditions to (12).

Taking logs and time-derivatives of V_i yields

$$r = \frac{\Pi_{Xi}}{V_i} + \frac{\dot{V}_i}{V_i},$$

which is a perfect-foresight, no-arbitrage condition for the equilibrium of the capital market. It requires that the rate of return to firm ownership equal the rate of return to a loan of size V_i. The rate of return to firm ownership is the ratio between profits and the firm's stock market value plus the capital gain (loss) from the stock appreciation (depreciation).

In symmetric equilibrium the demand curve (5) yields that the cost of entry is $\beta \frac{Y}{N}$. The corresponding demand for labor in entry is $L_N = \dot{N} \beta \frac{Y}{N}$. The case $V > \beta \frac{Y}{N}$ yields an unbounded demand for labor in entry, $L_N = +\infty$, and is ruled out since it violates the general equilibrium conditions. The case $V < \beta \frac{Y}{N}$ yields $L_N = -\infty$, which means that the non-negativity constraint on L_N binds and $\dot{N} = 0$. Free-entry requires $V = \beta \frac{Y}{N}$. Using the price strategy (31), the rate of return to entry becomes (13).

6.2 The economy's resources constraint and balanced trade

I now show that the household's budget constraint reduces to the economy's labor market clearing condition, and that this condition contains the balanced trade condition stating that the economy trades labor services for oil. Starting from (2), recall that $A = NV$ and $rV = \Pi_X + \dot{V}$. Substituting into

(2) yields

$$\dot{N}V = N\Pi_X + L + \tau E + \Pi_E - Y.$$

Observing that $N\Pi_X = NPX - L_X - L_Z - (P_E + \tau) E$, $NPX = Y$ and $\Pi_E = P_E E - L_E - P_O O$, this becomes

$$L = \dot{N}V + L_X + L_Z + L_E + P_O O.$$

Now recall that the free entry condition yields that total employment in entrepreneurial activity is $L_N = \dot{N}V$. Finally, let $L_O = P_O O$ denote the amount of labor exchanged for oil, and substitute this balanced trade condition into the expression above to write

$$L = L_N + L_X + L_Z + L_E + L_O.$$

This says that the small open economy allocates labor to five activities: creation of new goods/firms, production of goods and reduction of production costs for existing firms, generation of energy and "extraction" of oil. Extraction is in quotation marks because it takes the form of an exchange of the domestic resource (labor) for the foreign one (oil). In other words, trade is the extraction technology available to a resource poor economy.

6.3 Proof of Lemma 1

Observe that

$$\epsilon_X^E \equiv -\frac{\partial \log E}{\partial \log (P_E + \tau)} = 1 - \frac{\partial \log S_X^E}{\partial \log (P_E + \tau)} = 1 - \frac{\partial S_X^E}{\partial (P_E + \tau)} \frac{P_E + \tau}{S_X^E}$$

so that $\epsilon_X^E \leq 1$ if

$$\frac{\partial S_X^E}{\partial (P_E + \tau)} = \frac{\partial}{\partial (P_E + \tau)} \left(\frac{(P_E + \tau) E}{(P_E + \tau) E + L_X} \right) \geq 0.$$

This in turn is true if

$$\begin{aligned} \left(1 - S_X^E\right) \frac{\partial ((P_E + \tau) E)}{\partial (P_E + \tau)} - S_X^E \frac{\partial L_X}{\partial (P_E + \tau)} &\geq 0 \\ \left(1 - S_X^E\right) \left[\frac{\partial ((P_E + \tau) E)}{\partial (P_E + \tau)} + \frac{\partial L_X}{\partial (P_E + \tau)} \right] - \frac{\partial L_X}{\partial (P_E + \tau)} &\geq 0. \end{aligned}$$

Recall now that total cost is increasing in $P_E + \tau$ so that

$$\frac{\partial\left(\left(P_E+\tau\right)E\right)}{\partial\left(P_E+\tau\right)} + \frac{\partial L_X}{\partial\left(P_E+\tau\right)} > 0.$$

It follows that

$$\frac{\partial L_X}{\partial\left(P_E+\tau\right)} \leq 0$$

is a sufficient condition for $\epsilon_X^E \leq 1$ since it implies that both terms in the inequality above are positive.

6.4 Proof of Proposition 2

Use (11) and the fact that $Y = Ly^*$ to rewrite (17) as

$$y^* = \frac{1+\tau\frac{E}{L}}{1-\beta\left(\rho-\lambda\right)}. \tag{32}$$

Then,

$$\begin{aligned}
\frac{dy^*}{d\tau} &= \frac{1}{1-\beta\left(\rho-\lambda\right)}\frac{d\left(\tau\frac{E}{L}\right)}{d\tau} \\
&= \frac{\frac{1}{L}}{1-\beta\left(\rho-\lambda\right)}\left[E+\tau\frac{\partial E}{\partial\tau}\right] \\
&= \frac{\frac{E}{L}}{1-\beta\left(\rho-\lambda\right)}\left[1+\tau\frac{\partial E}{\partial\tau}\frac{1}{E}\right] \\
&= \frac{\frac{E}{L}}{1-\beta\left(\rho-\lambda\right)}\left[1+\frac{\tau}{P_E+\tau}\frac{\partial\log E}{\partial\log\left(P_E+\tau\right)}\right] \\
&= \frac{\frac{E}{L}}{1-\beta\left(\rho-\lambda\right)}\left[1-\frac{\tau}{P_E+\tau}\epsilon_X^E\right].
\end{aligned}$$

This expression is positive if $\epsilon_X^E \leq 1$. If, in contrast, $\epsilon_X^E > 1$ this expression changes sign at

$$1-\frac{\tau}{P_E+\tau}\epsilon_X^E = 0.$$

Observing that $y^*(0) = y^*(\infty)$ ensures that this equation has a solution and that y^* is a hump-shaped function of τ.

6.5 Proof of Proposition 4

Taking logs of (24) yields

$$\log T(t) = \theta \log Z_0 + \theta \int_0^t \hat{Z}(s)\, ds + \frac{1}{\epsilon - 1} \log N(t).$$

Using the definition $n \equiv Ne^{-\lambda t}$, the expression for g^* in (25) and adding and subtracting $\hat{Z}^*$ from $\hat{Z}(t)$, I obtain

$$\log T(t) = \theta \log Z_0 + g^* t + \theta \int_0^t \left[\hat{Z}(s) - \hat{Z}^*\right] ds + \frac{1}{\epsilon - 1} \log n(t).$$

Using (18), (23) and the definition of Δ I rewrite the third term as

$$\begin{aligned}
\theta \int_0^t \left(\hat{Z}(s) - \hat{Z}^*\right) ds &= \theta \frac{\alpha\theta(\epsilon - 1)}{\epsilon} \int_0^t \left(\frac{y^*}{n(s)} - \frac{y^*}{n^*}\right) ds \\
&= \gamma \int_0^t \left(\frac{n^*}{n(s)} - 1\right) ds \\
&= \gamma\Delta \int_0^t e^{-\nu s} ds \\
&= \frac{\gamma\Delta}{\nu}\left(1 - e^{-\nu t}\right),
\end{aligned}$$

where

$$\gamma \equiv \theta \frac{\alpha\theta(\epsilon - 1)}{\epsilon} \frac{y^*}{n^*} = \theta \frac{\theta(\epsilon - 1)}{\epsilon} \frac{\phi\alpha - \rho}{\frac{1 - \theta(\epsilon - 1)}{\epsilon} - \rho\beta}.$$

Using (23) and the definition of Δ I rewrite the last term as

$$\begin{aligned}
\frac{1}{\epsilon - 1} \log n(t) &= \frac{1}{\epsilon - 1} \log \frac{n^*}{1 + \Delta e^{-\nu t}} \\
&= \frac{1}{\epsilon - 1} \log n_0 + \frac{1}{\epsilon - 1} \log \frac{\frac{n^*}{n_0}}{1 + \Delta e^{-\nu t}} \\
&= \frac{1}{\epsilon - 1} \log n_0 + \frac{1}{\epsilon - 1} \log \frac{1 + \Delta}{1 + \Delta e^{-\nu t}}.
\end{aligned}$$

These results yield (26).

6.6 Proof of Proposition 5

Observe that

$$\frac{d}{d\tau}\left(\log\frac{y^*}{c^*}\right)<0\Leftrightarrow\frac{dy^*}{d\tau}<y^*\frac{S_X^E}{P_E+\tau}$$

since

$$\frac{d\log c^*}{d\tau}=\frac{dC_X}{d\left(P_E+\tau\right)}\frac{P_E+\tau}{C_X}\frac{d\left(P_E+\tau\right)}{d\tau}\frac{1}{P_E+\tau}=\frac{S_X^E}{P_E+\tau}.$$

Now use (32), (11) and $Y=Ly^*$ to rewrite the second inequality as

$$\begin{aligned}\frac{E}{L}\left[1-\frac{\tau}{P_E+\tau}\epsilon_X^E\right] &< \frac{E}{L}\frac{1-\beta\left(\rho-\lambda\right)}{\frac{\epsilon-1}{\epsilon}}\\ 1-\frac{1}{\epsilon}-\frac{\epsilon-1}{\epsilon}\frac{\tau}{P_E+\tau}\epsilon_X^E &< 1-\beta\left(\rho-\lambda\right)\\ -\frac{\epsilon-1}{\epsilon}\frac{\tau}{P_E+\tau}\epsilon_X^E &< \frac{1}{\epsilon}-\beta\left(\rho-\lambda\right).\end{aligned}$$

The right-hand side of this expression is positive because $\frac{1}{\epsilon}-\beta\left(\rho-\lambda\right)>0$ since the feasibility constraint $\frac{1}{\epsilon}-\beta\rho>0$ holds. The left-hand side is negative. Therefore, y^*/c^* is decreasing in τ for all $\tau\geq 0$.

6.7 Proof of Proposition 6

I first use (26) and the definition of Δ to write (3) as

$$\begin{aligned}\log u^*\left(t\right) &= \log\frac{y^*}{P_Y\left(t\right)}\\ &= \log\frac{\epsilon-1}{\epsilon}+\log\frac{y^*}{c^*}+\log T\left(t\right)\\ &= \log\left(\frac{\epsilon-1}{\epsilon}Z_0^\theta n_0^{\frac{1}{\epsilon-1}}\right)+\log\frac{y^*}{c^*}+g^*t\\ &\quad+\frac{1}{\epsilon-1}\log\frac{1+\Delta}{1+\Delta e^{-\nu t}}+\frac{\gamma\Delta}{\nu}\left(1-e^{-\nu t}\right).\end{aligned}$$

Without loss of generality, I set

$$\frac{\epsilon-1}{\epsilon}Z_0^\theta n_0^{\frac{1}{\epsilon-1}}=1$$

and obtain (27). I then substitute this expression into (1) and write

$$\begin{aligned} U^* &= \int_0^\infty e^{-(\rho-\lambda)t}\left[\log\frac{y^*}{c^*}+g^*t\right]dt \\ &\quad +\frac{\gamma\Delta}{\nu}\int_0^\infty e^{-(\rho-\lambda)t}\left(1-e^{-\nu t}\right)dt \\ &\quad +\frac{1}{\epsilon-1}\int_0^\infty e^{-(\rho-\lambda)t}\log\frac{1+\Delta}{1+\Delta e^{-\nu t}}dt. \end{aligned}$$

The first and second integrals have straightforward closed form solutions. (The third is solvable as well, but it entails a very complicated expression containing the hypergeometric function and is not worth using since it adds no insight and does not simplify the algebra in the analysis below.) Hence, I obtain (28).

6.8 Proof of Proposition 7

Let $y_0 = y^*(0)$ denote y^* evaluated at $\tau = 0$. Observe then that:

$$\Delta(0) = \frac{y^*(0)}{y_0} - 1 = 0;$$

$$\Delta(\infty) = \frac{y^*(\infty)}{y_0} - 1 = \frac{\frac{\epsilon-1}{\epsilon}}{1-\beta(\rho-\lambda)-\frac{\epsilon-1}{\epsilon}} > 0;$$

$$U^*(0) = \frac{1}{\rho-\lambda}\left[\log\frac{y^*(0)}{c^*(0)}+\frac{g^*}{\rho-\lambda}\right] > 0;$$

$$\begin{aligned} U^*(\infty) &= \frac{1}{\rho-\lambda}\left[\log\frac{y^*(\infty)}{c^*(\infty)}+\frac{g^*}{\rho-\lambda}+\frac{\gamma\Delta(\infty)}{\rho-\lambda+\nu}\right] \\ &\quad +\frac{1}{\epsilon-1}\int_0^\infty e^{-(\rho-\lambda)t}\log\frac{1+\Delta(\infty)}{1+\Delta(\infty)e^{-\nu t}}dt \\ &= -\infty. \end{aligned}$$

Note that the restriction $U^*(0) > 0$ makes sense (and does not require special assumptions beyond $\log\frac{y^*(0)}{c^*(0)} > 0$), while $U^*(\infty) = -\infty$ follows from that fact that $y^*(\infty)$ is finite and $c^*(\infty) = \infty$. Next, use the properties derived in Proposition 6 and note that $\frac{dU^*}{d\tau} > 0$ iff

$$\frac{d}{d\tau}\left(\log\frac{y^*}{c^*}\right) + \frac{\gamma}{\rho-\lambda+\nu}\frac{d\Delta}{d\tau} + \frac{\rho-\lambda}{\epsilon-1}\int_0^\infty \frac{e^{-(\rho-\lambda)t}}{1+\Delta}\frac{1-e^{-\nu t}}{1+\Delta e^{-\nu t}}\frac{d\Delta}{d\tau}dt > 0.$$

Now observe that $U^*(\infty) = -\infty$ means that the function U^* eventually must be decreasing. Moreover, the second and third terms in the inequality above are always positive. The first term is negative. It follows that $\frac{dU^*}{d\tau}$ changes sign exactly once. Therefore, U^* is hump-shaped in τ iff the inequality holds in a neighborhood of $\tau = 0$, otherwise it is always decreasing in τ.

Now observe that $\frac{dU^*}{d\tau}\mid_{\tau=0} > 0$ requires

$$\begin{aligned}\frac{d\log c^*}{d\tau} &< \left(\frac{1}{1+\Delta} + \frac{\gamma}{\rho-\lambda+\nu}\right)\frac{d\Delta}{d\tau} \\ &\quad + \frac{\rho-\lambda}{\epsilon-1}\frac{1}{1+\Delta}\frac{d\Delta}{d\tau}\int_0^\infty e^{-(\rho-\lambda)t}\frac{1-e^{-\nu t}}{1+\Delta e^{-\nu t}}dt,\end{aligned}$$

which one can rewrite as

$$\frac{\frac{d\log c^*}{d\tau}}{\frac{d\log y^*}{d\tau}} < 1 + \frac{\gamma}{\rho-\lambda+\nu} + \frac{\rho-\lambda}{\epsilon-1}\int_0^\infty e^{-(\rho-\lambda)t}\frac{1-e^{-\nu t}}{1+\Delta e^{-\nu t}}dt$$

since

$$\frac{d\Delta}{d\tau}\frac{1}{1+\Delta} = \frac{d\log y^*}{d\tau}.$$

Letting $\tau \to 0$,

$$\frac{\frac{\epsilon}{\epsilon-1}}{y^*(0)} < 1 + \frac{\gamma}{\rho-\lambda+\nu} + \frac{\rho-\lambda}{\epsilon-1}\int_0^\infty e^{-(\rho-\lambda)t}\left(1-e^{-\nu t}\right)dt.$$

This yields

$$\frac{\epsilon}{\epsilon-1}\left[\frac{1}{\epsilon} - \beta(\rho-\lambda)\right] < \frac{\gamma+\frac{\nu}{\epsilon-1}}{\rho-\lambda+\nu},$$

which I can rewrite

$$-\frac{\epsilon}{\epsilon-1}\beta(\rho-\lambda) < \frac{\gamma(\epsilon-1)-\rho+\lambda}{\rho-\lambda+\nu}.$$

This inequality holds if

$$\gamma\left(\epsilon-1\right)>\rho-\lambda.$$

Using the definitions of γ and g^*, and a little bit of algebra, this inequality reduces to

$$g^*>\rho\frac{1-\theta\left(\epsilon-1\right)}{\epsilon-1},$$

which I can rewrite

$$\frac{g^*}{\rho}+\theta>\frac{1}{\epsilon-1}.$$

Notes

[1]This literature has grown so rapidly and extensively that any attempt at summarizing it here would do injustice to the many contributors. It is probably more productive to refer the reader to the recent reviews by [1], [18] and, in particular, [7] and [20].

[2]One reason is that incorporating environmental externalities and resource scarcity increases dramatically the complexity of growth models. As a consequence, the early attempts have focussed mostly on first-generation models of endogenous innovation. A relatively small literature that developed recently has started to push the frontier harder and generate novel insights concerning the energy-growth relation. Two other papers that deserve particular mention are [19] and [2]. They build models that are close in spirit to what I do here. As for [6], the main difference is that I use a model of endogenous innovation without the scale effect to study the welfare effects of energy taxes.

[3][21] and [15] have recently shown that these models have profound implications for the analysis of taxation.

[4]This result is derived under the restriction that the government uses only one instrument in an environment where the optimal policy would, in fact, require several.

[5]Both assumptions can be relaxed at the cost of complicating the analysis without adding insight; see Section 5.

[6]This ensures that the government balances the budget without introducing feedback effects that are not the focus of this paper.

[7]I omit a term representing preference for environmental quality. The reason is that it is not necessary to develop my argument about energy taxes since I focus on the response of the decentralized market equilibrium and not on the socially optimal policy.

[8]For a detailed discussion of the microfoundations of a spillovers function of this class and of how it generates endogenous growth without the scale effect, see [17].

[9]It is also possible to think of this as a small open economy that takes as given the world interest rate. Since the model has the property that the domestic interest rate jumps to its steady state level, given by the domestic discount rate, as long the small open economy has the same discount rate as the rest of the world the equilibrium discussed in the paper displays the same properties as an equilibrium with free financial flows.

[10]See [8] and, in particular, [16] for a more detailed discussion of the microfoundations of this assumption.

[11]For simplicity I ignore the non-negativity constraint on $\dot{N}$. I can do so without loss of generality because population growth implies that the mass of firms eventually grows all the time. See [13] for a discussion of this property.

[12]See [16] for a detailed discussion of this property in Schumpeterian models of endogenous growth.

[13]I illustrate only this case to keep the figure clean and to highlight the novel aspect of my results. The reader can easily see that if the path of $\log u^*$ with endogenous technological change does not overtake the baseline case $\tau = 0$, then the welfare effect of the tax must be negative – exactly as in the case of exogenous technological change. The next Proposition provides conditions that guarantee that the case in the figure occurs.

[14]It is worth sketching how one might incorporate technical change in the energy sector in this model. The most obvious way is to model the energy sector exactly as the manufacturing sector, namely an endogenous mass of non-competitive firms (with a much higher eleasticity of substitution to capture the notion that energy inputs are more homogenous than consumption goods) that carry out their own cost-reducing R&D. A variant of this approach is to allow for learning by doing in energy production, instead of R&D, to generate ongoing cost reduction. Under some additional assumptions this approach allows one to dispense with non-competitive pricing, but this at best is only a minor simplification. The main problem with these approaches is that the price of energy becomes time-varying, a feature that

complicates a great deal the analysis of dynamics because one cannot work with constant expenditure on consumption goods. (To see this, observe that (17) does not admit a constant soultion if P_E is not constant.) This feature, of course, stems from the fact that I model (realistically, in my view) manufacturing and energy as vertically related sectors. One way to bypass this problem is to follow, e.g., [19] or [2] and model manufacturing and energy as parallel sectors that supply a representative final producer. The main benefit of this approach is that under some additional assumptions one can isolate the manufacturing sector from the dynamics of the price of energy since energy does not enter directly the manufacturers' production technology. However, it has the cost that one needs to track time-varying sectoral shares so that in the end one cannot escape the trade-off between realism and tractability. Whether these approaches are desirable depends on what one wants to accomplish with the model. Since I am interested in welfare, being able to compute anlytically the model's transition path is a *major* benefit, well worth the cost of shutting down technical change in the energy sector. Giving up this property is worth only if in exchange I obtain a *major* gain in realism or insight. In my judgment none of the alternatives that I or, to my knowledge, the rest of the literature have experimented with delivers such a benefit.

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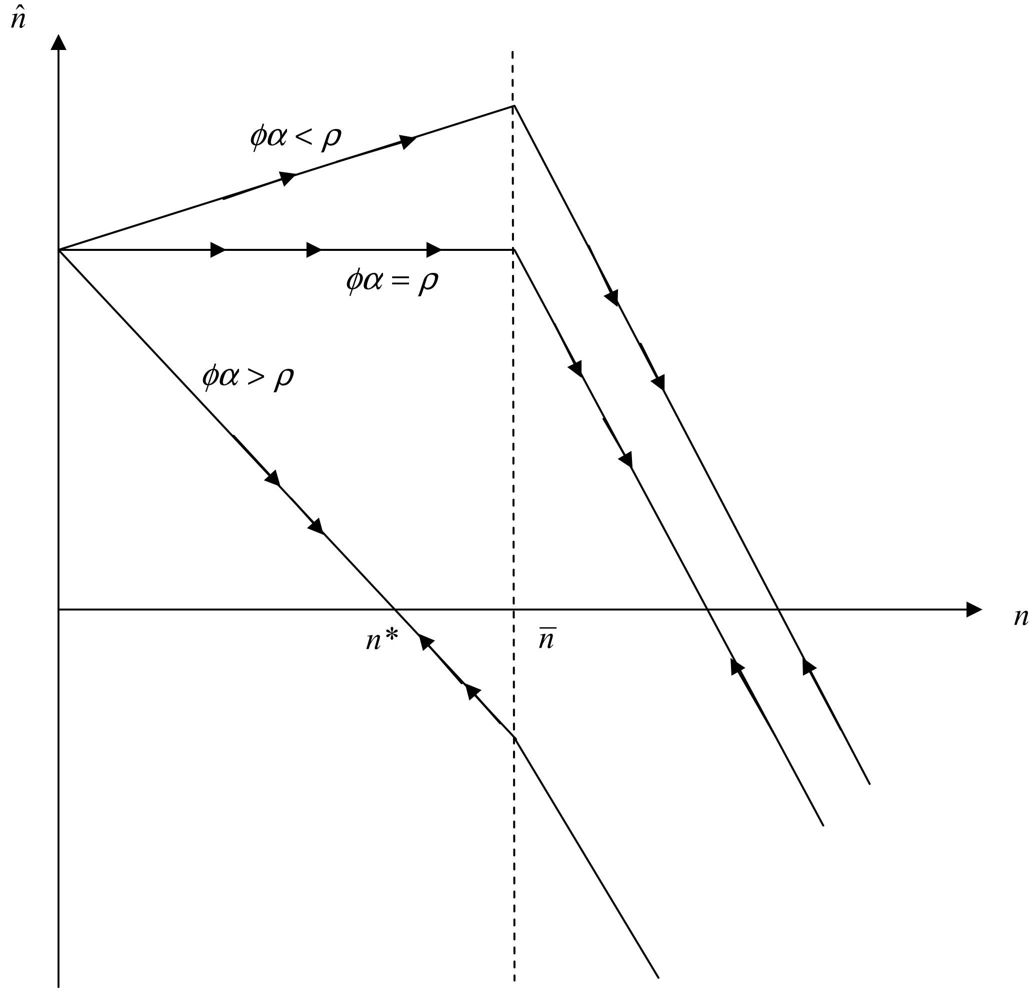

Figure 1: General Equilibrium Dynamics

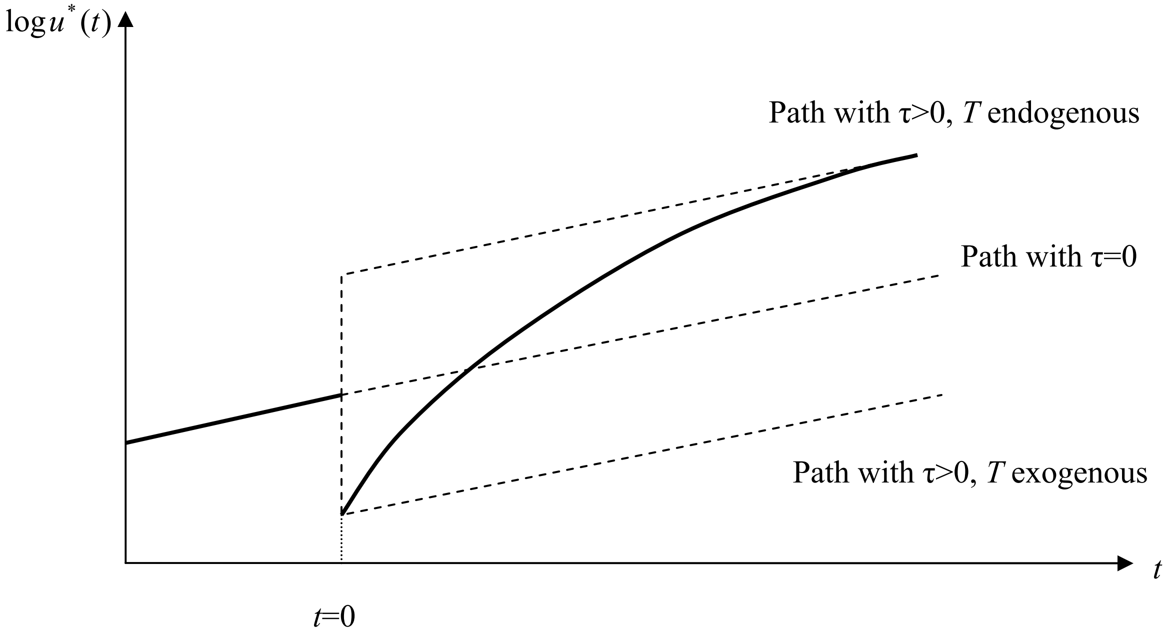

Figure 2: The path of consumption

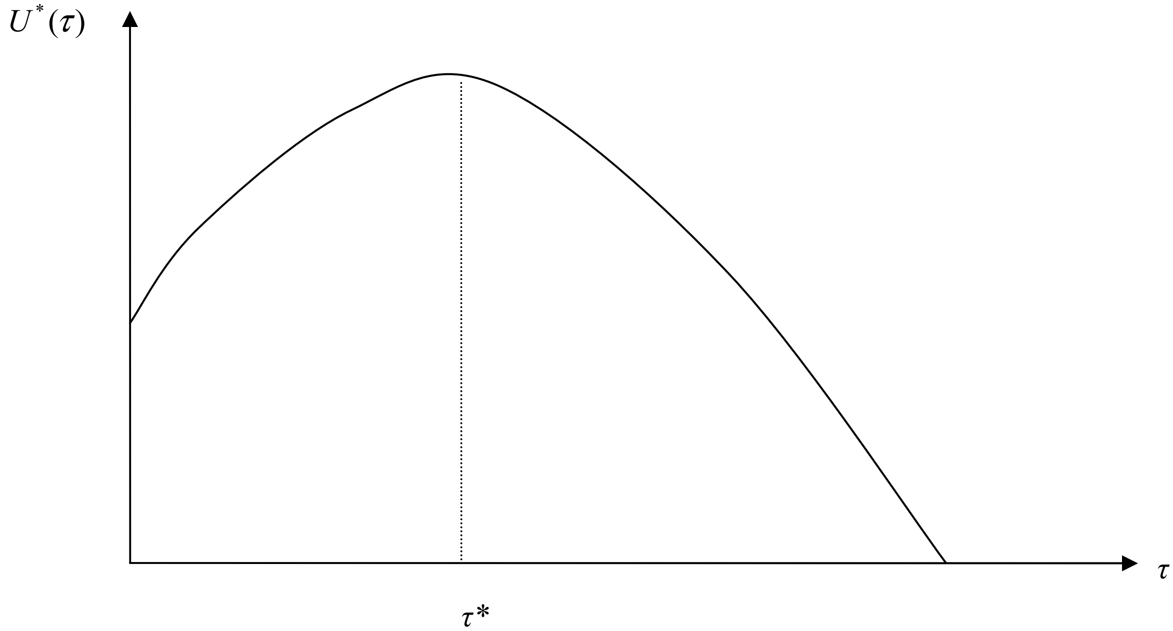

Figure 3: The energy tax and welfare