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# Credit quantity and credit quality: Bank competition and capital accumulation

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#### Abstract

In this paper we show that bank competition has an intrinsically ambiguous impact on capital accumulation. We further show that it is also responsible for the emergence of development traps in economies that otherwise would be characterized by unique equilibria. These results explain the conflicting evidence emerging from the recent empirical studies of the effects of bank competition on economic growth. We obtain them developing a dynamic, general equilibrium model of capital accumulation where banks operate in a Cournot oligopoly. More banks lead to a higher quantity of credit available to entrepreneurs, but also to diminished incentives to offer relationship services that improve the likelihood of success of investment projects. We also show that conditioning on one key parameter resolves the theoretical ambiguity: in economies where intrinsic market uncertainty is high (low), less (more) competition leads to higher capital accumulation.

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#### 1. Introduction

Recent empirical work has documented multiple, conflicting effects of bank competition on the real economy. Some contributions find evidence that bank competition leads to more credit availability, more firm entry and more growth (e.g., Black and Strahan [11], Beck, Demirguc-Kunt, and Maksimovic [5], Cetorelli and Gambera [18], Cetorelli and Strahan [19], Bertrand, Schoar and Thesmar [9]). Others find instead that credit quality and availability may be higher in less competitive environments (e.g., Petersen and Rajan [42], Shaffer [45], Cetorelli and Gambera [18], Bonaccorsi and Dell'Ariccia [12], Zarutskie [48]). The evidence thus seems to indicate that bank competition matters for capital accumulation and growth, but also that there is ambiguity about the sign of the relationship.

To the best of our knowledge, the literature still lacks a theoretical model of economic growth with a fully specified banking sector able to generate the contrasting predictions that the evidence suggests. In this paper, we attempt to do just that. We develop a dynamic general equilibrium model with oligopolistic banks that yields innovative insights on the effect of banking market structure on capital accumulation and economic growth, and important refinements to the associated normative prescriptions.

In our model banks compete both in gathering savings from households and in lending to entrepreneurs. Entrepreneurs are ex-ante identical but subject to idiosyncratic risk that could lead them to default. In this environment, banks can extend *arm's length* loans, that is, outright loans that simply specify the amount, the interest rate and the repayment date. A bank that does so provides savers an insurance service — holding a large portfolio of loans, it diversifies away idiosyncratic risk and thus offers a safe return — but leaves unchanged the probability of default of individual entrepreneurs. Alternatively, the bank can extend a loan and, at a cost, provide additional services that increase the likelihood of entrepreneurial success. We define these more sophisticated loans as *relationship* loans. Banks that extend them do not simply diversify idiosyncratic risk away, they also reduce it at the source by making entrepreneurs better.

Our main assumption is that the benefit of the relationship services is not fully appropriable: since the services provided improve the performance of the entrepreneur as a whole, banks that extend to the same entrepreneur simple arm's length loans would earn the higher risk-adjusted return on his projects without sustaining the cost of providing the services. There is thus a free-riding problem that affects the banks' incentives to provide relationship services. Our model studies how such incentives regulate the equilibrium provision of such services, and how the equilibrium depends on structural parameters.<sup>3</sup>

We show that the degree of competition among banks plays a fundamental role in determining the provision of relationship services, with more intense competition diminishing banks' incentives to provide them. The model, therefore, identifies two channels through which competition affects the *quantity* and *quality* of credit, that is, the aggregate volume of credit and the efficiency with which the economy transforms such credit into capital. The first channel features a complementarity relationship between the provision of the relationship services and the quantity of credit offered to a client: since the services make the borrower more likely to succeed, banks are

<sup>&</sup>lt;sup>3</sup> We are certainly not the first to stress the problem of appropriability in models of banking. This issue is central, for instance in Campbell and Kracaw [16] and is also explicitly mentioned in Thakor [47, p. 303]. Both models are based on banks resolving an ex-ante problem of private information, while we deal with an ex-post contribution that eliminates the project's risk. The difference however is not substantial. As shown in Petersen and Rajan [42], the problem of free riding and potential under-provision of relationship loans does not hinge on the ex-ante resolution of informational frictions.

willing to offer larger loans to such borrowers. This novel property implies that more competition reduces the provision of relationship services and results into a smaller volume of credit and lower productivity of investment projects, which jointly result into lower capital formation. The second channel is well known: more competition yields lower spreads between interest rates on loans and the rate of return to saving and thus reduces the deadweight loss due to banks' exercise of market power.

When we take into account the endogenous feedbacks between the banking sector and the rest of the economy we expose an intrinsically ambiguous effect of bank competition on the path of capital accumulation, and the model offers indications on how to resolve such ambiguity. More precisely, we show that perfect competition is the banking market structure that maximizes long-run income if the benefits from relationship services is relatively small because the idiosyncratic risk affecting entrepreneurs is negligible (the likelihood of default is small to begin with). At the opposite end, monopoly maximizes long-run income when banks' services have a relatively large impact on aggregate investment because idiosyncratic risk is substantial. In intermediate scenarios the market structure that maximizes long-run development is an oligopoly. The model's main insight, therefore, is that bank competition plays a fundamental role in the process of capital accumulation, but this role is only understood concurrently to that played by other fundamentals.

We also find that banking market structure *per se* can give rise to a development trap in environments where the fundamentals would otherwise be consistent with unique, high-income equilibria in which banks offer relationship services and lend efficiently. Interestingly, the market structure that allows the economy to escape from the trap may not be the one that maximizes steady-state income outside of the trap. The possibility for development traps determined by banking market structure is another potential explanation for the ambiguous evidence documented in the literature. It also emphasizes the complexity of the task faced by regulators in a dynamic environment where the effect of competition on banking practices varies with the economy's level of development.

Two strands of literature related to our study have developed in recent years. The first focuses on the role that banks play in promoting economic growth. This line of research, part of the broader debate on the importance of finance, has solidified the view that a more developed and efficient banking sector has positive effects on real economic activity.<sup>4</sup> The second, more in the tradition of banking studies and corporate finance, applies standard industrial organization arguments to predict that more competition leads to lower interest rates on loans and a larger volume of credit. However, more nuanced claims, recognizing additional roles fulfilled by banks other then the outright extension of loans, suggest that more competition may lead to worse credit practices and perhaps even lower credit availability.<sup>5</sup> While these contributions are deep in the analysis of the banking industry, they abstract from broader considerations for aggregate economic variables.

Given our objective of linking theoretically bank competition to capital accumulation and growth, building a dynamic general equilibrium model strikes us as the natural thing to do. It turns out that virtually all of the insights and predictions of our model derive from the explicit

<sup>&</sup>lt;sup>4</sup> This literature is vast. For exhaustive reviews see Levine [34] and Cetorelli [17].

<sup>&</sup>lt;sup>5</sup> For more theoretical arguments on one side and the other of this debate see, Rajan [41], Petersen and Rajan [42], Marquez [36], Dell'Ariccia and Marquez [20], Hauswald and Marquez [25], Boot and Thakor [13], Boyd and De Nicolo' [15].

general equilibrium setup. <sup>6</sup> The fact that banks face a well-defined downward sloping demand for credit — which we derive from the equilibrium conditions in the production sector — and an upward sloping supply of deposits — which we derive from households' intertemporal decision between current and future consumption — leads naturally to feedback effects that enrich our understanding of the role of bank competition. Take for instance the basic result that more competition reduces banks' incentives to produce relationship services. In our approach it is also the case that when more banks compete for the households' savings, they offer them higher interest rates, which in turn leads to a higher supply of savings and therefore to a higher supply of credit. But a higher credit supply *raises* banks' incentives to provide the relationship services, since — all else equal — the incidence of the cost sustained to provide such services is lower. At the same time, if this translates into a higher amount of capital that goes into production, the return on capital decreases, and this *weakens* their incentives. Clearly, the overall effect of a change in the number of banks on the aggregate volume of credit and on the provision of relationship services is far from being straightforward and the model allows us to sort out the various channels and obtain sharp predictions that can reconcile the theory with the evidence.<sup>7</sup>

Our paper is also related to the well-established literature modeling credit market frictions and aggregate economic activity. This literature, normally associated with the earlier contributions by Bernanke and Gertler [7], Kiyotaki and Moore [32], and Bernanke, Gertler and Gilchrist [8], has developed general equilibrium models where financial market frictions emerge from the existence of agency problems between borrowers and lenders. These models have enriched the structure of otherwise standard general equilibrium frameworks and are routinely applied in simulation exercises and in the formulation of monetary policy. However, they suffer from an important limitation, namely, that the macroeconomic effects of credit frictions arise solely from the activity of non-financial firms, while intermediaries play a passive role. In Gertler and Kiyotaki's own words: "... [this] literature with financial frictions emphasized credit market constraints on non-financial borrowers and treated intermediaries largely as a veil" (Gertler and Kiyotaki [23, p. 3]). While our model does not pretend to approximate the richness in structure of the models developed in this literature, credit intermediaries in our framework play a very *active* role, having a decision rule that yields the profit-maximizing quantity-quality combination as a function of credit market structure, and for given market structure, of the level of capital accumulation. 9

## 2. The economy: preferences and technology

The economy is populated by overlapping generations living for two periods. Each generation consists of a continuum of mass one of identical individuals. Population is thus constant. Each

<sup>&</sup>lt;sup>6</sup> Guzman [24] analyzes bank competition in a general equilibrium framework but focuses only on the canonical inefficiencies associated with market power and consequently finds that perfect competition maximizes capital accumulation.

That a general equilibrium approach is important is confirmed in a number of theoretical studies that have focused specifically on bank competition and financial stability. As stated in Allen and Gale [1]: "In simple partial-equilibrium models, it is possible to generate a negative trade-off between competition and financial stability. However, ... the nature of the trade-off [...] is more complicated than was first thought". Indeed, significant qualifiers to this statements have been presented in general equilibrium models such as, e.g., Allen and Gale [2] and Boyd, De Nicolo' and Smith [14].

<sup>&</sup>lt;sup>8</sup> Gertler and Kiyotaki [23] itself is one of the recent attempts in this literature to build more structure around the activity of intermediaries.

<sup>&</sup>lt;sup>9</sup> In fact, if we modified our set up by introducing an aggregate shock component (for example, a TFP shock), banks' decision rule would also yield the optimal quantity—quality combination contingent on the cycle. In particular, the model would predict pro-cyclical credit quantity and credit quality.

young agent is endowed with no capital and with one unit of labor. When old, the agent does not work and lives off his savings. We abstract from labor-leisure choice so that young agents supply their entire labor endowment in the market.

Let  $c_t$  and  $c_{t+1}$  be consumption at time t and t+1 for a representative member of generation t. The agent maximizes

$$U(c_t, c_{t+1}) = u(c_t) + u(c_{t+1}) = c_t^{\alpha} + c_{t+1}^{\alpha}, \quad \alpha < 1$$
 (1)

subject to:

$$c_t = W_t - s_t$$

$$c_{t+1} = s_t r_{t+1},$$

where  $s_t$  is the amount of saving at time t and  $r_{t+1}$  is the rate of return on saving.<sup>10</sup> All of our results obtain with a generic utility function but we shall work with the power function form to streamline the exposition.

Substitution of the two constraints into (1) yields directly that the solution to the maximization problem is the saving supply schedule

$$r_{t+1} = h(S_t; W_t) = \left\lceil \frac{S_t}{W_t - S_t} \right\rceil^{\frac{1-\alpha}{\alpha}},\tag{2}$$

where we use the assumption that there is a mass one of identical young agents to write the function in terms of aggregate savings,  $S_t$ .

On the production side of our economy, there exists a representative, competitive firm producing a homogeneous final good with a standard neoclassical production function that satisfies the Inada conditions,

$$Y_t = F(K_t, L_t) = K_t^{\gamma} L_t^{1-\gamma}, \quad 0 < \gamma < 1$$
 (3)

where Y, K and L are, respectively, output, capital and labor. Since labor supply is inelastic, in equilibrium  $L_t = 1$ . Therefore, in our analysis we can work with the intensive-form version of (3), which we denote  $f(\cdot) = K_t^{\gamma}$ . All of our results obtain with a generic neoclassical production function.

The competitive final producer's profit maximization problem yields the following demand schedules for capital and labor:

$$R_t = f'(K_t) = \gamma K_t^{\gamma - 1},\tag{4}$$

$$W_{t} = f(K_{t}) - K_{t} f_{K}(K_{t}) = (1 - \gamma) K_{t}^{\gamma},$$
(5)

where R is the rental rate on capital and W is the wage rate.

 $<sup>^{10}</sup>$  We set the discount factor equal to one because it plays no essential role in our analysis. Also, as we show in the next section, in this model banks make positive profits. To account for them we assume that banks are owned by savers. That is, agents save by both depositing and purchasing equity shares of banks. Formally,  $s_t = d_t + e_t$ , where  $d_t$  is deposits and  $e_t$  is equity capital. Banks in turn use both deposits and equity capital to supply credit. A standard arbitrage argument requires that the rate of return to deposit be equal to the rate of return to equity, i.e.,  $r_{t+1}^d = r_{t+1}^e$ . We denote this common rate of return  $r_{t+1}$ . Banks' profits are thus part of the resources that old agents use to finance consumption. The reason is that the definition of return to equity says  $r_{t+1}^e V_{it} = \prod_{it+1} + V_{it+1} - V_{it}$ , where  $\prod_{it+1}$  is the profit generated by bank i and  $V_{it}$  is the price of an equity share in bank i.

<sup>&</sup>lt;sup>11</sup> See, e.g., Barro and Sala-i-Martin [4] for a list of its properties.

#### 3. Investment activity and the credit market

Since our main goal is to model the role of credit market competition in the dynamics of capital accumulation, we design a credit sector that is derived from first principles yet sufficiently streamlined to allow tractability and meaningful identification of the channels through which its market structure affects the other variables of the model. In this section we discuss the conceptual framework and show how the defining characteristic of our model — the existence of a free-riding problem that weakens banks' incentives to engage in sophisticated lending practices — arises from few, simple assumptions.

## 3.1. The framework

INVESTMENT. We model investment as the linear transformation of final output into capital. There are no divisibility constraints in the size of an investment project. We are therefore envisioning the standard capital-theoretic structure where agents demand resources to invest and all potential suppliers are aware of such demand. To fix terminology, we assume that the agents that operate the investment technology are the young agents, and we refer to them as entrepreneurs. Because they have zero endowment, entrepreneurs need credit to undertake investment. Once entrepreneurs receive credit, they transform it in units of capital, which they then supply as an input to the final producers operating the technology in (3).

Entrepreneurs are ex-ante identical. The linear technology that transforms credit into capital for entrepreneur  $i \in [0, 1]$  at time t is  $K_{it} = \vartheta_i X_{it}$ , where  $X_{it}$  is the total credit obtained by i at time t, and  $\vartheta_i$  is a random variable, i.i.d. across time and across entrepreneurs, which takes value one with probability  $\theta$  and zero with probability  $(1 - \theta)$ . Accordingly, each entrepreneur succeeds with probability less than one and if he fails the expected liquidation value is zero. For simplicity,  $\vartheta_i$  is invariant in the size of the project, its density function is known and its realizations are observable.

BANKS. In this economy banks collect savings from old agents and lend to the population of young entrepreneurs. Since there are no divisibility constraints or costs of advertising the need for funds, banks face simultaneously the entire population of borrowers and can supply credit at any scale they deem optimal.

Because the distribution of the idiosyncratic liquidity shocks is known, a bank can at the very least supply credit to the whole population of entrepreneurs and diversify away the project-level idiosyncratic risk. This provision of insurance is one of the basic functions that the literature typically assigns to intermediaries. We call this strategy *arm's length* lending to mean that it implies the extension of just outright loans.

However, by virtue of having a broader scale and scope than individual investors, banks are in a position to provide additional lending services to cater to the liquidity needs of the entrepreneurs, should these arise. We define a loan with these additional services a *relationship* loan. What we have in mind is that the bank can provide a contingent liquidity line, that is a more flexible lending product, which in reality materializes as, for instance, loan commitments or credit lines. <sup>13</sup> The provision of these relationship services to entrepreneur *i* costs the bank

<sup>12</sup> Since we impose no frictions to diversification, this same outcome could also obtain through direct finance. We come back to this point later on, when we show that this is in fact a limiting case of our model.

<sup>13</sup> Overdraft agreements, charge cards, factoring agreements, etc., are other bank products that can add cash-flow flexibility to entrepreneurs.

 $\beta$  units of output. However, providing such services enhances the likelihood of success of the entrepreneur by endowing him with needed cash-flow flexibility while facing the uncertainty implied by the random variable  $\vartheta_i$ .<sup>14</sup>

If at least one bank offers these additional services to the entrepreneur, we assume that all uncertainty dissipates and the project succeeds with probability one. In the event that multiple banks provide the services to the same entrepreneur there is no multiplicative effect on his ex post return, just a multiplication of costs.

INTUITION. Abstracting from ex-ante differences in quality, or ex-post actions, what we are after is that even the most skilled and best intentioned entrepreneur faces a positive risk of default due to the inherently uncertain environment in which the investment project takes place.

We could offer different interpretations of this feature of business activity, but for the sake of clarity we focus on the fact that entrepreneurs face liquidity shocks during their investment process: throughout their activity, they have inflows and outflows of payments (e.g., receivables, payables, payroll, etc.). Mismatches in the timing of such flows could make an otherwise profitable project insolvent. In addition, market events beyond individual control may force entrepreneurs to undertake unexpected, and costly, changes that exceed the originally budgeted cost of the project. Hence, uncertainty about the actual realization of these factors can lead entrepreneurs to run into liquidity problems, insolvency and consequently default, even if the projects maintain a positive net present value. We do not model explicitly this process but capture it instead with the random variable  $\vartheta_i$ .

The management of liquidity is an essential aspect of real-world corporate finance and planning. Focusing on liquidity needs we therefore emphasize a crucial aspect of firm performance and model the banks' unique role in supporting it.

Our modeling of the bank-entrepreneur relationship highlights that by providing relationship services the bank produces an *improvement in the technology* that transforms credit into capital. Therefore, our banks provide entrepreneurs not only with credit but also with what effectively amounts to a complementary input — the relationship services — that affects the ex-post rate of return on their projects.

There is ample empirical evidence that bank liquidity services improve firms' performance. For instance, James [28] shows that firms announcing the reception of bank liquidity commitments experience abnormal rises of their stock prices. This evidence indicates that markets assign a significant value added to this bank service in their assessment of firms' future performance. The impact on firms' equity value is confirmed in many other contributions (e.g., Lummer and McConnell [35], James and Weir [29], Hoshi, Kashyap and Scharfstein [27]). The intuition is quite clear: access to contingent liquidity is a form of insurance for the entrepreneur in the bad state of the world (see, e.g., Gatev and Strahan [22]), which allows entrepreneurs to reduce their overall cost of funding by reducing their risk premium (see, e.g., Shockley and Thakor [46]).

Our focus on liquidity provision captures a defining characteristic of banks and builds on Holmstrom and Tirole [26] and, especially, Kashyap, Rajan and Stein [31]. Both contributions put the liquidity needs of the firms at the center of the analysis and in both cases banks are indeed

<sup>14</sup> There are other ways to formalize the provision of relationship services that can improve firms' performance. For example, a bank may take an active role in the management of the enterprise itself, through the provision of advisory and consulting services but also by having own executives as members of firms' board of directors. For instance, in the 1990s and early 2000s, about a quarter of S&P non-financial companies had a banker on board (Kroszner and Strahan [33], Santos and Rumble [44]).

characterized as unique institutions able to provide liquidity services in addition to satisfying firms' standard funding needs. The main ingredient in the Kashyap Rajan and Stein [31] is the need for firms to access flexible, contingent funding throughout the life span of the project. We capture this with the random variable  $\vartheta_i$  in the technology that transforms credit into capital. The second ingredient is the existence of an overhead cost to provide such service. In their model it is the amount of funds that the bank needs to keep as cash to meet the potential contingent demand of funding. In our model, this overhead is captured by the parameter  $\beta$ .

#### 3.2. The free-riding problem

An important feature of our setup is that the provision of relationship services is entrepreneur-specific. Equally important is that in providing this input the bank improves the performance of the borrower but that there is no full appropriability of such benefit. In this sense, the provision of the relationship services has clear characteristics of non-rivalry and non-excludability. Non-rivalry stems from the fact that liquidity services have a beneficial effect on the entrepreneurial activity as a *whole*: access to the liquidity contingent line reduces the likelihood of the entrepreneur's default, hence it affects the value of all the claims of any entrepreneur's creditor, not just those of the bank that provided the liquidity facility. Indeed, in a recent paper, Ongena, Roscovan, Song and Werker [37] provide evidence that the announcement of a bank credit commitment reduces the credit spread on *bonds* issued by the firm. This is direct evidence that receiving the liquidity services by a bank makes overall entrepreneurial activity less risky, so that also other creditors of the firms can benefit from it.

This improvement in the overall performance of the entrepreneur gives rise to the incentive to free ride: other banks that are simultaneously lending at arm's length to the same entrepreneur would experience the higher return on their investment without having sustained the entrepreneur-specific relationship cost.

If the bank could impose an exclusive contract on each entrepreneur, the decision to provide the relationship input would simply require comparing its net return with the arm's length strategy. However, exclusivity seems overly restrictive in financial contracts. Bisin and Guaitoli [10], for instance, argue that exclusive contracts require strong assumptions on the ability of banks to monitor the entrepreneur's potential trades with other institutions. Because of this intrinsic difficulty in observing and verifying such possible trades, exclusive contracts would also be difficult to enforce in a court of law. And, as Bisin and Guaitoli also argue, markets for unsecured loans, which is what we have in our model, "do not seem to operate even implicitly through exclusive contracts" (p. 307).

In our setup the bank would want to impose an exclusive contract but it would also realize that the entrepreneur has no incentive to either accept or live by the rules of the agreement. Accepting simultaneously additional loans allows him to scale up operations in the expectation of certain success. This actually fits observation. Firms normally do use more than one bank at the time. Ongena and Smith [38], for instance, report that across twenty European countries, firms borrowing from one bank is the exception rather than the rule: "Across all countries, the mean number of relationships is 5.6 and the median is 3" (p. 29). 15

Moreover, since entrepreneurs' idiosyncratic risk is invariant to the size of the investment, there are no risk externalities that would discourage other banks from extending such loans

<sup>&</sup>lt;sup>15</sup> More broadly, Detragiache, Garella and Guiso [21] write: "Much of the theory of banking relationships implicitly assumes that firms have only one bank, but empirical evidence shows that this is not always the case" (p. 1155).

to the same entrepreneur. Notice in fact that the incentives for other banks to lend to the same entrepreneur is even stronger knowing that he is receiving relationship services from one bank. In addition, threatening withdrawal of the liquidity support upon observance of the entrepreneur taking another loan is not a viable strategy: once the relationship loan is issued, it is in the best interest of the bank to follow through with the commitment to provide liquidity, since it affects its own profitability. This is actually what we observe in reality: in some cases credit lines are written with clauses related to the observation of firms' actions and/or changes in firms' conditions that free the bank from its obligation; yet, in practice such clauses are hardly ever invoked (see, e.g. Avery and Berger [3], Berger and Udell [6], Roberts and Sufi [43]).

To sum up, in our model a bank decides whether to extend credit with or without providing additional relationship services knowing that other banks are simultaneously formulating the same decision on the same population of borrowers. The model studies how the market structure of the banking sector regulates the incentives that banks face in making this decision.

# 3.3. Timing of events

Before turning to this crucial component of the model, it is useful to summarize the timing of events. At time t old agents of generation t-1, who have saved resources to finance time t consumption, supply their savings to banks. Entrepreneurs (the population of young agents of generation t) demand credit from the banking sector. Banks simultaneously determine their lending strategy, that is, whether to provide the "relationship input" in addition to the loan itself. Entrepreneurs obtain the loans and, perhaps, the additional services. They succeed or fail in transforming credit into capital. The successful entrepreneurs add to the aggregate capital stock, which is then used to produce the final good. Given total output  $Y_t$ , a fraction represents the compensation for the successful entrepreneurs, which is used to pay back bank loans. Banks pay savers who consume the payment at time t+1. A fraction of output  $Y_t$  is the labor income of young agents of generation t who, according to their preferences, decide how much to consume and how much to save. Their savings are then intermediated by banks to generate credit supply for entrepreneurs of generation t+1.

#### 4. The choice of quantity and quality of credit

In this section we begin our technical analysis by showing how our conceptual apparatus translates into a specific maximization problem for the individual bank. We then derive first-order conditions and use them to characterize the bank's behavior. In the following two sections we then use these results to characterize the general equilibrium dynamics of the economy.

## 4.1. Notation and preliminary considerations

Let  $i \in [0, 1]$  denote a loan applicant from the mass of entrepreneurs, and let j = 1, ..., N denote one of the banks in operation. We take the number of banks as exogenous. This assumption is based primarily on the observation that, in contrast to most other industries where the default is that market structure and competitive conduct evolve endogenously, banking has historically been heavily regulated. This is true both in the U.S. and in other countries. Hence, it is plausible to consider banking market structure as exogenous when studying its effect on the real

economy. <sup>16</sup> Having said this, however, after presenting our main results we shall also study the model's equilibrium under free entry.

Recall that banks serve simultaneously the entire population of borrowers and they supply credit at any scale they deem optimal. At the same time they also determine whether to offer additional relationship services to each entrepreneur that receives a loan. In this setup, and in light of our objective to study how banking market structure affects capital accumulation, it is quite natural to model banks as Cournot competitors that determine the optimal scale of operations and the lending strategy taking into account that their choice affects the aggregate amount of credit raised from the households and the overall likelihood that entrepreneurs receive relationship services, and that together these two factors determine the aggregate amount of capital that is supplied to the final producers. The advantage of the Cournot approach is the well-known property that monopoly and (pure) competition are the extremes of a well-understood range of outcomes with respect to the number of banks: monopoly applies for N = 1, competition for  $N \to \infty$ , oligopoly in between.

It turns out, moreover, that in our setup the decision of a bank to offer the relationship services can be framed exactly as in the public finance literature on the voluntary provision of a public good, especially Palfrey and Rosenthal [39] and [40], so that our use of the Cournot model merges seamlessly with the Nash equilibria studied in that literature.<sup>17</sup> Palfrey and Rosenthal [39] show the existence of a symmetric mixed strategy equilibrium as solution to the participation game, with agents contributing with probability p and abstaining with probability 1-p to the construction of a public good. They model a general setup where the number of agents needed to build the public good can be greater than one. Our model, where only one bank providing the financial services is sufficient to change the probability of success of the entrepreneur, and therefore the ex-post return on the investment, is a special case of their setup. However, our agents (banks) make the additional, simultaneous decision about the scale of operations (lending), which in Palfrey and Rosenthal [39] is assumed away by positing an exogenously given project (public good) size. The equivalence of our setup to that of Palfrey and Rosenthal [39] tells us that the main insight driving our model — that banks face a freerider problem and solve it by adopting a mixed strategy, whereby they randomize their choice to provide relationship services to each individual entrepreneur — rests on a solid theoretical foundation.

In what follows, we construct the typical bank's objective function starting from the problem of bank j regarding entrepreneur i. We then aggregate up to the entire population of entrepreneurs and finally derive the bank's profit function. To maximize clarity, at this stage of the analysis we shall use the subscript j to denote the choice variables of bank j and the subscript q to denote the choice variables of bank  $q \neq j$ . In the computation of the Nash equilibrium we shall impose symmetry and drop the subscripts.

<sup>&</sup>lt;sup>16</sup> For instance, before the process of deregulation initiated in the mid 1970s, the U.S. banking industry had been effectively partitioned, since the nineteenth century, within state boundaries, and even within states there were significant restrictions to bank expansion: at the beginning of the 1970's, 38 states prohibited bank branching within a state (unit banking states) or imposed significant limitations to branching. At the same time, banks were completely prohibited from acquiring banks outside the state in which they were headquartered (see, e.g., Jayaratne and Strahan [30]).

<sup>&</sup>lt;sup>17</sup> We are extremely grateful to Huseyin Yildirim for helping us spot and clarify the connection of our setup to Palfrey and Rosenthal [39,40].

#### 4.2. The typical bank's problem

With probability  $p_j$  bank j offers to entrepreneur i relationship services in addition to a loan. With probability  $1-p_j$ , instead, bank j extends just an arm's length loan to entrepreneur i. Two outcomes are then possible: with probability  $\prod_{q\neq j}(1-p_q)$  none of the other banks provide the relationship services; with probability  $1-\prod_{q\neq j}(1-p_q)$  at least one of the other banks provides the services.

Then, using (2) and (4), substituting total credit X for S in the first one, and dropping the time subscript without loss of generality, the bank's expected profit from issuing to entrepreneur i a loan of size  $x_{ji}$  is

$$\begin{split} \pi_{ji} &= p_j \left( \gamma K^{\gamma - 1} x_{ji} - \beta \right) + (1 - p_j) \prod_{q \neq j} (1 - p_q) \theta \gamma K^{\gamma - 1} x_{ji} \\ &+ (1 - p_j) \bigg[ 1 - \prod_{q \neq j} (1 - p_q) \bigg] \gamma K^{\gamma - 1} x_{ji} - \bigg[ \frac{X}{W - X} \bigg]^{\frac{1 - \alpha}{\alpha}} x_{ji}. \end{split}$$

The first term says that with probability  $p_j$  the bank's loan  $x_{ji}$  earns the marginal product of capital but that the bank bears the cost  $\beta$  of guaranteeing the project's success. The second line says that the bank's loan  $x_{ji}$  earns the marginal product of capital with probability  $(1-p_j)\prod_{q\neq j}(1-p_q)\theta$ , which is the product of the probability that no banks supply relationship services times the "primitive" probability of success  $\theta$ . The third term captures free riding, i.e., without bearing the relationship cost  $\beta$  the bank's loan  $x_{ji}$  earns the marginal product of capital with probability  $(1-p_j)[1-\prod_{q\neq j}(1-p_q)]$ , which is the probability that at least one of the other banks supplies the relationship services and thereby guarantees the project's success. The fourth line says that to generate the loan  $x_{ji}$  the bank needs to compensate savers according to their desired rate of return. <sup>18</sup>

If we aggregate over the mass of applicants, the bank's total profit is

$$\pi_{j} = \int_{0}^{1} p_{j} \left[ \gamma K^{\gamma - 1} x_{ji} - \beta \right] di + \int_{0}^{1} (1 - p_{j}) \prod_{q \neq j} (1 - p_{q}) \theta \gamma K^{\gamma - 1} x_{ji} di$$

$$+ \int_{0}^{1} (1 - p_{j}) \left[ 1 - \prod_{q \neq j} (1 - p_{q}) \right] \gamma K^{\gamma - 1} x_{ji} di - \int_{0}^{1} \left[ \frac{X}{W - X} \right]^{\frac{1 - \alpha}{\alpha}} x_{ji} di.$$
 (6)

Therefore, defining  $x_j \equiv \int_0^1 x_{ji} di$ , observing that  $(1 - p_j) \prod_{q \neq j} (1 - p_q) = \prod_q (1 - p_q)$ , and rearranging terms, we obtain

$$\pi_j = \left\{ \gamma K^{\gamma - 1} \left[ 1 - (1 - \theta) \prod_q (1 - p_q) \right] - \left[ \frac{X}{W - X} \right]^{\frac{1 - \alpha}{\alpha}} \right\} x_j - p_j \beta. \tag{7}$$

This expression allows us to concentrate on two choice variables only: the total amount of lending  $x_i$  that bank j does and the probability  $p_j$  with which it provides the additional services to

<sup>18</sup> Recall that the investment technology that transforms credit into capital is linear and that entrepreneurs are atomistic price takers subject to a zero profit conditions. Hence, they have an infinitely elastic demand for credit.

the individual entrepreneur. As said earlier, since we are characterizing the banks' strategic interaction in a Cournot model, the individual bank takes into account the effect of its own actions on the amount of savings X raised from the households and lent to entrepreneurs, the probability  $1 - \prod_q (1 - p_q)$  that an entrepreneur receives relationship services, and consequently the amount of capital K supplied to the production firms.

To appreciate fully how the individual bank contributes to K in this environment, we combine terms in (6) and write

$$\int_{0}^{1} \left[ 1 - \prod_{q} (1 - p_q) \right] x_{ji} \, di = x_{j}^{rel}$$

as the fraction of total credit of bank j loaned to entrepreneurs that received relationship services, while

$$\int_{0}^{1} \prod_{q} (1 - p_q) x_{ji} di = x_j^{arm}$$

is the fraction loaned by bank j to entrepreneurs that did not receive relationship services. Accordingly, we have

$$x_j^{rel} = \left[1 - \prod_q (1 - p_q)\right] x_j$$

and

$$x_j^{arm} = \prod_q (1 - p_q) x_j,$$

where  $x_j = x_j^{rel} + x_j^{arm}$  is the total amount of credit extended by bank j. Then,

$$\begin{split} K &= \sum_{j} x_{j}^{rel} + \theta \sum_{j} x_{j}^{arm} \\ &= \sum_{j} \left[ 1 - \prod_{q} (1 - p_{q}) \right] x_{j} + \theta \sum_{j} \prod_{q} (1 - p_{q}) x_{j} \\ &= \sum_{j} \left[ 1 - (1 - \theta) \prod_{q} (1 - p_{q}) \right] x_{j}. \end{split}$$

We then denote

$$m \equiv 1 - (1 - \theta) \prod_{q} (1 - p_q)$$
 (8)

and since

$$X = \sum_{j} x_{j},$$

we can write

$$K = mX, (9)$$

where m is a reduced-form measure of how effectively the economy transforms credit into capital. This measure is based on the intensity with which banks provide relationship services. Therefore, we can think of m as the aggregate *endogenous efficiency* of investment. If  $p_q = 1$   $\forall q$ , then m = 1 and there is no waste of credit. If  $p_q = 0$   $\forall q$ , then  $m = \theta$  and there is the maximum possible waste of credit.

# 4.3. The bank's optimal choice of $x_i$ and $p_i$

We are now ready to solve the bank's maximization problem. Eqs. (2), (4), (8) and (9) allow us to write the bank's problem (7) in compact notation as follows:

$$\max_{x_j, p_j} [mR(mX) - r(X)]x_j - p_j\beta \quad \text{s.t. (2) and (4)}.$$

The first-order condition with respect to  $x_i$  is

$$mR - r + \left(m^2 \frac{\partial R}{\partial X} - \frac{\partial r}{\partial X}\right) x_j = 0.$$
 (10)

The first-order condition with respect to  $p_i$  is

$$p_{j} = \begin{cases} 0 & \text{for } x_{j} [R + m \frac{dR}{dK} X] \frac{dm}{dp_{j}} < \beta, \\ ? & \text{for } x_{j} [R + m \frac{dR}{dK} X] \frac{dm}{dp_{j}} = \beta, \\ 1 & \text{for } x_{j} [R + m \frac{dR}{dK} X] \frac{dm}{dp_{j}} > \beta. \end{cases}$$

$$(11)$$

It is useful to discuss these conditions separately.

Eq. (10) describes the behavior of a bank with market power in the output (oligopolist) and input (oligopsonist) markets. To highlight what this implies, we rewrite it as

$$\frac{R}{r} = \underbrace{\frac{1}{m}}_{\text{inverse of credit efficiency}} \cdot \underbrace{\frac{1 + \frac{x_j}{X} \frac{1}{\varepsilon_r}}{1 - \frac{x_j}{X} \frac{1}{\varepsilon_R}}}_{\text{exercise of market power}},$$
(12)

where

$$\varepsilon_r \equiv \frac{\partial X}{\partial r} \frac{r}{X} = \frac{\alpha}{1 - \alpha} \frac{W - X}{W} \tag{13}$$

and

$$\varepsilon_R \equiv -\frac{\partial X}{\partial R} \frac{R}{X} = \frac{1}{1 - \gamma} \tag{14}$$

are, respectively, the elasticity of saving supply and credit demand derived from (2) and (4). These elasticities capture the property that our banks internalize the effects of their individual quantity decisions on the total quantity of credit, which in turn affects the interest rates on loans and savings (recall: deposits plus bank equity capital). A novel feature of (12) is the presence of the inverse of the credit efficiency term m, which says that the more banks offer relationship services, the smaller is the spread between interest rates on loans and the return to saving. The reason is that when less credit is wasted, the economy accumulates more capital, and the corresponding lower marginal product of capital results into entrepreneurs' lower willingness to pay for loans.

Eq. (11) states that when the marginal benefit of providing the relationship service is smaller than the marginal cost the bank does not provide it at all, i.e., it sets  $p_j = 0$ . Conversely, if the marginal benefit of providing the service exceeds the marginal cost it is optimal to provide it to all its clients irrespective of free riding considerations, i.e., the bank sets  $p_j = 1$ . The middle line says that when the marginal benefit of providing the service equals the cost, the bank wants  $0 < p_j < 1$  but is indifferent to the specific value of  $p_j$ , which in equilibrium is determined by the *simultaneous* solution of the two first-order conditions. <sup>19</sup> Observe that

$$m\frac{dR}{dK}X = m\frac{dR}{dX}\frac{dX}{dK}X = m\frac{dR}{dX}\frac{1}{m}X = \frac{dR}{dX}\frac{X}{R}R = -\frac{R}{\varepsilon_R}.$$

Using this result and (4), we can rewrite the indifference condition as

$$\underbrace{x_{j}}_{\text{total credit issued}} \cdot \underbrace{\gamma K^{\gamma - 1}}_{\text{interest rate on loans}} \cdot \underbrace{\left(1 - \frac{1}{\epsilon_{R}}\right)}_{\text{contribution to capital via contribution to credit efficiency}} \cdot \underbrace{\left(1 - \theta\right) \prod_{q \neq j} (1 - p_{q})}_{\text{contribution to aggregate prob}} = \beta. \tag{15}$$

The bank's marginal benefit from providing the relationship services is decreasing in  $p_q$  for all  $q \neq j$ , once again capturing the role of free riding, and increasing in the bank's overall scale of operations  $x_j$ . Note also that the condition can hold iff  $0 < \theta < 1$  and  $\beta > 0$ . Intuitively, if either  $\theta = 0$  or  $\theta = 1$  the service does not matter and it is optimal to set  $p_j = 0$ . Similarly, if  $\beta = 0$  the benefit is always larger than the cost and it is optimal to set  $p_j = 1$  regardless of what the other banks do. It is worth stressing that the indifference condition (15) shows that the incentive to free ride arises because bank j can set  $p_j = 0$  and still reap the higher return if another bank provides the relationship service. To see this, start from a situation where the condition holds with equality and  $p_j > 0$ . Let the probability  $p_q$ ,  $q \neq j$  rise. The left-hand side of (15) falls and bank j switches to  $p_j = 0$ . The intuition is that the higher probability of some other bank providing the service reduces the marginal benefit to bank j of providing the service itself and induces bank j to free ride.

## 5. The banking sector's symmetric Nash equilibrium

We impose symmetry and characterize the Nash equilibrium of the banking sector by solving simultaneously (12) and (15). We first rewrite them as:

$$\frac{\gamma (mX)^{\gamma-1}}{(\frac{X}{W-X})^{\frac{1-\alpha}{\alpha}}} = \frac{1}{m} \frac{N + \frac{1-\alpha}{\alpha} \frac{W}{W-X}}{N-1+\gamma},$$
$$\frac{X}{N} \gamma^2 (mX)^{\gamma-1} (1-\theta) (1-p)^{N-1} = \beta.$$

<sup>&</sup>lt;sup>19</sup> It is straightforward to check that, abstracting from the quantity choice, this is indeed the indifference condition for the provision of the public good discussed by Palfrey and Rosenthal [39].

Then use (8) to obtain:

$$1 - (1 - \theta)(1 - p)^{N} = \left[\frac{X^{1 - \gamma} (\frac{X}{W - X})^{\frac{1 - \alpha}{\alpha}}}{\gamma} \frac{N + \frac{1 - \alpha}{\alpha} \frac{W}{W - X}}{N - 1 + \gamma}\right]^{\frac{1}{\gamma}},\tag{16}$$

$$X = \left[ \frac{\beta N}{\gamma^2} \frac{[1 - (1 - \theta)(1 - p)^N]^{1 - \gamma}}{(1 - \theta)(1 - p)^{N - 1}} \right]^{\frac{1}{\gamma}}.$$
 (17)

We graph these two functions in (X, p) space in Fig. 1. To fix ideas, we refer to (16) as the "lending curve" since it yields the optimal lending volume given the banks' probability of providing the relationship service. Similarly, we refer to (17) as the "relationship curve" since it yields the optimal probability of undertaking relationship lending given the banks' lending volume. The equilibrium is the intersection of the two curves. The following two lemmas state formally the properties of the two curves. Since we are interested in the role of competition, we highlight the role of three structural parameters: N, because it is our measure of competition in banking;  $\theta$ , because it regulates crucially the role of N;  $\beta$ , because it determines whether relationship lending is profitable in the first place.

**Lemma 1.** Denote the right-hand side of (16) as F(X; W, N). The lending curve in (X, p) space is a function

$$p^L(X; W, N, \theta) = \begin{cases} 0, & 0 \leq X \leq X_0^L(W, N, \theta), \\ 1 - [\frac{F(X; W, N)}{1 - \theta}]^{\frac{1}{N}}, & X_0^L(W, N, \theta) < X < X_1^L(W, N), \\ 1, & X \geq X_1^L(W, N), \end{cases}$$

where:

- $X_0^L(W, N, \theta) = \arg \operatorname{solve}\{\theta = \digamma(X; W, N)\};$
- $X_1^L(W, N) = \arg \text{solve}\{1 = F(X; W, N)\};$
- in the region  $X_0^L(W, N, \theta) < X < X_1^L(W, N)$  the following holds:
  - $p_X^L(X; W, N, \theta) > 0;$
  - $p_{XX}^L(X; W, N, \theta) < 0;$
  - $p^L(X; W, N, \theta)$  increasing in  $W, N, \theta$ .

**Proof.** The non-negativity constraint  $p \ge 0$  binds whenever

$$p = 1 - \left[\frac{F(X; W, N)}{1 - \theta}\right]^{\frac{1}{N}} < 0.$$

Therefore, there exists a value  $X_0^L(W, N, \theta)$  defined by

$$1 - \left\lceil \frac{F(X; W, N)}{1 - \theta} \right\rceil^{\frac{1}{N}} = 0 \quad \Rightarrow \quad F(X; W, N) = \theta$$

and such that p = 0 for  $X \leq X_0^L(W, N, \theta)$ . The constraint  $p \leq 1$  binds whenever

$$p = 1 - \left[\frac{F(X; W, N)}{1 - \theta}\right]^{\frac{1}{N}} > 1.$$

Therefore, there exists a value  $X_1^L(W, N)$  defined by

$$1 - \left\lceil \frac{F(X; W, N)}{1 - \theta} \right\rceil^{\frac{1}{N}} = 1 \quad \Rightarrow \quad F(X; W, N) = 0$$

and such that p=1 for  $X\geqslant X_1^L(W,N)$ . The other properties follow directly from differentiating with respect to  $X,W,N,\theta$ .  $\square$ 

Eq. (16) describes the aggregate amount of credit X that banks wish to issue given their relationship strategy p. For values  $p \in (0,1)$  the curve is increasing because as banks provide more services, they wish to lend more since lending becomes safer and generates a higher rate of return. The curve is convex because as banks want to extend more credit, because they provide more services, they must also pay a higher interest rate to savers. An important property of this curve is that when the wage rises (because the previous period capital stock is larger), savers demand a lower interest rate and the banks' profit margin rises. Accordingly, they lend more for any given relationship strategy p.

**Lemma 2.** Denote the right-hand side of (17) as  $\Omega(p; N, \theta, \beta)$ . The relationship curve in (X, p) space is a function

$$p^R(X;N,\theta,\beta) = \begin{cases} 0, & 0 \leqslant X \leqslant X_0^R(N,\theta,\beta), \\ \Omega^{-1}(p;N,\theta,\beta), & X > X_0^R(N,\theta,\beta), \end{cases}$$

where:

- $X_0^R(N,\theta,\beta) = \left[\frac{\beta N}{\nu^2} \frac{\theta^{-\gamma}}{1-\theta}\right]^{1/\gamma};$
- in the region  $X > X_0^R(N, \theta, \beta)$  the following holds:
  - $p_X^R(X; N, \theta, \beta) > 0;$
  - $-\ p_{XX}^R(X;N,\theta,\beta)<0;$
  - $-\lim_{X\to\infty} p^R(X; N, \theta, \beta) = 1;$
  - $-p^{R}(X; N, \theta, \beta)$  increasing in N,  $\beta$ , U-shaped in  $\theta$ .

**Proof.** Study (17) in (p, X) space and then plot its inverse in (X, p) space. Direct differentiation establishes how the curve shifts with  $N, \beta, \theta$ .  $\square$ 

Eq. (17) describes the relationship strategy p that banks wish to adopt given the aggregate amount of credit X. The first property to note is that if aggregate credit is too low, the amount of credit x of the individual bank is too small and the bank cannot cover the cost of providing services. Accordingly, banks choose p = 0. Only when aggregate credit is sufficiently large banks provide services.<sup>20</sup> In the region where p > 0, the curve is monotonically increasing because as aggregate credit gets larger banks spread the cost of providing services on larger loans. If N > 1, it converges asymptotically to p = 1. The concavity of the function reflects the fact that the bank's benefit from providing services depends on its contribution to credit efficiency m and

<sup>&</sup>lt;sup>20</sup> This result doe *not* depend on the fact that the screening cost is independent of loan size. An extension to the case of size-dependent cost, which yields the same qualitative result that we present here, is available on request.

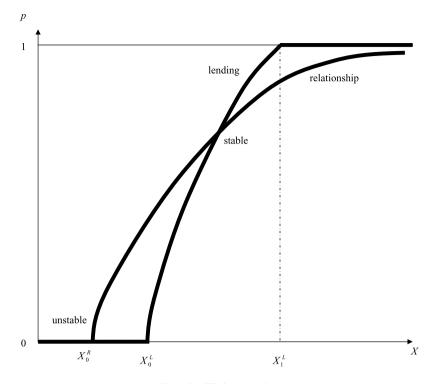


Fig. 1. Equilibrium p and X.

through it to K. Since the interest rate on loans decreases with K, an increase in credit X that induces an increase in p is subject to diminishing returns. Note that because the marginal benefit of providing services depends only on the interest rate on loans, the relationship curve does not depend on the wage W.

The equilibrium of the banking sector is the intersection of the lending curve (16) and the relationship curve (17). Consider Fig. 1. The curves intersect only once for positive p. This intersection is stable in that a deviation with higher X for the same p leads banks to reduce X and thus return to the intersection point. The point  $X_0^R(N, \theta, \beta)$  is also a possible equilibrium, since it yields the optimal amount of credit given p = 0, but it is unstable and thus can be ignored.

Inspection of Fig. 1 reveals that there exists a critical value of the wage  $W_0(N, \theta, \beta)$  such that for  $W \leq W_0(N, \theta, \beta)$  the equilibrium is  $(X_0^L, 0)$  since the two curves do no intersect for p > 0. The value of  $W_0(N, \theta, \beta)$  follows from solving

$$X_0^L(W, N, \theta) = X_0^R(N, \theta, \beta).$$

This is the first aggregate implication of the model and highlights the role of the cost  $\beta$  of providing services.

**Remark 1.** For  $W \le W_0(N, \theta, \beta)$ , the banking sector is unable to afford services. Only if  $W > W_0(N, \theta, \beta)$  relationship lending and the associated higher efficiency is profitable. In this case the equilibrium has the property that both the quantity and the quality of credit increase in W because increases in W shift the lending curve (16) up and yield a movement along the relationship curve (17). As  $W \to \infty$ ,  $X \to \infty$  while  $p \to 1$ .

The following proposition summarizes these insights.

**Proposition 3.** The equilibrium of the credit market is represented by two functions

$$X(W;N,\theta,\beta) = \begin{cases} X_0^L(W,N,\theta), & 0 \leqslant W \leqslant W_0(N,\theta,\beta), \\ X^*(W;N,\theta,\beta), & W > W_0(N,\theta,\beta), \end{cases}$$

and

$$p(W; N, \theta, \beta) = \begin{cases} 0, & 0 \leqslant W \leqslant W_0(N, \theta, \beta), \\ p^*(W; N, \theta, \beta), & W > W_0(N, \theta, \beta), \end{cases}$$

with the following properties:

- $W_0(N, \theta, \beta)$  is increasing in N,  $\theta$ ,  $\beta$  with
  - $-\lim_{N\to\infty} W_0(N,\theta,\beta) = \infty,$
  - $-\lim_{\theta\to 1} W_0(N,\theta,\beta) = \infty,$
  - $-\lim_{\beta\to\infty} W_0(N,\theta,\beta) = \infty.$
- $X_0^L(W, N, \theta)$  is increasing in W with

$$X_0^L(W_0(N,\theta,\beta);N,\theta) = X^*(W_0(N,\theta,\beta);N,\theta,\beta).$$

- $X_W^*(W; N, \theta, \beta) > 0$  with  $\lim_{W \to \infty} X^*(W; N, \theta, \beta) = \infty$ .
- $p_W^*(W; N, \theta, \beta) > 0$  with  $\lim_{W \to \infty} p^*(W; N, \theta, \beta) = 1$ .

The next three remarks highlight equilibrium properties related to bank competition.

**Remark 2.** The number of banks N has an ambiguous effect on the equilibrium values  $p^*$  and  $X^*$ .

Inspection of Fig. 1 provides the intuition for this property. When N rises and the market becomes more competitive, profit margins shrink and banks increase the volume of credit they issue, given the relationship strategy p. This effect is captured by the downward shift of the lending curve (16). On the other hand, banks also wish to do less relationship lending since the marginal benefit from providing financial services shrinks. This effect is captured by the downward shift of the relationship curve (17). As one can see, these shifts yield an ambiguous effect of the number of banks on both the individual probability of providing services p and total lending X.

**Remark 3.** Since  $X_0^L(W; N, \theta)$  is increasing in N, a larger number of banks delays the onset of relationship lending in the sense that the more competitive is the banking sector, the higher is the volume of saving (due to a higher W) that triggers the provision of relationship services.

The reason is that with lower profit margins, the indifference condition (15) holds only if the overall market is larger so that the individual bank lends more. When W crosses the threshold  $W_0(N, \theta, \beta)$  we start seeing the channels that our model identifies: competition reduces margins and tends to raise total credit, but it also reduces the incentive to provide services, the efficiency of credit, and therefore the banks' willingness to lend.

**Remark 4.** It is possible to have two intersections, in which case the rightmost one is stable and the other one unstable. This pattern yields multiple equilibria because the point  $X_0^L(W; N, \theta)$  is feasible and locally stable.

The most interesting consequence of this configuration is that as the wage grows, the lending curve (16) shifts down and eventually becomes tangent to the relationship curve (17). It is then possible to have a discontinuous jump to the interior stable equilibrium with p > 0. In the analysis below we focus on the case of a unique equilibrium because it is simpler and captures fully the model's main insight.

The analysis in the previous section has characterized how banks' individual decisions about screening drive the aggregate efficiency term

$$m = 1 - (1 - \theta)(1 - p)^N$$
.

This term plays a critical role in determining capital accumulation since

$$K = mX$$
.

It is therefore useful to restate Proposition 3 in terms of these two variables in order to highlight how they depend on the wage and the model's parameters. The main advantage of this exercise is that it yields directly the equation that governs the aggregate dynamics of the model.

To this end, we rewrite (16)–(17) as:

$$m = \left[ \frac{\left(\frac{K}{m}\right)^{1-\gamma} \left(\frac{K}{m-\frac{K}{m}}\right)^{\frac{1-\alpha}{\alpha}}}{\gamma} \frac{N + \frac{1-\alpha}{\alpha} \frac{W}{W - \frac{K}{m}}}{N - 1 + \gamma} \right]^{\frac{1}{\gamma}}, \tag{18}$$

$$K = \left[ \frac{\beta N}{\gamma^2} \frac{m}{(1 - \theta)^{\frac{1}{N}} (1 - m)^{\frac{N-1}{N}}} \right]^{\frac{1}{\gamma}}.$$
 (19)

These two loci have properties that are isomorphic to those of the (16)–(17) curves studied in Fig. 1. We refer to them as the "efficiency curve" and the "accumulation curve", respectively. We can then construct Fig. 2 and obtain the following result.

**Proposition 4.** The equilibrium level of credit market efficiency and the equilibrium amount of capital that the economy builds within each period are represented by two functions

$$K(W; N, \theta, \beta) = \begin{cases} K_0^A(W; N, \theta), & 0 \leqslant W \leqslant W_0(N, \theta, \beta), \\ K^*(W; N, \theta, \beta), & W > W_0(N, \theta, \beta), \end{cases}$$

and

$$m(W; N, \theta, \beta) = \begin{cases} \theta, & 0 \leq W \leq W_0(N, \theta, \beta), \\ m^*(W; N, \theta, \beta), & W > W_0(N, \theta, \beta), \end{cases}$$

with the following properties:

- $W_0(N, \theta, \beta)$  is increasing in N,  $\theta$ ,  $\beta$  with
  - $-\lim_{N\to\infty} W_0(N,\theta,\beta) = \infty,$
  - $-\lim_{\theta\to 1} W_0(N,\theta,\beta) = \infty$
  - $-\lim_{\beta\to\infty}W_0(N,\theta,\beta)=\infty.$

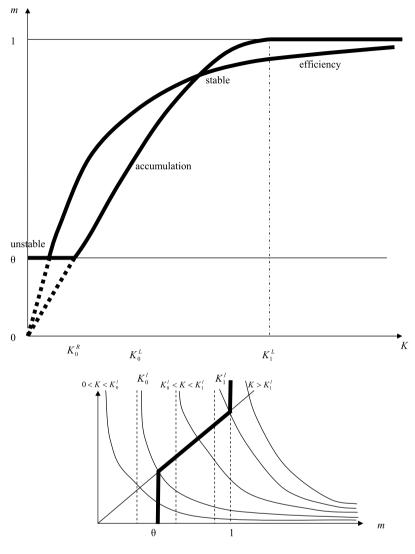


Fig. 2. Equilibrium m and K.

- $K_0^A(W; N, \theta)$  is increasing in W.  $K_0^A(W_0(N, \theta, \beta); N, \theta) = K^*(W_0(N, \theta, \beta); N, \theta, \beta)$ .
- $K_W^*(W; N, \theta, \beta) > 0$  with  $\lim_{W \to \infty} X^*(W; N, \theta, \beta) = \infty$ .
- $m_W^*(W; N, \theta, \beta) > 0$  with  $\lim_{W \to \infty} m^*(W; N, \theta, \beta) = 1$ .

**Proof.** The graph in the lower panel of Fig. 2 shows the construction of the efficiency curve in the upper panel. Denote the right-hand side of (18) as  $\Upsilon(m; K, W, N)$ . This is a monotonically decreasing function of m with a vertical asymptote at

$$m = \frac{K}{W}.$$

This function is also increasing in K. Thus as K rises,  $\Upsilon(m; K, W, N)$  shifts up and traces the 45° line, thereby generating a function  $m^A(K; W, N)$  increasing in K. The constraint  $m \ge \theta$  (i.e.,  $p \ge 0$ ) binds whenever  $K \le K_0^A(W, N, \theta)$  defined by

$$\Upsilon(\theta; K_0^A, W, N) = \theta.$$

Similarly, the constraint  $m \le 1$  (i.e.,  $p \le 1$ ) binds whenever  $K \ge K_1^A(W, N)$  defined by

$$\Upsilon(1; K_1^A, W, N) = 1.$$

The function  $m^A(K; W, N)$  is decreasing in W since  $\Upsilon(m; K, W, N)$  is decreasing in W.

Constructing the efficiency curve  $m^E(K; N, \theta, \beta)$  simply requires studying the function (19) in (K, m) space and then plotting it in (m, K) space. Note that there exists a value  $K_0^A(N, \theta, \beta)$  such that for  $K \leq K_0^A(N, \theta, \beta)$  we have  $m = \theta$ .

The equilibrium is the intersection of the two curves. The effect of the wage follows from the downward shift of the accumulation curve. If the wage is too low, that is, if  $W \leq W_0$ , where  $W_0$  is defined by  $K_0^A(W, N, \theta) = K_0^E(N, \theta, \beta)$ , the equilibrium is  $m = \theta$  and  $K_0^A(W, N, \theta)$ . If instead  $W > W_0$ , the interior equilibrium generates two functions  $K^*(W; N, \theta, \beta)$  and  $m^*(W; N, \theta, \beta)$  both increasing in W since as W rises the accumulation curve shifts down and yields a movement along the efficiency curve. As  $W \to \infty$  we have that  $K \to \infty$  and  $M \to M$ .

Proposition 4 yields directly the equation governing the general equilibrium path of the economy. The effect of the number of banks N on credit efficiency m is in principle ambiguous, but this follows directly from Remark 2. However, referring to Fig. 2, we see that if the shift of the efficiency curve (19) dominates over the shift of the accumulation curve (18) the effect of an increase in N is to reduce m. Inspection of the two equations suggests that, in fact, this property is likely to hold. The reason is that the efficiency curve (19) shifts down without bound, while the accumulation curve (18) shifts with the term

$$\frac{N + \frac{1-\alpha}{\alpha} \frac{mW}{mW - K}}{N - 1 + \gamma},$$

which is bounded above. Thus, while we cannot rule out that starting from small values of N the initial effect of increasing N is to raise m, we can fully expect that as N grows very large its effect on m becomes negative.

An interesting way of interpreting this property is to think of 1 - m as the *losses-to-loans ratio*. Then, the prediction of the model is that (too much) competition raises the losses-to-loans ratio. This prediction is consistent with the empirical evidence presented by Shaffer [45], who documents a negative relationship between the number of banks operating in a market and the losses-to-loans ratio.

**Remark 5.** The scenario where banks choose not to provide relationship services is a natural limiting case of the model where credit can also flow through direct finance.

Since there are no frictions in diversification, idiosyncratic risk could also be diversified away through direct finance of investment projects. This is actually a nice feature of the model that explains under what conditions more sophisticated financial intermediation — i.e., banks fulfilling this function *and* providing relationship services — emerges and how the market structure of the credit sector affects its evolution.

#### 6. Aggregate capital accumulation

We now study the aggregate implications of the model. We first construct the difference equation characterizing the economy's path of capital accumulation and then derive the paper's main comparative statics results.

## 6.1. Dynamics

Recall that the wage is an increasing function of the lagged capital stock,  $W_t = W(K_{t-1})$ ; see Eq. (5). Accordingly, there is a value  $K_{t-1} = K_0(N, \theta, \beta)$  such that  $W(K_0) = W_0(N, \theta, \beta)$ . Recall that  $W_0(N, \theta, \beta)$  is increasing in  $N, \theta, \beta$ . Therefore,  $K_0(N, \theta, \beta)$  is increasing in  $N, \theta, \beta$ . Proposition 4 then yields

$$K_{t+1} = \begin{cases} K_0^A(W(K_t); N, \theta), & 0 \leqslant K_t \leqslant K_0(N, \theta, \beta), \\ K^*(W(K_t); N, \theta, \beta), & K_t > K_0(N, \theta, \beta). \end{cases}$$

This implies that there are two regions of the state-space wherein banking is, respectively, fully inefficient and only partially inefficient. Once  $K_t$  passes the threshold  $K_0(N, \theta, \beta)$ , the economy moves to a higher capital accumulation trajectory because banks reach the minimum scale necessary to make relationship lending profitable. The following proposition states these properties formally.

**Proposition 5.** The economy's general equilibrium is described by the first-order difference equation

$$K_{t+1} = \Phi(K_t; N, \theta, \beta), \tag{20}$$

where

$$\Phi = \begin{cases} K_0^A(W(K_t); N, \theta), & 0 \leq K \leq K_0(N, \theta, \beta), \\ K^*(W(K_t); N, \theta, \beta), & K > K_0(N, \theta, \beta). \end{cases}$$

The function  $\Phi(K;\cdot)$  is continuous, differentiable everywhere except at the point  $K=K_0(N,\theta,\beta)$ , and exhibits the following properties which ensure that there exists at least one non-trivial steady state  $K_{ss}>0$ :

- $\Phi(0) = 0$ ;
- $\Phi_K(\cdot) > 0$ ;
- $\lim_{K\to 0} \Phi_K(K) = \infty$ ;
- $\lim_{K\to\infty} \Phi_K(K) = 0$ .

The trajectory marked in bold in Fig. 3 illustrates the dynamics of the economy. Note that because of the threshold  $K_0(N, \theta, \beta)$  multiple steady states may emerge. This is an important result since it is exactly the number of banks N that determines whether this happens.

**Remark 6.** Even with standard primitives that would normally guarantee a well-behaved dynamic transition to a unique steady state with relationship lending, the market structure of the banking sector can yield a development trap with no relationship lending.

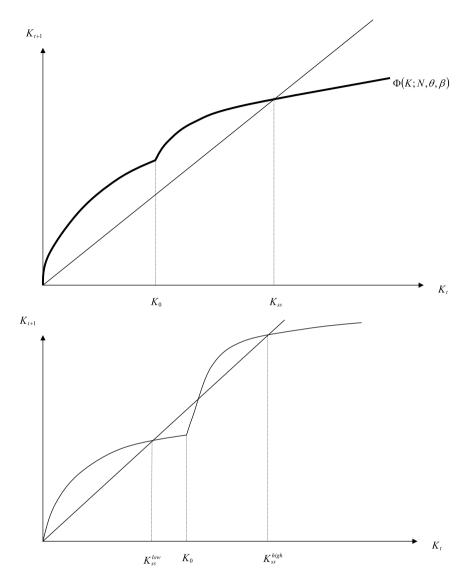


Fig. 3. General equilibrium dynamics.

Inspection of Fig. 3 shows that conditional on being in the lower steady state with no relationship lending, a change in N can remove the steady state with no relationship lending by shifting the curve  $\Phi(K_t; N, \theta, \beta)$  above the 45° line for all  $K_t \leq K_0(N, \theta, \beta)$  and thereby putting the economy on a path that leads to the steady state with relationship lending. Yet, conditional on being on the upper trajectory with relationship lending, the number of banks that maximizes steady-state income can be different from the one that allows escaping from the trap.

It is useful to be specific. Removing the trap requires that the number of banks satisfies

$$K_0^A(W(K_0); N, \theta) > K_0(N, \theta, \beta). \tag{21}$$

This condition says nothing about the level of N that makes the curve  $K^*(W(K_t); N, \theta, \beta)$  as high as possible, which is what is required to maximize the steady-state level of capital produced by equilibria with relationship lending. In fact, the change in N required to remove the trap can very well shift the  $K^*(W(K_t); N, \theta, \beta)$  curve *down*, thereby reducing steady-state capital.

Note also that since both sides of (21) are increasing in N, the sign of the change in N required to satisfy it, starting from some arbitrary level of N, is ambiguous. Thus, it is quite possible to have that escaping the trap requires an increase in competition while maximizing steady-state income requires a decrease. If so, the model's implication is that the optimal number of banks is contingent on  $K_t$ . A specific example makes this discussion more concrete.

Suppose there exists a regulator who sets the number of banks. Suppose also that (21) yields a threshold  $N_{trap}$  such that for  $N > N_{trap}$  (21) holds while for  $N \le N_{trap}$  it does not. Suppose, finally, that there exists a number of banks  $N_{max}$  that maximizes the level of capital in the steady state with p > 0 and such that for  $N_{max}$  the no-trap condition (21) fails, i.e.,  $N_{max} < N_{trap}$ . Then the regulator should set  $N > N_{trap}$  to remove the trap and keep it there until the economy has accumulated  $K_t > K_0(N_{max}, \theta, \beta)$ . Once the economy passes this threshold, the regulator can set  $N = N_{max}$  because the economy is out of the basin of attraction of the underdevelopment trap associated to  $N_{max}$ .

It is possible to construct several examples with similar features. The general insight is that regulating the banking sector through direct control of the number of banks is a complex exercise that requires detailed information. The analysis of the steady state with relationship lending that we undertake next underscores this point.

#### *6.2. The steady state*

For simplicity we focus on the case where the function  $\Phi(K_t; N, \theta, \beta)$  has a unique steady state with relationship lending. The steady state value of the capital stock,  $K_{ss}(N, \theta, \beta)$ , is the solution of the system:

$$m = \left[ \frac{\left(\frac{K}{m}\right)^{1-\gamma} \left(\frac{\frac{K}{m}}{(1-\gamma)K^{\gamma} - \frac{K}{m}}\right)^{\frac{1-\alpha}{\alpha}}}{\gamma} \frac{N + \frac{1-\alpha}{\alpha} \frac{(1-\gamma)K^{\gamma}}{(1-\gamma)K^{\gamma} - \frac{K}{m}}}{N - 1 + \gamma} \right]^{\frac{1}{\gamma}}, \tag{22}$$

$$K = \left[ \frac{\beta N}{\gamma^2} \frac{m}{(1 - \theta)^{\frac{1}{N}} (1 - m)^{\frac{N-1}{N}}} \right]^{\frac{1}{\gamma}}.$$
 (23)

The graph in the lower panel of Fig. 4 shows the construction of the accumulation curve that we use in the upper panel. Denote the right-hand side of (22) as  $\Psi(m; K, N)$ . This is a monotonically decreasing function of m with a vertical asymptote at

$$m = \frac{K^{1-\gamma}}{1-\gamma}.$$

The function is also increasing in K so that as K rises,  $\Psi(m; K, N)$  shifts up and traces the 45° line, thereby generating a function  $m_{ss}^A(K; N)$  increasing in K. The constraint  $m \ge \theta$  (i.e.,  $p \ge 0$ ) binds whenever  $K \le K_0^L(N, \theta)$  defined by

$$\Psi(\theta; K_0^A, N) = \theta.$$

Similarly, the constraint  $m \le 1$  (i.e.,  $p \le 1$ ) binds whenever  $K \ge K_1^A(N)$  defined by

$$\Psi(1; K_1^A, N) = 1.$$

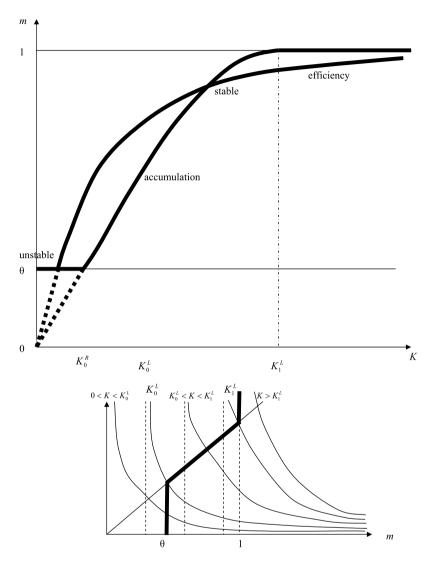


Fig. 4. Steady-state equilibrium m and K.

We then obtain the kinked accumulation curve in the figure. The efficiency curve is the same as in Fig. 2.

We have two types of solutions. If (23) is above (22) for all values of  $m \ge \theta$ , the equilibrium is given by (22) evaluated at  $m = \theta$ . The more interesting case is when (23) and (22) intersect for  $m \in (\theta, 1)$ .

# 6.3. Effect of the model's parameters on the steady state

As mentioned earlier, the main parameters of interest are N,  $\theta$  and  $\beta$ . A change in the cost of supplying relationship services  $\beta$  has an obvious and unambiguous effect on the steady state

levels of both m and K. A decrease in  $\beta$  shifts up the efficiency curve but it does not affect the accumulation curve, thus leading to both higher m and K.

Since our main focus is the role of competition, we want to study how this equilibrium changes with the number of banks. From what gathered so far, the role of N is intrinsically ambiguous. When N increases, both (23) and (22) shift down, capturing the fact that more competition reduces the interest rate spread and generates more credit, given banks' choice of p, while it reduces bank's incentives to provide relationship services, given the size of the credit market X.

However, this ambiguity can be resolved if we investigate further the role of N in conjunction with the role of  $\theta$ . This parameter measures the baseline level of risk for entrepreneurs and thus is a good indicator of agents' need to tackle investment frictions in the model.

In order to clarify this relationship we begin first by looking at the steady state at the extreme cases N = 1 and  $N \to \infty$ . Consider first (22)–(23) when N = 1:

$$m = \Psi(m; K, 1) \equiv \left[ \frac{\left(\frac{K}{m}\right)^{1-\gamma} \left(\frac{K}{(1-\gamma)K^{\gamma} - \frac{K}{m}}\right)^{\frac{1-\alpha}{\alpha}}}{\gamma} \frac{1 + \frac{1-\alpha}{\alpha} \frac{(1-\gamma)K^{\gamma}}{(1-\gamma)K^{\gamma} - \frac{K}{m}}}{\gamma} \right]^{\frac{1}{\gamma}}, \tag{24}$$

$$K = \left[\frac{\beta}{\gamma^2} \frac{m}{1-\theta}\right]^{\frac{1}{\gamma}}.$$
 (25)

Note that the efficiency curve no longer converges asymptotically to m = 1 for  $K \to \infty$  but instead admits a finite value of K that yields m = 1. The reason is that the monopoly bank does not face the free riding problem that dampens the incentives to provide relationship services faced by oligopolistic banks. In particular, the monopoly bank converges to a steady state with m = 1 if the accumulation curve (24) cuts the m = 1 line to the right of the point where the efficiency curve (25) cuts it. This requires that the parameters satisfy:

$$K_{ss}(1) > \left\lceil \frac{\beta}{\gamma^2} \frac{1}{1-\theta} \right\rceil^{1/\gamma},$$

where

$$K_{ss}(1) = \arg \text{solve}\{1 = \Psi(1; K, 1)\}.$$
 (26)

In the following, it is useful (albeit not necessary) to assume that this condition holds.

Consider now (22)–(23) when  $N \to \infty$ :

$$m = \Psi(m; K, \infty) \equiv \left[ \frac{\left(\frac{K}{m}\right)^{1-\gamma} \left(\frac{\frac{K}{m}}{(1-\gamma)K^{\gamma} - \frac{K}{m}}\right)^{\frac{1-\alpha}{\alpha}}}{\gamma} \right]^{1/\gamma}, \tag{27}$$

$$m = \theta \quad \forall K.$$
 (28)

The second line captures the property that when the number of banks is too large, banks choose p = 0. In this case then, the equilibrium is at the intersection of the accumulation curve (28) with the line  $m = \theta$ , that is,

$$K_{ss}(\infty, \theta) = \arg \operatorname{solve} \{ \theta = \Psi(\theta; K, \infty) \}.$$
 (29)

We are now ready to establish the connection with the parameter  $\theta$ , which, as said, governs the importance of relationship lending. Observe first that for  $\theta = 1$  Eqs. (29) and (26) yield  $K_{ss}(\infty, 1) > K_{ss}(1)$ . That is, when we shut down the model's friction, and thus make relationship lending unimportant, the equilibrium with the monopolistic distortion of the quantity of

credit is inferior to the one without it. In fact, since for  $\theta=1$  we have m=1 regardless of the number of banks, it is clear that to maximize long-run output we should let  $N\to\infty$ . Consider now  $\theta\to 1$ , that is, entrepreneurial risk is not exactly zero but still so low that relationship lending is almost irrelevant. Again, we obtain that competition raises steady-state capital because it eliminates the deadweight losses associated with banks' market power. In other words, the relation between N and  $K_{ss}$  is monotonically increasing and the number of banks that maximizes steady-state capital is  $N^*\to\infty$ .

Things change drastically when we move to the opposite end of the spectrum. The case  $\theta = 0$  is trivial since it implies shutting down the credit market altogether. We thus focus on  $\theta \to 0$ , which means that entrepreneurial risk is so high that relationship lending is crucial because the losses from arms' length lending are too large and outweigh the benefits of eliminating market power. In this case,  $K_{ss}$  is monotonically decreasing in N and the number of banks that maximizes steady-state capital is  $N^* = 1$ .

The intuition driving the extreme cases, which we illustrate in Fig. 5, suggests that for intermediate values of  $\theta$ , a hump-shaped relation should emerge yielding that the number of banks that maximizes steady-state capital is a finite value  $N^* \in (1, \infty)$ . In other words, oligopoly banking strikes the best possible balance between the deadweight losses from banks' market power and the benefits of relationship lending.<sup>21</sup>

#### 6.4. Free entry

Suppose there exists an exogenous, fixed cost of operating a bank  $\kappa > 0$ . In the symmetric equilibrium, the zero-profit condition yields

$$\pi(W; N, \theta, \beta) = \kappa$$

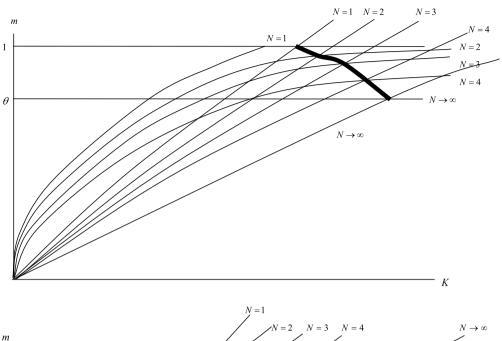
where the left-hand side is the profit flow defined in Eq. (7) evaluated at the bank's optimum and imposing symmetry.

It is straightforward to show that  $\pi_W > 0$ . To see how  $\pi(W; N, \theta, \beta)$  depends on N, we start from N = 1 and, for simplicity, we treat N as a continuous variable (i.e., we abstract from the integer constraint). A monopoly bank engaging in relationship lending is viable if  $\pi(W; 1, \theta, \beta) > \kappa$  for  $W > W_0(1, \theta, \beta)$ . Assume that this condition holds. Note that  $\pi(W; N, \theta, \beta) \to 0$  as  $N \to \infty$ . It follows that  $\pi(W; N, \theta, \beta)$  must intersect the line  $\kappa$  at least once. Restricting attention, for simplicity, to the case in which  $\pi(W; N, \theta, \beta)$  is monotonic in N, we obtain that there exists a unique, stable (in the Nash sense) solution  $N(W; \theta, \beta, \kappa)$  such that  $N_W > 0$ ,  $N_\theta > 0$ ,  $N_\beta < 0$ ,  $N_\kappa < 0$ .

The dynamics of capital accumulation follow from the mapping

$$K_{t+1} = \Phi(W(K_t); N, \theta, \beta)$$

while one can envision entry regulation as a policy tool to promote capital accumulation, there are related policy instruments. For instance, a monopolist with an imposed ceiling on lending rates — provided that it still warrants choosing relationship lending in equilibrium — could be an improvement over the oligopolistic equilibrium. Similarly, some forms of tax subsidization of the provision to relationship services may constitute an improvement. We thank a referee for making these suggestions. More in general, the promotion of explicit mechanisms among banks to coordinate on the provision of relationship services would lead to higher capital accumulation.



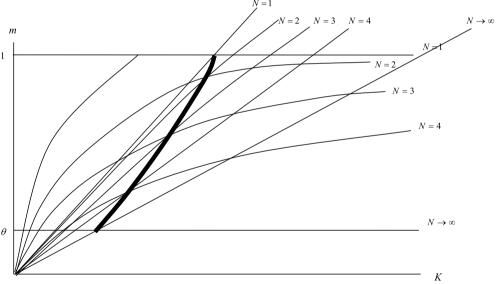


Fig. 5. Effect of N when  $\theta$  is high and low.

that we constructed in Proposition 5. All we need to do is use the solution  $N(W; \theta, \beta, \kappa)$  and write

$$K_{t+1} = \Phi(W(K_t); N(W(K_t); \theta, \beta, \kappa), \theta, \beta) \equiv \Gamma(K_t; \theta, \beta, \kappa).$$

The new mapping  $\Gamma$  incorporates the endogenous response of the number of banks to changes in profitability induced by the changes in the aggregate capital stock. Qualitatively, this feedback changes things according to how it affects the derivative  $\Gamma_K$ . More precisely, note that

$$\Gamma_K = \Phi_K + \Phi_N \cdot N_K.$$

Thus, as long as  $\Phi_N > 0$  allowing for free entry does not change the qualitative properties of the transition path. Only for  $\Phi_N < 0$  the free-entry feedback changes the qualitative results because it is possible for capital accumulation to attract banks to the point where banks reduce relationship lending. If this happens, the function  $\Gamma$  can become hump-shaped.

A non-monotonic mapping between  $K_{t+1}$  and  $K_t$  is not a problem *per se*. As long as the  $\Gamma(K_t)$  intersects the 45° degree line with local slope inside the unit circle, even when negative, the steady state  $K^*$  is stable. It is only when the free-entry feed-back is so strong that it bends the function  $\Gamma(K_t)$  so much that it acquires a local slope  $\Gamma'(K^*) < -1$  that things become different and more complex. The reason is that the steady state  $K^*$  loses stability and a cycle of period two appears. As is well known from the literature on non-linear deterministic systems, especially bifurcation theory, once this happens complex dynamics become possible.

This is an extremely interesting extension that captures important interactions and generates rich results. However, to fully do it justice would require us to get into the details of the dynamics of deterministic cycles — something that cannot be done properly in a page or two. We therefore leave the issue to future work. There is a fairly large and, especially now, vibrant literature on credit cycles and we see our model as capable of making a novel contribution to that literature. We think, however, that working out such contribution in sufficient detail would take this paper beyond its current scope.

#### 7. Conclusion

We have presented a dynamic general equilibrium model of capital accumulation in which oligopolistic banks serve as financial intermediaries between savers and entrepreneurs. Banks play an active role in determining the equilibrium dynamics. They provide insurance services to savers through the diversification of idiosyncratic risk and, when profitable, in addition to funds provide to entrepreneurs relationship services that enhance their likelihood of success. However, banks that do not offer these services, and extend to the same entrepreneur simple arm's length loans, earn the higher risk-adjusted return without having sustained the cost of providing the services. There is thus a free-riding problem and under provision of services. The model allows us to study in detail these issues and to characterize how competition affects the general equilibrium path of the economy.

With this contribution we fill a gap in the theoretical literature on finance and growth, a literature that has recognized the importance of banks in fostering economic growth but has not explored in depth the role played by the market structure of the banking industry. Perhaps the reason is that conventional wisdom suggests that perfect competition — price taking behavior due to a large number of banks — should be the optimal market structure. However, the available empirical evidence, and existing models of financial intermediation, paint a more nuanced picture, suggesting the existence of multiple channels through which banking market structure affects growth that result into an ambiguous relationship.

A particularly valuable feature of our model is that it is extremely parsimonious: we do not make special assumptions and have only two free parameters that affect the role of the number of banks — the function characterizing idiosyncratic risk in entrepreneurial activity and the cost to provide relationship services. Therefore, our main ingredients are simply the partial appropriability of the benefits of relationship activity and oligopolistic rivalry among banks. We also stress the importance of a dynamic, general equilibrium approach, which allows us to obtain a rich set of new macroeconomic results.

First, we show that bank competition has an intrinsically ambiguous impact on aggregate economic variables. Without conditioning on other covariates it is just not clear whether a more competitive banking sector leads to better outcomes. This theoretical insight explains the apparent lack of consistent results from the empirical evidence.

Second, we show that in environments where entrepreneurial idiosyncratic risk is especially severe (mild), we predict higher (lower) capital accumulation under less (more) competitive banking market structures. In intermediate cases the market structure that maximizes long-run development is an oligopoly. If idiosyncratic risk evolves along the development path, then these results imply a further dynamic dimension to the determination of optimal banking market structure that the literature has ignored.

Third, we show that development traps may emerge just because of banking market structure. This is a very important result for the following reasons: a) It means that we could have otherwise identical economies, including the same banking market structure, exhibiting significantly different levels of income and/or growth trajectories — a property that gives us additional clues about what drives the inconclusive empirical results emerging in the literature; b) The normative implication for a banking regulator is that the optimal market structure to escape from the trap is not the optimal market structure to achieve the highest levels of economic development — another insight pointing at the necessarily dynamic nature of banking regulation; c) It implies a severe criticism of policies regarding emerging markets, where the traditional prescription is that in order to achieve income convergence it is necessary to simply adopt the same institutional and regulatory environments prevalent *today* in developed countries — a policy that proposed without deeper qualifications is ineffective at best and very damaging at worst.

While we emphasize the strengths of our approach, we are also aware of its limitations. Our approach has allowed us to explore the dynamic general equilibrium properties of the model to a great extent, while at the same time working out in full detail the microeconomics of the banks' lending strategy. As stressed in the introduction, our banks are active agents whose decisions directly affect the economy-wide equilibrium path. To obtain our rich spectrum of results, however, we have modeled the relationship between banks and firms in a relatively stylized fashion. Extensions to more fully articulated dynamic aspects of the bank-firm contract space could unveil additional features of the complex and ever-important nexus between finance and the real economy. Similarly, and as pointed out earlier, allowing for bank free-entry dynamics would make the model richer and capable to address the important issue of credit cycles. We leave these tasks to future research.

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