

# Growth on a finite planet: resources, technology and population in the long run

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Published online: 2 July 2015  
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**Abstract** We study the interactions between technological change, resource scarcity and population dynamics in a Schumpeterian model with endogenous fertility. We find a steady state in which population is constant and determined by resource scarcity while income grows exponentially. If labor and resources are substitutes in production, income and fertility dynamics are stable and the steady state is the global attractor of the system. If labor and resources are complements, income and fertility dynamics are unstable and drive the economy towards either demographic explosion or collapse. We calibrate the model numerically to match past US data on fertility and land scarcity, obtaining future scenarios for the current century and quantifying the response of fertility and productivity to exogenous shocks.

**Keywords** Endogenous innovation · Resource scarcity · Population growth · Fertility choices

**JEL codes** E10 · L16 · O31 · O40

## 1 Introduction

More than two centuries after the publication of Thomas Malthus' (1798) *Essay on the Principle of Population*, understanding the interactions between economic growth, resource scarcity and population remains a central aim of scholars in different fields of social sciences.

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**Electronic supplementary material** The online version of this article (doi:[10.1007/s10887-015-9118-z](https://doi.org/10.1007/s10887-015-9118-z)) contains supplementary material, which is available to authorized users.

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The debate revolves around two fundamental questions: (1) whether a larger population is good or bad for human development and welfare (Birdsall and Sinding 2001; Kelley 2001) and (2) how population growth changes with economic conditions (Kremer 1993; Wang et al. 1994). There is an emerging consensus that both subjects should be tackled at the same time: assessing the consequences of a growing population requires considering the feedback effects of resource scarcity on fertility (Bloom and Canning 2001). A prominent example is the literature on unified growth theory (UGT), which seeks to explain the historical phases of development, from the Malthusian Stagnation to the Industrial Revolution and then the current regime of sustained growth of per capita incomes (Galor and Weil 2000; Galor 2005, 2011). In the benchmark UGT model, the central mechanism behind economy-environment interactions is that population growth affects natural resource scarcity and labor productivity, while income dynamics affects fertility and thus future population growth.

In this paper, we investigate the mechanism linking resource scarcity, incomes and population in a Schumpeterian model of endogenous growth. Our theory delivers an interaction between income dynamics and fertility but, differently from UGT, the feedback effect of resource scarcity operates through resource prices and incomes, and may have opposite directions depending on the degree of substitutability between labor and resources in the production of intermediate goods. In fact, our main results hinge on a ‘resource price effect’ that is specific to our model, as we discuss below. Our general aim is to build a theory of economy-environment interactions capable of addressing one of the main future challenges for modern industrialized economies: how to sustain innovation-driven income per capita growth in a habitat—Planet Earth—that has finite carrying capacity of people. We tackle this issue by studying under what circumstances population-resource dynamics generate a steady state where income per capita grows at a constant (endogenous) rate while population stabilizes at a constant (endogenous) level.

In our model firms produce intermediate goods using labor and a resource in fixed aggregate supply—e.g., land—while households make fertility choices according to utility maximization. We employ a Schumpeterian model of endogenous growth in which horizontal and vertical innovations coexist: firms producing differentiated goods undertake R&D to increase their total factor productivity while outside entrepreneurs design new products and set up new firms in order to serve the market (Peretto 1998; Dinopoulos and Thompson 1998). This class of models has received substantial empirical support in recent years (Laincz and Peretto 2006; Ha and Howitt 2007; Madsen 2008; Madsen et al. 2010; Madsen and Ang 2011) and is particularly useful in addressing our research question because it predicts that the effect of endowments on growth is only temporary. Specifically, product proliferation, i.e., net entry, sterilizes the (strong) scale effect in steady state because it fragments the aggregate market into submarkets whose size does not increase with the size of the endowments. In our analysis with endogenous fertility, the elimination of the (strong) scale effect generates steady states in which population is constant and does not affect productivity growth.

Our first result is that there exists a steady state in which income per capita grows at a constant rate and population is constant. Importantly, the existence of this steady state is not due to specific assumptions on fertility preferences but rather to the price effects generated by the degree of substitutability between labor and resources in the production of goods: if we impose unit elasticity of input substitution, such steady state disappears and the long-run growth rate of population is constant. Our second result is that the elasticity of substitution between labor and resources determines the stability properties of the steady state. More precisely, if labor and resources are substitutes, the economy converges to the steady state with constant population for any initial condition. The reason is that population growth reduces the resource-labor ratio but, due to substitutability, the resource price rises

only moderately. As a consequence, resource income *declines* over time relative to labor income, the fertility rate decreases and population stabilizes in the long run. If labor and resources are complements, instead, the steady state with constant population is a separating threshold: if the resource is either initially scarce or abundant, population diverges and we have demographic explosion or collapse. The reason is that complementarity generates a *self-reinforcing* feedback effect: starting from the steady state, a rise in population yields a resource price increase that raises resource income, boosting fertility and thereby population growth. If, instead, the deviation from the steady state is toward resource abundance—i.e., a drop in population—the resource price response induces lower income and further population decline.

We stress that feedback effects from resource income per capita to fertility decisions are neutralized in the special case of unit elasticity, i.e., when labor and resources are neither complements nor substitutes. This conclusion is relevant in three respects. First, it implies that departing from the hypothesis of Cobb–Douglas technology can be crucial for long-term predictions on fertility and growth, whereas most related studies assume a unit elasticity between labor and fixed factors (e.g., [Hansen and Prescott 2002](#); [Lucas 2002](#); [Doepke 2004](#)). Second, the result is important from an empirical perspective since the recent cross-country evidence rejects the hypothesis of unit elasticity between labor and land (or labor and natural resources treated as fixed factors: see [Ashraf et al. 2008](#); [Weil and Wilde 2009](#)). Third, our prediction that constant long-run population growth rates only emerge in the Cobb–Douglas case captures a simple, yet often neglected idea: the hypothesis of exponential population growth in the long run is clearly at odds with the fact that Planet Earth has a finite carrying capacity of people.

We complete our analysis by presenting three applications of the model. First, we study the fertility response to exogenous income shocks, stressing the consistency between our qualitative results and recent evidence on the fertility–income relationship ([Brückner and Schwandt 2014](#)). Second, we calibrate the model to match the 1960–2012 data on birth rates and land scarcity in the US, obtaining transitional dynamics that are consistent with the actual co-evolution of birth rates and income ([Jones and Tertilt 2008](#)). Third, using the in-sample calibration, we construct a reference equilibrium path for the 1960–2100 period. This allows us to use the US economy as a laboratory to quantify the potential consequences of a future demographic shock affecting the US from the year 2025 onwards.<sup>1</sup>

With respect to the existing literature on population–resource interactions, our analysis differs in both ends and means. In UGT models, consumption goods are produced by means of human capital and land, population growth affects labor productivity, and the resulting income dynamics determine fertility rates. Our analysis provides different insights for three main reasons. First, UGT assigns a central role to human capital whereas our theory focuses on the fertility response to resource prices: the two mechanisms are not mutually exclusive and each of them explains different stylized facts that characterized the 20th century.<sup>2</sup> Second, both the nature and the role of technological change are different. In our model, vertical innovations are Hicks-neutral with respect to labor and land so that technical change is

<sup>1</sup> Given the lack of global data, using a specific country as a laboratory allows us to investigate the empirical and quantitative properties of the model. We show in Sect. 6 below that our model can be applied to a single economy with a minimum departure from the hypothesis of a closed system.

<sup>2</sup> More specifically, with respect to TFP growth our model predicts that fertility and TFP growth are positively associated along the transition (both decline). Within our framework this outcome is consistent and intuitive: lower fertility slows down the growth of market size, which reduces the incentive for R&D. In UGT, instead, fertility and TFP growth are negatively correlated and productivity growth is driven by human capital accumulation, from which we abstract. We thank an anonymous referee for pointing this out.

consistent with constant population in the long run; in UGT, instead, technical change is land-augmenting and lifts the economy out of the Malthusian trap by allowing population to grow when the subsistence consumption constraint is binding. Third, our main results hinge on a ‘resource price effect’ that is specific to our model: the response of fertility to increased resource scarcity may be positive or negative, depending on the strength of the increase in the market resource price relative to the wage rate. Notably, the resource price effect is also neglected in the parallel literature on industrial take-offs (e.g., [Lucas 2002](#)) because the existing models either abstract from the resource market or, when they allow for a resource market, assume a unit elasticity of substitution between labor and resources.<sup>3</sup>

Considering alternative approaches, there is an established literature on bio-economic systems that seeks to explain the rise and fall of civilizations by modeling the relation between population dynamics and resource availability in closed systems (e.g., islands) where natural resources’ regeneration interacts with human harvesting. These models generate rich dynamics, including feast–famine oscillating paths and/or environmental crises that can eventually drive human society to extinction ([Taylor 2009](#)), and have been calibrated to replicate the collapse of Easter Island and similar historical episodes ([Brander and Taylor 1998](#); [Basener and Ross 2005](#); [Good and Reuveny 2009](#)). However, they neglect a fundamental element in the functioning of modern societies: endogenous, innovation-driven productivity growth.

A strand of literature that comes close to our quest for more comprehensive predictions for future growth in a finite habitat is that on sustainable development, since it embraces a forward-looking perspective by definition ([Pezzey 1992](#)). At the conceptual level, sustainability analysis is motivated by the concern that future intergenerational conflicts will hinge on three issues: the scarcity of primary inputs, the environmental damage caused by economic activity, and the further pressure exerted by population growth on both resource scarcity and environmental quality. Formal theories, however, insist on the first two issues—in particular, exhaustible resources ([Smulders 2005](#)) and pollution externalities ([Xepapadeas 2005](#))—and typically neglect the interdependence of scarcity and fertility. Despite its obvious relevance for sustainability, only a few contributions formally analyze the fertility-scarcity interaction ([Schäfer 2014](#); [Bretschger 2013](#)).<sup>4</sup>

## 2 The model

There are two main groups of agents. The first is a representative household who purchases a homogeneous consumption good, supplies labor services and a natural resource (e.g., land) in competitive markets, accumulates wealth in the form of financial assets, and, crucially, makes reproduction decisions. The consumption good is produced by final firms assembling differentiated intermediate inputs, each variety of which is supplied by one monopolistic firm. These intermediate firms are the second main group of actors in the model since productivity growth stems from their decisions concerning two types of innovations. First, the mass of intermediate firms increases due to the costly development of new product lines (horizontal innovation). Second, each intermediate firm undertakes in-house R&D to increase its own

<sup>3</sup> A recent exception is [Strulik and Weisdorf \(2008\)](#), which investigates how the price of agricultural goods, determined in a two-sector economy by land scarcity and learning-by-doing activities, affects population growth and economic growth.

<sup>4</sup> [Schäfer \(2014\)](#) builds a model of directed technological change in which skill-biased technological change induces a decline in population growth and a transitory increase in the depletion rate of natural resources. [Bretschger \(2013\)](#) considers poor substitution (complementarity) between labor and an exhaustible resource in a Romer-style model of endogenous growth that exhibits the strong scale effect.

productivity (vertical innovation). The interplay between horizontal and vertical innovations allows the economy to grow in steady state at a constant endogenous rate that is independent of factor endowments (Peretto 1998; Dinopoulos and Thompson 1998; Peretto and Connolly 2007). Connolly and Peretto (2003) studied the role of endogenous fertility in this framework. Our analysis extends the model to include privately-owned natural resources and varying degrees of substitutability between labor and resource inputs, exploiting the tractable framework developed by Peretto (2012).<sup>5</sup>

## 2.1 Households

The law of motion of adult population postulates that childhood lasts for one instant and then the child becomes an adult worker:

$$\dot{L}(t) = B(t) - d \cdot L(t), \quad (1)$$

where  $L$  is the mass of adults,  $B$  is the mass of children and  $d > 0$  is the exogenous and constant death rate. Children consume the same homogeneous good as adults but do not work. Consumption and reproduction decisions are endogenous and reflect the intertemporal choices of a single representative household who maximizes

$$U \equiv \int_0^\infty \log u(t) e^{-\rho t} dt, \quad (2)$$

where  $u(t)$  is instantaneous utility at time  $t$ , and  $\rho > 0$  is the discount rate. Instantaneous utility depends on consumption per adult,  $C_L$ , consumption per child,  $C_B$ , the mass of adults and the mass of children, according to

$$\log u(t) = \mu \log (C_L(t) L(t)^\eta) + (1 - \mu) \log (C_B(t) B(t)^\eta), \quad (3)$$

where the parameters  $\mu$  and  $\eta$  are both positive and below unity. This specification of preferences postulates that the decision maker cares about utility of adults and utility of children with weights  $\mu$  and  $1 - \mu$ , and that each individual derives utility from own individual consumption as well as from the mass of individuals with weight  $\eta$ . The fact that  $C_B$  is part of the decision problem introduces a trade-off in reproduction choices between the mass of children and consumption expenditure per child.<sup>6</sup> For future reference, we rewrite (3) as

$$\log u(t) = \log C_L(t)^\mu C_B(t)^{1-\mu} + \log L(t)^\eta + \log b(t)^{\eta(1-\mu)}, \quad (4)$$

where  $b(t) \equiv B(t)/L(t)$  is the *gross fertility rate*, and  $\eta(1 - \mu)$  is the associated weight in household utility.

Each adult supplies inelastically one unit of labor to the production sector of the economy. The household is also endowed with  $\Omega$  units of a non-exhaustible natural resource (e.g., land) that it supplies inelastically to manufacturing firms. The wealth constraint reads

$$\dot{A}(t) = r(t) A(t) + w(t) L(t) + p(t) \Omega - P_C(t) [C_L(t) L(t) + C_B(t) B(t)], \quad (5)$$

<sup>5</sup> Peretto (2012) studies the effects on income, growth and welfare of a shock to the natural resource endowment in a model with constant population.

<sup>6</sup> The original version of this paper (Peretto and Valente 2013) did not consider child consumption as an argument in the utility function. We thank an anonymous referee for suggesting this extension that produces the same qualitative results but allows us to incorporate a quantity-quality tradeoff in the spirit of Unified Growth Theory. Our results are nonetheless robust to alternative preference specifications as discussed in Sect. 5.4.

where  $A$  is assets holding,  $r$  is the rate of return on assets,  $w$  is the wage rate,  $p$  is the market price of the resource and  $P_C$  is the price of consumption. The household maximizes welfare (2) subject to the dynamic laws (1) and (5), using consumption and fertility levels as control variables and taking all prices as given.

## 2.2 Final sector

In the final sector, perfectly competitive firms produce the consumption good by means of manufactured intermediate inputs using the technology

$$C(t) = \left( \int_0^{N(t)} X_i(t)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}, \quad (6)$$

where  $C$  is total output of the homogeneous good,  $N$  is the mass of varieties of intermediate inputs,  $X_i$  is the quantity of the  $i$ -th variety, and  $\epsilon > 1$  is the elasticity of substitution between pairs of intermediate goods. Final producers maximize profits taking the prices and the mass of varieties as given. The resulting demand for each intermediate good is

$$P_{X_i}(t) = \frac{P_C(t) C(t)}{\int_0^{N(t)} X_i(t)^{\frac{\epsilon-1}{\epsilon}} di} \cdot X_i(t)^{-\frac{1}{\epsilon}}, \quad (7)$$

where  $P_{X_i}$  is the price of the  $i$ th variety.

## 2.3 Intermediate production and vertical innovation

Each variety of intermediate input is supplied by a monopolist. The intermediate firm  $i$  operates the production technology

$$X_i(t) = Z_i(t)^\theta \cdot F(L_{X_i}(t) - \phi, R_i(t)), \quad (8)$$

where  $X_i$  is output,  $L_{X_i}$  is overall labor employed in production,  $\phi > 0$  is a fixed operating cost in units of labor (henceforth, fixed operating cost),  $R_i$  is the resource input, and  $F(\cdot, \cdot)$  is a standard production function homogeneous of degree one in its main arguments.<sup>7</sup> Importantly,  $F(\cdot, \cdot)$  may exhibit an elasticity of input substitution below or above unity. Whether labor and the resource are complements or substitutes matters for our results and we will discuss all possible scenarios, including the case of unit elasticity where  $F(\cdot, \cdot)$  is Cobb–Douglas.

The productivity of the firm depends on the stock of firm-specific knowledge,  $Z_i$ , with elasticity  $\theta \in (0, 1)$ . Importantly, this firm-specific productivity term is Hicks-neutral with respect to labor and the resource. The stock of firm-specific knowledge increases according to

$$\dot{Z}_i(t) = \alpha K(t) L_{Z_i}(t), \quad (9)$$

where  $L_{Z_i}$  is labor employed in R&D. The productivity of labor in R&D depends on the exogenous parameter  $\alpha > 0$  and on the stock of public knowledge,  $K$ . Public knowledge accumulates as a result of spillovers across firms within the intermediate sector: when one firm generates a new idea, it also generates non-excludable knowledge that benefits the R&D

<sup>7</sup> The fixed operating cost,  $\phi$ , ties product proliferation to population growth, as discussed in detail in [Peretto and Connolly \(2007\)](#). Section 4.2 clarifies how  $\phi$  influences the interaction between the rates of horizontal and vertical innovations.

of other firms according to the spillover function<sup>8</sup>

$$K(t) = \int_0^{N(t)} \frac{1}{N(t)} Z_i(t) di. \quad (10)$$

Considering a monopolistic firm that starts to produce in instant  $t$ , the present discounted value of its net cash flow is

$$V_i(t) = \int_t^\infty \Pi_i(v) e^{-\int_t^v [r(v') + \delta] dv'} dv, \quad (11)$$

where  $\Pi_i$  is the instantaneous profit,  $r$  is the instantaneous interest rate and  $\delta > 0$  is the instantaneous death rate of firms.<sup>9</sup> In each instant, the firm chooses the cost-minimizing combination of rival inputs,  $L_{X_i}$  and  $R_i$ , and the output level  $X_i$  that maximize static profits  $\Pi_i$  subject to the demand schedule (7) of final producers. Given this choice, the monopolist then determines the time path of R&D employment  $L_{Z_i}$  that maximizes present-value profits (11) subject to the R&D technology (9), taking as given the other firms' innovation paths. The solution to this problem is described in the Appendix and yields the maximized value of the firm given the time path of the mass of firms.

## 2.4 Horizontal innovation: entry

New firms enter the intermediate sector as time passes. Outside entrepreneurs hire labor to perform R&D activities that develop new varieties of intermediates, and then set up firms to serve the market. We assume that for each entrant, denoted  $i$  without loss of generality, the labor requirement translates into a sunk entry cost (henceforth, entry cost) that is proportional to the value of production of the new good when it enters the market,  $P_{X_i} X_i$ . Denoting by  $L_{N_i}$  the labor employed in start-up activity, the entry cost is  $w L_{N_i} = \beta P_{X_i} X_i$ , where  $\beta > 0$  is a parameter representing technological opportunity. This assumption captures the notion that entry requires more effort the larger the anticipated volume of production.<sup>10</sup>

The value of the firm entering the market at time  $t$  equals the maximized present-value net cash flow  $V_i(t)$  because, once in the market, the firm solves an intertemporal problem identical to that of the generic incumbent. Free entry, therefore, requires

$$V_i(t) = \beta P_{X_i}(t) X_i(t) = w(t) L_{N_i}(t) \quad (12)$$

for each entrant.

## 3 Equilibrium conditions

The intertemporal choices of households and the profit-maximizing behavior of firms characterize the equilibrium path of the economy. This section describes consumption and fertility decisions, the dynamics of innovation rates and the relevant market-clearing conditions.

<sup>8</sup> Specification (10) is the simplest form of spillover function that eliminates the strong scale effect in models of this class, and exhibits sound microfoundations as discussed in [Peretto and Smulders \(2002\)](#).

<sup>9</sup> The main role of the instantaneous death rate is to avoid the asymmetric dynamics and associated hysteresis effects that arise when entry entails a sunk cost. Such unnecessary complications would distract attention from the main point of the paper.

<sup>10</sup> Our assumption on the entry cost can be rationalized in several ways and does not affect the generality of our results. [Peretto and Connolly \(2007\)](#), in particular, discuss alternative formulations of the entry cost that yield the same qualitative properties for the equilibrium dynamics of the mass of firms that we exploit here.

### 3.1 Consumption and fertility choices

From (4), the Hamiltonian for the household problem takes the general form

$$H \equiv \log u(C_L, C_B, L, b) + \lambda_A \cdot \dot{A} + \lambda_L \cdot \dot{L},$$

where  $(C_L, C_B, b)$  are the control variables and  $\lambda_A$  and  $\lambda_L$  are the marginal shadow values of asset holdings and household size, respectively. The necessary conditions for maximization are derived in Appendix and yield three key relationships. The first is the standard Euler equation

$$\frac{\dot{P}_C(t)}{P_C(t)} + \frac{\dot{C}(t)}{C(t)} = r(t) - \rho \quad (13)$$

which governs the dynamics of total consumption expenditure. The second relationship is the static condition determining fertility choice at instant  $t$ ,

$$\eta(1 - \mu) + \lambda_L(t) B(t) = 1 - \mu, \quad (14)$$

which equates the current utility value of generating children to the current utility cost of providing consumption to them.<sup>11</sup> The third relationship is the dynamic law governing the gross fertility rate over time: by combining (14) with the relevant co-state equation for the marginal shadow value  $\lambda_L$ , we obtain

$$\frac{\dot{b}(t)}{b(t)} = \underbrace{\frac{b(t)}{(1 - \mu)(1 - \eta)} \cdot \left[ \eta + \frac{w(t)L(t) - P_C(t)C(t)}{P_C(t)C(t)} \right]}_{\text{Rate of return from generating future adults}} - \rho. \quad (15)$$

Expression (15) asserts that the fertility rate increases over time when the anticipated rate of return from generating future adults exceeds the utility discount rate  $\rho$ . The term in square brackets in (15) emphasizes two components of this rate of return. The first is the elasticity parameter  $\eta$ , reflecting the direct utility benefit of expanding the future mass of adults. The second component is a ‘private productivity gain’—namely, the anticipated net gain for the household of generating a future worker—measured by the (rate of) excess of household labor income,  $wL$ , over household consumption expenditures,  $P_C C$ . The private productivity gain is the crucial channel whereby firms’ productivity and resource scarcity affect the dynamics of the fertility rate in our model.

### 3.2 Production and innovation rates

In the intermediate sector, the solution to the typical firm’s problem yields a *symmetric equilibrium*: as shown in the Appendix, each monopolist charges the same price  $P_{X_i} = P_X$  and produces the same quantity of intermediate good,  $X_i = X$ . Combining this result with the zero-profit condition in the final sector, the total value of intermediates equals total consumption expenditure, with each monopolist capturing the same share  $1/N$  of the goods’ total market value:

$$P_X(t) X(t) = \frac{P_C(t) C(t)}{N(t)}. \quad (16)$$

<sup>11</sup> In the left hand side of (14), the current utility value to the household of generating children represented by direct utility benefits (from (4), the term  $\eta(1 - \mu)$  is the elasticity of utility to the fertility rate) plus the shadow value of increasing current population,  $\lambda_L B$ . The right hand side of Eq. (14) is the current utility cost, represented by the elasticity of utility to child consumption,  $1 - \mu$ .



Accordingly, the output quantities of the final and the intermediate sectors are linked through the equilibrium relationship

$$C(t) = N(t)^{\frac{\epsilon}{\epsilon-1}} X(t). \quad (17)$$

Considering employment in the intermediate sector, we can write the aggregate quantities of labor employed in production and vertical R&D as  $L_X = NL_{X_i}$  and  $L_Z = NL_{Z_i}$ , respectively. Similarly, total resource use in manufacturing production equals  $R = NR_i$ . The typical firm's knowledge stock is  $Z_i = Z$  for each  $i \in [0, N]$  and evolves according to

$$\frac{\dot{Z}(t)}{Z(t)} = \alpha \frac{L_Z(t)}{N(t)}. \quad (18)$$

Since the value of an intermediate firm's production is  $P_X X = P_C C/N$ , the free-entry condition (12) implies  $V_i N = \beta P_C C$ . Using this result, and denoting total employment in start-up operations by  $L_N = (\dot{N} + \delta N) \cdot L_{N_i}$ , the net increase in the mass of firms equals

$$\frac{\dot{N}(t)}{N(t)} = \frac{w(t)}{\beta P_C(t) C(t)} \cdot L_N(t) - \delta. \quad (19)$$

The rates of vertical and horizontal innovation in (18) and (19) are interdependent through the no-arbitrage condition that the associated returns must be equal (see Appendix).

### 3.3 Market clearing conditions

Resource market clearing requires that total resource use in intermediates production,  $R = NR_i$ , be equal to the available endowment:  $R = \Omega$ . Accordingly, the household's resource income equals  $p\Omega$ .

Labor market clearing requires  $L = L_X + L_Z + L_N$ . Asset market equilibrium requires that the value of the household's portfolio equal the value of the securities issued by firms:  $A = NV_i = \beta P_C C$ . Substituting this condition into the wealth constraint (5), and using the saving rule (13), we obtain

$$P_C(t) C(t) = \frac{1}{1 - \beta\rho} [w(t) L(t) + p(t) \Omega]. \quad (20)$$

This expression says that the ratio of household consumption expenditure to the household's income from labor and land,  $wL + p\Omega$ , is constant over time.

## 4 General equilibrium dynamics

For clarity, we split the analysis of dynamics in two parts. First, we study the interaction of population and resource scarcity (Sect. 4.1). Next, we describe the interaction between horizontal and vertical innovations in determining productivity growth (Sect. 4.2). A crucial characteristic of the resulting dynamics is the existence of a steady state displaying constant population associated to constant growth of (real) consumption per capita. Henceforth, we take labor as the numeraire and set  $w(t) \equiv 1$  in each instant  $t$ . This normalization implies, inter alia, that an increase in  $p$  represents an increase in the resource price relative to the wage rate.

## 4.1 Fertility and resource scarcity

In this subsection we characterize the interactions between population and resource scarcity as a dynamic system involving two variables: the resource endowment per capita and the fertility rate. The resource endowment per capita,  $\omega(t) \equiv \Omega/L(t)$ , is a state variable that is given at time zero but is subsequently driven by fertility choices via the dynamics of population. We derive the dynamical system in two steps. In Sect. 4.1.1, we treat the value of  $\omega(t)$  as given at time  $t$  and derive the equilibrium values of the resource price and consumption expenditure per capita. Building on this result, in Sect. 4.1.2 we derive the two-by-two system that describes the joint dynamics of  $\omega(t)$  and  $b(t)$ .

### 4.1.1 Fertility, expenditure and resource price

Denoting consumption expenditure per capita by  $y \equiv P_C C/L$ , the equilibrium condition (20) becomes

$$y(t) = \frac{1 + p(t)\omega(t)}{1 - \beta\rho}, \quad (21)$$

which says that consumption expenditure per capita is proportional to the sum of labor income per capita, 1, and resource income per capita,  $p(t)\omega(t)$ . Resource income per capita, in turn, is determined by the equilibrium between the demand for the resource by firms and the household's supply. From the firms' conditional demand for the resource, we obtain (see Appendix)

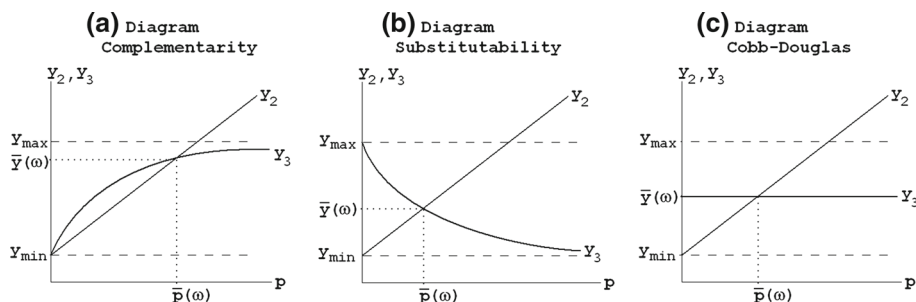
$$p(t)\omega(t) = \frac{\epsilon - 1}{\epsilon} \cdot S(p(t)) \cdot y(t), \quad (22)$$

where  $S(p) \in (0, 1)$  is the cost share of resource use—i.e., the ratio between total resource rents paid by firms to resource owners and the total variable costs of manufacturing production—and is a function of the resource price. This expression specifies how expenditure decisions determine resource income through the endogenous resource price. Specifically, the cost share of resource use  $S(p)$  is increasing or decreasing in the resource price depending on the elasticity of input substitution (see Appendix):

$$\frac{\partial S(p)}{\partial p} \begin{cases} > 0 & \text{if } (L_{X_i}, R_i) \text{ are complements;} \\ < 0 & \text{if } (L_{X_i}, R_i) \text{ are substitutes;} \\ = 0 & \text{if } F(\cdot, \cdot) \text{ is Cobb–Douglas.} \end{cases} \quad (23)$$

The intuition for result (23) is that, under complementarity (substitutability), the resource demand per unit of labor is relatively rigid (elastic) and an increase in the resource price raises (reduces) the share of firm's costs for resource use relative to wage payments. This *cost-share effect* plays a crucial role in our results.

For a given level of the resource endowment per capita,  $\omega(t)$ , Eqs. (21) and (22) form a static system in two unknowns that determines the equilibrium levels of  $p(t)$  and  $y(t)$ . Figure 1 describes graphically the equilibrium determination, showing that that equilibrium expenditure per capita always falls within the interval  $y(\omega) \in (y_{\min}, y_{\max})$ , where  $y_{\min} = \frac{1}{1-\rho\beta}$  and  $y_{\max} = \frac{1}{(1/\epsilon)-\rho\beta}$  (see Appendix). Importantly, consumption expenditure per capita responds differently to variations in the resource endowment per capita  $\omega(t)$  depending on



**Fig. 1** Equilibrium determination in Proposition 1. The loci  $y_2$  and  $y_3$  (21) and (22), respectively, and the equilibrium determines  $\bar{y}(\omega)$  and  $\bar{p}(\omega)$ . See the proof of Proposition 1 in Appendix for details

whether labor and resources are complements or substitutes.<sup>12</sup> The following Proposition summarizes the relevant comparative-statics effects.

**Proposition 1** *Given  $\omega$ , there exists a unique pair*

$$\{p^*(\omega), y^*(\omega)\}$$

*determining the equilibrium levels of consumption expenditure per capita and the resource price. The marginal effects of an increase in  $\omega$  are:*

- (i) *Complementarity:*  $\partial p^*(\omega) / \partial \omega < 0$ ,  $\partial y^*(\omega) / \partial \omega < 0$ ;
- (ii) *Substitutability:*  $\partial p^*(\omega) / \partial \omega < 0$ ,  $\partial y^*(\omega) / \partial \omega > 0$ ;
- (iii) *Cobb–Douglas:*  $\partial p^*(\omega) / \partial \omega < 0$ ,  $\partial y^*(\omega) / \partial \omega = 0$ .

Proposition 1 establishes two key results. First, the effect of an increase in the resource endowment per capita  $\omega$  on the equilibrium resource price  $p^*$  is always negative. Second, the effect of  $\omega$  on equilibrium consumption expenditure per capita  $y^*$  is negative (positive) if labor and resources are complements (substitutes). The reason is that an increase in  $\omega$  raises or reduces *resource income* per capita,  $p\omega$ , depending on the elasticity of input substitution. Under complementarity, resource demand is relatively inelastic and an increase in resource supply generates a drastic—that is, more than one-for-one—reduction of the price. Consequently, resource income per capita  $p\omega$  falls, driving down consumption expenditure per capita  $y$ . Under substitutability, resource demand is relatively elastic and the increase in  $\omega$  generates a mild reduction in the resource price, which implies a positive net effect on resource income per capita and thereby higher consumption expenditure per capita. In the special Cobb–Douglas case, the price and quantity effects exactly compensate each other so that resource income per capita and expenditure per capita are not affected by scarcity:  $\partial (p\omega)^* / \partial \omega = 0$  and  $\partial y^* / \partial \omega = 0$ .

These results play a key role in determining the equilibrium path of the economy: the qualitative characteristics of the transitional dynamics change depending on how income reacts to increased resource scarcity. We address this point by exploiting the instantaneous equilibrium defined in Proposition 1 to determine the joint dynamics of  $\omega(t)$  and  $b(t)$ .

<sup>12</sup> In graphical terms, an increase in  $\omega$  implies that the locus  $y_2$ —which represents Eq. (21)—rotates counter-clockwise whereas the locus  $y_3$ —which represents Eq. (22)—is unaffected. Consequently, an increase in  $\omega$  induces a decline in  $\bar{y}(\omega)$  under complementarity (diagram (a)), an increase in  $\bar{y}(\omega)$  under substitutability (diagram (b)), and no effect on  $\bar{y}(\omega)$  in the Cobb–Douglas case (diagram (c)).

### 4.1.2 Dynamic system

Since the total resource endowment is fixed, resources per capita decline as population grows: from (1), the dynamics of  $\omega(t)$  obey the differential equation

$$\dot{\omega}(t) = \omega(t) \cdot (d - b(t)). \quad (24)$$

The dynamics of the fertility rate are governed by Eq. (15): the marginal return from generating future workers may be re-expressed in terms of expenditure per capita as

$$\frac{\dot{b}(t)}{b(t)} = \left[ \frac{1}{(1 - \eta) \cdot y^*(\omega(t))} - 1 \right] \cdot \frac{b(t)}{1 - \mu} - \rho, \quad (25)$$

where the properties of the equilibrium relationship  $y^*(\omega)$  are those established in Proposition 1. The system formed by (24) and (25) allows us to analyze the general equilibrium dynamics of the resource-population ratio and the fertility rate. Before studying in detail the properties of this system, we complete the description of the general equilibrium dynamics by considering innovation rates and productivity growth.

## 4.2 Innovations and productivity growth

In this model real final output is equal to real consumption. Accordingly, the growth rate of the economy,  $G(t)$ , is (see Appendix)

$$G(t) = \underbrace{\left\{ \theta \cdot \frac{\dot{Z}(t)}{Z(t)} + \frac{1}{\epsilon - 1} \cdot \frac{\dot{N}(t)}{N(t)} \right\}}_{\text{TFP growth rate}} + \underbrace{\left[ \frac{\dot{y}(t)}{y(t)} - S(p(t)) \cdot \frac{\dot{p}(t)}{p(t)} \right]}_{\text{Transitional resource-income effect}}, \quad (26)$$

where the term in curly brackets represents the growth rate of total factor productivity (TFP) determined by vertical and horizontal innovations. The ‘transitional resource-income effect’ in square brackets, instead, captures possible unbalanced dynamics among expenditures per capita, resource price and the wage rate. If the economy reaches a balanced growth equilibrium where both expenditure per capita and the resource price are proportional to the wage rate (normalized to unity), we have  $\dot{y}(t) = \dot{p}(t) = 0$  and the term in square brackets becomes zero. Out of such steady states, however, the contribution of the transitional dynamics of resource income to the overall real growth rate can be substantial. Moreover, we can distinguish between a first component,  $\dot{y}/y$ , which captures the role of expenditure growth in raising resource demand, and a second component,  $S(p) \cdot \dot{p}/p$ , which represents the scarcity drag, i.e., the increase in the resource price due to the growing resource demand. We study the quantitative importance of these components in Sect. 6 below.

The costly development of vertical and horizontal innovations is profitable only if the firm’s volume of production is large enough: there thus exist thresholds of market size below which vertical innovation or horizontal innovation, or both, are inactive because firms cannot obtain a rate of return equal to the prevailing interest rate in the economy. These thresholds, which we discuss in the Appendix<sup>13</sup>, play an important role in the characterization of early development phases—where “no-innovation traps” plausibly arise—but have no crucial bearing on the present analysis, which focuses on the future behavior of an economy that has already transited to “modern” production. Consequently, we henceforth assume that the values of the relevant

<sup>13</sup> The proof of Lemma 2 in Appendix proves the existence of threshold levels in firm size determining regions of the phase space where vertical and/or horizontal innovations shut down.

parameters and the initial conditions ( $L(0)$ ,  $N(0)$ ) are such that both vertical and horizontal innovations are active.

The following Lemma establishes that, in equilibrium, the rates of vertical and horizontal innovation are jointly determined by two variables: *firm size*, denoted by  $x \equiv P_X X = P_C C/N$ , and the interest rate.

**Lemma 2** *Along the equilibrium path, the rates of vertical and horizontal innovation are, respectively:*

$$\frac{\dot{Z}(t)}{Z(t)} = x(t) \cdot \frac{\alpha\theta(\epsilon - 1)}{\epsilon} - r(t) - \delta, \quad (27)$$

$$\frac{\dot{N}(t)}{N(t)} = \frac{1}{\beta} \left[ \frac{1 - \theta(\epsilon - 1)}{\epsilon} - \frac{1}{x(t)} \cdot \left( \phi - \frac{r(t) + \delta}{\alpha} \right) \right] - \rho - \delta. \quad (28)$$

The behavior of firm size,  $x(t)$ , is governed by the differential equation

$$\dot{x}(t) = \frac{\alpha\phi - r(t) - \delta}{\alpha\beta} - \frac{1 - \theta(\epsilon - 1) - \beta\epsilon(r(t) + \delta)}{\beta\epsilon} \cdot x(t). \quad (29)$$

According to Eq. (29), the evolution of firm size depends on the equilibrium path of the interest rate. The interest rate, in turn, follows the dynamics of aggregate market size, that reflect households' consumption and fertility choices: from (1) and (13), we have

$$r(t) = \rho + \frac{\dot{y}(t)}{y(t)} + b(t) - d. \quad (30)$$

These results highlight the functioning of the modern economy captured by our model structure: the path of the interest rate carries all the information that firms need in order to choose paths of vertical and horizontal R&D that are consistent with the evolution of the size of the market for manufacturing goods. The path of market size, in turn, depends on the evolution of the economy's resource base, that is, on the path of population.

Before analyzing population dynamics, we characterize the behavior of productivity growth when the economy converges to a steady state where expenditure per capita, population and the resource price are constant. Eqs. (26) and (30) imply that in such a steady state, the interest rate equals  $r(t) = \rho$  and the economy's real growth rate equals the TFP growth rate. Then, the following result holds:

**Proposition 3** *Suppose that  $\lim_{t \rightarrow \infty} \dot{y}(t) = \lim_{t \rightarrow \infty} \dot{p}(t) = \lim_{t \rightarrow \infty} \dot{L}(t) = 0$ . Then, the net rate of horizontal innovation is zero and income growth is exclusively driven by vertical innovation:*

$$\lim_{t \rightarrow \infty} \frac{\dot{N}(t)}{N(t)} = 0 \text{ and } \lim_{t \rightarrow \infty} G(t) = \theta \cdot \lim_{t \rightarrow \infty} \frac{\dot{Z}(t)}{Z(t)}. \quad (31)$$

Provided that parameters satisfy  $\rho + \delta < \frac{\alpha\phi\theta(\epsilon-1)}{1-\beta\epsilon(\rho+\delta)}$ , the growth rate is strictly positive:

$$\lim_{t \rightarrow \infty} G(t) = \theta \frac{\theta(\epsilon - 1)[\alpha\phi - (\rho + \delta)]}{1 - \theta(\epsilon - 1) - \beta\epsilon(\rho + \delta)} - \theta(\rho + \delta) > 0. \quad (32)$$

Proposition 3 establishes two main results concerning equilibria with constant population. First, as discussed in detail in Peretto (1998) and Peretto and Connolly (2007), steady-state economic growth is exclusively driven by vertical innovation: the process of entry enlarges the mass of goods until the gross entry rate matches the firms' death rate. Consequently, the mass of firms is constant and each firm invests a constant amount of labor in vertical R&D. The second result is that steady-state real income growth is independent of factor

endowments because net entry eliminates the strong scale effect. This property allows the economy to exhibit equilibria in which population is constant but real income per capita grows at a constant, endogenous rate. The next section addresses this point in detail.

## 5 Population, resources and technology

This section characterizes the equilibrium path of population and derives the main results of this paper. Population dynamics determine the supply of labor and the extent of resource scarcity at each point in time. An important property of the model is that the path of population can be studied in isolation from market size and innovation rates: system (24) and (25) fully captures the interactions between fertility and resource scarcity, and generates the equilibrium paths of population and resource use underlying the dynamics of aggregate market size. Then, as explained in Sect. 4.2, aggregate market size and the interest rate induced by population dynamics determine the evolution of firm size and, ultimately, total factor productivity growth.

The next subsection studies the joint dynamics of fertility and resource endowment per capita, characterizing the steady state with constant population. The stability properties of the steady state crucially depend on the elasticity of substitution between labor and the natural resource in manufacturing production: Sect. 5.2 discusses strict complementarity and strict substitutability, whereas Sect. 5.3 considers the special Cobb–Douglas case.

### 5.1 Steady state with constant population

Consider a steady state  $(\omega^{ss}, b^{ss})$  in which both the resource per capita and the fertility rate are constant. Imposing  $\dot{\omega} = 0$  and  $\dot{b} = 0$  in the dynamic system (24) and (25), we obtain:

$$b^{ss} = d, \quad (33)$$

$$b^{ss} = \rho(1 - \mu) \frac{(1 - \eta) \cdot y^{ss}(\omega^{ss})}{1 - (1 - \eta) \cdot y^{ss}(\omega^{ss})}. \quad (34)$$

Equation (33) is the obvious requirement of zero net fertility for constant population. Equation (34) determines steady-state endowment per capita  $\omega_{ss}$  and thereby defines the stationary value of expenditure per capita  $y^{ss}$  that is consistent with the fertility rate  $b^{ss} = d$ . Concerning the existence of a steady state with positive fertility rate  $b^{ss} > 0$ , it can be shown that  $\eta > \rho\beta$  guarantees  $(1 - \eta) \cdot y^{ss} < 1$  and, hence, a positive right hand side in (34). In the remainder of the analysis, we impose this sufficient, though not necessary, parameter restriction.<sup>14</sup>

From Proposition 1, the steady-state equilibrium  $(\omega^{ss}, b^{ss})$  also implies a stationary value for the resource price, which we denote by  $p^{ss}$ . Concerning expenditure and population levels, we have:

<sup>14</sup> The proof of Proposition 1 (see Appendix) shows that equilibrium expenditure per capita is always bounded by  $y(\omega) \in (y_{\min}, y_{\max})$ , where  $y_{\min} = \frac{1}{1 - \rho\beta}$  and  $y_{\max} = \frac{1}{(1/\epsilon) - \rho\beta}$ . Consequently, to guarantee that the term in brackets is always positive it is sufficient to assume  $\frac{1}{1 - \eta} > y_{\max}$ , which necessarily holds as long as  $\eta > \rho\beta$ . It is possible to consider alternative cases where  $\eta \leq \rho\beta$  but this would complicate the phase-diagram analysis without much gain in terms of economic insight.

**Proposition 4** Assume  $\eta > \rho\beta$ . Then, there exists a steady state where expenditure per capita and population are, respectively:

$$y^{ss} = \frac{1}{1-\eta} \cdot \frac{d}{d + \rho(1-\mu)} > 0, \quad (35)$$

$$L^{ss} = \frac{p^{ss}}{y^{ss} \cdot (1-\beta\rho) - 1} \cdot \Omega > 0. \quad (36)$$

Recall that by Proposition 3, given the constant values  $(p^{ss}, y^{ss}, b^{ss})$ , real income growth equals the constant rate of vertical innovation. An important characteristic of this steady state is that  $y^{ss}$  and  $L^{ss}$  are independent of technology. From (35), expenditure per capita depends solely on preferences and demographic parameters: neither the endowment of the natural resource,  $\Omega$ , nor total factor productivity play any role. From (36), population is proportional to the resource endowment but remains independent of technology, while real income per capita grows at the endogenous rate (32). Therefore, we have a steady state with the property that resource scarcity limits the population level, but where real income grows at an endogenous rate driven by technological change. Before pursuing this property further, we need to assess whether and under what circumstances the economy converges to such steady state.

## 5.2 Dynamics under substitutability and complementarity

The stability of the steady state with constant population depends on the input elasticity of substitution in the intermediate sector. We thus have three main cases: complementarity, substitutability and unit elasticity (Cobb–Douglas). In this subsection we concentrate on strict complementarity and strict substitutability. The phase diagrams for these cases, shown in Fig. 2, yield the following result.

**Proposition 5** Under substitutability, the  $\dot{b} = 0$  locus is increasing and cuts the  $\dot{\omega} = 0$  locus from below so that  $(\omega^{ss}, b^{ss})$  is saddle-path stable. Consequently, the steady state with constant population is the global attractor of the system. Under complementarity, the  $\dot{b} = 0$  locus is decreasing and cuts the  $\dot{\omega} = 0$  locus from above so that  $(\omega^{ss}, b^{ss})$  is an unstable node or focus. Consequently, the steady state with constant population is a separating threshold: if the resource is initially scarce (abundant) relative to labor, the economy experiences demographic explosion (collapse).

Proposition 5 establishes that the economy converges to the steady state with constant population if labor and the resource are substitutes in production. Under complementarity, instead, such steady state is unstable and the economy follows diverging equilibrium paths leading to population explosion or collapse depending on the relative scarcity of the resource at time zero.<sup>15</sup>

The intuition for these results follows from the effects of resource scarcity on *resource income per capita* established in Proposition 1. First, consider the case of substitutability: the equilibrium trajectory lies along the saddle path depicted in Fig. 2, diagram (a). Suppose that the resource is initially abundant, that is,  $\omega_0 > \omega^{ss}$ . The initial equilibrium level of the fertility rate  $b(0)$  lying along the stable arm of the saddle exceeds the death rate  $d = b^{ss}$

<sup>15</sup> We report the details on the uniqueness of the equilibrium path in Appendix. For brevity, we focus on the case in which the steady-state loci intersect and the steady state exists. Nonetheless, global dynamics are well defined and the equilibrium path is unique even when the loci do not intersect: in such cases (which are essentially slight extensions of the Cobb–Douglas scenarios studied in Sect. 5.3) the system converges either to exponential population growth or to population collapse, depending on the parameters.

so that population grows and  $\omega$  declines. As resources per capita shrink, the resource price,  $p$ , rises during the transition. Crucially, when labor and resources are substitutes, the price effect due to increasing land scarcity is not very strong and the economy experiences *falling* resource income per capita,  $p\omega$ , and, consequently, a falling fertility rate (cf. Proposition 1). Symmetrically, if the resource is relatively scarce at time zero,  $\omega_0 < \omega^{ss}$ , net fertility is initially negative, population shrinks during the transition and  $\omega$  rises while  $p$  falls; since the price effect is weak, resource income per capita  $p\omega$  rises, driving the fertility rate up. In both cases, the transition ends when the fertility rate equals the death rate,  $d = b^{ss}$ . Hence, under substitutability the steady state with constant population is the global attractor of the system because population growth generates resource income dynamics that yield *self-balancing feedback effects*: as resource scarcity tightens, the resource price rises, but less than one for one with the endowment, so that resource income per capita falls.

Now consider the case of complementarity in Fig. 2, diagram (b). In this scenario, the steady state is not stable because the resource income effect is reversed. If the resource is initially scarce,  $\omega_0 < \omega^{ss}$ , the dynamics exacerbate scarcity because, as population growth reduces  $\omega$ , the resource price  $p$  rises more than one for one yielding a *rise* of resource income per capita  $p\omega$  and a rise in fertility (cf. Proposition 1). This implies a feedback effect whereby population grows faster and drives the economy further away from the steady state. Resource per capita  $\omega$  then tends asymptotically to zero as the economy experiences a demographic explosion. Symmetrically, if the resource is relatively abundant at time zero,  $\omega_0 > \omega^{ss}$ , population shrinks and the increase in  $\omega$  *reduces* resource income per capita via strong reductions in the resource price  $p$ , yielding a negative effect on fertility. Hence, under complementarity, the steady state is not the global attractor of the system because population growth generates resource income dynamics that yield *self-reinforcing feedback effects* on fertility.

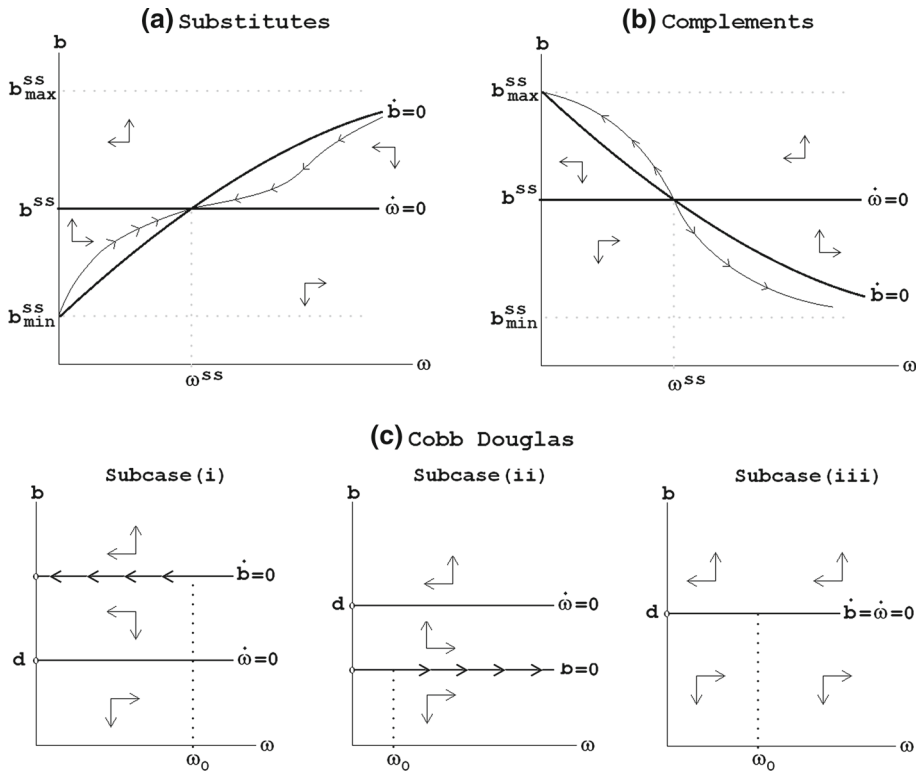
The mechanism generating extinction under complementarity is quite different from that suggested by bio-economic models in which collapse is due to over-exploitation of the natural resource base—see, e.g., D'Alessandro (2007) and, especially, Taylor (2009). In contrast to these stories, the demographic collapse in our model is due to an excessive *scarcity of manpower* that prevents the economy from taking advantage of the natural resource base. This situation is self-reinforcing because the low resource income per capita yields below-replacement fertility and further population decline. Moreover, as we highlighted in the discussion of the dynamics of the innovation rates, the collapse of the population eventually results in the shutting down of R&D activity and ultimately of modern manufacturing production itself.

Also, our results are novel with respect to those of Unified Growth Theory because the qualitative dynamics described in Fig. 2 are generated by a price effect that does not arise if there is no resource market—as in Galor and Weil (2000)—or, if there is, when labor and resources exhibit a unit elasticity of substitution, as in Lucas (2002). To make this point transparent, we now turn to the Cobb–Douglas case and show that the steady state with constant population is indeed created by the resource price effect.

### 5.3 The special Cobb–Douglas case

When the intermediate sector's technology takes the Cobb–Douglas form, the steady state with constant population does not exist and the model predicts that population grows or shrinks forever at a constant rate. The proof follows from Proposition 1. A unit elasticity of input substitution implies that neither expenditures nor the fertility rate are affected by variations in resources per capita. Consequently, the  $\dot{\omega} = 0$  and  $\dot{b} = 0$  loci become horizontal





**Fig. 2** Phase diagrams of system (24) and (25). Under substitutability and complementarity—graphs (a), (b), respectively – the locus  $\dot{b} = 0$  is a monotonous function determining a unique steady state with constant population. In the Cobb–Douglas case, depending on the parameter values, population grows exponentially in subcase (i), declines exponentially in subcase (ii), is constant in the special subcase (iii)

straight lines. The properties of the dynamic system (24) and (25) fall in three subcases: (i) the  $\dot{b} = 0$  locus lies below the  $\dot{\omega} = 0$  locus, (ii) the  $\dot{b} = 0$  locus lies above the  $\dot{\omega} = 0$  locus, or (iii) the two loci coincide.

Figure 2, panel c, describes the phase diagram in all subcases (see the Appendix for details). The common characteristic is that, given the initial condition  $\omega(0) = \omega_0$ , the fertility rate lies along the  $\dot{b} = 0$  locus at time zero.<sup>16</sup> In subcase (i), the economy moves along the  $\dot{b} = 0$  locus and population grows at a constant exponential rate during the whole transition, implying that the resource endowment per capita shrinks to zero asymptotically. Subcase (ii) is specular: the economy moves along the  $\dot{b} = 0$  locus with permanently declining population and no transitional dynamics in the fertility rate. In subcase (iii), the parameters are such that the equilibrium fertility rate exactly coincides with the exogenous mortality rate. However, this steady state is different from the one arising with non-unit input elasticity as there are no interactions between resource scarcity and fertility over time: the economy maintains the initial resource endowment per capita  $\omega_0$  forever.

<sup>16</sup> All explosive paths yielding  $b(t) \rightarrow \infty$  or  $b(t) < 0$  at some finite date are ruled out by standard arguments: they either violate the transversality condition or the household's budget constraint.

## 5.4 Remarks

### 5.4.1 Robustness

Our main results concerning the existence and stability properties of the steady state with constant population are robust to alternative specifications of fertility preferences. In the current analysis, the cost of child bearing cost is determined by the assumed trade-off between consumption per adult and consumption per child. The original version of this paper ([Peretto and Valente 2013](#)) obtains the same predictions by assuming, instead, that child bearing entails a fixed time cost of reproduction which reduces total labor supply and thereby the household labor income. More generally, the model predictions do not hinge on the nature of child bearing costs but rather on the fact that the response of fertility to increased resource scarcity is determined by variations in resource income per capita. For example, the ‘resource price mechanism’ driving the results would not arise in the absence of a resource market with effective property rights.

### 5.4.2 Constant long-run population under complementarity

In the previous analysis, the economy permanently diverges from the steady state with constant population if labor and resources are complements. However, minimal extensions of the model may generate constant long-run population even under complementarity. In the original version of this paper ([Peretto and Valente 2013](#)), for example, we assume that a fraction of the per capita endowment of the resource cannot be exploited for production purposes—e.g., part of the economy’s total land must be devoted to residential use. The existence of a minimum resource requirement allows the economy to avoid demographic explosion even under complementarity. The reason is that, as the resource per capita moves close to the minimum threshold level, the land price signals congestion and fertility rates are subject to an enhanced preventive check that always stabilizes the population level before it grows too large. Moreover, this mechanism also operates with Cobb–Douglas technology, implying that the standard result of exponential population growth is a rather special case.

### 5.4.3 Technological change

We assumed that the technological change driving long-run growth—i.e., vertical innovations—is Hicks-neutral with respect to labor and land. It is possible to introduce land-augmenting technological change, as in UGT, but doing so would complicate the model without adding insight to this paper’s research question. Since we focus on the future prospects for economic growth, we do not need to postulate a bias of technological change which may instead be relevant to study, e.g., the industrial take-off or the escape from a Malthusian trap.

### 5.4.4 Related empirical evidence

Our emphasis on the cases of complementarity and substitutability seems relevant from an empirical perspective since recent cross-country evidence rejects the hypothesis of unit elasticity between labor and land (or between labor and natural resources interpreted as fixed factors: see [Ashraf et al. 2008](#); [Weil and Wilde 2009](#)). In particular, most contributions find high estimated values for the elasticity of substitution—e.g., [Weil and Wilde \(2009\)](#) report estimates ranging from 1.6 to 8.0. Concerning the joint dynamics of population and

income, however, there is no empirical work that attempts at testing the impulse-response mechanism between fertility and resource income that characterizes our model.<sup>17</sup> A recent study that points in a similar direction is Brückner and Schwandt (2014): although their analysis abstracts from feedback effects induced by the elasticity of substitution, they show that positive income shocks raise population growth via increased fertility. We explain further the link between our theory and Brückner and Schwandt's (2014) empirical findings Sect. 6.1 below.

## 6 Exogenous shocks and quantitative analysis

This section presents three applications of our model. First, we study the fertility response to exogenous income shocks (Sect. 6.1), showing that the core mechanism of our theory is consistent with recent evidence on the fertility-income relationship (Brückner and Schwandt 2014). Second, in order to check the plausibility of the theoretical predictions, we apply the model to the United States by calibrating the parameters to match the 1960–2012 data on birth rates and land scarcity (Sect. 6.2). In this exercise, we use the US economy as a laboratory: the simulation yields a reference future equilibrium path for the 1960–2100 period, using the parameters of the in-sample calibration for 1960–2012. This application of the model to a single economy only requires a minimal departure from the hypothesis of a perfectly closed system. Third, we extend the benchmark simulation for the US to study, both qualitatively and quantitatively, the consequences of a demographic shock (Sect. 6.3).

### 6.1 Fertility response to income shocks

In a recent paper, Brückner and Schwandt (2014) document the fertility effects of income shocks using state-of-the-art dynamic panels for a large number of countries. Importantly, they use shocks to the world oil price as an instrument in the identification procedure, obtaining the result—crucial for our purposes—that positive *exogenous* income shocks raise population growth via increased fertility. While Brückner and Schwandt (2014) do not specify a theoretical model, their empirical result appears to address the core mechanism of our model, i.e., the fertility response to income variations for given values of the fundamental demographic parameters. We illustrate how by considering a parameter shock that modifies the steady state land-to-population ratio exclusively through the resource income channel.

Suppose that labor and land are substitutes. The economy is initially in the steady state with constant population, and there is a permanent unexpected rise in  $\epsilon$ . Recalling expression (22), this shock produces a *ceteris paribus* increase in the amount of resource income. In the phase diagram of system (24) and (25), the shock implies a counter-clockwise rotation of the  $\dot{b} = 0$  locus whereas the  $\dot{\omega} = 0$  locus is unchanged.<sup>18</sup> As shown in Fig. 3, diagram (a), the shock yields a new steady state in which the long-run fertility rate  $b^{ss}$  is the same as before

<sup>17</sup> This is partly due to lack of data on land rents for many countries, including the most developed economies. The same problem of data availability constrains the methods for estimating the elasticity of substitution (see Weil and Wilde 2009).

<sup>18</sup> Formally, the reason for the shift is that a higher  $\epsilon$  raises  $b_{\max}^{ss}$ , i.e., the horizontal asymptote of the  $\dot{b} = 0$  locus. In economic terms, variations in the parameters governing resource demand change resource income and thereby the opportunity cost of fertility—which means modifying the  $\dot{b} = 0$  locus—while the same variations leave the natural law of resource depletion (24) unaffected.

but the long-run level of land per capita  $\omega^{ss}$  is lower ( $\omega'' < \omega'$ ). The intuition is that the positive income shock lowers the opportunity cost of fertility and initially drives the current birth rate above replacement. In the long run, instead, population is stabilized again as the price effect generated by the increased scarcity offsets the net private gains from population expansion.

The sequence of events triggered by the shock is as follows. First, given  $\omega$ , the positive shock to resource demand yields a higher resource price  $p$ . Second, the resulting upward jump in resource income raises expenditure per capita  $y$ . Third, the fertility rate  $b$  initially reacts to the higher expenditure with an upward jump on the saddle path leading to the new steady state. Fourth, higher expenditure and slower population growth increase firm size, inducing entry of new firms as well as more vertical R&D investment by existing firms: the higher rates of horizontal and vertical innovations raise TFP growth both in the short and in steady state.

Following the same logical order, the graphs reported in Fig. 3, panel b, provide a quantitative assessment of both the initial and the transitional effects of the income shock. We obtain these diagrams assuming an initial steady state with  $\epsilon = 2.20$  and a shock to  $\epsilon = 2.46$ . All the other parameter values are the same as in the calibration that we discuss in the next subsection.

Beyond its illustrative purpose, the numerical exercise delivers an additional result. Under the assumed parameters, the growth rates of TFP and real output converge to the new steady state level following qualitatively different transitional paths. Specifically, after the initial jumps, TFP growth declines monotonically whereas real output growth exhibits a hump-shaped path—i.e., it keeps on increasing for a while after the shock, reaches a peak and then converges from above to the long-run level.<sup>19</sup> Driving this difference is the behavior of the transitional resource-income effect, see Eq. (26), which is strictly negative during the transition because we have  $\dot{y}/y < 0$  and  $\dot{p}/p > 0$ . This effect becomes smaller and smaller in absolute value as time passes, as  $y$  and  $p$  approach the respective steady states. In our calibration, this effect dominates the transitional TFP slowdown in the short-medium run, determining the hump-shaped path of real income growth. We will encounter a similar effect in the next subsection, where we apply the model to the US economy and interpret this mechanism as a “resource-income drag” linked to the transitional decline of population growth.

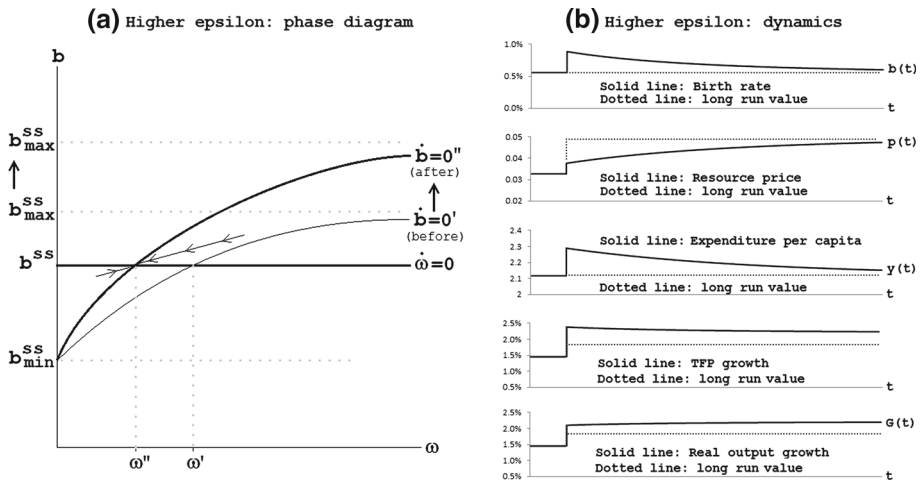
## 6.2 Birth rates and land scarcity in US

This subsection performs a numerical simulation of the model to study the joint dynamics of birth rates and land scarcity in the United States. As a first step, we introduce a minor modification to the theory which allows us to reinterpret the model as one of a single economy that is subject to migratory inflows. We then calibrate the model to match the 1960–2012 data on birth rates and land scarcity in the US, obtaining a reference future equilibrium path for the whole present century.

Like in other industrialized economies, the age-adjusted fertility rate in the US is already below the replacement level<sup>20</sup>: total births are only slightly above total deaths, and a relevant

<sup>19</sup> In Fig. 3, panel b, the pre-shock growth rate of both TFP and real output is 1.45% and the new long-run growth rate, represented by the dotted lines, is 1.83%. The last graph shows that  $G(t)$  reaches a peak above 2% and then converges to 1.83% only in the very long run: the convergence speed of real output growth is rather slow relative to the other variables, which makes the hump-shaped transitional path difficult to show in Fig. 3, panel b.

<sup>20</sup> According to World Bank (2014) data, in each year within the 2011–2013 period, the average fertility rate in the US is 1.9 children per woman, strictly below the replacement ratio of 2.1.



**Fig. 3** Effects of an exogenous income shock generated by an increase in the value of  $\epsilon$ . See Sect. 6.1 for details on each graph

component of population growth is represented by positive net immigration. In order to capture this aspect, we re-define the law of motion of population as

$$\dot{L}(t) = b(t) \cdot L(t) - d \cdot L(t) + v \cdot L(t) \quad (37)$$

where  $v$  is the *net migration rate*, measuring the ratio between immigrants, net of emigrants, and domestic population at each point in time. Assuming that  $v$  is an exogenous constant capturing an average value, our model yields exactly the same results as before with a small, though empirically relevant difference: the steady state is now characterized by a fertility rate equal to the difference between the death rate and the migration rate

$$b^{ss} = d - v.$$

In this situation, population is constant but the birth rate is strictly below the death rate as long as net immigration is positive,  $v > 0$ . Infact, by comparing (1) with (37), we can define  $\bar{d} \equiv d - v$  and re-obtain the original model in which  $\bar{d}$  replaces  $d$  in all our analytical results. In particular, the steady state  $b^{ss} = \bar{d}$  is the long run equilibrium of the economy under substitutability, as we have shown in Sect. 5.2. In the remainder of this section, this setup will be our benchmark model. In particular, we will interpret variable  $b(t)$  as the US crude birth rate, and we identify the fixed factor ‘natural resource’ with total available land in the US.

Considering the US economy, the two empirical facts that are relevant to our analysis are well known. First, starting from the peak reached in the late 1950s, the birth rate declined substantially during the 1960s, experiencing a sharp drop during the oil crises of the 1970s and then fluctuating slightly above 1 % until nowadays. Second, land per capita declined during the same period as population density grew by 70 % between 1960 and 2011. These dynamics would correspond, in our model, to an equilibrium path featuring a joint decline in both  $b(t)$  and  $\omega(t)$ , a transitional behavior that only arises under *substitutability* between land and labor.<sup>21</sup> In order to address this point quantitatively, we calibrate the parameters of

<sup>21</sup> Assuming  $\omega_0 > \omega^{ss}$ , the only equilibrium path that is consistent with a joint decline in both  $b(t)$  and  $\omega(t)$  is the superior branch of the saddle in Fig. 2, graph (a), which refers to the case of substitutability.

system (24) and (25) in order to match the past trends in birth rates and land per capita. The net migration and death rates are set equal to  $v = 0.30\%$  and  $d = 0.85\%$ , in line with the recently observed averages. The steady state is thus characterized by

$$\lim_{t \rightarrow \infty} b(t) = \bar{d} = d - v = 0.55\%.$$

For the intermediate sector, we assume the CES production function

$$F(L_{X_i} - \phi, R_i) = \left[ \psi \cdot (L_{X_i} - \phi)^{\frac{\tau-1}{\tau}} + (1 - \psi) \cdot R_i^{\frac{\tau-1}{\tau}} \right]^{\frac{\tau}{\tau-1}},$$

where  $\psi \in (0, 1)$  is the labor share, and  $\tau$  is the input elasticity of substitution. Given this technology, the cost-share function  $S(p)$ , as defined in expression (22), reads

$$S(p) = \frac{(1 - \psi)^\tau p^{1-\tau}}{\psi^\tau w^{1-\tau} + (1 - \psi)^\tau p^{1-\tau}}.$$

Fixing a set of benchmark values  $\rho = 2\%$ ,  $\eta = 0.6$ ,  $\psi = 0.85$ , and  $\beta = 0.03$ , we calibrate the remaining parameters ( $\epsilon$ ,  $\mu$ ,  $\tau$ ) to ensure: (i) the existence of the steady state  $b^{ss} = 0.55\%$ ; (ii) the feasibility of an initial equilibrium value  $b(0) \approx 2.4\%$ , consistent with the US birth rate in 1960; (iii) a convergence speed to the steady state consistent with the rate of fertility decline observed between 1960 and 2012 in the US. Among the combinations satisfying these requirements, we choose  $\epsilon = 2.46$ ,  $\mu = 0.95$ ,  $\tau = 4$  (see the Appendix for further details).

Figure 4, diagram (a), describes the components of population growth, and superimposes the actual 1960–2012 data series on the simulated paths, covering the 1960–2100 period. According to the simulation, the birth rate will fall below the death rate by the end of the present century: therefor, population growth is only due to immigration. The simulated paths of land price, land per capita, and land income per capita are reported in Fig. 4, diagram (b), with an ex-post normalization (1960 = 1) for each variable that facilitates the comparison among the respective growth rates.<sup>22</sup> Substitutability implies that while  $p(t)$  grows, land income per capita  $p(t)\omega(t)$  declines. The closeness of the simulated path  $\omega(t)$  to the data of US land per capita confirms the good average matching between simulated and observed population growth rates in the 1960–2012 period.<sup>23</sup> The lack of data on land rents prevented us from constructing a comparable ‘actual time series’ for land income per capita.<sup>24</sup>

We complete the simulation by setting the technological parameters ( $\alpha, \theta, \delta, \phi$ ) so as to obtain a long-run growth rate of output equal to  $G^{ss} = 1.83\%$ .<sup>25</sup> Figure 4, diagram (c), draws the resulting equilibrium path and further distinguishes among TFP and transitional components of output growth. Total factor productivity declines because the effect of entry/horizontal innovations—i.e., the term  $(\epsilon - 1)^{-1} \cdot \dot{N}/N$  in Eq. (26)—vanishes quickly whereas the TFP growth due to vertical innovations,  $\theta \cdot \dot{Z}/Z$ , grows over time but at slow pace. Importantly, the overall output growth rate

<sup>22</sup> The actual values at time zero (i.e., year = 1960) obtained from the simulation are  $p(0) = 0.0152$ ,  $\omega(0) = 95.28$  and, consequently,  $p(0)\omega(0) = 1.45$ .

<sup>23</sup> The series of land per capita (1960–2012) appearing in Fig. 4, graph (b), is the inverse of population density as reported by the World Bank (2014).

<sup>24</sup> The only related evidence is the 1990–2011 series of natural resource incomes compiled by the World Bank (2014), which shows that ‘forest land rents’ have been capturing a declining share of total US income during the last two decades—a trend that is consistent with our model’s predictions under substitutability.

<sup>25</sup> Specifically, we set  $\alpha = 2.5$ ,  $\theta = 0.07$ ,  $\phi = 1$  and  $\delta = 0.1\%$ .

$G(t)$  increases over time despite the TFP slowdown, according to the mechanism noted in Sect. 6.1. The present context allows us to interpret this phenomenon as follows.

When labor and land are substitutes, the transitional resource-income effect in Eq. (26) operates like a *resource-income drag* in the short run. Infact, when population grows relatively fast, the resource price grows quickly, and a relatively high value of  $\dot{p}/p > 0$  keeps the output growth rate low in the short run. As time passes, population growth slows down and the same mechanism makes the transitional resource-income effect smaller in absolute value, pushing up the overall output growth rate during the whole transition—cf. Fig. 4, diagram (c). The fact that the resource-income drag may be quantitatively relevant for transitional output growth is an interesting result that should deserve careful empirical scrutiny.

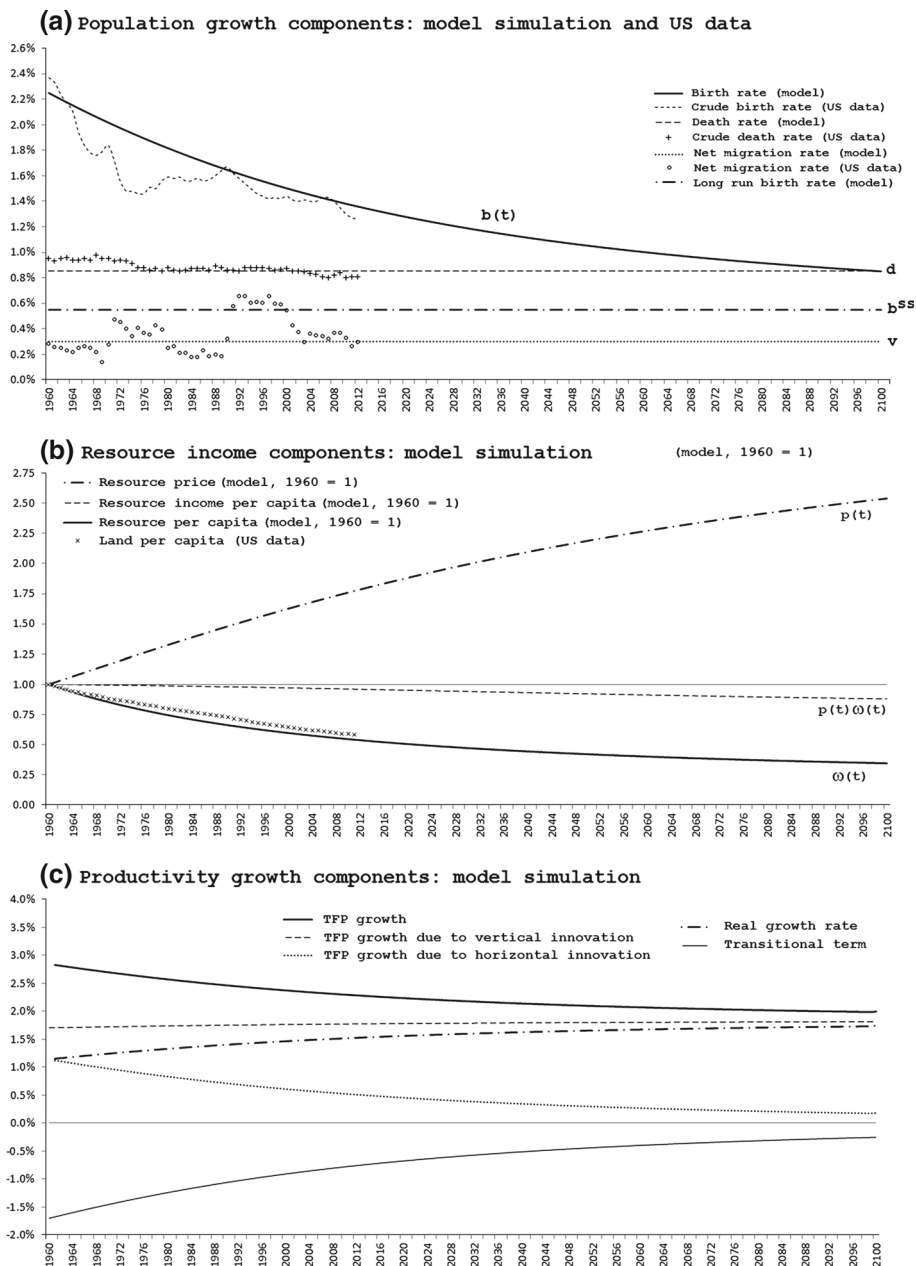
### 6.3 Demographic shock

This section exploits the benchmark simulation of the US economy to study the consequences of a future reduction in the US migration rate. In qualitative terms, the model yields clear predictions: considering the phase diagram of system (24) and (25), a permanent reduction of the average migration rate,  $v$ , induces an upward shift of the  $\dot{\omega} = 0$  locus like the one depicted in Fig. 5, graph (a). Differently from the income shock studied in Sect. 6.1, the migration shock modifies both the fertility rate  $b^{ss}$  and the level of per capita resource  $\omega^{ss}$  in the long run. More precisely, the drop in net migration generates a sequence of (i) lower population growth rates and (ii) higher birth rates relative to the pre-shock situation, regardless of whether the economy is initially in the steady state.<sup>26</sup> The intuition for result (i) is that a drop in the migration rate is equivalent to an increase in the death rate: the reduction of  $v$  directly reduces population growth and thus raises land per capita in the long run—e.g., from  $\omega'$  to  $\omega'' > \omega'$  in Fig. 5, graph (a). The intuition for result (ii) is that lower migration induces labor scarcity and thereby a higher net private benefit from fertility. It follows that the sudden drop in net immigration does not translate into an equivalent drop in the population growth rate because it is partially offset by an increase in the domestic birth rate.

In order to analyze future reductions in the US migration rate, we assume that, at the time of the shock, the economy is placed to the right of the initial steady state, i.e., land per capita is above the pre-shock steady state  $\omega'$  in Fig. 5, graph (a). The qualitative features of the transitional dynamics depend on how far land per capita is from  $\omega'$  when the shock hits. If it is relatively close (e.g.,  $\omega = \omega_a$ ), the transition features *reversion*: the previously declining birth rate jumps upwards, and keeps on increasing thereafter. If it is relatively far (e.g.,  $\omega = \omega_b$ ), instead, the birth rate *overshoots* upward and converges to the new long-run value from above.

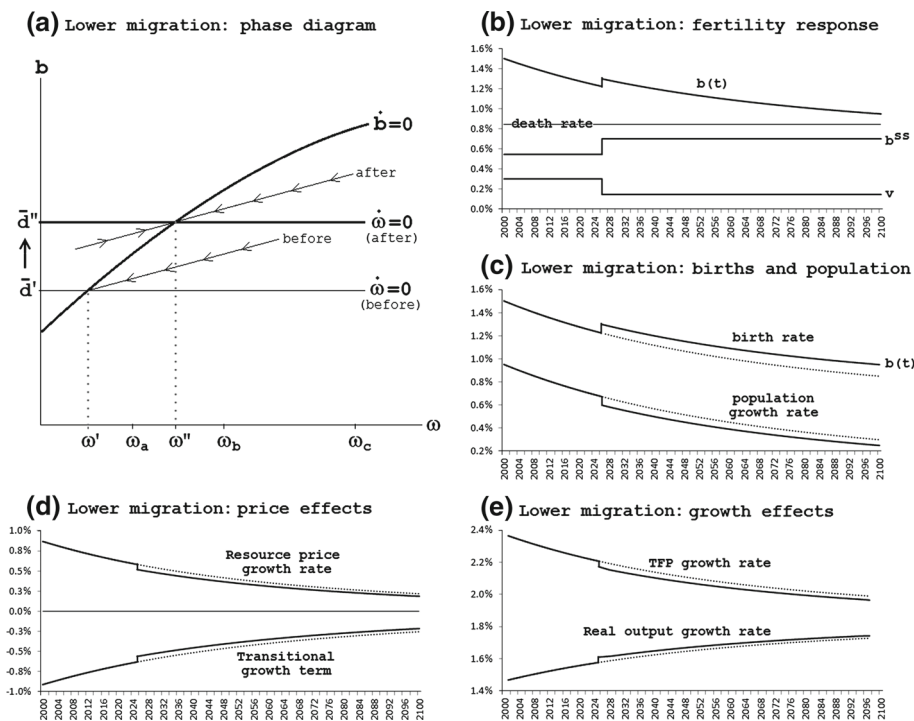
In our quantitative analysis, we consider the benchmark simulation of the US economy and assume that the net migration rate halves—i.e.,  $v$  declines from 0.30 to 0.15 %—from year 2025 onwards. At the time of the shock, land per capita is far from our reference steady

<sup>26</sup> To prove this statement, consider Fig. 5, graph (a), and suppose that the economy is initially placed along any given point along the saddle path leading to the initial steady state  $(\bar{d}', \omega')$ . After the shock, the birth rate jumps upward on the new saddle path leading to the new steady state  $(\bar{d}'', \omega'')$  given the pre-shock level of resource per capita  $\omega'$ . In such a point, the gap between the after-shock fertility rate and  $\bar{d}''$  is smaller than the gap between the pre-shock fertility rate and  $\bar{d}'$ , which implies that the after-shock population growth rate is lower than the before-shock population growth rate.



**Fig. 4** Model simulation over the 1960–2100 period and US data over the 1960–2012 period. See Sect. 6.2 for details on each graph





**Fig. 5** Model simulation: the effects of an exogenous permanent reduction in the US net migration rate from the year 2025 onwards ( $v$  falls from 0.30 to 0.15 %). See Sect. 6.3 for details on each graph

state so that we observe the overshooting effect<sup>27</sup>: the birth rate and the population growth rate jump in opposite directions when the shock hits, and decline over time afterwards, as shown in Fig. 5, graphs (b), (c). Considering the resource price and expenditure per capita, neither  $p$  nor  $y$  change their levels at the time of the shock but their respective growth rates do react to the demographic change: as shown in Fig. 5, graph (d), slower population growth implies slower growth in the resource price. As regards real output growth, the demographic shock affects  $G(t)$  during the transition but not in the long run. During the transition, output growth is subject to two contrasting effects. On the one hand, slower growth in the resource price implies a smaller transitional resource-income effect (cf. Fig. 5, graph (d)) and therefore a positive effect on output growth. On the other hand, the shock implies slower TFP growth during the transition: reduced population growth slows down the growth of market size  $Ly$ , which reduces the incentives to innovate and yields a fall in the rate of entry of new firms (see Fig. 5, graph (e)). In our simulation, the positive transitional effect dominates although real output growth is only slightly higher: at the time of the shock,  $G(t)$  jumps from 1.58 to 1.61 %.

We can consider this analysis as an example of quantitative analysis suggesting further applications. In particular, the model may be fruitfully extended to include, e.g., endogenous migratory flows, differences in fertility preferences between locals and foreigners, inequality in land distribution across resident households.

<sup>27</sup> In terms of the analytical phase diagram in Fig. 5, graph (a), the US economy exhibits a land per capita equal to  $\omega_c$  so that the *pre-shock* birth rate is still above the *long-run after-shock* birth rate  $\bar{d}''$ . The resulting transitional dynamics exhibit overshooting but are slightly different from the transitional path that would arise starting from point  $\omega_b$  (which is associated to a *pre-shock* birth rate strictly below the *long-run after-shock* birth rate  $\bar{d}''$ ).

## 7 Conclusion

This paper investigated the dynamic interdependence of resource scarcity, income and population in a Schumpeterian model with endogenous fertility. The analysis offers the following results. When labor and resources are strict complements or strict substitutes in production, the increase in resource scarcity induced by population growth generates price effects that modify income per capita yielding opposite feedback effects on fertility. These price effects create a steady state in which population is constant, income per capita grows at a constant endogenous rate, and population size is independent of technology. Under substitutability, this steady state is the system's global attractor. Under complementarity, instead, it is a separating threshold and the population level follows diverging paths: higher (lower) resource scarcity generated by the growth (decline) of population increases (decreases) income per capita and fertility rates, implying self-reinforcing feedback effects that drive the economy towards demographic explosion (extinction).

The paper thus proposes a theory of the *population level* which is consistent with the fact that Planet Earth has a finite carrying capacity of people. This basic characteristic of closed systems is not captured by balanced growth models that feature exponential population growth in the long run. Nonetheless, our theory can be exploited to perform a quantitative analysis of real-world economies. Given the lack of global data, we focused on the US economy as a laboratory to analyze the joint dynamics of population, income, land scarcity and total factor productivity. Future quantitative research may exploit the benchmark model to include, e.g., endogenous migratory flows, preference heterogeneity and inequality in land distribution across households.

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