



# Resource abundance, growth and welfare: A Schumpeterian perspective<sup>☆</sup>

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## ABSTRACT

This paper takes a new look at the long-run implications of resource abundance. It develops a Schumpeterian model of endogenous growth that incorporates an upstream resource-intensive sector and yields an analytical solution for the transition path. It then derives conditions under which, as the economy's endowment of a natural resource rises, (i) growth accelerates and welfare rises, (ii) growth decelerates but welfare rises nevertheless, and (iii) growth decelerates and welfare falls. Which of these scenarios prevails depends on the response of the natural resource price to an increase in the resource endowment. The price response determines the change in income earned by the owners of the resource (the households) and thereby the change in their expenditure on manufacturing goods. Since manufacturing is the economy's innovative sector, this income-to-expenditure effect links resource abundance to the size of the market for manufacturing goods and drives how re-source abundance affects incentives to undertake innovative activity.

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## 1. Introduction

This paper takes a new look at the long-run effects of resource endowments through the lens of modern Schumpeterian growth theory. Using a model of the latest vintage that yields a closed-form solution for the transition path, it derives conditions under which, as the economy's endowment of a natural resource rises, growth accelerates and welfare rises, conditions under which growth decelerates but welfare rises nevertheless, and conditions under which growth decelerates and welfare falls.

Which of these scenarios prevails depends on how the pattern of factor substitution determines the response of the natural resource price to an increase in the resource endowment. The price response determines the change in income earned by the owners of the resource (the households) and thereby the change in their expenditure on manufacturing goods. Since manufacturing is the economy's innovative sector, this income-to-expenditure effect links resource abundance to the size of the market for manufacturing goods and drives how resource abundance affects incentives to undertake innovative activity.

The model has two factors of production in exogenous supply, labor and a natural resource, and two sectors, primary production (or resource processing) and manufacturing. I define resource abundance as the endowment of the natural resource relative to labor. The primary sector uses labor to process the raw natural resource; the manufacturing sector uses labor and the processed natural resource to produce differentiated consumption goods. Because both sectors use labor, its reallocation from manufacturing to primary production drives the economy's adjustment to an increase in resource abundance. The manufacturing sector is technologically dynamic: firms and entrepreneurs undertake R&D to learn how to use factors of production more efficiently and to design new products. Importantly, the process of product proliferation fragments the aggregate market into sub-markets whose size does not increase with the size of the endowments and thereby sterilizes the scale effect. This means that the effect of resource abundance on growth is only *temporary*. The resulting structure is extremely tractable and yields a closed-form solution for the transition path.

Substitution between labor and resource inputs matters because it determines the price elasticity of demand for the natural resource. Inelastic demand means that the price has to fall drastically to induce the market to absorb the additional quantity; elastic demand means that the adjustment requires a mild drop in the price. Importantly, the two cases are part of a continuum because I use technologies with factor substitution that changes with prices. Now, if the economy exhibits substitution, demand is elastic, the price effect is mild, the quantity effect dominates, and resource income rises, spurring more spending on manufacturing goods and a temporary growth acceleration. If, instead,

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the economy exhibits complementarity, demand is inelastic, the price effect is strong, resource income falls, and we have a temporary growth deceleration. Whether the economy experiences a growth acceleration or deceleration, however, is not sufficient to determine what happens to welfare, since, given technology, the lower resource price makes consumption goods cheaper. Assessing the welfare effect of the change in the endowment ratio, therefore, requires resolving the trade-off between short- and long-run effects.

My main result is that I identify threshold values of the equilibrium price of the natural resource—and therefore threshold values of the endowment ratio—that yield the following sequence of scenarios as we gradually raise the endowment ratio from tiny to very large.

1. Growth accelerates and welfare rises. This happens when the resource price is high because the endowment ratio is low. In this situation, demand is elastic, the quantity effect dominates over the price effect, and, consequently, resource income rises. In other words, starting from a situation of scarcity, the increase in the endowment ratio generates a growth acceleration associated to an initial jump up in consumption that yields higher welfare.
2. Growth decelerates but welfare rises nevertheless. That is, the rise in the endowment ratio causes a growth deceleration that is offset by an initial jump up in consumption. This happens when the endowment ratio is between the two thresholds and, correspondingly, the resource price is in its intermediate range. Relative to the previous case, demand becomes inelastic and the price effect dominates over the quantity effect, with the result that resource income falls.
3. Growth decelerates and welfare falls. This happens when the resource price is low because the endowment ratio is high. In this case, demand is inelastic, the price effect dominates over the quantity effect, and resource income falls. Differently from the previous case, the fall in resource income is now sufficiently large to cause the growth deceleration to dominate over the initial jump up in consumption or to cause the initial jump in consumption to be down.

There is a large and growing body of literature studying the consequences of natural-resource abundance. Within this literature my paper is most closely related to theoretical models focusing on the reallocation of resources away from the manufacturing sector and towards the resource sector, a reallocation that is sometimes termed “Dutch disease” in the literature. Dutch-disease models often assume that the manufacturing sector is characterized by learning by doing or other externalities, so these models, like mine, have the potential to induce a fall in the growth rate as well as a fall in welfare.<sup>1</sup>

Relative to this literature my model differs in three fundamental dimensions. First, I work with a closed economy rather than an open economy. Second, I model the productivity growth process in the manufacturing sector through investments in R&D. Third, I introduce vertical integration between the manufacturing sector and the resource sector by making (processed) natural resources an input in the production of the manufacturing good. Note that from the closed economy assumption it follows that there is a distinction between resource endowments (the physical quantities of natural resources) and resource wealth (its market value, or price times quantity). In open-economy models the price is usually taken as given so a change in the endowment is entirely isomorphic to a change in wealth.

From these differences follow major differences in implications. In my model an increase in resource *wealth* is never associated with a decline in the growth rate, nor it can reduce welfare. This is because, due to closed economy assumption, an increase in resource wealth increases demand for domestic manufacturing goods and thus the incentives to engage in R&D. What can reduce growth and welfare is

an increase in the resource *endowment*, if it precipitates a sufficiently large fall in the price and hence in resource wealth (again, this mechanism is absent in other models where the economy takes the price as given). However, not all declines in resource wealth following increases in endowments are associated with declines in welfare. Even if resource wealth and the subsequent growth rate decline, welfare can increase because of the vertical-integration feature of the model: the more abundant endowment reduces production costs for manufacturing consumption goods.

The closed economy assumption also implies that my model speaks to a different set of facts relative to the Dutch disease literature and most of the empirical literature on the so-called resource curse. Indeed the latter tends to use natural-resource exports as a share of GDP (or sometimes per capita) as its proxy for resource abundance (clearly my theory has no predictions for the effects of resource exports) and variation in prices is seen as as important and as exogenous a source of variation as variation in physical endowments.<sup>2</sup> Instead, my theory is applicable to investigating the effects of new resource discoveries in closed economies or, possibly, of huge new discoveries in the world as a whole.

Success stories broadly consistent with my scenario 1 abound. Wright and Czelusta (2007), for example, argue that the United States overtook the United Kingdom and become the world leader in terms of GDP per worker-hour precisely at the time—roughly 1890–1913—when it become the world’s dominant producer of virtually every major industrial mineral of the era. They conclude that “the condition of abundant resources was a significant factor in shaping, if not propelling, the U.S. path to world leadership in manufacturing” (p. 185). A modern-day case that provides a more specific example of a growth acceleration is Chile. Wright and Czelusta (p. 196) report that in the 1990s it grew at about 8.5% per year and argue that the mining industry was central to this performance, accounting for about 8.5% of GDP and about half of its total exports over the period. Notably, Chile accounts for 35% of world copper production and is a major producer of several other minerals.

It is harder to find examples of failure stories consistent with my scenarios 2–3. The reason is that the literature rarely disentangles prices and quantities to the level of detail needed to isolate the income effect driving my theory. Consequently, it is often impossible to infer from a particular narrative whether the driving factor underlying the growth deceleration is the relative price effect that my theory emphasizes or the other mechanisms discussed in, e.g., the literature on the natural resource curse.

The paper’s organization is as follows. Section 2 sets up the model. Section 3 constructs the general equilibrium of the market economy. Section 4 discusses the key properties of the equilibrium that drive the paper’s main results. Section 5 derives the main results. It first studies the conditions under which an increase in the resource endowment results into a growth deceleration or acceleration and then studies the conditions under which it yields lower or higher welfare. It also discusses interesting implications for our reading of the empirical literature. Section 6 concludes.

## 2. The model

### 2.1. Overview

The basic model that I build on is developed in Peretto and Connolly (2007). A representative household supplies labor services in a competitive market. It also borrows and lends in a competitive market for financial assets. The household values variety and buys as many differentiated consumption goods as possible. Manufacturing firms hire labor to produce differentiated consumption goods, undertake R&D, or,

<sup>1</sup> See, e.g., Corden and Neary (1982), Corden (1984), Krugman (1987), van Wijnbergen (1984), Younger (1992), and Torvik (2001).

<sup>2</sup> See Sachs and Warner (1995, 2001), Alexeev and Conrad (2009), Brunneschweiler and Bulte (2008), van der Ploeg and Poelhekke (2010), van der Ploeg (forthcoming).

in the case of entrants, set up operations. Production of consumption goods also requires a processed resource, which is produced by competitive suppliers using labor and a raw natural resource. The introduction of this upstream primary sector is the main innovation of this paper. The economy starts out with a given range of goods, each supplied by one firm. Entrepreneurs compare the present value of profits from introducing a new good to the entry cost. Once in the market, firms establish in-house R&D facilities to produce cost-reducing innovations. As each firm invests in R&D, it contributes to the pool of public knowledge and reduces the cost of future R&D. This allows the economy to grow at a constant rate in steady state.

## 2.2. Households

The representative household has a constant mass  $L$  of identical members, each one endowed with one unit of labor. It maximizes

$$U(t) = \int_t^\infty e^{-\rho(s-t)} \log u(s) ds, \quad \rho > 0 \quad (1)$$

subject to the flow budget constraint

$$\dot{A} = rA + WL + p\Omega + \Pi_M - Y, \quad (2)$$

where  $\rho$  is the discount rate,  $A$  is assets holding,  $r$  is the rate of return on assets,  $W$  is the wage rate,  $L$  is labor supply since there is no preference for leisure, and  $Y$  is consumption expenditure. In addition to asset and labor income, the household receives rents from ownership of the endowment,  $\Omega$ , of a natural resource whose market price is  $p$  and dividend income from resource-processing firms,  $\Pi_M$ .<sup>3</sup> The household takes these terms as given.

Instantaneous utility in Eq. (1) is

$$\log u = \log \left[ \int_0^N \left( \frac{X_i}{L} \right)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}, \quad \epsilon > 1 \quad (3)$$

where  $\epsilon$  is the elasticity of product substitution,  $X_i$  is the household's purchase of each differentiated good, and  $N$  is the mass of goods (the mass of firms) existing at time  $t$ .

The solution for the optimal expenditure plan is well known. The household saves according to

$$r = r_A \equiv \rho + \frac{\dot{Y}}{Y} \quad (4)$$

and taking as given this time-path of expenditure maximizes Eq. (3) subject to  $Y = \int_0^N P_i X_i di$ . This yields the demand schedule for product  $i$ ,

$$X_i = Y \frac{P_i^{-\epsilon}}{\int_0^N P_i^{1-\epsilon} di}. \quad (5)$$

With a continuum of goods, firms are atomistic and take the denominator of Eq. (5) as given; therefore, monopolistic competition prevails and firms face isoelastic demand curves.

## 2.3. Manufacturing: production and innovation

The typical firm produces with the technology

$$X_i = Z_i^\theta \cdot F_X(L_{X_i} - \phi, M_i), \quad 0 < \theta < 1, \quad \phi > 0 \quad (6)$$

where  $X_i$  is output,  $L_{X_i}$  is production employment,  $\phi$  is a fixed labor cost,  $M_i$  is processed resource use (henceforth “materials” for short),  $Z_i$  is the firm's stock of firm-specific knowledge, and  $F_X(\cdot)$  is a standard neoclassical production function homogeneous of degree one in its arguments. Technology (Eq. (6)) gives rise to total cost

$$W\phi + C_X(W, P_M) Z_i^{-\theta} X_i, \quad (7)$$

where  $C_X(\cdot)$  is a standard unit-cost function homogeneous of degree one in the wage  $W$  and the price of materials  $P_M$ .

The firm accumulates knowledge according to the R&D technology

$$\dot{Z}_i = \alpha K L_{Z_i}, \quad \alpha > 0 \quad (8)$$

where  $\dot{Z}_i$  measures the flow of firm-specific knowledge generated by an R&D project employing  $L_{Z_i}$  units of labor for an interval of time  $dt$  and  $\alpha K$  is the productivity of labor in R&D as determined by the exogenous parameter  $\alpha$  and by the stock of public knowledge,  $K$ .

Public knowledge accumulates as a result of spillovers. When one firm generates a new idea to improve the production process, it also generates general-purpose knowledge which is not excludable and that other firms can exploit in their own research efforts. Firms appropriate the economic returns from firm-specific knowledge but cannot prevent others from using the general-purpose knowledge that spills over into the public domain. Formally, an R&D project that produces  $\dot{Z}_i$  units of proprietary knowledge also generates  $\dot{Z}_i$  units of public knowledge. The productivity of research is determined by some combination of all the different sources of knowledge. A simple way of capturing this notion is to write

$$K = \int_0^N \frac{1}{N} Z_i di,$$

which says that the technological frontier is determined by the average knowledge of all firms.<sup>4</sup>

## 2.4. The primary or resources sector

In the primary sector competitive firms hire labor,  $L_M$ , to extract and process natural resources,  $R$ , into materials,  $M$ , according to the technology

$$M = F_M(L_M, R), \quad (9)$$

where the function  $F_M(\cdot)$  is a standard neoclassical production function homogeneous of degree one in its arguments. The associated total cost is

$$C_M(W, p)M, \quad (10)$$

where  $C_M(\cdot)$  is a standard unit-cost function homogeneous of degree one in the wage  $W$  and the price of resources  $p$ .

This is the simplest way to model the primary sector for the purposes of this paper. Materials are produced with labor and a natural resource. The natural resource is in fixed endowment and earns rents. The primary sector competes for labor with the manufacturing sector. This captures the fundamental inter-sectoral allocation problem faced by this economy.

## 3. Equilibrium of the market economy

This section constructs the symmetric equilibrium of the manufacturing sector. It then characterizes the equilibrium of the primary sector.

<sup>3</sup> As is clear from this assumption, to keep the model as simple as possible, I posit a non-exhaustible resource like land. For a generalization of the model to the case of a renewable (or exhaustible) resource, see Prasertsom (2010).

<sup>4</sup> For a detailed discussion of the microfoundations of a spillovers function of this class, see Peretto and Smulders (2002).

Finally, it imposes general equilibrium conditions to determine the aggregate dynamics of the economy. The wage rate is the numeraire, i.e.,  $W \equiv 1$ .

### 3.1. Partial equilibrium of the manufacturing sector

The typical manufacturing firm is subject to a death shock. Accordingly, it maximizes the present discounted value of net cash flow,

$$V_i(t) = \int_t^\infty e^{-\int_t^s [r(v) + \delta] dv} \Pi_i(s) ds, \quad \delta > 0$$

where  $e^{-\delta t}$  is the instantaneous probability of death. Using the cost function (Eq. (7)), instantaneous profits are

$$\Pi_{X_i} = [P_i - C_X(1, P_M) Z_i^{-\theta}] X_i - \phi - L_{Z_i},$$

where  $L_{Z_i}$  is R&D expenditure.  $V_i$  is the value of the firm, the price of the ownership share of an equity holder. The firm maximizes  $V_i$  subject to the R&D technology (Eq. (8)), the demand schedule (Eq. (5)),  $Z_i(t) > 0$  (the initial knowledge stock is given),  $Z_j(t')$  for  $t' \geq t$  and  $j \neq i$  (the firm takes as given the rivals' innovation paths), and  $Z_j(t') \geq 0$  for  $t' \geq t$  (innovation is irreversible). The solution of this problem yields the (maximized) value of the firm given the time path of the number of firms.

To characterize entry, I assume that upon payment of a sunk cost  $\beta P_i X_i$ , an entrepreneur can create a new firm that starts out its activity with productivity equal to the industry average.<sup>5</sup> Once in the market, the new firm implements price and R&D strategies that solve a problem identical to the one outlined above. Hence, a free entry equilibrium requires  $V_i = \beta P_i X_i$ .

Appendix A shows that the equilibrium thus defined is symmetric and is characterized by the factor demands:

$$L_X = Y \frac{\epsilon-1}{\epsilon} (1 - S_X^M) + \phi N; \quad (11)$$

$$M = Y \frac{\epsilon-1}{\epsilon} \frac{S_X^M}{P_M}, \quad (12)$$

where

$$S_X^M \equiv \frac{P_M M_i}{C_X(W, P_M) Z_i^{-\theta} X_i} = \frac{\partial \log C_X(W, P_M)}{\partial \log P_M}.$$

Associated to these factor demands are the returns to cost reduction and entry, respectively:

$$r = r_Z \equiv \alpha \left[ \frac{Y \theta (\epsilon-1)}{\epsilon N} - \frac{L_Z}{N} \right] - \delta; \quad (13)$$

$$r = r_N \equiv \frac{1}{\beta} \left[ \frac{1}{\epsilon} - \frac{N}{Y} \left( \phi + \frac{L_Z}{N} \right) \right] + \hat{Y} - \hat{N} - \delta. \quad (14)$$

Note how both rates of returns depend positively on firm size  $Y/N$ .

### 3.2. General equilibrium

Competitive resource-processing firms produce up to the point where  $P_M = C_M(1, p)$  and demand factors according to:

$$R = S_M^R \frac{M P_M}{p} = Y \frac{\epsilon-1}{\epsilon} \frac{S_X^M S_M^R}{p}, \quad (15)$$

$$L_M = (1 - S_M^R) M P_M = Y \frac{\epsilon-1}{\epsilon} S_X^M (1 - S_M^R), \quad (16)$$

where

$$S_M^R \equiv \frac{\partial \log C_M(W, p)}{\partial \log p}$$

and I have used Eqs. (11) and (12) to obtain the expressions after the second equality sign. These factor demands yield that the competitive resource firms make zero profits.

Equilibrium of the primary sector requires  $R = \Omega$ . One can thus think of Eq. (15) as the equation that determines the price of the natural resource, and therefore resource income for the household, given the level of economic activity, measured by expenditure on consumption goods  $Y$ .

The remainder of the model consists of the household's budget constraint (Eq. (2)), the labor demands (Eqs. (11) and (16)), the returns to saving, cost reduction and entry in Eqs. (4), (13) and (14). The household's budget constraint becomes the labor market clearing condition (see Appendix A for the derivation):

$$L = L_N + L_X + L_Z + L_M,$$

where  $L_N$  is aggregate employment in entrepreneurial activity,  $L_X + L_Z$  is aggregate employment in production and R&D operations of existing firms and  $L_M$  is aggregate employment of resources-processing firms. Assets market equilibrium requires equalization of all rates of return (no-arbitrage),  $r = r_A = r_Z = r_N$ , and that the value of the household's portfolio equals the value of the securities issued by firms,  $A = NV = \beta Y$ .

### 3.3. Dynamics

Substituting  $A = \beta Y$  into Eq. (2) and using the rate of return to saving Eq. (4), I obtain

$$\frac{Y - L - p\Omega}{Y} = \beta\rho.$$

Then, letting  $y \equiv \frac{Y}{L}$  denote expenditure per capita and  $\omega \equiv \frac{\Omega}{L}$  denote the endowment ratio, I can use this expression, the demand for the resource (Eq. (15)) and the condition  $R = \Omega$  to study the instantaneous equilibrium  $(p^*, y^*)$  as the intersection of the two curves:

$$y = \frac{1 + p\omega}{1 - \beta\rho}; \quad (17)$$

$$y = \frac{1}{1 - \beta\rho - \frac{\epsilon-1}{\epsilon} S_M^R(p) S_X^M(p)}. \quad (18)$$

The first describes how resource income determines expenditure on consumption goods; the second how expenditure drives demand for the factors of production and thereby determines resource income.

The interpretation of this solution is that the interaction of the resource and manufacturing goods markets with the expenditure decision of the household determines the endogenous but *constant* values of all of the relevant nominal variables (given my choice of numeraire  $W \equiv 1$ ). As in first-generation endogenous growth models, this property yields at any point in time a constant interest rate and simplifies drastically the overall dynamics, the difference is that here the constant interest rate does not yield a jump to the steady state but a smooth transition.

To see this, note that given the pair  $(p^*, y^*)$ ,  $Y^* = L y^*$  is constant and the Euler equation (Eq. (4)) yields  $r^* = \rho$ . Assuming that the economy is always in the region where firms invest in vertical R&D (see

<sup>5</sup> See Etro (2004) and, in particular, Peretto and Connolly (2007) for a more detailed discussion of the microfoundations of this assumption.

Appendix A for details), this result allows me to solve Eqs. (8) and (13) for

$$\hat{Z} = \alpha \frac{L_Z}{N} = \frac{Y^* \alpha \theta (\epsilon - 1)}{N \epsilon} - \rho - \delta. \quad (19)$$

Substituting this result into Eq. (14) then yields

$$\hat{N} = \frac{1}{\beta} \left[ \frac{1 - \theta (\epsilon - 1)}{\epsilon} - \left( \phi - \frac{\rho + \delta}{\alpha} \right) \frac{N}{Y^*} \right] - (\rho + \delta).$$

The general equilibrium of the model thus reduces to a single differential equation in the mass of firms. The economy converges to

$$N^* = \frac{\frac{1 - \theta (\epsilon - 1)}{\epsilon} - (\rho + \delta) \beta}{\phi - \frac{\rho + \delta}{\alpha}} Y^*, \quad (20)$$

which substituted into Eq. (19) yields

$$\hat{Z}^* = \frac{\phi \alpha - (\rho + \delta)}{\frac{1 - \theta (\epsilon - 1)}{\epsilon} - (\rho + \delta) \beta} \frac{\theta (\epsilon - 1)}{\epsilon} - (\rho + \delta). \quad (21)$$

This steady-state growth rate is independent of the endowments  $L$  and  $\Omega$  because there is no scale effect.

It is insightful to use the value  $N^*$  in Eq. (20) and rewrite the differential equation for  $N$  as

$$\hat{N} = \nu \left( 1 - \frac{N}{N^*} \right), \quad \nu \equiv \frac{1 - \theta (\epsilon - 1)}{\beta \epsilon} - (\rho + \delta) \quad (22)$$

which is a logistic (see, e.g., Banks, 1994) with growth coefficient  $\nu$  and carrying capacity  $N^*$ . It has the solution

$$N(t) = \frac{N^*}{1 + e^{-\nu t} \left( \frac{N^*}{N_0} - 1 \right)}, \quad (23)$$

where  $N_0$  is the initial condition.

The interpretation of this extremely tractable structure is that at any point in time the equilibrium of factors market and the consumption/saving decision of the household determine the size of the market for manufacturing goods  $Y^*$ . This, in turn, determines the carrying capacity coefficient  $N^*$  in the logistic equation characterizing the equilibrium proliferation of products in the economy.

#### 4. Properties of the equilibrium: technology, the path of consumption and welfare

Recall that the variable  $y$  studied in the previous section is not consumption per capita but expenditure per capita. To get consumption per capita,  $y$  needs to be divided by the economy's CPI,

$$P_Y = \left[ \int_0^N p_j^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}}.$$

Accordingly, since manufacturing firms set prices at a markup  $\frac{\epsilon}{\epsilon-1}$  over marginal cost (see Appendix A), in symmetric equilibrium the flow of consumption in the utility function (Eq. (3)), is

$$\frac{y}{P_Y} = \frac{\epsilon-1}{\epsilon} \frac{y^*}{c^*} N^{\frac{1}{\epsilon-1}} Z^{\theta},$$

where

$$c^* \equiv C_X(1, C_M(1, p^*)).$$

Consequently, we have

$$\log u^* = \log \left( \frac{\epsilon-1}{\epsilon} \frac{y^*}{c^*} Z^{\theta} N^{\frac{1}{\epsilon-1}} \right).$$

We can reinterpret the utility function (Eq. (3)) as a production function for a final homogenous good assembled from intermediate goods, so that  $u$  is a measure of output, and define aggregate TFP as

$$T \equiv Z^{\theta} N^{\frac{1}{\epsilon-1}}. \quad (24)$$

The key property of the model, then, is that the analytical solution for the path  $N(t)$  yields the analytical solution for the path  $T(t)$ .

To see this, observe first that taking logs and time derivatives Eq. (24) yields

$$\hat{T}(t) = \theta \hat{Z}(t) + \frac{1}{\epsilon-1} \hat{N}(t),$$

where  $\hat{Z}(t)$  is given by Eq. (19) and  $\hat{N}(t)$  by Eq. (22). In steady state we thus have

$$\hat{T}^* = \theta \hat{Z}^* \equiv g^*,$$

which is independent of the endowments  $L$  and  $\Omega$ , and thus of  $\omega$ . Moreover, according to Eq. (20) in steady state we have

$$\frac{Y^*}{N^*} = \frac{Y_0}{N_0} \Rightarrow \frac{Y^*}{Y_0} = \frac{N^*}{N_0}.$$

We can thus define

$$\Delta^* \equiv \frac{N^*}{N_0} - 1 = \frac{Y^*}{Y_0} - 1,$$

which is the percentage change in expenditure that the economy experiences in response to changes in fundamentals and/or policy parameters. In the interpretation proposed earlier, it fully summarizes the effects of such changes on the scale of economic activity and, in particular, on the economy's carrying capacity for firms/products. The mechanism underlying my results, therefore, is an impulse-response dynamics where  $\Delta^*$  is the (permanent) impulse and the logistic equation (Eq. (23)) governs the response. The following proposition states the result formally.

**Proposition 1.** Let  $\log u^*(t)$  and  $U^*$  be, respectively, the instantaneous consumption index (3) and the welfare function (1) evaluated at  $y^*$ . Then, a path starting at time  $t=0$  with initial condition  $N_0$  and converging to the steady state  $N^*$  is characterized by:

$$\log T(t) = \log T_0 + g^* t + \left( \frac{\gamma}{\nu} + \frac{1}{\epsilon-1} \right) \Delta^* (1 - e^{-\nu t}), \quad (25)$$

where

$$\gamma \equiv \theta \frac{\alpha \theta (\epsilon - 1)}{\epsilon} \frac{\phi - \frac{\rho + \delta}{\alpha}}{\frac{1 - \theta (\epsilon - 1)}{\epsilon} - (\rho + \delta) \beta}.$$



Therefore,

$$\log u^*(t) = \log \frac{y^*}{c^*} + g^*t + \left( \frac{\gamma}{\nu} + \frac{1}{\epsilon-1} \right) \Delta^* (1 - e^{-\nu t}), \quad (26)$$

where without loss of generality  $\frac{\epsilon-1}{\epsilon} T_0 = 1$ . Upon integration this yields

$$U^* = \frac{1}{\rho} \left[ \log \left( \frac{y^*}{c^*} \right) + \frac{g^*}{\rho} + \mu \Delta^* \right], \quad (27)$$

where

$$\mu \equiv \frac{\gamma + \frac{\nu}{\epsilon-1}}{\rho + \nu}.$$

**Proof.** See Appendix A.  $\square$

The way the model's mechanism works, then, is straightforward. First, the lack of scale effects implies that the endowment shock studied in this paper does not affect steady-state TFP growth  $g^*$ . Second, the endowment shock perturbs the equilibrium of the resource market and causes the resource price  $p^*$  to fall. This in turn has two effects: it yields a change in the household resource income  $p^*\Omega$ , which, through the intertemporal consumption-saving choice, shows up as an instantaneous change in expenditure per capita  $y^*$ , and a fall in the CPI due to the fall of the cost of production of consumption goods  $c^*$ . The change in expenditure is seen by firms and entrepreneurs as a change in market size  $Y^*$  that triggers changes in the level and composition of R&D activity and thereby a temporary deviation of the economy's growth rate of TFP  $g(t)$  from its steady-state value  $g^*$ . The temporary change in the growth rate of TFP drives the temporary deviation of the rate of decay of the economy's CPI from its steady-state value and thereby the temporal evolution of consumption.

**Proposition 1** pulls all these effects together as follows: the transitional component of the TFP operator in Eq. (25) summarizes the cumulated gain/loss due to above/below steady-state cost reduction and product variety expansion; the expression for flow utility (Eq. (26)) shows separately the initial jump due to  $y^*/c^*$  and the smooth evolution due to  $T$ ; the expression for (27) then shows how upon integration the path of utility collapses to a single number whose value changes with  $\Omega$  through two channels: steady-state (real) expenditure calculated holding technology constant, i.e., the initial jump in consumption, and the transitional acceleration/deceleration of TFP relative to the steady-state path.

## 5. Resource abundance, growth and welfare

This section begins with a discussion of the conditions under which a change in the endowment ratio raises or lowers consumption expenditure so that the market for manufacturing goods expands or contracts. It then shows how the interaction of the initial change in consumption and the transition dynamics after the shock produce a change in welfare whose sign can be assessed analytically.

### 5.1. Expenditure and prices

The first step in the assessment of the effects of resource abundance is to use Eqs. (17) and (18) to characterize expenditure and prices. The following property of the demand functions (12) and (15) is useful.

**Lemma 2.** Let:

$$\begin{aligned} \epsilon_X^M &\equiv -\frac{\partial \log M}{\partial \log P_M} = 1 - \frac{\partial \log S_X^M}{\partial \log P_M} = 1 - \frac{\partial S_X^M}{\partial P_M} \frac{P_M}{S_X^M}, \\ \epsilon_M^R &\equiv -\frac{\partial \log R}{\partial \log p} = 1 - \frac{\partial \log S_M^R}{\partial \log p} = 1 - \frac{\partial S_M^R}{\partial p} \frac{p}{S_M^R}. \end{aligned}$$

Then,

$$\frac{\partial (S_M^R(p) S_X^M(p))}{\partial p} = \Gamma(p) \frac{S_M^R(p) S_X^M(p)}{p}, \quad (28)$$

where

$$\Gamma(p) \equiv \left( 1 - \epsilon_X^M(p) \right) S_M^R(p) + 1 - \epsilon_M^R(p).$$

**Proof.** See Appendix A.  $\square$

$\Gamma(p)$  is the elasticity of  $S_M^R(p) S_X^M(p)$  with respect to  $p$ . According to Eq. (12), therefore, it is the elasticity of the demand for the resource  $R$  with respect to its price  $p$ , holding constant expenditure per capita  $y$ . It thus captures the partial equilibrium effects of price changes in the resource and materials markets for given market size and regulates the shape of the income relation. Differentiating Eq. (18), rearranging terms and using Eq. (15) yields

$$\frac{d \log y(p)}{dp} = \frac{\frac{\epsilon-1}{\epsilon} \frac{d(S_M^R(p) S_X^M(p))}{dp}}{1 - \beta \rho - \frac{\epsilon-1}{\epsilon} S_M^R(p) S_X^M(p)} = \omega \Gamma(p),$$

which says that the effect of changes in the resource price on expenditure on manufacturing goods depends on the overall pattern of substitution that is reflected in the price elasticities of materials and resource demand and in the resource share of materials production costs. To see the pattern most clearly, pretend for the time being that  $\Gamma(p)$  does not change sign with  $p$ . I comment later on how allowing  $\Gamma(p)$  to change sign for some  $p$  makes the model even more interesting. The following proposition states the results formally, Fig. 1 illustrates the mechanism.

**Proposition 3.** Suppose that  $\Gamma(p)$  is positive, zero or negative for all  $p$ . Then, there are three cases.

1. *Complementarity.* This occurs when  $\Gamma(p) > 0$  and the income relation (18) is a monotonically increasing function of  $p$  with domain  $p \in [0, \infty)$  and codomain  $y \in [y^*(0), y^*(\infty))$ , where

$$y^*(0) = \frac{1}{1 - \beta \rho - \frac{\epsilon-1}{\epsilon} S_M^R(0) S_X^M(0)},$$

$$y^*(\infty) = \frac{1}{1 - \beta \rho - \frac{\epsilon-1}{\epsilon} S_M^R(\infty) S_X^M(\infty)}.$$

Then there exists a unique equilibrium  $(p^*(\omega), y^*(\omega))$  with the property:

$$\begin{aligned} p^*(\omega) : (0, \infty) &\rightarrow (\infty, 0), \quad \frac{dp^*(\omega)}{d\omega} < 0 \quad \forall \omega; \\ y^*(\omega) : (0, \infty) &\rightarrow [y^*(\infty), y^*(0)], \quad \frac{dy^*(\omega)}{d\omega} < 0 \quad \forall \omega. \end{aligned}$$

2. *Cobb–Douglas-like economy.* This occurs when  $S_M^R$  and  $S_X^M$  are exogenous constants,  $\Gamma(p) = 0$  and the income relation (18) is the flat line

$$y = \frac{1}{1 - \beta \rho - \frac{\epsilon-1}{\epsilon} S_M^R S_X^M} \equiv y_{CD}^*.$$

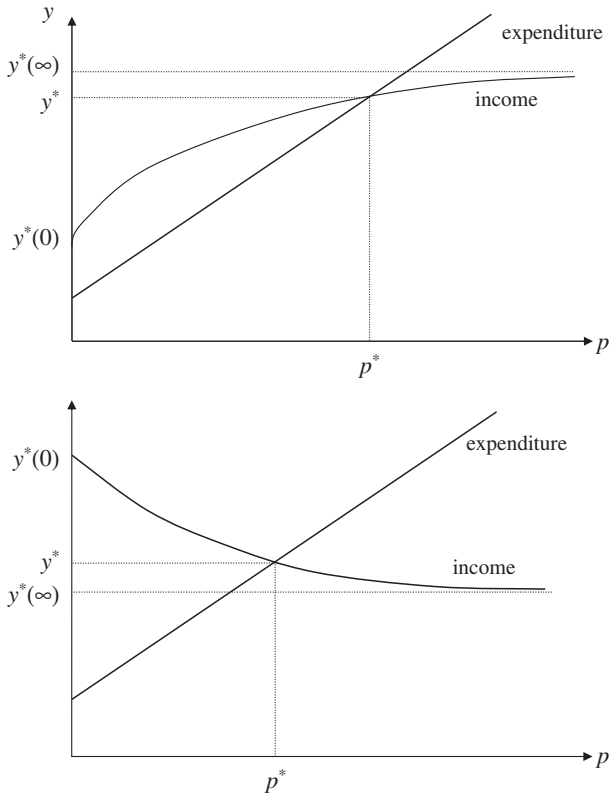


Fig. 1. General equilibrium: monotonic income relation.

Then there exists a unique equilibrium  $(p_{CD}^*(\omega), y_{CD}^*(\omega))$  with the property:

$$p_{CD}^*(\omega) : (0, \infty) \rightarrow (\infty, 0), \quad \frac{dp^*(\omega)}{d\omega} < 0 \quad \forall \omega.$$

3. *Substitution.* This occurs when  $\Gamma(p) < 0$  and the income relation (18) is a monotonically decreasing function of  $p$  with domain  $p \in [0, \infty)$  and codomain  $y \in (y^*(\infty), y^*(0)]$ , where

$$y^*(\infty) = \frac{1}{1 - \beta\rho - \frac{\epsilon - 1}{\epsilon} S_M^R(\infty) S_X^M(\infty)},$$

$$y^*(0) = \frac{1}{1 - \beta\rho - \frac{\epsilon - 1}{\epsilon} S_M^R(0) S_X^M(0)}.$$

Then there exists a unique equilibrium  $(p^*(\omega), y^*(\omega))$  with the property:

$$p^*(\omega) : (0, \infty) \rightarrow (\infty, 0), \quad \frac{dp^*(\omega)}{d\omega} < 0 \quad \forall \omega;$$

$$y^*(\omega) : (0, \infty) \rightarrow [y^*(\infty), y^*(0)], \quad \frac{dy^*(\omega)}{d\omega} > 0 \quad \forall \omega.$$

**Proof.** See Appendix A.  $\square$

I refer to the second case as the Cobb–Douglas-like economy because this seemingly special configuration, requiring  $\epsilon_M^R = 1 + (1 - \epsilon_X^M) S_M^R$ , is in fact quite common in the literature as it occurs when both technologies are Cobb–Douglas and  $\epsilon_X^M = \epsilon_M^R = 1$ .

Proposition 3 says that the effect of resource abundance on the resource price is *always* negative, while the effect on expenditure

changes sign according to the substitution possibilities between labor and materials in manufacturing and between labor and the natural resource in materials production. This brings us to the observation that if  $\Gamma(p)$  changes sign for some  $p$ , the model generates endogenously a switch from substitution to complementarity. Fig. 2, where the income relation (Eq. (18)) is a hump-shaped function of  $p$ , illustrates this case.

The pattern is best captured by looking at the properties of the function  $\Gamma(p)$ . Using the definitions in Lemma 2,

$$\begin{aligned} \frac{d\Gamma(p)}{dp} = & -\frac{d\epsilon_X^M(P_M)}{dP_M} \frac{dP_M}{dp} S_M^R(p) \\ & + (1 - \epsilon_X^M(P_M)) (1 - \epsilon_M^R(p)) - \frac{d\epsilon_M^R(p)}{dp}. \end{aligned}$$

This derivative is negative if the following two conditions hold:

- the elasticities  $\epsilon_X^M$  and  $\epsilon_M^R$  are increasing in  $P_M$  and  $p$ , respectively;
- the terms  $1 - \epsilon_X^M$  and  $1 - \epsilon_M^R$  have opposite sign.

The hump-shaped income relation in Fig. 2 then obtains if  $\Gamma(0) > 0$  and  $\Gamma(\infty) < 0$ . The second condition says that if demand in one sector is elastic, say  $1 < \epsilon_X^M$ , then demand in the other sector is inelastic,  $1 > \epsilon_M^R$ . Below I provide an example of how one can use CES technologies to construct an economy where these conditions hold. Here I discuss the general property.

**Proposition 4.** Suppose that there exists a price  $\bar{p}$  where  $\Gamma(p)$  changes sign, from positive to negative, so that the income relation (18) is a hump-shaped function of  $p$  with domain  $p \in [0, \infty)$  and codomain  $y \in [y^*(0), y^*(\infty))$  or  $y \in [y^*(\infty), y^*(0))$ , where

$$y^*(0) = \frac{1}{1 - \beta\rho - \frac{\epsilon - 1}{\epsilon} S_M^R(0) S_X^M(0)},$$

$$y^*(\infty) = \frac{1}{1 - \beta\rho - \frac{\epsilon - 1}{\epsilon} S_M^R(\infty) S_X^M(\infty)}.$$

Then there exists a unique equilibrium  $(p^*(\omega), y^*(\omega))$  with the property:

$$p^*(\omega) : (0, \infty) \rightarrow (\infty, 0), \quad \frac{dp^*(\omega)}{d\omega} < 0 \quad \forall \omega;$$

$$y^*(\omega) : (0, \infty) \rightarrow [y^*(\infty), y^*(0)], \quad \frac{dy^*(\omega)}{d\omega} \begin{cases} > 0 & \omega < \bar{\omega} \\ < 0 & \omega > \bar{\omega} \end{cases},$$

Where  $\bar{\omega}$  is the value of  $\omega$  such that  $p^*(\bar{\omega}) = \bar{p}$ .

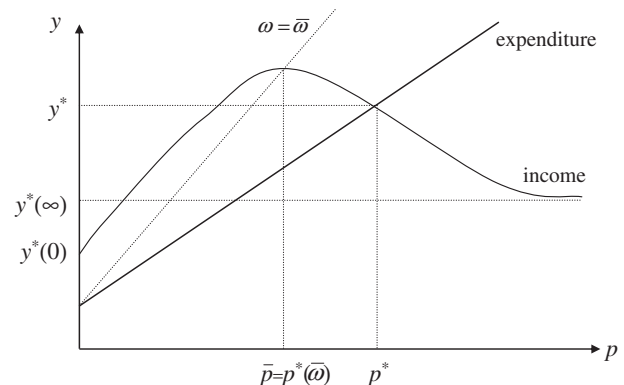


Fig. 2. General equilibrium: hump-shaped income relation.

**Proof.** See Appendix A.  $\square$

The main message of this analysis is that resource abundance raises expenditure, and thereby results into a larger market for manufacturing goods, when the economy exhibits overall substitution between labor and resources (processed and raw) in the manufacturing of consumption goods and the processing of the natural resource into materials. Conversely, when the economy exhibits overall complementarity resource abundance results into a smaller market for manufacturing goods. More importantly, whether the economy exhibits substitution or complementarity depends on equilibrium prices and thus on the endowment ratio itself. In other words, there are solid reasons to expect that the effect of the endowment ratio on the path of consumption is non-monotonic.

## 5.2. The path of consumption and welfare

For concreteness, consider an economy in steady state  $(p^*, y^*)$  and imagine an increment  $d\omega$  in its endowment ratio. Then, by construction we have

$$\Delta^* = \frac{y^*(\omega) + \frac{dy^*(\omega)}{d\omega}}{y^*(\omega)} - 1 = \frac{1}{y^*(\omega)} \frac{dy^*(\omega)}{d\omega}$$

and we can use the results of the previous section in a straightforward manner. Let us look first at the initial effect on consumption.

**Proposition 5.** *The impact effect of the change in the endowment ratio is*

$$\frac{d \log \left( \frac{y^*(\omega)}{c^*(\omega)} \right)}{d\omega} = [\Gamma(p^*(\omega)) - \Psi(p^*(\omega))] \omega \frac{dp^*(\omega)}{d\omega}, \quad (29)$$

where

$$\Psi(p^*(\omega)) \equiv \frac{\epsilon}{(\epsilon-1)y^*(\omega)} = \kappa - S_M^R(p^*(\omega)) S_X^M(p^*(\omega)),$$

$$\kappa \equiv \frac{\epsilon}{\epsilon-1} (1 - \beta\rho) > 1.$$

**Proof.** See Appendix A.  $\square$

Eq. (26), Proposition 5 and the result in Proposition 3 that  $\frac{dp^*(\omega)}{d\omega} < 0$  in all cases (complementarity, Cobb–Douglas, substitution) then yield three possible configurations for the path of consumption:

1.  $\Gamma(p^*(\omega)) < 0 < \Psi(p^*(\omega))$ . The initial jump is up and then we have a growth acceleration. Welfare rises.
2.  $0 < \Gamma(p^*(\omega)) < \Psi(p^*(\omega))$ . The initial jump is up and then we have a growth deceleration. The welfare change is ambiguous since we have an intertemporal trade-off.
3.  $0 < \Psi(p^*(\omega)) < \Gamma(p^*(\omega))$ . The initial jump is down and then we have a growth deceleration. Welfare falls.

Fig. 3 illustrates these possibilities. Case 2 highlights the need to look at welfare directly to resolve the ambiguity.

**Proposition 6.** *The welfare effect of a change in the endowment ratio is*

$$\frac{dU^*(\omega)}{d\omega} = \frac{1}{\rho - \lambda} [(1 + \mu)\Gamma(p^*(\omega)) - \Psi(p^*(\omega))] \omega \frac{dp^*(\omega)}{d\omega}. \quad (30)$$

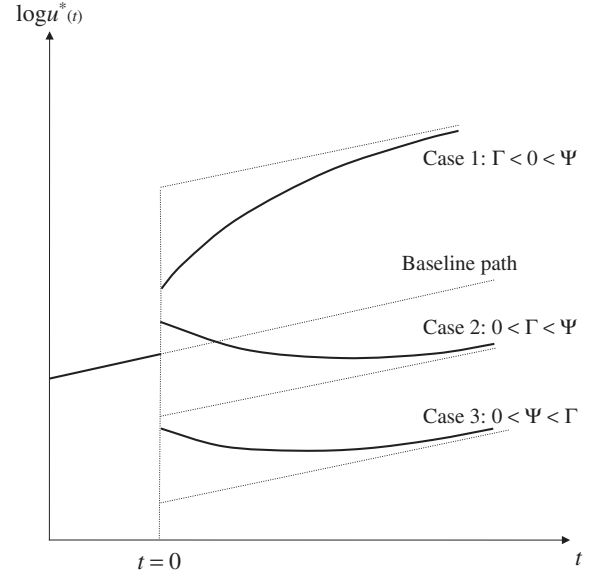


Fig. 3. The path of consumption.

**Proof.** See Appendix A.  $\square$

We can then characterize four scenarios for welfare.

1. The rise of the endowment ratio generates a growth acceleration associated to an initial jump up in consumption when

$$(1 + \mu)\Gamma(p^*(\omega)) < \Gamma(p^*(\omega)) \leq 0 < \Psi(p^*(\omega)).$$

In this case welfare rises unambiguously because the whole path of utility after the shock lies above the pre-shock path.<sup>6</sup>

2. The rise of the endowment ratio causes a growth deceleration, but the deceleration is offset by the fact that resource abundance reduces prices and raises the level of consumption. This happens when

$$0 < \Gamma(p^*(\omega)) < (1 + \mu)\Gamma(p^*(\omega)) < \Psi(p^*(\omega)).$$

In this case as well welfare rises.

3. The rise of the endowment ratio generates a growth deceleration that dominates over the initial jump up in consumption when

$$0 < \Gamma(p^*(\omega)) < \Psi(p^*(\omega)) < (1 + \mu)\Gamma(p^*(\omega)).$$

In contrast to the previous case, welfare now falls.

4. The rise of the endowment ratio generates a growth deceleration associated to an initial fall of consumption when

$$0 < \Psi(p^*(\omega)) < \Gamma(p^*(\omega)) < (1 + \mu)\Gamma(p^*(\omega)).$$

This is the worst-case scenario in which welfare clearly falls because the whole path of utility after the shock is below the pre-shock one.

To map these scenarios into values of the endowment ratio itself it is useful (albeit not necessary) to impose more structure.

<sup>6</sup> Notice that this includes the Cobb–Douglas economy since in that case  $1 = \epsilon_X^M = \epsilon_M^R$  and growth does not respond to  $\omega$  at all while lower prices yield higher utility.



### 5.3. The role of $\omega$ : a CES economy

Consider the following technologies:

$$X_i = Z_i^\theta [\psi_X (L_{X_i} - \phi)^{\sigma_X} + (1 - \psi_X) M_i^{\sigma_X}]^{\frac{1}{\sigma_X}}, \quad \sigma_X \leq 1;$$

$$M = [\psi_M L_M^{\sigma_M} + (1 - \psi_M) R^{\sigma_M}]^{\frac{1}{\sigma_M}}, \quad \sigma_M \leq 1.$$

From the associated variable unit-cost functions (see [Appendix A](#)) one derives (recall that  $W \equiv 1$ ):

$$S_X^M = \frac{1}{1 + \left( \frac{\psi_X}{1 - \psi_X} \right)^{\frac{1}{1 - \sigma_X}} p^{\frac{\sigma_X}{1 - \sigma_X}}}; \quad \epsilon_X^M = 1 + \frac{\sigma_X}{1 - \sigma_X} (1 - S_X^M);$$

$$S_M^R = \frac{1}{1 + \left( \frac{\psi_M}{1 - \psi_M} \right)^{\frac{1}{1 - \sigma_M}} p^{\frac{\sigma_M}{1 - \sigma_M}}}; \quad \epsilon_M^R = 1 + \frac{\sigma_M}{1 - \sigma_M} (1 - S_M^R).$$

As is well known, the CES contains as special cases the linear production function ( $\sigma = 1$ ) wherein inputs are perfect substitutes, the Cobb–Douglas ( $\sigma = 0$ ) wherein the elasticity of substitution between inputs is equal to 1, and the Leontief ( $\sigma = -\infty$ ) wherein inputs are perfect complements.

Let:

$$\Gamma(\omega) \equiv \Gamma(p^*(\omega)) = (1 - \epsilon_X^M(p^*(\omega))) S_M^R(p^*(\omega)) + 1 - \epsilon_M^R(p^*(\omega));$$

$$\Psi(\omega) \equiv \Psi(p^*(\omega)) = \kappa - S_M^R(p^*(\omega)) S_X^M(p^*(\omega)).$$

Observe that if  $\sigma_X > 0$  and  $\sigma_M > 0$ , then  $\Gamma(p) < 0$  for all  $p$  and growth always accelerates. If  $\sigma_X < 0$  and  $\sigma_M < 0$ , instead,  $\Gamma(p) > 0$  for all  $p$  and growth always slows down. Interestingly, if  $\sigma_X$  and  $\sigma_M$  have opposite signs we can capture how the economy moves from one case to the other as the endowment ratio changes. Specifically, let  $\sigma_X > 0$  and  $\sigma_M < 0$  so that the manufacturing sector exhibits gross substitution between labor and materials while the resource sector exhibits gross complementarity between labor and raw resources. Recall that this means that demand for processed resources, i.e., materials, in the manufacturing sector is elastic, while demand for raw resources in the primary sector is inelastic.

It is then straightforward to obtain the following three cases.

1. Welfare rises because growth accelerates and the initial jump in consumption is up when

$$\Gamma(\omega) \leq 0 \Leftrightarrow 0 < \omega < \bar{\omega}.$$

2. Welfare rises because the initial jump up in consumption dominates the growth deceleration when

$$0 < (1 + \mu)\Gamma(\omega) < \Psi(\omega) \Leftrightarrow \bar{\omega} < \omega < \tilde{\omega}.$$

3. Welfare falls because the initial jump up in consumption is dominated by the growth deceleration when

$$0 < \Psi(\omega) < (1 + \mu)\Gamma(\omega) \Leftrightarrow \tilde{\omega} < \omega < \infty.$$

[Fig. 4](#) illustrates this analysis. [Appendix A](#) establishes formally the properties of the curves  $\Gamma(\omega)$  and  $\Psi(\omega)$  used in the figure to find the threshold values of the endowment ratio. Here I focus on the economics.

First, and most important: Why does the overall pattern of substitution/complementarity matter so much for growth and welfare? Because it regulates the reduction in the price  $p$  required for the economy to absorb the extra endowment of  $\Omega$  relative to  $L$ . Moreover, the primary and manufacturing sectors are vertically related so the adjustment involves the interdependent responses of  $P_M$  to the increase in supply of processed resources and of  $p$  to the increase in supply of raw resources. Now, inelastic demand means that the price has to fall drastically to induce the market to absorb the additional quantity. Hence, if demand is inelastic in both sectors the overall adjustment requires drastic drops in both  $P_M$  and  $p$ . What matters is that the drastic fall of  $p$  results in a fall of resources income  $p\omega$  which depresses expenditure  $y$ . This is the case  $\Gamma > 0$ . In contrast, if demand is elastic in both sectors the overall adjustment requires mild drops in prices. The crucial difference is that in this case  $\Gamma < 0$  so that resources income  $p\omega$  rises because the quantity effect dominates the price effect.

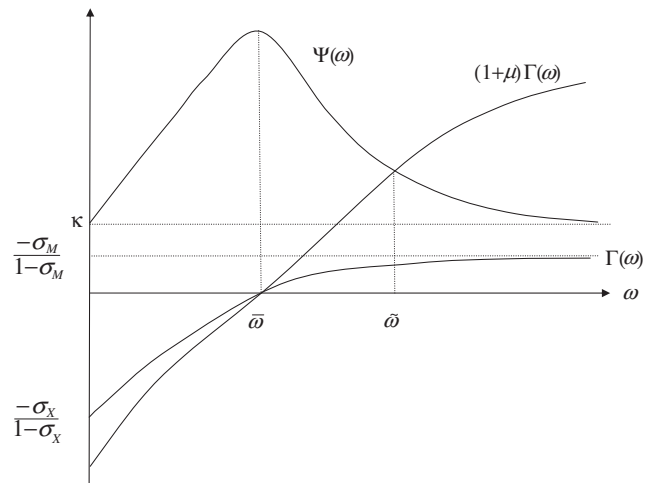
With this intuition in hand, we can now interpret the analytical results of the model. The function  $\Gamma(\omega)$  starts out negative and increases monotonically, changing sign at  $\bar{\omega}$  and converging to its positive upper bound as  $\omega \rightarrow \infty$ . To see this, write

$$\Gamma'(\omega) = \frac{d\Gamma(p^*)}{dp^*} \frac{dp^*}{d\omega} > 0 \quad \forall \omega$$

and observe that this economy satisfies the conditions for  $d\Gamma(p)/dp < 0 \forall p$ , namely:  $\epsilon_X^M > 1$  and increasing in  $P_M$ ;  $\epsilon_M^R < 1$  and increasing in  $p$ .

The function  $\Psi(\omega)$  is hump-shaped with a peak exactly at  $\bar{\omega}$ , where  $\Gamma(\omega)$  changes sign. The reason is that this is the value where the derivative of  $S_M^R(p) S_X^M(p)$  with respect to  $p$  equals zero. Notice also that  $\Gamma(\omega) < \Psi(\omega)$  for all  $\omega$  (see [Appendix A](#) for the proof) so that Case 3 from [Fig. 3](#) and Scenario 4 from the analysis of welfare no longer apply because initial consumption cannot fall.

The resulting pattern is the following: The endowment ratio is initially very low and prices are very high, that is,  $\omega \rightarrow 0 \Rightarrow p \rightarrow \infty \Rightarrow P_M = C_M(1, p) \rightarrow \infty$ . Under these conditions,  $\Gamma(0) < 0 < \Psi(0)$  so that an increase in  $\omega$  produces a growth acceleration associated to a jump up in initial consumption. Intuitively, this says that in a situation of extreme scarcity and extremely high price a helicopter drop of natural resource is good. As the relative endowment grows, the overall pattern of substitution changes. In particular,  $\Gamma(\omega)$  changes sign at  $\omega = \bar{\omega}$  and we enter the region where we get a “curse of natural resources” because an increase in resource abundance yields a decrease in expenditure on manufacturing goods that triggers a slowdown of TFP growth. This, “curse”, however, is not really a curse since we are to the left of  $\tilde{\omega}$  and the slowdown is



**Fig. 4.** The endowment ratio and equilibrium outcomes.

associated to an initial jump up in consumption that dominates in the intertemporal trade-off. As we keep moving to the right and enter the next region,  $\bar{\omega} < \omega < \infty$ , the growth deceleration is again associated to an initial jump up in consumption but now the deceleration dominates in the intertemporal trade-off and welfare falls. It is here that we have a curse.

In the analysis above I set  $\sigma_M < 0$  and  $\sigma_X > 0$  because resource economists see poor substitution in the resource-intensive sector and good substitution in manufacturing as the empirically plausible configuration. The model, however, provides a complete characterization for any configuration of substitution/complementarity across the two sectors. It is useful to look at one that generates the initial fall in consumption ruled out under  $\sigma_M < 0$  and  $\sigma_X > 0$ . Recall that if  $\sigma_M < 0$  and  $\sigma_X < 0$  we have  $\Gamma(p) > 0$  for all  $p$  and the “curse” always occurs because  $dy^*/d\omega < 0$  for all  $\omega$ . Interestingly, in this case the  $\Gamma(\omega)$  and  $\Psi(\omega)$  curves are both monotonically increasing in  $\omega$  with  $\Gamma(0) = 0$ ,  $\Psi(0) = \kappa - 1$ ,  $\Gamma(\infty) = -\sigma_M/(1 - \sigma_M)$ , and  $\Psi(\infty) = \kappa$ . Consequently, if we assume  $\kappa < -\sigma_M/(1 - \sigma_M)$  the curves intersect and there exists a threshold value of  $\omega$  such that for  $\omega$  larger than that value  $\Psi(\omega) < \Gamma(\omega)$ . This is the condition for Case 3 from Fig. 3 and Scenario 4 from the analysis of welfare where initial consumption falls. The intuition for this property is that by setting  $\sigma_X < 0$  we impose complementarity in manufacturing as well and we exacerbate its adverse effects throughout the economy’s vertical production structure. When  $\sigma_X > 0$ , in contrast, substitution in manufacturing softens the adverse effect experienced by the resource processing sector and precludes the initial fall of consumption.

It is useful to close this section with some remarks on the generality of the analysis. CES technologies yield a highly tractable structure. However, they are restrictive in that they yield that within each sector demand is either always inelastic or always elastic. To obtain the economy-wide cross over from complementarity to substitution one then needs to exploit the sectoral composition effect and posit that  $\sigma_X$  and  $\sigma_M$  have opposite sign. Nevertheless, the results of Section 5.2 do not require this assumption. Inspection of the function  $\Gamma(\omega)$  suggests that to obtain  $\Gamma(0) < 0$  and  $\Gamma(\infty) > 0$ , and therefore the threshold  $\bar{\omega}$ , all that are needed are technologies that deliver price elasticities of demand that start out below one and turn larger than one as the price of the good rises.

#### 5.4. The labor reallocation

It is insightful to characterize the economy’s reallocation of labor across sectors in detail. Using Eq. (16) we have

$$\frac{L_M}{L} = y \frac{\epsilon - 1}{\epsilon} S_X^M (1 - S_M^R).$$

It is then easy to show that (see Appendix A)

$$\frac{d}{d\omega} \left( \frac{L_M}{L} \right) > 0 \quad \forall \omega,$$

so that, intuitively, resource abundance yields a reallocation of labor from manufacturing to primary production. This result implies that

$$\frac{L_X}{L} + \frac{L_Z + L_N}{L} = 1 - \frac{L_M}{L}$$

falls to its new steady state value when  $\omega$  increases.

This is an interesting property as it says that there is a reallocation of labor from production of manufacturing goods to production of materials, but that this reallocation is *not necessarily* associated to a TFP slowdown. This is a fundamental difference between this model and models that generate the curse of natural resources by tying productivity growth to manufacturing employment through learning by doing mechanisms.

Since the inter-sectoral reallocation is instantaneous, the dynamics that drive the time path of TFP take place within manufacturing. Eqs. (4), (14) and the formulation of the entry cost yield

$$\frac{L_Z + L_N}{L} = \frac{L_Z}{L} + \frac{\dot{N}}{L} \cdot \beta \frac{Y}{N}.$$

Recall that  $Y$  jumps on impact to its steady-state value  $Y^* = Ly^*$  while  $N$  is predetermined and does not jump. Then, using Eq. (23) at any time  $t$  we have

$$\frac{L_Z + L_N}{L} = y^* \left[ \frac{1}{\epsilon} - \beta(\rho + \delta) - \frac{\phi}{1 + e^{-\nu t} \Delta^*} \frac{N^*}{Y^*} \right],$$

where we know from the analysis of Section 3 that  $\frac{N^*}{Y^*}$  is independent of  $\omega$ . Thus, when the economy experiences a growth acceleration because  $y^*$  rises, the R&D share of employment jumps up and converges from above to a permanently higher value, while the ratio  $\frac{L_X}{L}$  jumps down and converges from below to a permanently lower value. The reverse happens when  $y^*$  falls and the economy experiences a growth deceleration.

## 6. Conclusion

The debate on whether natural resource abundance is good or bad for the long-run fortunes of the economy is almost as old as economics itself. To contribute to this debate—and expand its scope—in this paper I developed a closed economy Schumpeterian model of R&D-driven endogenous growth that incorporates an upstream resource-intensive sector. The model yields the analytical solution for the transition path and thus allows one to resolve the intertemporal trade-off and determine whether the effects of natural resource abundance on initial income and growth yield an increase or a decrease in welfare. I found that resource abundance has non-monotonic effects on growth and welfare. More precisely, following the classic procedure for the construction of growth regressions (Barro and Sala-i-Martin, 2004), I showed that average growth over a time interval and the associated welfare level can be represented as hump-shaped functions of resource abundance.

The non-monotonic effect of resource abundance on welfare suggests that there is an optimal value of the endowment ratio. Since for simplicity the model considers a non-exhaustible resource that provides a constant flow of productive services—land, for example—and assigns diffuse ownership of it to atomistic households, it is not immediate to extract policy implications from this optimality result. It is plausible to envision institutional arrangements that regulate the supply of the resource services in order to prevent the economy from moving into the downward sloping part of the welfare curve. But this would require changing drastically the model’s assumptions about ownership and owners’ freedom of use of the resource, something surely interesting but beyond the scope of this paper. Extending the model to the case of exhaustible or renewable resources—where it is more natural to think of supply decisions that regulate the ratio of resource services to labor services available to producers—is another promising and intuitive way to pursue further this insight (for an example, see Prasertsom, 2010).

## Appendix A

### A.1. The decisions of firms and entrepreneurs

Consider the Current Value Hamiltonian

$$CVH_i = [P_i - C_X(1, P_M) Z_i^{-\theta}] X_i - \phi - L_{Z_i} + z_i \alpha K L_{Z_i},$$

where the firm's knowledge,  $Z_i$ , is the state variable, R&D investment,  $L_{Z_i}$ , and the product's price,  $P_i$ , are the control variables,  $z_i$  is the shadow value of  $Z_i$  and the firm takes public knowledge,  $K$ , as given.

Since the Hamiltonian is linear, one has three cases. The case  $1 > z_i \alpha K$  implies that the value of the marginal unit of knowledge is lower than its cost. The firm, then, does not invest. The case  $1 < z_i \alpha K$  implies that the value of the marginal unit of knowledge is higher than its cost. Since the firm demands an infinite amount of labor to employ in R&D, this case violates the general equilibrium conditions and is ruled out. The first order conditions for the interior solution are given by equality between marginal revenue and marginal cost of knowledge,  $1 = z_i \alpha K$ , the constraint on the state variable, Eq. (8), the terminal condition,

$$\lim_{s \rightarrow \infty} e^{-\int_t^s [r(v) + \delta] dv} z_i(s) Z_i(s) = 0,$$

and a differential equation in the costate variable,

$$r + \delta = \frac{\dot{z}_i}{z_i} + \theta C_X(1, P_M) Z_i^{-\theta-1} \frac{X_i}{z_i},$$

that defines the rate of return to R&D as the ratio between revenues from the knowledge stock and its shadow price plus (minus) the appreciation (depreciation) in the value of knowledge. The revenue from the marginal unit of knowledge is given by the cost reduction it yields times the scale of production to which it applies. The price strategy is

$$P_i = C_X(1, P_M) Z_i^{-\theta} \frac{\epsilon}{\epsilon-1}. \quad (31)$$

Peretto (1998, Proposition 1) shows that under the restriction  $1 > \theta(\epsilon-1)$  the firm is always at the interior solution, where  $1 = z_i \alpha K$  holds, and equilibrium is symmetric.

The cost function (7) gives rise to the conditional factor demands:

$$L_{X_i} = \frac{\partial C_X(W, P_M)}{\partial W} Z_i^{-\theta} X_i + \phi;$$

$$M_i = \frac{\partial C_X(W, P_M)}{\partial P_M} Z_i^{-\theta} X_i.$$

Then, Eq. (31), symmetry and aggregation across firms yield Eqs. (11) and (12).

Also, in symmetric equilibrium  $K=Z=Z_i$  yields  $\dot{K}/K = \alpha L_Z/N$ , where  $L_Z$  is aggregate R&D. Taking logs and time derivatives of  $1 = z_i \alpha K$  and using the demand curve (Eq. (5)), the R&D technology (Eq. (8)) and the price strategy (Eq. (32)), one reduces the first-order conditions to Eq. (13).

Taking logs and time-derivatives of  $V_i$  yields

$$r + \delta = \frac{\Pi_{X_i}}{V_i} + \frac{\dot{V}_i}{V_i},$$

which is a perfect-foresight, no-arbitrage condition for the equilibrium of the capital market. It requires that the rate of return to firm ownership equal the rate of return to a loan of size  $V_i$ . The rate of return to firm ownership is the ratio between profits and the firm's stock market value plus the capital gain (loss) from the stock appreciation (depreciation).

In symmetric equilibrium the demand curve (Eq. (5)) yields that the cost of entry is  $\beta \frac{Y}{N}$ . The corresponding demand for labor in entry is

$$L_N = (\dot{N} + \delta N) \beta \frac{Y}{N}.$$

The case  $V > \beta \frac{Y}{N}$  yields an unbounded demand for labor in entry,  $L_N = +\infty$ , and is ruled out since it violates the general equilibrium

conditions. The case  $V < \beta \frac{Y}{N}$  yields  $L_N = -\infty$ , which means that the non-negativity constraint on  $L_N$  binds and  $\dot{N} = -\delta N$ , which implies negative net entry due to the death shock. Free-entry requires  $V = \beta \frac{Y}{N}$ . Using the price strategy (Eq. (31)), the rate of return to entry becomes Eq. (14).

## A.2. The economy's resources constraint

I now show that the household's budget constraint reduces to the economy's labor market clearing condition. Starting from Eq. (2), recall that  $A = NV$  and  $(r + \delta)V = \Pi_X + \dot{V}$ . Substituting into Eq. (2) yields

$$\dot{N}V = N\Pi_X + L + p\Omega + \Pi_M - Y.$$

Observing that  $N\Pi_X = NPX - L_X - L_Z - P_MM$ ,  $NPX = Y$ ,  $\Pi_M = P_MM - L_M - pR$ ,  $R = \Omega$ , and that the free entry condition yields that total employment in entrepreneurial activity is  $L_N = NV$ , this becomes

$$L = L_N + L_X + L_Z + L_M.$$

## A.3. Detailed derivation of the logistic equation for $N(t)$

For simplicity, in the text I focussed on the case in which firms always invest in vertical R&D. More generally, taking into account the non-negativity constraint on  $L_Z$ , the equation getting the rate of vertical innovation in symmetric equilibrium is

$$\hat{Z} = \alpha \frac{L_Z}{N} = \begin{cases} \frac{Y^* \alpha \theta (\epsilon-1)}{N} - \rho - \delta & N < \bar{N} \\ 0 & N \geq \bar{N} \end{cases},$$

where

$$\bar{N} \equiv Y^* \frac{\alpha \theta (\epsilon-1)}{(\rho + \delta) \epsilon}.$$

Substituting this result into Eq. (14) yields

$$\hat{N} = \begin{cases} \frac{1}{\beta} \left[ \frac{1-\theta(\epsilon-1)}{\epsilon} - \left( \phi - \frac{\rho + \delta}{\alpha} \right) \frac{N}{Y^*} \right] - (\rho + \delta) & N < \bar{N} \\ \frac{1}{\beta} \left[ \frac{1}{\epsilon} - \phi \frac{N}{Y^*} \right] - (\rho + \delta) & N \geq \bar{N} \end{cases},$$

which converges to

$$N^* = \begin{cases} \frac{\frac{1-\theta(\epsilon-1)}{\epsilon} - (\rho + \delta)\beta}{\phi - \frac{\rho + \delta}{\alpha}} Y^* & \frac{1-\theta(\epsilon-1)}{\epsilon} - (\rho + \delta)\beta < \frac{\theta(\epsilon-1)}{(\rho + \delta)\epsilon} \\ \frac{\frac{1}{\epsilon} - (\rho + \delta)\beta}{\phi} Y^* & \frac{1-\theta(\epsilon-1)}{\epsilon} - (\rho + \delta)\beta \geq \frac{\theta(\epsilon-1)}{(\rho + \delta)\epsilon} \end{cases}.$$

These solutions exist only if the feasibility condition  $\frac{1}{\epsilon} > (\rho + \delta)\beta$  holds. The interior steady state with both vertical and horizontal R&D requires the additional conditions  $\alpha\phi > \rho + \delta$  and

$$(\rho + \delta)\beta + \frac{\theta(\epsilon-1)}{\epsilon} < \frac{1}{\epsilon} < (\rho + \delta)\beta + \frac{\alpha\phi}{\rho + \delta} \frac{\theta(\epsilon-1)}{\epsilon}.$$

As discussed in detail in Peretto (1998) and Peretto and Connolly (2007), models of this class have well-defined dynamics also when one

of the two R&D activities shuts down because it is return-dominated by the other. The conditions imposed here simply ensure that in steady state the arbitrage condition that the return to vertical innovation be equal to the return to horizontal innovation holds for positive values of both  $L_Z$  and  $L_N$ .

#### A.4. Proof of Proposition 1

Taking logs of Eq. (24) yields

$$\log T(t) = \theta \log Z_0 + \theta \int_0^t \hat{Z}(s) ds + \frac{1}{\epsilon-1} \log N(t).$$

Adding and subtracting  $\hat{Z}^*$  from  $\hat{Z}(t)$  yield

$$\log T(t) = \theta \log Z_0 + g^*t + \theta \int_0^t [\hat{Z}(s) - \hat{Z}^*] ds + \frac{1}{\epsilon-1} \log N(t).$$

Using Eq. (19) the integral becomes

$$\begin{aligned} \theta \int_0^t (\hat{Z}(s) - \hat{Z}^*) ds &= \theta \frac{\alpha\theta(\epsilon-1)}{\epsilon} \int_0^t \left( \frac{Y^*}{Y(s)} - \frac{Y^*}{N^*} \right) ds \\ &= \gamma \int_0^t \left( \frac{N^*}{N(s)} - 1 \right) ds, \end{aligned}$$

where

$$\gamma \equiv \theta \frac{\alpha\theta(\epsilon-1)}{\epsilon} \frac{Y^*}{N^*} = \theta \frac{\alpha\theta(\epsilon-1)}{\epsilon} \frac{\phi - \frac{\rho + \delta}{\alpha}}{1 - \theta(\epsilon-1) - \rho\beta}.$$

Finally, Eq. (23) and the definition of  $\Delta^*$  yield

$$\begin{aligned} \theta \int_0^t (\hat{Z}(s) - \hat{Z}^*) ds &= \gamma \int_0^t e^{-\nu s} \left( \frac{N^*}{N_0} - 1 \right) ds \\ &= \frac{\gamma}{\nu} \Delta^* (1 - e^{-\nu t}). \end{aligned}$$

Using this result and Eq. (23) again yields

$$\log T(t) = \log T_0 + g^*t + \frac{\gamma\Delta^*}{\nu} (1 - e^{-\nu t}) + \frac{1}{\epsilon-1} \log \frac{1 + \Delta^*}{1 + e^{-\nu t}\Delta^*}.$$

Next consider

$$\log u^*(t) = \log \frac{\epsilon-1}{\epsilon} + \log \left( \frac{y^*}{c^*} \right) + \log T(t)$$

and use the expression just derived to write

$$\log u^*(t) = \log \frac{y^*}{c^*} + g^*t + \frac{\gamma\Delta^*}{\nu} (1 - e^{-\nu t}) + \frac{1}{\epsilon-1} \log \frac{1 + \Delta^*}{1 + e^{-\nu t}\Delta^*}.$$

Substituting this expression into Eq. (1) yields

$$\begin{aligned} U^* &= \int_0^\infty e^{-\rho t} \log u^*(t) dt \\ &= \int_0^\infty e^{-\rho t} \left[ \log \left( \frac{y^*}{c^*} \right) + g^*t \right] dt \\ &\quad + \frac{\gamma}{\nu} \Delta^* \int_0^\infty e^{-\rho t} (1 - e^{-\nu t}) dt \\ &\quad + \frac{1}{\epsilon-1} \int_0^\infty e^{-\rho t} \log \frac{1 + \Delta^*}{1 + e^{-\nu t}\Delta^*} dt. \end{aligned}$$

The first and second integrals have closed form solutions; the third has a complicated solution involving the hypergeometric function. For

my purposes, it is more useful to work with the following approximation. Since in general  $\log(1+x) \approx x$ , I can rewrite

$$\log \frac{1 + \Delta^*}{1 + e^{-\nu t}\Delta^*} = \log(1 + \Delta^*) - \log(1 + e^{-\nu t}\Delta^*) = \Delta^* (1 - e^{-\nu t}),$$

which yields Eqs. (25), (26) and

$$\begin{aligned} U^* &= \int_0^\infty e^{-(\rho-\lambda)t} \left[ \log \left( \frac{y^*}{c^*} \right) + g^*t \right] dt \\ &\quad + \left( \frac{\gamma}{\nu} + \frac{1}{\epsilon-1} \right) \Delta^* \int_0^\infty e^{-(\rho-\lambda)t} (1 - e^{-\nu t}) dt, \end{aligned}$$

which upon integration yields Eq. (27).

#### A.5. Proof of Lemma 2

Differentiating and manipulating terms yield:

$$\begin{aligned} \frac{\partial (S_X^M S_M^R)}{\partial p} &= \frac{\partial S_X^M}{\partial p} S_M^R + \frac{\partial S_M^R}{\partial p} S_X^M \\ &= \frac{\partial S_X^M}{\partial P_M} \frac{P_M}{S_X^M} \cdot \frac{\partial P_M}{\partial p} \frac{p}{P_M} \cdot \frac{S_X^M S_M^R}{p} + \frac{\partial S_M^R}{\partial p} \frac{p}{S_M^R} \cdot \frac{S_M^R S_X^M}{p} \\ &= \left[ \frac{\partial S_X^M}{\partial P_M} \frac{P_M}{S_X^M} \cdot \frac{\partial P_M}{\partial p} \frac{p}{P_M} + \frac{\partial S_M^R}{\partial p} \frac{p}{S_M^R} \right] \frac{S_M^R S_X^M}{p} \\ &= \left[ \frac{\partial S_X^M}{\partial P_M} \frac{P_M}{S_X^M} \cdot \frac{\partial C_M}{\partial p} \frac{p}{C_M} + \frac{\partial S_M^R}{\partial p} \frac{p}{S_M^R} \right] \frac{S_M^R S_X^M}{p}. \end{aligned}$$

Recalling that

$$\begin{aligned} \frac{\partial S_X^M}{\partial P_M} \frac{P_M}{S_X^M} &= 1 - \epsilon_X^M, \\ \frac{\partial S_M^R}{\partial p} \frac{p}{S_M^R} &= 1 - \epsilon_M^R, \\ \frac{\partial C_M}{\partial p} \frac{p}{C_M} &= S_M^R \end{aligned}$$

and substituting into the expression above yields Eq. (28).

#### A.6. Proof of Proposition 3

Refer to Fig. 1. In the case of complementarity, depicted in the upper panel, we have the following pattern. For  $\omega \rightarrow 0$  the expenditure line (Eq. (17)) is almost, but not quite, flat and intersects the income relation (Eq. (18)) for  $p \rightarrow \infty$  and  $y \rightarrow y^*(\infty)$ . As  $\omega$  grows, the expenditure line rotates counterclockwise and the intersection shifts left, tracing the income relation. We thus obtain that both  $p^*$  and  $y^*$  fall. As  $\omega \rightarrow \infty$ , the expenditure line becomes vertical and the intersection occurs at  $p \rightarrow 0$  and  $y \rightarrow y^*(0)$ .

In the case of substitution in the lower panel, we have a similar pattern with the difference that the income relation has negative slope so that  $y^*$  increases as the expenditure line rotates counterclockwise. Specifically, for  $\omega \rightarrow 0$  the expenditure line is almost, but not quite, flat and intersects the income relation for  $p \rightarrow \infty$  and  $y \rightarrow y^*(\infty)$ . As  $\omega$  grows, the expenditure line rotates counterclockwise and the intersection shifts left yielding that  $p^*$  falls while  $y^*$  rises. As  $\omega \rightarrow \infty$ , the expenditure line becomes vertical and the intersection occurs at  $p \rightarrow 0$  and  $y \rightarrow y^*(0)$ .

#### A.7. Proof of Proposition 4

Refer to Fig. 2. The proof is essentially the same as above. The only difference is that as the expenditure line rotates it traces the hump-

shaped income relation yielding that  $p^*$  always falls while  $y^*$  rises for  $0 < \omega < \bar{\omega}$  and falls for  $\bar{\omega} < \omega < \infty$ .

#### A.8. Proof of Proposition 5

Observe first that

$$\frac{d \log \left( \frac{y}{c} \right)}{d\omega} = \frac{d \log y}{d\omega} - \frac{d \log c}{d\omega}.$$

Now,

$$\frac{d \log y}{d\omega} = \frac{d \log y}{dp} \frac{dp}{d\omega} = \omega \Gamma(p) \frac{dp}{d\omega}$$

and

$$\begin{aligned} \frac{d \log c}{d\omega} &= \frac{1}{C_X} \frac{dC_X}{d\omega} = \frac{1}{C_X} \frac{dC_X}{dP_M} \frac{dP_M}{dp} \frac{dp}{d\omega} \\ &= \frac{P_M}{C_X} \frac{dC_X}{dP_M} \frac{dp}{dp} \frac{1}{P_M} \frac{dp}{d\omega} \\ &= \frac{S_X^M S_M^R}{p} \frac{dp}{d\omega}. \end{aligned}$$

Hence,

$$\frac{d \log \left( \frac{y}{c} \right)}{d\omega} = \left( \Gamma - \frac{S_X^M S_M^R}{p\omega} \right) \omega \frac{dp}{d\omega}.$$

which using Eq. (12) becomes Eq. (30).  $\kappa > 1$  follows from the feasibility condition  $\frac{1}{\epsilon} > (\rho + \delta)\beta$ .

#### A.9. Proof of Proposition 6

Observe that by construction

$$\frac{d\Delta^*}{d\omega} = \frac{1}{y^*} \frac{dy^*}{d\omega}.$$

Differentiation of Eq. (27) then yields

$$\begin{aligned} \frac{dU^*}{d\omega} &= \frac{1}{\rho} \left[ \frac{d \log \left( \frac{y^*}{c^*} \right)}{d\omega} + \mu \frac{1}{y^*} \frac{dy^*}{d\omega} \right] \\ &= \frac{1}{\rho} \left[ (1 + \mu) \frac{d \log y^*}{d\omega} - \frac{d \log c^*}{d\omega} \right]. \end{aligned}$$

Using the expressions calculated above and rearranging terms yield Eq. (31).

#### A.10. The $\Gamma(\omega)$ and $\Psi(\omega)$ curves used in Fig. 4

The  $\Gamma(\omega)$  curve. Dropping stars to simplify notation, I have

$$\Gamma(\omega) = \left[ -\frac{\sigma_X}{1-\sigma_X} (1 - S_X^M(p(\omega))) + \frac{\sigma_M}{1-\sigma_M} S_M^R(p(\omega)) - \frac{\sigma_M}{1-\sigma_M} \right].$$

Proposition 4 yields that  $p(0) = \infty$  and  $p(\infty) = 0$ . Therefore, for  $\omega \rightarrow 0$ :

$$S_M^R(0) = \lim_{p \rightarrow \infty} \frac{1}{1 + \left( \frac{\psi_M}{1-\psi_M} \right)^{\frac{1}{1-\sigma_M}} p^{\frac{\sigma_M}{1-\sigma_M}}} = 1.$$

Also,

$$P_M(0) = \lim_{p \rightarrow \infty} \left[ \psi_M^{\frac{1}{1-\sigma_M}} + (1-\psi_M)^{\frac{1}{1-\sigma_M}} p^{\frac{\sigma_M}{1-\sigma_M}} \right]^{\frac{\sigma_M-1}{\sigma_M}} = \infty,$$

so that

$$S_X^M(0) = \lim_{P_M \rightarrow \infty} \frac{1}{1 + \left( \frac{\psi_X}{1-\psi_X} \right)^{\frac{1}{1-\sigma_X}} P_M^{\frac{\sigma_X}{1-\sigma_X}}} = 0.$$

Consequently,

$$\Gamma(0) = \frac{-\sigma_X}{1-\sigma_X} < 0.$$

In contrast, for  $\omega \rightarrow \infty$ :

$$S_M^R(\infty) = \lim_{p \rightarrow 0} \frac{1}{1 + \left( \frac{\psi_M}{1-\psi_M} \right)^{\frac{1}{1-\sigma_M}} p^{\frac{\sigma_M}{1-\sigma_M}}} = 0.$$

Also,

$$P_M(\infty) = \lim_{p \rightarrow 0} \left[ \psi_M^{\frac{1}{1-\sigma_M}} + (1-\psi_M)^{\frac{1}{1-\sigma_M}} p^{\frac{\sigma_M}{1-\sigma_M}} \right]^{\frac{\sigma_M-1}{\sigma_M}} = \psi_M^{-\frac{1}{\sigma_M}},$$

so that

$$S_X^M(\infty) = \frac{1}{1 + \left( \frac{\psi_X}{1-\psi_X} \right)^{\frac{1}{1-\sigma_X}} \psi_M^{-\frac{\sigma_X}{1-\sigma_X}} p^{\frac{\sigma_X}{1-\sigma_X}}}.$$

Consequently,

$$\Gamma(\infty) = \frac{-\sigma_M}{1-\sigma_M} > 0.$$

The next step is to show that the curve is monotonically increasing:

$$\begin{aligned} \Gamma'(\omega) &= \underbrace{\frac{\sigma_X}{1-\sigma_X} \cdot \frac{dS_X^M}{dP_M}}_{+} \cdot \underbrace{\frac{dP_M}{dp}}_{+} \cdot \underbrace{\frac{dp}{d\omega}}_{-} \cdot S_M^R \\ &\quad + \underbrace{\left[ -\frac{\sigma_X}{1-\sigma_X} (1 - S_X^M) + \frac{\sigma_M}{1-\sigma_M} \right]}_{-} \cdot \underbrace{\frac{dS_M^R}{dp}}_{+} \cdot \underbrace{\frac{dp}{d\omega}}_{-} \\ &> 0. \end{aligned}$$

Finally, by continuity there exists a value  $\bar{\omega}$  where  $\Gamma(\bar{\omega}) = 0$ . The  $\Psi(\omega)$  curve. Again dropping stars, I have

$$\Psi(\omega) = \kappa - S_M^R(p(\omega)) S_X^M(p(\omega)).$$



The limiting behavior at 0 and  $\infty$  is straightforward. The previous calculations yield:

$$\Psi(0) = \kappa - S_M^R(0)S_X^M(0) = \kappa;$$

$$\Psi(\infty) = \kappa - S_M^R(\infty)S_X^M(\infty) = \kappa.$$

Next observe that

$$\begin{aligned}\Psi'(\omega) &= \frac{d(S_M^R(p(\omega))S_X^M(p(\omega)))}{dp(\omega)} \frac{dp(\omega)}{d\omega} \\ &= \Gamma(\omega) \frac{S_M^R(p(\omega))S_X^M(p(\omega))}{p(\omega)} \frac{dp(\omega)}{d\omega},\end{aligned}$$

so that the curve is hump-shaped with its maximum exactly at the value  $\bar{\omega}$  where  $\Gamma(\omega)$  changes sign.

*The threshold values.* Observe that  $\kappa > 1 > \frac{-\sigma_M}{1-\sigma_M}$ . It is evident from Fig. 2 then that  $\Gamma(\omega) < \Psi(\omega) \forall \omega$ . It follows that there is only one relevant intersection, of  $\Psi(\omega)$  with  $(1+\mu)\Gamma(\omega)$ , that yields the threshold value  $\tilde{\omega}$  such that  $\bar{\omega} < \tilde{\omega}$ .

#### A.11. The reallocation

The expression for the share of employment in the primary sector and the expressions derived in the proofs above yield

$$\begin{aligned}\frac{d}{d\omega} \left( \frac{L_M}{L} \right) &= \frac{\epsilon-1}{\epsilon} \left[ \frac{dy}{d\omega} S_X^M (1-S_M^R) + y \frac{dS_X^M}{d\omega} - \frac{d(S_X^M S_M^R)}{d\omega} \right] \\ &= \frac{\epsilon-1}{\epsilon} \left[ \frac{\Gamma}{\Psi} S_X^M (1-S_M^R) + 1 - \epsilon_X^M - \Gamma \right] \omega \frac{dp}{d\omega}.\end{aligned}$$

Recall that  $\frac{dp}{d\omega} < 0$ . Then

$$\frac{d}{d\omega} \left( \frac{L_M}{L} \right) > 0 \quad \forall \omega$$

because the term in brackets is

$$\frac{\Gamma}{\Psi} (S_X^M - \kappa) + 1 - \epsilon_X^M < 0$$

since  $\kappa > 1$  and  $1 - \epsilon_X^M < 0$  under the assumption that manufacturing exhibits substitution between labor and materials.

## References

- Alexeev, M., Conrad, R., 2009. The elusive curse of Oil. *The Review of Economics and Statistics* 91, 586–598.
- Banks, R.B., 1994. *Growth and Diffusion Phenomena*. Springer-Verlag, Berlin Heidelberg.
- Barro, R.J., Sala i Martin, X., 2004. *Economic Growth*. MIT university Press, Cambridge MA.
- Brunneschweiler, C.N., Bulte, E.H., 2008. The resource curse revisited and revised: a tale of paradoxes and Red herrings. *Journal of Environmental Economics and Management* 55, 248–264.
- Corden, W.M., Neary, J.P., 1982. Booming sector and de-industrialization in a small open economy. *The Economic Journal* 92, 825–848.
- Corden, W.M., 1984. Booming sector and Dutch disease economics: survey and consolidation. *Oxford Economic Papers* 36, 359–380.
- Etro, F., 2004. Innovation by leaders. *The Economic Journal* 114, 281–303.
- Krugman, P., 1987. The narrow moving band, the Dutch disease, and the economic consequences of Mrs. Thatcher: notes on trade in the presence of dynamic economies of scale. *Journal of Development Economics* 27, 41–55.
- Peretto, P.F., 1998. Technological Change and Population Growth. *Journal of Economic Growth* 3, 283–311.
- Peretto, P.F., Connolly, M., 2007. The Manhattan metaphor. *Journal of Economic Growth* 12, 329–350.
- Peretto, P.F., Smulders, S., 2002. Technological distance, growth and scale effects. *Economic Journal* 112, 603–624.
- Prasertsom, N., 2010. Endogenous growth and renewable resources under different management regimes, manuscript. Duke University, Department of Economics.
- van der Ploeg, F., 2011. Natural Resources: Curse or Blessing? *Journal of Economic Literature* 49 (2), 366–420.
- van der Ploeg, F., Poelhekke, S., 2010. The pungent smell of “Red herrings”: subsoil assets, rents, volatility, and the resource curse. *Journal of Environmental Economics and Management* 60, 44–55.
- Sachs, J.D., Warner, A.M., 1995. Natural resource abundance and economic growth. NBER WP 5398.
- Sachs, J.D., Warner, A.M., 2001. The curse of natural resources. *European Economic Review* 45, 827–838.
- Torvik, R., 2001. Learning by doing and the Dutch disease. *European Economic Review* 45, 285–306.
- Younger, S.D., 1992. Aid and the Dutch disease: macroeconomic management when everybody loves you. *World Development* 20, 1587–1597.
- van Wijnbergen, S., 1984. The Dutch disease: a disease after all? *The Economic Journal* 94, 41–55.
- Wright, G., Czelusta, J., 2007. Resource-based growth past and present. In: Lederman, D., Maloney, W.F. (Eds.), *Natural Resources: Neither Curse Nor Destiny*. Stanford University Press.