

# VERTICAL MERGER and SPECIFIC INVESTMENTS: A Tale of the Second Best

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*Abstract:* For decades economists and regulatory authorities have debated whether vertical mergers improve welfare. This paper revisits this question in a new setting and reaches new conclusions. We consider investments in specific assets and build a realistic model of an industry - with multiple upstream and downstream firms. We ask how merger affects all downstream firms' investments, not just the investment internalized by a merger. We show any vertical merger distorts all investments. Relative to socially efficient levels, a merged firm increases its investment in internal supply and decreases investment in external supply. Other firms' investments are also skewed. Hence, there is a problem of the second best. When contracts are incomplete, a merger can reduce transactions costs and bargaining distortions for the merged pair. However, the merger creates new investment distortions that can lead to an overall loss in welfare. The results counter basic arguments against regulation of vertical merger.

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## I. Introduction

Economists, lawyers, and judges have debated for decades whether vertical mergers are efficient or restraints to trade. In this paper, we examine this question in a new setting and reach new conclusions. We consider vertical merger when downstream firms can invest in assets specific to suppliers. Our environment, with multiple firms and potentially multiple specific investments, captures a reality. Firms invest, often strategically, in vertical supply relations.<sup>1</sup> We find that, even when downstream firms compete for inputs and inputs are allocated efficiently, vertical merger can lead to inefficient investments. These conclusions counter a basic argument against regulation of vertical mergers.

This paper considers the effect of vertical merger on all specific investments, not just those internalized by a merger. A main insight from the theory of the firm literature is that vertical mergers can enhance welfare by reducing transactions costs. Williamson (1979) argues that, in particular, when contracts are incomplete, vertical merger of two firms eliminates opportunistic behavior and leads to efficient specific investments.<sup>2</sup> Rather than consider two isolated firms, we build a realistic model of an industry - with multiple downstream buyers and multiple upstream suppliers. Buyers can make specific investments in several suppliers at once. We find that when any pairs of firms are vertically merged, investments are inefficient. The merger strengthens a buyer's incentive to invest in an internal supply unit and reduces its incentive to invest in an external supplier. Other firms also skew their investments. Hence, the paper reveals a new problem of the second best. While vertical merger may eliminate distortions from hold-up for a pair of firms, it creates potentially many other investment distortions. Whether overall welfare improves depends on the relative magnitudes these distortions.

Our theoretical analysis reveals new strategic interactions among upstream and downstream firms. In our multilateral setting, we identify *supply-stealing* and *supply-freeing effects* of specific

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<sup>1</sup>See, for example, Cawthorne (1995) and Uzzi (1996) for the textile industry, Nichiguchi (1994) for automobiles, and Saxenian (1994) for software. Spencer and Qiu (2001) analyze relationship specific investments within Japanese keiretsu.

<sup>2</sup>In Grossman and Hart's (1986) treatment, there remains a possibility of distortions from ex post bargaining in a merged firm, since the managers of each unit will continue to bargain. In our analysis a merger eliminates one of these decision makers, and hence there is no ex post bargaining within a merged firm.

investments. A downstream firm that increases its investment in a supplier increases its own expected demand for that supplier. It therefore *steals supply* from other buyers. The expected price of this supplier will rise. A downstream firm that decreases an investment in a supplier *frees supply* for other buyers. The expected price of this seller will fall. Because of these effects, we show that a buyers' investments in the same supplier are strategic substitutes and investments in different suppliers are strategic complements.

We show how a merged firm takes advantage of the supply stealing effects. It "skews" its investments: It increases its investment in its internal seller and decreases investment in external sellers. It thereby raises the demand for its supply unit and its expected revenues. We also identify the strategic response of other firms to the merger and solve for equilibrium investments. Because investments are strategic complements and substitutes, a merger not only skews the merged firm's investments, all firms' investments are skewed in equilibrium.

Beyond the theoretical insights, our analysis informs the vertical merger policy debate. One side of the debate, which we call the "Chicago school," maintains that vertical mergers can only be efficiency-enhancing. They cannot lead to harmful vertical foreclosure, where a merged firm does not supply outside buyers or raises input prices.<sup>3</sup> Rather, merger can lead to internal efficiency gains. Any increases in internal procurement and decreases in outside sales reflects these gains, since a merged firm will sell to an outside buyer whenever that buyer has a higher value for the inputs.<sup>4</sup> On the other side of the debate, a series of papers finds that vertical merger can reduce welfare when downstream firms compete in the output market. Merger does indeed change incentives to sell inputs to rival manufacturers [Salinger (1988), Ordoover, Saloner, Salop (1990), Hart and Tirole (1990)].<sup>5</sup>

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<sup>3</sup>Another issue in the debate concerns the impact on the final goods market. Consumers may also suffer from reduced supply and increased prices.

<sup>4</sup>In particular, Bork (1978) argues that merger can only lead to internal efficiency gains, and if there are such gains, the purchasing unit will optimally favor its internal supply source (p. 207). In his discussion of *Ford Motor Co. v. United States*, in which the United States Supreme Court prevented Ford from purchasing a spark plug supplier, Bork (pg. 236) writes that "The structure of an industry supplying the automotive industry will be whatever is most efficient for the automotive industry. There can be nothing wrong in the automobile manufacturers acquiring all of their suppliers. The decision to make oneself or to buy from others is always made on the basis of difference in cost and effectiveness, criteria the law should permit the manufacturer to apply without interference."

<sup>5</sup>When, for example, firms are Cournot competitors in the final goods market, a unit of supply translates into an additional unit of final output capacity, and can harm the profits of a merged manufacturer-supplier. A large literature now explains reasons why competition in the output market could change a merged firm's incentives to

We present here an argument that vertical merger is harmful even when firms do not compete in the output market. That is, we consider a Chicago school world where a merged firm will sell to an outside buyer whenever that buyer has a higher value for the inputs. Moreover, merger can lead to efficiency gains for a pair of firms. In our analysis, we add the feature that there are multiple firms and each firm can make specific investments that affect the value of inputs from different suppliers. We find that vertical merger can be inefficient because it changes firms' incentives to invest in specific assets. A buyer that merges with a supplier may increase its investment in its internal supplier and decrease its investment in an external supplier, relative to the efficient levels of investments.<sup>6</sup> This "skewing" of investments increases the merged firm's expected value of internal supply relative to external supply. This enhanced value in turn increases the expected revenues the merged supply unit will earn from outside buyers. Since the supply unit receives higher expected revenues, the skewing of investments increases the merged firm's profits. A merged firm will, ex post, purchase more from its internal supply unit. But the initial investments, which changed the relative value of suppliers' inputs, are not socially optimal.

Our analysis combines themes of three literatures. The literature on incomplete contracting considers how ownership affects incentives to invest in specific assets (e.g., Williamson (1979), Grossman and Hart (1986), Hart and Moore (1990), Bolton and Whinston (1993)).<sup>7</sup> This literature highlights how specific assets affects firms' ex post bargaining positions. A second literature on vertical merger, cited above, considers how merger affects the supply of inputs and input prices. A third literature, on networks, considers links or relationships among multiple agents [e.g., Jackson and Wolinsky (1996), Bala and Goyal (2000), Kranton and Minehart (2001)]. Ingredients from each literature appear in our modeling. We consider how vertical merger affects input prices and sales when downstream firms invest in specific assets to potentially multiple suppliers.

In our list of references, Bolton and Whinston's (1993) analysis demands particular attention.

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supply to a rival. See, for example, Ma (1997) and Church & Gandal (2000) and Choi and Yi (2000).

<sup>6</sup>In a model with one buyer and two suppliers, Segal and Whinston (2000) examine how an exclusive dealing contract affects investment incentives. Their problem is different, but they distinguish between external and internal investments in a related way.

<sup>7</sup>Recent work includes Inderst and Wey (2003) and De Fontenay and Gans (2004). Like Hart and Moore (1990), these papers use the Shapley value to divide ex-post surplus.

They consider vertical merger in a model with two downstream firms and one upstream firm where a downstream firm can make a specific investment in the upstream firm. They find that a merged firm may overinvest in the asset specific to the supplier and the other downstream firm may underinvest in the asset specific to the supplier. Our analysis generalizes these important results and finds new effects.

First, we allow for more firms and more specific investments. A downstream firm can invest in any upstream firm. The possibility that a single firm can choose to make several specific investments reflects “real-world” industrial structures where manufacturers can have dealings with many suppliers.<sup>8</sup> In this multilateral setting, we identify new strategic interactions between buyers’ investments. These interactions lead to different results about equilibrium vertical merger. We discuss these differences in Section III, part D. A main finding is that equilibrium patterns of vertical merger depend on the entire network of firms’ investments.

A second contribution of our model is methodological. In our framework, it is straightforward to model any number of upstream and downstream firms (Section IV). Most models of vertical merger do not have this flexibility. Rather than using an extensive form game of competition for inputs that might depend in an idiosyncratic way on the number of firms, we require prices and allocations to satisfy a minimal “supply equals demand” condition. These prices can be easily determined in markets with any number of firms. In Kranton and Minehart (2000), we showed that these “competitive” prices are individually rational and pairwise stable and that market allocations are efficient. We also showed that there is a range of competitive prices. Using the range of competitive prices, we are able to evaluate how our results depend on the division of bargaining power between upstream firms and downstream firms.

The paper proceeds as follows. In the next section we illustrate the basic intuition of our model. We present a numerical example with two downstream firms and two suppliers that shows our main results: a merged firm over-invests in assets specific to its internal supplier and under-invests in assets specific to external suppliers. Firms have an incentive to merge to benefit

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<sup>8</sup>The network structure of many industries has been widely documented. See Kranton and Minehart (2000) for a discussion of various cases. Bolton and Whinston (1993) also consider a model with two upstream firms and two downstream firms. However, for this analysis they restrict a downstream firm to invest in and potentially buy from a single supplier so that the investments are not multilateral.

from manipulating these specific investments. Each additional merger reduces welfare, and merger occurs in equilibrium. In Section III we extend our results to a general model of two downstream firms and two suppliers. Section IV considers greater numbers of firms. We conclude in Section V where we discuss the application of our analysis to regulation.

## II. A Numerical Example for Two Buyers and Two Sellers

There are two downstream units, which we will call buyers, and two upstream units, which we will call sellers. Each seller has the capacity to produce one indivisible unit of an input at zero marginal cost, and each buyer demands one indivisible unit of an input. Buyers can make specific investments in sellers, and an investment increases the expected value of purchasing from that seller. Values are random, reflecting, for example, uncertainty in final consumers' demand for different goods.<sup>9</sup> Buyers make investments in anticipation of future demand for inputs. This assumption captures many industries where investments are of longer duration than the day-to-day or month-to-month fluctuations in consumer demand.<sup>10</sup>

### A. A Merger Game

We consider a three-stage non-cooperative game. In the first stage, firms make merger decisions. In the second stage, buyers invest in assets specific to sellers.<sup>11</sup> The investments determine the distributions of buyers' valuations for sellers' inputs. In the final stage, buyers learn their valuations and compete to obtain inputs from sellers.

This set-up assumes firms can not use long-term contingent contracts to assign investments, future prices, or allocations of goods. The model thus embodies the standard Grossman and

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<sup>9</sup>A buyer has the capacity to produce one unit of output using one input from either seller 1 or seller 2. Depending on the realization of final demand, the buyer prefers to use one input from seller 1 or one input from seller 2. One interpretation is that each buyer may offer several goods in the final output markets, or several varieties of the same good (e.g. different drugs or different models of cars) and have demand for the different inputs depending on the ultimate realization of consumer demand. We will assume that buyers' values are i.i.d, so that buyers are not rivals in the output market.

<sup>10</sup>Examples include pharmaceuticals and cable television which we discuss in the conclusion, as well as the garment industry (Uzzi, 1996), electronics and engineering (Nishiguchi, 1994, and Lorenz, 1989), and toys ("The Puppet-master of Toytown," p.88, *Economist*, September 6, 1997).

<sup>11</sup>Later, we discuss the alternative possibility that sellers invest in assets specific to buyers.

Hart (1986) and Hart and Moore (1990) incomplete contracts framework: agents must make investments before uncertainty is resolved, and contingent contracts are not possible.

We consider vertical mergers of single buyers and sellers. In keeping with Grossman and Hart (1986), Hart and Moore (1990), we assume that ownership does not directly affect the production technology. That is, a merged seller can produce an input for its merged buyer or another buyer. Ownership will affect investment incentives, however, and we will contrast the investments of merged buyers and independent buyers. We assume that a merged buyer has claim to both its revenues and its merged seller's revenues. Therefore, when a buyer  $i$  is merged with seller  $j$ , buyer  $i$  chooses investments to maximize the joint profits of buyer  $i$  and seller  $j$ . When a buyer  $i$  is not merged, it maximizes only its own profits.

Stage One: Firms simultaneously decide whether or not to merge vertically. At the end of this stage, the ownership structure is common knowledge. We will consider, without loss of generality, mergers between buyer 1 and seller 1, and mergers between buyer 2 and seller 2. We assume a pair will merge if and only if the merged firm's profits exceeds the sum of their individual un-merged profits, given the merger decision of the other pair.<sup>12</sup> Three possible ownership structures can emerge. In *no merger*, no firms are merged. In *partial merger*, one pair is merged. In *full merger*, both pairs are merged. For partial merger, without loss of generality, we will consider that buyer 1 and seller 1 are merged.

Stage Two: Given the ownership structure, buyers simultaneously choose specific investments to sellers. At the end of this stage, all investments are common knowledge. Let  $0 \leq g_{ij} \leq 1$  denote buyer  $i$ 's investment in seller  $j$ . With this notation we capture a buyer's investment in any seller, be it a merged seller or an external seller. The cost to buyer  $i$  of this investment is  $c(g_{ij})$  where  $c' > 0$  and  $c'' \geq 0$ . The lines in Figure 1 below depict the specific investments between buyers and sellers.

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<sup>12</sup>The results, therefore, do not depend on how gains from merger are split between the buyer and seller.

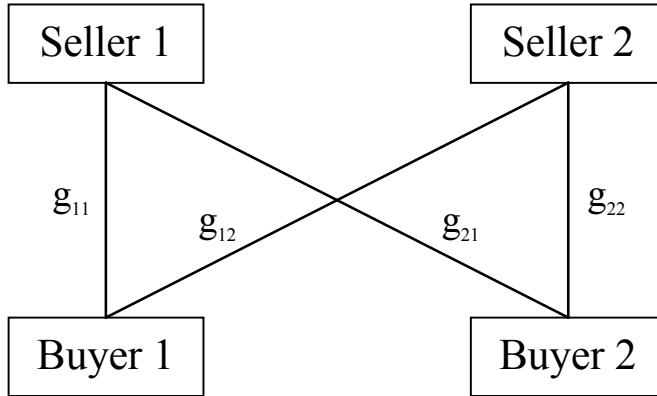


Figure 1. Specific Investments between Downstream and Upstream Firms

Each specific investment determines the probability that a buyer will have positive value for an input from that seller in stage 3.<sup>13</sup> Let  $v_{ij}$  be buyer  $i$ 's value of purchasing an input from seller  $j$ . Let  $g_{ij}$  be the probability that  $v_{ij} = v$ , and  $(1 - g_{ij})$  the probability that  $v_{ij} = 0$  where  $v > 0$ .

Stage Three: Buyers' valuations of goods are realized, and production and exchange takes place. Rather than posit an extensive form game of competition for inputs, we consider general prices and allocations of goods. We consider prices and allocations that are competitive in that they satisfy a minimal "supply and demand" condition: no buyer and seller could negotiate a better deal and participation in the market is voluntary. Formally, these payoffs are pairwise stable and individually rational.<sup>14</sup> Input prices  $p = (p_1, p_2)$  where  $p_i$  is the price of the input of seller  $i$ , and the allocation of inputs are pairwise stable if no buyer and seller could negotiate a different price or allocation that would be better for both. Allocations of inputs are then efficient: buyer 1(2) purchases from seller 1(2) whenever  $v_{11} + v_{22} \geq v_{12} + v_{21}$ . Otherwise buyer 1(2) purchases from seller 2(1). The set of pairwise stable price vectors forms a lattice – there is a minimum and a maximum price vector.<sup>15</sup> To understand these prices, suppose, for example, ex post valuations for the buyers are  $v_{11} = v_{22} = v, v_{12} = v_{21} = 0$ , as pictured in panel (a) of Figure 2. In this case, neither buyer and seller has any outside options, and any prices between 0 and  $v$  would be pairwise stable. The minimum price vector is  $p_1 = 0, p_2 = 0$ , and the maximum

<sup>13</sup>For simplicity,  $v_{ij}$  is assumed to take on only two possible values. Later, we will relax this assumption.

<sup>14</sup>The prices and allocations that are pairwise stable and individually rational give the core outcomes of this game. For the theory of core outcomes in assignment games and related results concerning the structure of payoffs, see Shapley and Shubik (1972), and Roth and Sotomayor (1990). Kranton and Minehart (2000) analyzes competitive prices in network games.

<sup>15</sup>For further discussion of this property, see Roth and Sotomayor (1990).

price vector is  $p_1 = v, p_2 = v$ . In another example, suppose the ex post valuations for the buyers are  $v_{11} = v_{21} = v, v_{12} = v_{22} = 0$ , as pictured in panel (b) of Figure 2. In this case, Seller 1 is a monopolist. Any price other than  $v$  for seller 1 would not be pairwise stable. (If seller 1 were to charge a buyer less than  $v$ , the other buyer would be willing to offer the seller a higher price). Hence, the minimum and maximum price vectors are the same:  $p_1 = v, p_2 = 0$ . Finally, consider  $v_{11} = v_{21} = v_{12} = v, v_{22} = 0$  as pictured in panel (c) of Figure 2. Here, the minimum price vector is  $p_1 = 0, p_2 = 0$ , and the maximum price vector is  $p_1 = v, p_2 = v$ . To see that these prices are pairwise stable, consider both sellers selling for some price  $p$  where  $0 \leq p \leq v$ . Both buyers are obtaining a good at a price of  $p$  and neither would want to offer the other seller a higher price. Both sellers are selling at  $p$ , so neither would want to offer a buyer a lower price.

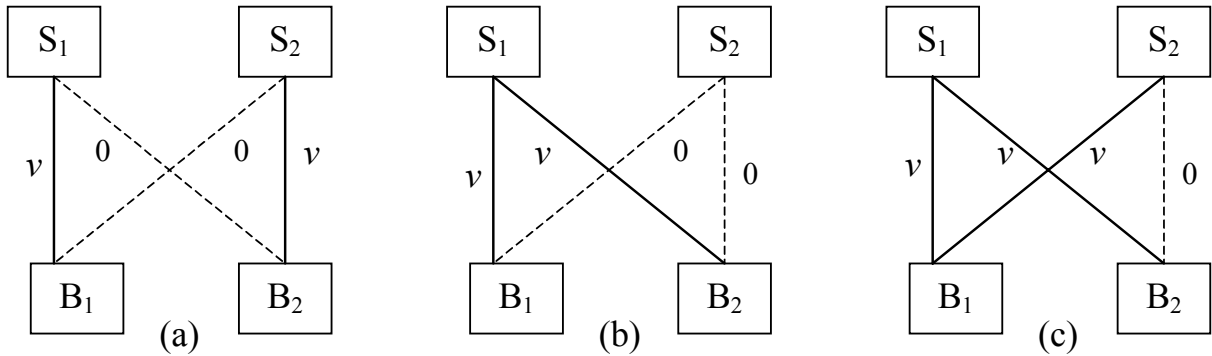


Figure 2. Different Realizations of Buyers' Valuations for Inputs from Different Sellers

We consider a convex combination of the minimum and maximum prices, where the weight on the minimum price vector,  $q$ , represents the “bargaining power” of buyers. If buyers have all of the bargaining power,  $q = 1$ , prices are given by the minimum price vector, and buyers earn all the surplus of exchange net of outside options. If  $q = 0$ , sellers have all of the bargaining power, and prices are given by the maximum price vector.

When a merged buyer purchases the internal input, the price is simply a transfer price. The transfer price insures that the input is allocated efficiently. A merged firm will sell the input to the other buyer only if it cannot do better by using the input itself.<sup>16</sup>

<sup>16</sup>The transfer price is the same as the price that would arise if the buyer and seller were not merged. That is,

These third stage prices and allocations give firms' second stage expected profits: third-stage expected revenues minus second-stage investment costs. Let  $\Pi_i^b$  denote buyer  $i$ 's profits and  $\Pi_j^s$  denote seller  $j$ 's expected profits. We have

$$\Pi_1^b = \alpha(qv) + \beta v - C(g_{11}) - C(g_{12}) \quad ((B))$$

$$\Pi_1^s = \rho(1 - q)v + \sigma v \quad ((S))$$

where  $\alpha, \beta, \rho,$  and  $\sigma$  are probabilities that depend on the investments and are given by

$$\begin{aligned} \alpha &= g_{11}g_{12}(1 - (1 - g_{21})(1 - g_{22})) + g_{11}(1 - g_{12})(1 - g_{21}(1 - g_{22})) \\ &\quad + (1 - g_{11})g_{12}(1 - (1 - g_{21})g_{22}) \\ \beta &= g_{11}g_{12}(1 - g_{21})(1 - g_{22}) \\ \rho &= g_{11}g_{12}(1 - (1 - g_{21})(1 - g_{22})) + g_{11}(1 - g_{12})(1 - g_{21}(1 - g_{22})) + (1 - g_{11})g_{12}g_{21} \\ &\quad + (1 - g_{11})(1 - g_{12})g_{21}(1 - g_{22}) \\ \sigma &= g_{11}(1 - g_{12})g_{21}(1 - g_{22}) \end{aligned}$$

To derive these payoffs, we determine the probabilities and prices for all possible realizations of buyers' valuations. For example, consider  $\Pi_1^b$  and the probabilities  $\alpha$  and  $\beta$ . The first term in  $\alpha$  gives the probability that buyer 1 values an input from either seller and buyer 2 values an input from at least one seller, as in panel (c) of Figure 2. In this case, the minimum price of both sellers is 0 and the maximum price is  $v$ . Hence the buyer earns  $qv$  in this event. The other terms in  $\alpha$  have similar interpretations.  $\beta$  is the probability that buyer 1 is a monopsonist; it is the only competitor for both sellers' inputs, and both the minimum and the maximum prices are zero. Hence, in this event, the buyer earns  $v$ . Consider next  $\Pi_1^s$  and the probabilities  $\rho$  and  $\sigma$ . The first term in  $\rho$  is the same as the first term in  $\alpha$ , i.e., the probability that buyer 1 values an input from either seller and buyer 2 values an input from at least one seller. Seller 1 earns  $(1 - q)v$  in this event.  $\sigma$  is the probability seller 1 is a monopolist, as pictured in panel (b) of Figure 2, and

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the division of bargaining power  $q$  is not changed by a merger. For internal procurement, a merged firm simply earns both the buyers share,  $q$ , and the seller's share,  $1 - q$ , of the marginal surplus of exchange.

hence earns  $v$ .

## B. Shared Bargaining Power Example

We now consider the outcome to our merger game for an example where buyers and sellers share the bargaining power equally. We set  $q = 1/2$ , so that the buyer and seller earn equal shares of the marginal surplus of exchange. As for other parameters, we let  $v = \frac{1}{2}$  and set  $C(g_{ij}) = g_{ij}^3$ .<sup>17</sup>

In this case, the efficient investments, that is, the investments that maximize the sum of all firms profits, are:<sup>18</sup>

$$\begin{aligned} g_{11} &= 0.317 & g_{12} &= 0.317 \\ g_{21} &= 0.317 & g_{22} &= 0.317 \end{aligned}$$

We now consider strategic investments and merger decisions and compare the outcome to these efficient levels of investments. We solve for perfect pure strategy Bayesian equilibria of this game. We analyze the game backwards. We consider second-stage equilibrium investments<sup>19</sup> for the three possible ownership structures: no merger, partial merger, and full merger. We then ask whether firms will merge.

**No merger.** When no firms are merged, buyer  $i$  chooses investments to maximize  $\Pi_i^b$ , taking as given the investments of buyer  $j$ . The first order conditions for the four investments form a

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<sup>17</sup>We use fixed values of  $v$  and  $q$  in order to compute an explicit equilibria. We discuss other values of the bargaining parameter  $q$  below. Changing the value of  $v$  does not have a significant impact on the results. The cost function is taken to be a cubic to insure that buyers invest in both sellers. A quadratic cost function yields qualitatively similar results when a slightly higher value of  $q$  is used.

<sup>18</sup>A straightforward numeric computation shows where  $\Pi_1^b + \Pi_1^s + \Pi_2^b + \Pi_2^s$  is maximized over  $(g_{11}, g_{12}, g_{21}, g_{22})$ . Welfare is 0.3592 at the maximum.

<sup>19</sup>That is, we solve for continuation equilibria for each possible outcome of the first stage of the game. For simplicity, we refer to these continuation equilibria as “equilibria.”

system of four equations.<sup>20</sup> This system yields a unique equilibrium<sup>21</sup> where

$$\begin{aligned} g_{11} &= 0.257 & g_{12} &= 0.257 \\ g_{21} &= 0.257 & g_{22} &= 0.257 \end{aligned}$$

The buyers' equilibrium profits are  $\Pi_1^b = \Pi_2^b = .069$  and the sellers' profits are  $\Pi_1^s = \Pi_2^s = 0.103$ . Here we see the distortions from the shared bargaining power  $q = 1/2$ . Buyers investments are not efficient. Buyers only earn 1/2 of the marginal surplus of an exchange, and, hence, they all underinvest in specific assets.

**Partial merger.** When buyer 1 and seller 1 are merged, buyer 1 chooses investments to maximize  $\Pi_1^b + \Pi_1^s$ . Buyer 2 is not merged and so chooses investments to maximize  $\Pi_2^b$ . As above, there is a unique equilibrium.<sup>22</sup> Here

$$\begin{aligned} g_{11} &= 0.370 & g_{12} &= 0.213 \\ g_{21} &= 0.239 & g_{22} &= 0.266 \end{aligned}$$

The merged firm profits are  $\Pi_1^b + \Pi_1^s = 0.181$ , buyer 2's profits are  $\Pi_2^b = .066$ , and seller 2's profits are  $\Pi_2^s = .093$ .

We see here main insights of our analysis. The merger eliminates the underinvestment distortion resulting from ex post bargaining (hold-up) between buyer 1 and seller 1. The merged firm together earns the full marginal surplus of exchange. However, compared to the efficient level of investment, buyer 1 invests more in its internal seller and less in seller 2. The firm has the incentive to skew its investments to raise the seller's revenues. It is easy to show that the

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<sup>20</sup>The system of equations is given by  $\frac{\partial \Pi_1^b}{\partial g_{11}} = 0$ ,  $\frac{\partial \Pi_1^b}{\partial g_{12}} = 0$ ,  $\frac{\partial \Pi_2^b}{\partial g_{21}} = 0$ , and  $\frac{\partial \Pi_2^b}{\partial g_{22}} = 0$  where  $\Pi_1^b$  and  $\Pi_2^b$  are the profit functions of the buyers.

<sup>21</sup>We solve the system of first order equations numerically using Maple. Maple identifies a unique solution in the relevant parameter range. We use analytical methods to determine that the solution is an equilibrium (the best reply functions are strictly concave). Details are available on request. To confirm the uniqueness of the equilibrium, we needed to confirm the completeness of Maple's numerical findings. For this purpose, we conferred with the algebraic geometer John Little. He employed analytical methods (Cox, Little, and O'Shea (1991)) to determine that the number of solutions to the system of first-order equations agreed with Maple's output. We also ruled out any corner solutions. Each of the other ownership structures is similarly analyzed.

<sup>22</sup>We solve the system of equations  $\frac{\partial(\Pi_1^b + \Pi_1^s)}{\partial g_{11}} = 0$ ,  $\frac{\partial(\Pi_1^b + \Pi_1^s)}{\partial g_{12}} = 0$ ,  $\frac{\partial \Pi_2^b}{\partial g_{21}} = 0$ , and  $\frac{\partial \Pi_2^b}{\partial g_{22}} = 0$ .

seller 1's revenues (Equation (S)) are increasing in  $g_{11}$  and decreasing in  $g_{12}$ . For example, the probability that seller 1 is a monopolist,  $\sigma$ , is higher when buyer 1 has a greater probability of demand for its input and less for seller 2's input. Overall, we find that  $\frac{\partial \Pi_1^s}{\partial g_{11}} > 0$  and  $\frac{\partial \Pi_1^s}{\partial g_{12}} < 0$ .<sup>23</sup>

The merger not only distorts the merged firm's investments, it distorts the independent buyer's investments. We see the supply stealing and supply freeing effects discussed in the introduction. By increasing  $g_{11}$  and decreasing  $g_{12}$  - that is, by skewing its investments - buyer 1 steals the supply of seller 1 from buyer 2 and frees the supply of seller 2. These effects change buyer 2's investment incentives. Because of the supply stealing effect, buyer 2 finds that investing in seller 1 is less profitable. Consider buyer 2's marginal return to its investment in seller 1,  $\frac{\partial \Pi_2^b}{\partial g_{21}}$ . We find that  $\frac{\partial \Pi_2^b}{\partial g_{21}}$  is decreasing in  $g_{11}$  and increasing in  $g_{12}$ . That is:  $\frac{\partial^2 \Pi_2^b}{\partial g_{21} \partial g_{11}} < 0$  and  $\frac{\partial^2 \Pi_2^b}{\partial g_{21} \partial g_{12}} > 0$ . Similarly, because of the supply freeing effect, buyer 2 finds that investing in seller 2 is more profitable. The marginal return to investing in seller 2,  $\frac{\partial \Pi_2^b}{\partial g_{22}}$ , is increasing in  $g_{11}$  and decreasing in  $g_{12}$ . That is:  $\frac{\partial^2 \Pi_2^b}{\partial g_{22} \partial g_{11}} > 0$  and  $\frac{\partial^2 \Pi_2^b}{\partial g_{22} \partial g_{12}} < 0$ . Thus, buyer 2 will skew its investments in favor of seller 2, even though the two firms are not merged.

**Full merger.** When both pairs of firms are merged, buyer 1 chooses investments to maximize  $\Pi_1^b + \Pi_1^s$ , and buyer 2 chooses investments to maximize  $\Pi_2^b + \Pi_2^s$ . As above, there is a unique equilibrium.<sup>24</sup> Here

$$\begin{aligned} g_{11} &= 0.379 & g_{12} &= 0.200 \\ g_{21} &= 0.200 & g_{22} &= 0.379 \end{aligned}$$

Each merged firm earns a profit of  $\Pi_i^b + \Pi_i^s = 0.170$ .

Here again we see how merger affects investments. Relative to the other ownership structures, both buyers further skew their investments. This skewing raises the probability that the internal seller earns a high price. Full merger involves greater skewing than any other ownership structure.

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<sup>23</sup>We find that  $\frac{\partial \Pi_1^s}{\partial g_{11}} > 0$  for all values of  $(g_{11}, g_{12}, g_{21}, g_{22})$  and that  $\frac{\partial \Pi_1^s}{\partial g_{12}} < 0$  in the relevant range of investments.

<sup>24</sup>We solve the system of equations  $\frac{\partial(\Pi_1^b + \Pi_1^s)}{\partial g_{11}} = 0$ ,  $\frac{\partial(\Pi_1^b + \Pi_1^s)}{\partial g_{12}} = 0$ ,  $\frac{\partial(\Pi_2^b + \Pi_2^s)}{\partial g_{21}} = 0$ , and  $\frac{\partial(\Pi_2^b + \Pi_2^s)}{\partial g_{22}} = 0$ .

### C. Equilibrium Ownership Structure

We now solve the first stage of the game to determine the equilibrium ownership structure. A pair will merge if and only if the merged firm's profits exceed the sum of their individual un-merged profits, given the merger decision of the other pair. We find there is a unique equilibrium: full merger. The merger game has the structure of a prisoner's dilemma. No matter the merger decision of the other pair, a buyer and seller can always increase their joint profits by merging.<sup>25</sup> That is, merger is a dominant strategy. Both pairs of buyers and sellers earn a higher joint profits under no merger, but only full merger is an equilibrium.

### D. Merger and Welfare: A Second Best Result

Comparing welfare under each ownership structure, we see a problem of the second best. Initially, when no firms are merged, buyers' investments are not efficient. Contracts are not complete, and in ex post bargaining, buyers do not earn the full surplus of exchange. Hence, buyers underinvest in specific assets. When one pair of firms merges, the merger reduces the distortions from ex post bargaining for the pair. However, the merger creates other investment distortions. Vertical merger thus involves a trade-off. Whether merger improves welfare or decreases welfare must depend on the relative magnitude of the hold-up problem and the incentives to skew investments. In this example, of the three ownership structures, no merger has the highest welfare and full merger has the lowest welfare.<sup>26</sup>

The tradeoff depends on the bargaining power. Here we considered shared bargaining power  $q = 1/2$ . For higher values of  $q$ , merger will also always reduce welfare. Buyers earn greater surplus of exchange, so the initial distortions from ex post bargaining are smaller. There is then less benefit to eliminating them by merger. For smaller values of  $q$ , initial distortions are higher and vertical merger may increase welfare. In all cases, however, merger will distort internal and external investments and so can never achieve full efficiency.

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<sup>25</sup>If the other pair is not merged, a buyer  $i$  and seller  $i$  increase their joint profits from .172 to 0.181 by merging. If the other pair is merged, a buyer  $i$  and seller  $i$  increase their joint profits from 0.159 to 0.170 by merging.

<sup>26</sup>The welfare  $\Pi_1^b + \Pi_1^s + \Pi_2^b + \Pi_2^s$  is 0.3436 with no merger, 0.3409 with partial merger, and 0.3407 with full merger.

Hence, contrary to the Chicago school's argument, vertical mergers can reduce welfare. In our model, merger does not distort incentives to sell inputs. Ex post, the allocation of inputs is efficient. But merged firms have an incentive to manipulate ex ante specific investments.

### III. A General Model for Two Buyers and Two Sellers

We now show our results in a general model of specific investments and valuations of inputs.

#### A. Buyers, Sellers, and Production Technology

As above, there are two buyers and two sellers. We now suppose the specific investment  $g_{ij}$  is any positive number in a (large) compact interval  $[0, \bar{g}]$  and for each  $g_{ij} \geq 0$  the valuation  $\tilde{v}_{ij}$  is a random variable with support in  $[0, \bar{v}]$ . The distribution function  $F_{ij}(v_{ij}; g_{ij})$  and its density function  $f_{ij}(v_{ij}; g_{ij})$  are twice continuously differentiable with respect to  $g_{ij}$  and  $v_{ij}$ .<sup>27</sup> The distributions  $F_{ij}(g_{ij}, \cdot)$  are independent for all  $i$  and  $j$ . We assume larger investments lead to higher valuations: for  $g'_{ij} > g_{ij}$ , the distribution  $F_{ij}(\cdot; g'_{ij})$  first order stochastically dominates  $F_{ij}(\cdot; g_{ij})$ . We assume further that  $f_{ij}(v_{ij}; g_{ij}) > 0$  for  $v_{ij} \in [0, \bar{v}]$  and that  $\partial F_{ij}(v_{ij}; g_{ij}) / \partial g_{ij} < 0$  on an open interval of  $v_{ij}$  for each  $g_{ij}$ .<sup>28</sup> Let  $\mathbf{g}_i = (g_{i1}, g_{i2})$  denote the vector of specific investments for buyer  $i$  and  $\mathbf{G} = (\mathbf{g}_1, \mathbf{g}_2)$  denote the *investment pattern*. Each buyer  $i$  incurs an investment cost  $C(\mathbf{g}_i) = c(g_{i1}) + c(g_{i2})$ , where  $c(0) = 0$  and  $c' > 0$ . Let  $\mathbf{v} = (v_{11}, v_{12}, v_{21}, v_{22})$  be a vector of realized valuations.

We again analyze a three-stage game. In the first stage, firms make merger decisions. In the second stage, buyers make specific investments in sellers. In the third stage, buyers learn their valuations and exchange takes place.

Further discussion is necessary only for the third stage. We again consider pairwise stable (core) outcomes. Allocations of inputs are efficient; buyer 1(2) purchases from seller 1(2) whenever  $v_{11} + v_{22} \geq v_{12} + v_{21}$ . Otherwise buyer 1(2) will purchase from seller 2(1). We again suppose the price of exchange is a convex combination of the minimum and maximum pairwise stable price

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<sup>27</sup>Formally, we define the distribution  $F_{ij}(v_{ij}, g_{ij})$  for  $v_{ij} \in [0, \infty)$ . However, as noted, the support of  $F_{ij}(\cdot, g_{ij})$  is always contained in  $[0, \bar{v}]$ .

<sup>28</sup>These positivity conditions guarantee strict inequalities in some of our results, but are not needed for most of them. The open interval where  $\partial F_{ij} / \partial g_{ij} < 0$  may depend on  $g_{ij}$ .

vectors, where  $q$  is the weight on the minimum price vector.<sup>29</sup> These price vectors are uniquely determined by the realization  $\mathbf{v}$  of buyers' valuations. To see how prices and revenues are formed, consider the following example. Suppose that the ex post valuations satisfy  $v_{11} + v_{22} \geq v_{12} + v_{21}$ . In the efficient allocation buyer 1(2) buys from seller 1(2), as pictured in Figure 3. The prices depend on whether buyer 2 prefers an input from seller 1 ( $v_{21} > v_{22}$ ), as pictured by the thick line in the figure. In this case, the minimum price vector is  $\mathbf{p}'$  where  $p'_1 = v_{21} - v_{22}$  and  $p'_2 = 0$ ; the price  $p'_1$  is just high enough so the buyer 2 would not want to pay seller 1 that price. The maximum price vector is  $\mathbf{p}''$  where  $p''_1 = v_{11} - \max\{v_{12} - v_{22}, 0\}$  and  $p''_2 = v_{22} - \max\{v_{21} - v_{11}, 0\}$ .<sup>30</sup> We take a convex combination  $\mathbf{p} = qp' + (1 - q)p''$  where  $q$  is the bargaining power of buyers.

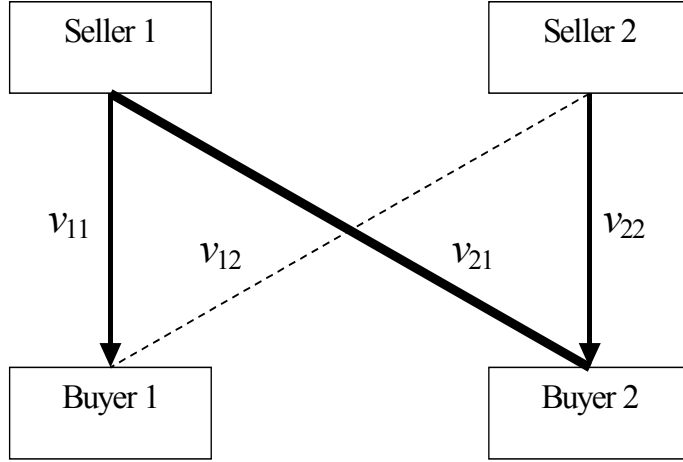


Figure 3. Buyer's Ex Post Valuations:  $v_{11} + v_{22} \geq v_{12} + v_{21}$  and  $v_{21} > v_{22}$

Let  $R_i^b(\mathbf{G})$  denote the expected revenues for buyer  $i$  and  $R_j^s(\mathbf{G})$  denote the expected revenues for seller  $j$ . As before, buyers and sellers expected revenues  $R_i^b(\mathbf{G})$  and  $R_j^s(\mathbf{G})$  depend on the levels

<sup>29</sup>A price vector  $(p_1, p_2)$  is *pairwise stable* for an efficient allocation if and only if when buyer  $i$  and seller  $j$  exchange a good, then  $0 \leq p_j \leq v_{ij}$  and  $v_{ij} - p_j \leq v_{ik} - p_k$  for  $k \neq j$ . For each efficient allocation, the set of competitive price vectors forms a lattice. In particular, there is a minimum and maximum pairwise stable price vector. If for some  $\mathbf{v}$ , several allocations are efficient, then the efficient allocations yield the same welfare and have the same set of pairwise stable price vectors. For these results, see Shapley and Shubik (1972), Roth and Sotomayor (1990), or Kranton and Minehart (2000).

<sup>30</sup>The maximum price vector is a buyer's valuation of the input (his maximum willingness to pay) minus a term that insures that the buyer does not prefer the input of the other seller. For example, suppose that  $v_{12} \geq v_{22}$ . Then  $v_{11} + v_{22} \geq v_{12} + v_{21}$  implies that  $v_{11} \geq v_{21}$ . We have  $p''_1 = v_{11} - (v_{12} - v_{22})$  and  $p''_2 = v_{22}$ . Buyer 2 pays his valuation, but buyer 1's price is adjusted downward from his valuation so that he is willing to buy the input from seller 1.

of specific investments. When a buyer  $i$  obtains a seller  $j$ 's good, buyer  $i$ 's revenues are  $v_{ij} - p_j$  and seller  $j$ 's revenues are  $p_j$ .<sup>31</sup> The prices depend on the realizations of buyers' valuations, and a firm's third stage revenues are an expectation taken with respect to the distribution of possible realizations of  $\mathbf{v}$ . The investments give the distribution of the valuations, and therefore give the probabilities of different events, such as the event described above. The appendix derives the full expressions for these expected revenues.

We solve the game backwards. A firm's expected profits in the game are its third stage revenues minus any second stage investment costs. Buyer  $i$ 's profits are  $\Pi_i^b(\mathbf{G}) \equiv R_i^b(\mathbf{G}) - C(\mathbf{g}_i)$  and seller  $j$ 's profits are  $\Pi_j^s(\mathbf{G}) \equiv R_j^s(\mathbf{G})$ . If buyer  $i$  and seller  $i$  are merged, their joint profits are  $\Pi_i^b(\mathbf{G}) + \Pi_j^s(\mathbf{G})$ . We consider second stage investments, then consider firms' merger decisions.

## B. Comparative Statics on Equilibrium Investments

We compare second-stage equilibrium investments for the three different ownership structures - no merger, partial merger, and full merger. A priori, it is not clear how to make such a comparison since there could be potentially many equilibrium investment patterns for each ownership structure.<sup>32</sup> We can, however, use the theory of monotone comparative statics (Milgrom and Roberts, 1990) to prove our results. Games where strategies are strategic complements are typical examples of supermodular games. Games where strategies are strategic substitutes can also be supermodular when the "reverse order" is taken on the strategy sets.

The bargaining parameter  $q$  is important in our analysis. We will use  $q = 1$  as a benchmark. In this case, buyers have all the bargaining power. We will show that unmerged buyers choose investments efficiently, and that merger reduces welfare. We then indicate how these results extend to smaller values of  $q$ .

In our game, when  $q = 1$ , investments by different buyers in different sellers are strategic complements and investments by different buyers in the same seller are strategic substitutes. For

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<sup>31</sup>As previously discussed, when a merged firm consumes its own input, the price is a transfer price that does not affect joint profits, but does insure that inputs are allocated efficiently.

<sup>32</sup>We can see this possibility in our basic model of Section II. We assumed that buyers' valuations were  $v_{ij} = \frac{1}{2}$  or  $v_{ij} = 0$ . If, instead, we had specified  $v_{ij} = 1.5$  or  $v_{ij} = 0$  and a quadratic cost function  $c(g_{ij}) = g_{ij}^2$ , the second-stage continuation game has three equilibria when no firms are merged. These computations are straightforward and are available on request.

example, consider buyers' investments in different sellers,  $g_{11}$  and  $g_{22}$ . We find that an increase in buyer 2's investment  $g_{22}$  causes the marginal return to buyer 1's investment  $g_{11}$  to rise; that is,  $\frac{\partial^2 \Pi_1^b(\mathbf{G})}{\partial g_{11} \partial g_{22}} > 0$ . We find further that  $\frac{\partial^2 [\Pi_1^b(\mathbf{G}) + \Pi_1^s(\mathbf{G})]}{\partial g_{11} \partial g_{22}} > 0$ , so that the investments  $g_{11}$  and  $g_{22}$  are strategic complements for both a merged and unmerged buyer. As another example, consider buyers' investments in the same seller,  $g_{11}$  and  $g_{21}$ . We have  $\frac{\partial^2 \Pi_1^b(\mathbf{G})}{\partial g_{11} \partial g_{21}} < 0$  and  $\frac{\partial^2 [\Pi_1^b(\mathbf{G}) + \Pi_1^s(\mathbf{G})]}{\partial g_{11} \partial g_{21}} < 0$  so that  $g_{11}$  and  $g_{21}$  are strategic substitutes for both a merged and unmerged buyer.

For independent buyers, these results are intuitive. They follow from supply stealing and supply freeing effects. As we saw in the example (for  $q = \frac{1}{2}$ ), when buyer 2 invests more in seller 2, it steals the supply of seller 2 from buyer 1. Since it demands a good from seller 2 more often, its investment also frees the supply of seller 1. Therefore, the marginal value of buyer 1's investment in seller 1 increases and the marginal value of investment in seller 2 decreases.

For merged buyers, these outcomes are less intuitive since a merged buyer also considers the profits of its merged seller. These latter effects may be quite different than for the buyer. For example, when buyers have all the bargaining power, seller 1 receives a positive price only when *both* buyers prefer its input ( $v_{11} > v_{12}$  and  $v_{21} > v_{22}$ ), and so an increase in  $g_{22}$  decreases the marginal value of  $g_{11}$  for seller 1. That is,  $\frac{\partial^2 \Pi_1^s(\mathbf{G})}{\partial g_{11} \partial g_{22}} < 0$ . We can still show, however, that for the merged firm the investments are complements; that is,  $\frac{\partial^2 \Pi_1^M(\mathbf{G})}{\partial g_{11} \partial g_{22}} > 0$ . When a merged firm obtains an input internally, the price is a transfer. The merged firm simply earns the value of the input. When  $g_{22}$  increases, buyer 1 procures the internal input from seller 1 more often, so the marginal return to  $g_{11}$  increases.

The next lemma shows that the second-stage investment game is supermodular for each ownership structure when  $q$  is sufficiently close to 1. We prove this lemma by establishing the signs of the cross partial derivatives of the merged and independent buyers' profit functions with respect to appropriate pairs of investments. The result follows from our first order stochastic dominance assumptions.<sup>33</sup>

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<sup>33</sup>Notice that we take the reverse order on the investments  $g_{12}$  and  $g_{21}$ . For example, to show supermodularity for buyer 1's own investments ( $g_{11}, -g_{12}$ ) requires showing that  $\frac{\partial^2 \Pi_1^b(\mathbf{G})}{\partial g_{11} \partial (-g_{12})} \geq 0$ , which is the same as showing that  $\frac{\partial^2 \Pi_1^b(\mathbf{G})}{\partial g_{11} \partial g_{12}} \leq 0$ . That is, buyer 1's own investments are strategic substitutes.

**Lemma 1.** *There is a  $\bar{q} < 1$  such that when  $q > \bar{q}$  and for each ownership structure, the investment game is supermodular in the investment strategies  $(g_{11}, -g_{12})$  for buyer 1 and  $(-g_{21}, g_{22})$  for buyer 2.*

**Proof:** All proofs are provided in the Appendix.

We now consider characteristics of equilibrium investment patterns. We will consider mergers between buyer  $i$  and seller  $i$ . We say that an investment pattern  $\mathbf{G}$  is *more skewed* than  $\mathbf{G}'$  if and only if  $g_{ii} \geq g'_{ii}$ , and  $g_{ij} \leq g'_{ij}$  for  $i, j \in \{1, 2\}$ . Our first result shows that for each ownership structure there is an equilibrium and the set of equilibria is defined by a most and least skewed investment pattern. The proposition is a straightforward application of Milgrom and Roberts (1990) and follows directly from the supermodularity of the investment game.

**Proposition 1.** *There is a  $\bar{q} < 1$  such that when  $q > \bar{q}$  and for each ownership structure, there exists a (pure) Nash equilibrium of the second-stage investment game. Moreover, there exist a most skewed and a least skewed equilibrium investment pattern.*

We now show that investments are always more “skewed” as the number of merged firms increases. We begin with a lemma. We parameterize the ownership structure by  $t \in \{N, P, F\}$  where  $N$  indicates no merger,  $P$  partial merger, and  $F$  full merger. The parameter  $t$  increases as the number of merged firms increases; i.e., we impose a partial ordering on  $t$  as follows:  $N \prec P \prec F$ . Firm  $i$  refers to either buyer  $i$  or the merged firm composed of buyer  $i$  and seller  $i$ , according to the ownership structure  $t$ . We have

**Lemma 2.** *There is a  $\bar{q} < 1$  such that when  $q > \bar{q}$  the marginal return to firm 1’s investment in  $g_{11}$  and in  $-g_{12}$  is weakly increasing in  $t$ , holding the other investments fixed. The marginal return to firm 2’s investment in  $g_{22}$  and in  $-g_{21}$  is weakly increasing in  $t$ , holding the other investments fixed.*

The lemma is proved by showing that internal investment benefits a seller and external investment hurts a seller:  $\frac{\partial \Pi_i^s}{\partial g_{ii}} > 0$  and  $\frac{\partial \Pi_i^s}{\partial g_{ij}} < 0$ . A merged firm  $i$  therefore earns a higher (lower) marginal

return from internal (external) investment than does an unmerged firm  $i$ .<sup>34</sup>

Our second result shows that equilibrium investments become more skewed the greater the number of merged firms and is again a straightforward application of Milgrom and Roberts [1990] given Lemma 2. We assume that  $q$  is sufficiently close to 1 so that all earlier results hold.

**Proposition 2.** *Consider the most skewed equilibrium when no firms are merged. The most skewed equilibrium investment pattern when one firm is merged is more skewed. And the most skewed equilibrium investment pattern when both firms are merged is yet more skewed. Consider the least skewed equilibrium pattern when no firms are merged. The least skewed equilibrium investment pattern when one firm is merged is more skewed. The least skewed equilibrium when both firms are merged is yet more skewed.*

We can explain the relationship between merger and skewed investments by the supply stealing and supply freeing effects. The effects yield a feedback; when one buyer skews its investments more, the other will as well. When buyer 1 merges with seller 1, it has an incentive to decrease its investment in seller 2 and increase its investment in its internal seller, seller 1. These investments steal the supply of seller 1 and free the supply of seller 2. Although nothing has changed in buyer 2's holdings, the marginal return to buyer 2's investments in seller 1(2) has decreased (increased). Buyer 2 will then also skew its investments. The equilibrium involves more skewed investments for both buyers.<sup>35</sup>

### C. Ownership Structures and Efficient Investments

Here we compare efficient investment patterns to the equilibrium investments under different ownership structures.

Efficient investment patterns balance the costs of the specific investments with the expected gains from exchange. By taking the efficient allocation of inputs for each realization of buyers' valuations  $\mathbf{v}$ , we can determine the maximal expected surplus from exchange,  $H(\mathbf{G})$ , for a given

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<sup>34</sup>When buyer  $j$  merges, the marginal returns to firm  $i$ 's investments do not change. This is consistent with the statement in the lemma that they weakly increase.

<sup>35</sup>This reaction is also an example of the LeChatelier Principle. For a discussion of Le Chatelier's principle, see Milgrom and Roberts (1994), Theorem 6 and Section III.

investment structure  $\mathbf{G}$ .<sup>36</sup> Let  $W(\mathbf{G})$  denote the welfare of  $\mathbf{G}$ ; that is,  $H(\mathbf{G})$  minus the investment costs:

$$W(\mathbf{G}) \equiv H(\mathbf{G}) - C(\mathbf{g}_1) - C(\mathbf{g}_2)$$

We say an investment pattern  $\mathbf{G}$  is *efficient* if and only if no other investment pattern yields strictly higher welfare.

We now show that merger reduces welfare in our game. The result is particularly stark when  $q = 1$ . In this case, when no firms are merged the efficient investment structure is an equilibrium of the second stage investment game. However, when any firms are merged, the efficient investment structure is not an equilibrium.

We have

**Proposition 3.** *In the no merger ownership structure, when  $q = 1$ , every efficient investment pattern is an equilibrium of the second stage investment game. That is, independent buyers make socially optimal investments in links.*

The result holds because in no merger, when  $q = 1$ , buyers' investment incentives are aligned with welfare. A buyer's marginal return to its investment is exactly equal to its marginal social value, given other buyers' investments. This outcome is a consequence of the competition for inputs. When buyers pay the minimum pairwise stable prices, the price a buyer pays is the incremental value the other buyer would place on obtaining the good. The purchasing buyer then earns the difference between its own value and the other buyer's foregone value - which is exactly the marginal return to welfare when it obtains the input. For example, consider the following realization of buyers' valuations:  $v_{11} + v_{22} \geq v_{12} + v_{21}$  and  $v_{21} > v_{22}$  as in Figure 3 above. In this case, in the efficient allocation, buyer 1(2) obtains a good from seller 1(2), but buyer 2 prefers an input from seller 1. As discussed above, in the minimum price vector, seller 1's price is just high enough so that buyer 2 is willing to forego purchasing from seller 1; that is,  $p_1 = v_{21} - v_{22}$ . Buyer 1's revenue is then  $v_{11} - p_1 = (v_{11} + v_{22}) - v_{21}$ . Welfare is  $v_{11} + v_{22}$ . Now consider social welfare when buyer 1 is not present (i.e., has no investments). Buyer 2 would purchase from seller

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<sup>36</sup>We have  $H(\mathbf{G}) = \iiint \max\{v_{11} + v_{22}, v_{12} + v_{21}\} \Pi f_{ij} dv_{ij}$ .

1 and welfare would be  $v_{21}$ . The change in welfare is  $(v_{11} + v_{22}) - v_{21}$ , which is exactly buyer 1's revenues in this event. Hence, when considering its investments, buyer 1 faces incentives that match social welfare.

Merged buyers' incentives, however, are not aligned with welfare. As we have shown previously, a merged buyer will skew its investments towards its internal seller. A merged buyer  $i$  chooses investments to maximize  $\Pi_i^b(\mathbf{G}) + \Pi_i^s(\mathbf{G})$ . As long as the seller's profits are strictly increasing in  $g_{ii}$  and decreasing in  $g_{ij}$ , buyer  $i$  will change its investments relative to the investments when not merged. We have<sup>37</sup>

**Proposition 4.** *When  $q = 1$  and at least one pair of firms is merged, efficient investment patterns are not equilibrium outcomes. At an efficient investment pattern, a merged firm could increase its profit by either increasing the internal investment or by decreasing the external investment.*

In our benchmark case, merger reduces welfare. When  $q = 1$ , buyers earn the full marginal surplus of exchange, and mergers introduce investment distortions that cause firms to skew their investments inefficiently towards their internal supply unit. When  $q < 1$ , there is a hold-up problem since sellers earn part of the marginal surplus of exchange. Merger can potentially enhance welfare by stimulating a firm to invest in its internal supply unit. We find however that when  $q$  is sufficiently close to 1, merger is still an inefficient outcome.<sup>38</sup>

**Corollary 1.** *There is a  $\bar{q} < 1$  such that when  $q > \bar{q}$ , there is an equilibrium investment pattern under no merger that yields strictly higher welfare than any equilibrium under partial or full merger.*

For values of  $q$  below  $\bar{q}$ , the initial investment distortion may become so significant that mergers enhance welfare. However, in every case, merger is a second-best solution. A merger may improve welfare by stimulating investment in an internal supplier. But because merger will depress investment in external suppliers, it can not achieve first-best investment levels.

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<sup>37</sup>The proposition assumes that the efficient investment pattern involves interior investments. If the efficient investment pattern involves corner solutions ( $g_{ij} = 0$ ), then the pattern could be an equilibrium when some firms are merged.

<sup>38</sup>We assume some standard regularity conditions that are detailed in the proof of this result.

## D. Equilibrium Ownership Structures

Because we have not imposed much structure on the distributions of buyers' valuations, the three-stage merger game may in general have many different equilibria. More than one ownership structure might be an equilibrium, and for a given ownership structure, there are possibly many equilibrium investment patterns. Indeed, because the game has strategic complementarities, the existence of multiple equilibria seems a likely outcome in general.

A key question is whether we can expect merger to arise in equilibrium.<sup>39</sup> Equilibrium of the no merger ownership structure requires that each pair of firms has higher joint profits under no merger than under partial merger. Consider an investment pattern that is a second-stage equilibrium under no merger. Next suppose that buyer 1 and seller 1 merge, and that buyer 2's investments do not change. Then clearly, buyer 1 and seller 1 can choose investments in such a way that their joint profits are at least as high as before they merged. Indeed, buyer 1 and seller 1 can generally do strictly better than before.<sup>40</sup> Therefore, if buyer 2 did not change its investments in response to the merger, then no merger would not be an equilibrium. However, by Proposition 2, buyer 2 does change its investments, skewing them towards seller 2. The next result shows that this skewing response hurts the merged firm. Hence, merger is not necessarily beneficial to the original pair buyer 1 and seller 1.

**Proposition 5.** *There is a  $\bar{q} < 1$  such that when  $q > \bar{q}$ , a merged firm's maximized profits are decreasing in any skewing by the other buyer. Hence, merger is not necessarily first best nor an equilibrium outcome.*

This Proposition tells us that if buyer 2's (optimal) investments in the alternative supplier is sufficiently large, then buyer 1 and seller 1 may not want to merge. No merger can be an equilibrium outcome, and indeed we see this possibility in examples in Section II.

We conclude this section by comparing our results to those of Bolton and Whinston (1993), focusing on how investments affect merger incentives. Bolton and Whinston consider vertical

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<sup>39</sup>As we showed in the last section, merger reduces welfare when  $q$  is sufficiently high.

<sup>40</sup>From lemma 2, the seller's first order conditions satisfy:  $\frac{\partial \Pi_1^s}{\partial g_{11}} \geq 0$  and  $\frac{\partial \Pi_1^s}{\partial g_{12}} \leq 0$ . If the no merger equilibrium was an interior equilibrium, so that  $\frac{\partial \Pi_1^b}{\partial g_{11}} = \frac{\partial \Pi_1^b}{\partial g_{12}} = 0$ , and if for example,  $\frac{\partial \Pi_1^i}{\partial g_{11}} > 0$ , then the merged firm can do strictly better by changing the investments.

merger in a model where two downstream firms can make a specific investment in the same upstream firm.<sup>41</sup> In our model, two downstream firms can invest in one or both of two upstream firms. This “network” feature of the model changes the insights of Bolton and Whinston (1993) in some important ways. In the previous section, we found that if one buyer skews its investments towards one supplier, then the other buyer responds by skewing its investments in the opposite direction towards the *other* supplier. This strategic complementarity is not present in Bolton and Whinston’s model because they consider investments in a single supplier.

An important consequence of this strategic complementarity is that vertical merger is not inevitable in our environment, and a no-merger ownership structure can be an efficient equilibrium. To see this, suppose that no firms are merged. If buyer 1 and seller 1 merge, then buyer 2 will skew its investments towards seller 2 and this could reduce the merged firm’s profits (Proposition 5). The intuition behind Proposition 5 is that when buyer 2 invests more in seller 2, the merged firm has to pay a higher price to obtain seller 2’s good. If this effect is sufficiently strong, buyer 1 and seller 1 will not merge. When buyers have all the bargaining power ( $q = 1$ ), we find examples that demonstrate the no merger ownership structure can be an efficient equilibrium.<sup>42</sup> This outcome contrasts with Bolton and Whinston’s result that vertical merger is inevitable when buyers have all the bargaining power, because merger always increases the profits of the merged firm.<sup>43</sup> This outcome depends on the assumption that the other buyer cannot make an investment in an alternative supplier.<sup>44</sup> In our model, however, buyer 2 can invest in an alternative supplier (seller 2). Our result that a merged firm’s profits depends on other buyers’ specific investments in other suppliers shows the value of considering a network of firms. In general, Proposition 5 demonstrates that in order to predict equilibrium ownerships patterns, we must consider the

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<sup>41</sup>They also consider a second model with two suppliers but limit each buyer to one specific investment. Because of this, the second model does not generate additional results that are comparable to ours. Hence, we focus on their basic model.

<sup>42</sup>See Section II.

<sup>43</sup>Their notion of equilibrium mergers (“quasi-stability”) is slightly different than ours, but both depend on increasing joint profits of the merged firm.

<sup>44</sup>In Proposition 5.2, Bolton and Whinston (1993) shows that a vertical merger must increase the joint profits of the merging firms. The reason is that the joint profits of a buyer 1 (buyer1) and the single seller (seller 1) do not depend on the investment of buyer 2. Any decrease in buyer 2’s investment reduces seller 1’s profit by exactly the same amount that it increases buyer 1’s profit. This result is also true in our model – if buyer 1 and seller 1 merge then their joint profit does not depend on buyer 2’s investment in seller 1. But their joint profit *does* depend on buyer 2’s investment in seller 2.

entire pattern of specific investments.

#### IV. Greater Numbers of Buyers and Sellers

We consider here whether the main insights from the two-buyer-two-seller case generalize to  $S \geq 2$  sellers and  $B \geq 3$  buyers.<sup>45</sup> As above we use  $q = 1$  (buyers have all the bargaining power) as a benchmark case, and we prove results for  $q$  sufficiently close to 1.

Our main result, that merger reduces welfare, continues to hold. When  $q = 1$  and no firms are merged, the efficient investment structure is an equilibrium of the second stage investment game. However, when any firms are merged, the efficient investment structure is not an equilibrium.

**Proposition 6.** *In the no merger ownership structure, when  $q = 1$ , every efficient investment pattern is an equilibrium of the second stage investment game. That is, independent buyers make socially optimal investments in links.*

As in our previous model, an unmerged buyers' revenues are exactly its marginal contribution to welfare. Therefore, unmerged buyers choose investments efficiently, given the investments of the other buyers.

As for the pattern of investments with more than two buyers, we find new incentives for merged firms to invest in external suppliers. A merged firm will still want to increase the investments specific to its internal supplier, but might want to increase or decrease investments specific to external suppliers. With more than two buyers, there are potentially two outside buyers that can compete with each other for the capacity of a merged firm. A merged buyer may want to increase its investment in outside sellers in order to stimulate this competition. Suppose, for example, that buyer 1 is merged with seller 1. When buyer 1 increases its investment in seller 2,  $g_{12}$ , it

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<sup>45</sup>As previously, buyer  $i$  makes specific investments  $g_{ij}$  which determine distribution functions  $F_{ij}(v_{ij}; g_{ij})$ . The vector of specific investments for a buyer  $i$  is  $\mathbf{g}_i = (g_{i1}, g_{i2}, \dots, g_{iS})$ , and  $\mathbf{G} = (\mathbf{g}_1, \mathbf{g}_2, \dots, \mathbf{g}_B)$  is the investment pattern. Each buyer  $i$  incurs an investment cost  $C(\mathbf{g}_i) = \sum_{j=1}^S c(g_{ij})$ , and  $\mathbf{v} = (v_{11}, v_{12}, v_{1S}, v_{21}, v_{22}, \dots, v_{BS})$  is the vector of realized valuations. Further discussion is necessary only for the first and third stages of the game. In the first stage, we consider mergers between buyer  $i$  and seller  $i$  where  $i \leq B$  and  $i \leq S$ . In the third stage, we consider pairwise stable (core) outcomes. For each realization of  $\mathbf{v}$ , the allocation of inputs is efficient. The price of exchange  $\mathbf{p} = (p_1, p_2, \dots, p_S)$  is a convex combination of the minimum and maximum pairwise stable price vectors where  $q$  is the weight on the minimum price vector. Here  $p_j$  is the price received by seller  $j$ . For more discussion of the core, see Kranton and Minehart (2000).

steals seller 2's supply from, say, buyers 2 and 3, as pictured in Figure 4. These two buyers may then both turn to seller 1 and compete more for its capacity. It is possible that the increased competition from outside buyers can more than offset the decrease in buyer 1's own demand for seller 1's capacity.<sup>46</sup>

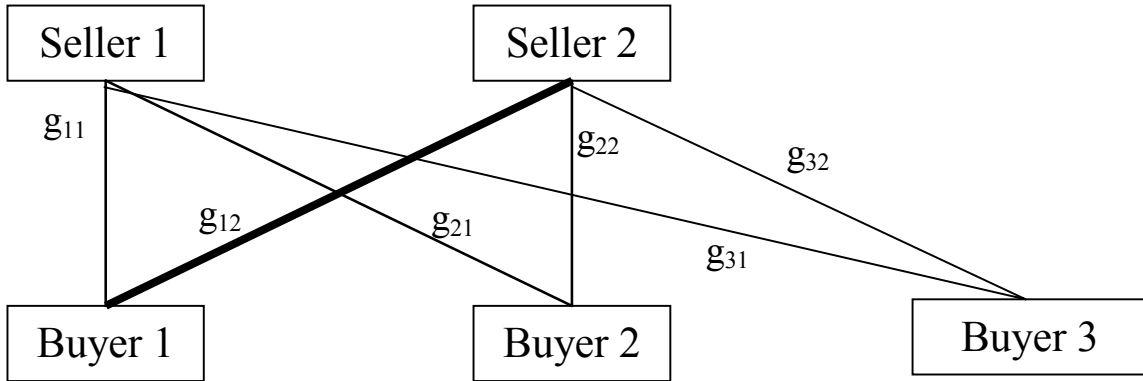


Figure 4: Specific Investments with Three Buyers and Two Sellers

We have<sup>47</sup>

**Proposition 7.** *When  $q = 1$  and at least one pair of firms is merged, efficient investment patterns are not equilibrium outcomes. At an efficient investment pattern, a merged firm could increase its profits by increasing the internal investment.*

**Corollary 2.** *There is a  $\bar{q} < 1$  such that when  $q > \bar{q}$ , there is an equilibrium investment pattern under no merger that yields strictly higher welfare than any equilibrium under partial or full merger.*

## V. Other Extensions

We discuss here whether our results would change if we relax different assumptions of our model.

<sup>46</sup>Because merger affects external investments in an ambiguous way, our skewing results (monotone comparative statics) of section B do not generalize to this model. However, we expect that in many examples, a merged buyer will reduce all or most of its external investments. The effect that leads a merged buyer to increase external investments is (loosely speaking) subtle.

<sup>47</sup>If the efficient investment pattern involves corner solutions ( $g_{ij} = 0$ ), then the efficient pattern could be an equilibrium even when a firm is merged. The proposition assumes that the efficient investment pattern involves interior investments.

In our basic model of two buyers and two sellers, we have assumed that buyers make investments. Alternatively, we might assume that sellers make investments or that both buyers and sellers make investments. If sellers make investments, then independent sellers will choose investments efficiently if they earn all the marginal surplus of exchange ( $q = 0$ ). (Proof of this result is available from the authors upon request.) Sellers earn all the surplus when buyers and sellers exchange inputs for the maximum pairwise stable prices. Merged sellers, however, would also consider the downstream unit's revenues, and therefore not invest efficiently. Merger would again reduce welfare. If buyers and sellers both make investments, then coordination problems can arise. A seller might not invest in a buyer because the buyer is not investing in the seller, and vice versa. We might avoid this problem by using a solution concept such as pairwise stability (Jackson and Wolinsky 1996). However, because it is impossible to award the full marginal surplus of an exchange to both the buyer and the seller, investments are not likely to be made efficiently even by independent firms.

Our results rely on competition for sellers' inputs, which arise in our model due to the capacity constraints of sellers. The fact that a seller can produce only one unit of an input underlies the competition in the input market. When two buyers both prefer a particular seller's input, only one will be able to obtain it. When buyers make specific investments, they anticipate this input market competition. The more a buyer invests in a seller, the more difficult it will be for another buyer to compete for the seller's input. This supply stealing effect (and the related supply freeing effect on the other seller) would not arise if sellers were not capacity constrained. However, if sellers have increasing marginal cost functions, versions of the supply stealing and supply freeing effects of specific investments should still obtain.

Finally, we consider how our results depend on the form of the costs for the specific investments. We have assumed that  $C(\mathbf{g}_i) = c(g_{i1}) + c(g_{i2})$ . More generally, a buyer's cost of investment could depend on the collective investments of all the buyers, and there could be cost interactions between a buyer's own investments. What matters for our results is that the investment game is supermodular.<sup>48</sup> With independent cost functions, the game is supermodular in part because

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<sup>48</sup>In our model, the maximization problems of firms need not be convex. Instead, as shown above, the comparative statics results are obtained using supermodularity.

the cross partial derivatives of the cost functions are zero. However, more general cost functions could also be compatible with supermodularity.

## VI. Conclusion

This paper analyzes vertical merger in a multiple firm setting. Its innovation is to consider downstream firms that make specific investments in several suppliers at once. In this setting, we find that a merged firm has the incentive to manipulate its investments to increase the revenues of its internal supply unit. In a model with two buyers and two sellers, we find that a merged firm will invest more in the internal supplier and less in the external supplier. Our analysis also uncovers the nature of the strategic response of firms to a merger. Manufacturers may strategically adjust their specific investments. When there are two buyers and two sellers, we find buyers' investments in the same seller are strategic substitutes and buyers' investments in different sellers are strategic complements. Hence, a merger by one buyer leads both buyers to skew their investments.

We find that merger is a second best solution to underinvestment in specific assets due to hold-up problems. A merger may improve welfare by stimulating investment in an internal supplier. But because merger also depresses investment in the external supplier, it does not achieve first-best investment levels. Furthermore, because of the strategic effects just described, other firms will also distort their investments in response to a merger. Whether or not merger leads to a gain in welfare depends on the relative magnitudes of the distortions. We provide a numerical example that welfare can fall even when the initial hold-up problem is significant.

This paper has implications for the regulation of vertical mergers. Historically, antitrust law has been concerned with the short-run effects of vertical mergers; that is, how merger directly affects the pricing and allocation of inputs. Recently regulatory authorities have recognized some long-run effects of mergers: that a merger can improve internal investments and lead to efficiency gains. General implications of merger for investments have not been of issue.

For example, consider the regulatory treatment in the United States of the wave of vertical mergers between pharmaceutical suppliers and pharmacy benefit managers (PBM's) in the

1990's.<sup>49</sup> These two types of firms match the buyers and sellers in our model; the pharmaceutical suppliers are the sellers, and the PBM's are the buyers - as they are intermediaries between the pharmaceutical suppliers and retailers/consumers. Like the buyers in our model, in the short-run the PBM's do not compete in the final goods market. Their client base is fixed; a consumer can obtain her pharmacy benefits only from the PBM that has contracted with her employer. The input market, the market for drugs, is competitive, and specific investments can increase the intrinsic value of a supplier's drugs (as a PBM investigates, for example, the efficacy and side effects of a particular treatment). Regulatory authorities were concerned that merged PBM's would favor their parent companies' drugs. The U.S. Federal Trade Commission (the FTC) ordered Merck-Medco in 1998, for example, "to take steps to diminish the effects of unwarranted preference that might be given to Merck's drugs over those of Merck's competitors."<sup>50</sup> The FTC's order however concerns the ex post distribution and pricing of drugs. It ignores the specific investments that could change a PBM's preference for a supplier's drugs.<sup>51</sup> Ex post, the PBM would optimally feature the parent company's drugs. But our analysis suggests the investments themselves may be inefficient.<sup>52</sup>

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<sup>49</sup>Merck purchased Medco (a PBM) in November 1993 for \$6.6 billion. Subsequently, SmithKline purchased Diversified Pharmaceutical Services (DPS) in May 1994 for \$2.3 billion and Eli Lilly purchased PCS Health Systems in November 1994 for \$4.1 billion. In 1994 other pharmaceutical companies formed strategic alliances with different PBM's. See Rangan and Bell (1998). These mergers have subsequently dissolved. For detailed discussions of mergers between pharmaceutical suppliers and pharmacy benefit management companies see, for example, Ragan and Bell (1998), Levy (1999, Chapters II and V), U.S. Food and Drug Administration Report (2001, Attachment G).

<sup>50</sup>U.S. Federal Trade Commission (1998, p.1). A similar consent agreement preceded the Eli Lilly Merger-PCS merger in 1995 (Federal Trade Commission, 1995).

<sup>51</sup>Medco took on several Merck managers with clinical expertise [Ragan and Bell (1998), pg. 17], and Merck-Medco invested \$120 million on information technology to develop health management protocols.

<sup>52</sup>A similar process may be at work in the regulation of cable television. From the mid 1980's through the 1990's there was a wave of vertical mergers between cable television operators and content providers. Chitty (2001) and Waterman and Weiss (1997) find that integrated cable operators carry their own content providers' programming more often than that of other providers. The U.S. Cable Television Consumer Protection and Competition Act of 1992 aimed to limit this by ensuring consumers' access to an array of programming (FCC (1992)).

## VII. Appendix

In various proofs we (1) take derivatives under integral signs and (2) apply the integration by parts formula  $\int xdy = xy | - \int ydx$ . A note at the end of the Appendix shows that these operations are valid for the equations we analyze.

### Buyers' and Sellers' Expected Revenues

We present the revenues for  $q = 1$  and  $q = 0$ . For other  $q$ , revenues are a convex combination of these revenue functions.

#### Case $q = 1$

When  $q = 1$ , for buyer 1 and seller 1 we have

$$R_1^b(\mathbf{G}) = \iiint\limits_{\mathbf{v} \geq \mathbf{0}} [\max\{v_{11} + v_{22}, v_{12} + v_{21}\} - \max\{v_{21}, v_{22}\}] f_{11}f_{12}f_{21}f_{22}dv_{11}dv_{12}dv_{21}dv_{22}$$

$$R_1^s(\mathbf{G}) = \int_0^\infty \int_{v_{22}}^\infty \int_0^\infty \int_{v_{12}}^\infty \min\{v_{21} - v_{22}, v_{11} - v_{12}\} f_{11}f_{12}f_{21}f_{22}dv_{11}dv_{12}dv_{21}dv_{22}$$

Similar expressions obtain for buyer 2 and seller 2. To see how we arrive at these formulas, suppose for example that  $v_{11} + v_{22} \geq v_{12} + v_{21}$ , so that in the efficient allocation buyer 1 purchases from seller 1. When  $q = 1$ , the buyer pays the minimum price. Buyer 1 pays a positive price only when buyer 2 prefers an input from seller 1: that is,  $v_{21} \geq v_{22}$ . Then the minimum price vector is  $\mathbf{p} = (v_{21} - v_{22}, 0)$ . The price  $p_1 = v_{21} - v_{22}$  is just high enough that buyer 2 would not want to pay seller 1 that price. Buyer 1's revenue is  $v_{11} - p_1 = v_{11} - (v_{21} - v_{22})$  in agreement with the formula above for  $R_1^b(\mathbf{G})$ . Seller 1's revenue is  $p_1 = v_{21} - v_{22}$  in agreement with the formula for  $R_1^s(\mathbf{G})$ . Suppose instead that  $v_{21} \leq v_{22}$ . The minimum price vector is now  $\mathbf{p} = (0, 0)$ . Buyer 1's revenue is  $v_{11}$  in agreement with the formula for  $R_1^b(\mathbf{G})$ . Seller 1's revenue is 0. This agrees with the formula for  $R_1^s(\mathbf{G})$  because the limits of integration impose the condition that  $v_{21} \geq v_{22}$ . An analogous argument shows that the revenue formulas are correct when  $v_{11} + v_{22} \leq v_{12} + v_{21}$  so that buyer 1(2) purchases from seller 2 (1).

#### Case $q = 0$

For buyer 1 and seller 1 we have

$$R_1^b(\mathbf{G}) = \int_0^\infty \int_0^\infty \int_{v_{22}}^\infty \int_{v_{21}}^\infty \min\{v_{12} - v_{22}, v_{11} - v_{21}\} f_{11} f_{12} f_{21} f_{22} dv_{11} dv_{12} dv_{21} dv_{22}$$

$$R_1^s(\mathbf{G}) = \iiint_{\mathbf{v} \geq \mathbf{0}} [\max\{v_{11} + v_{22}, v_{12} + v_{21}\} - \max\{v_{12}, v_{22}\}] f_{11} f_{12} f_{21} f_{22} dv_{11} dv_{12} dv_{21} dv_{22}$$

Similar expressions obtain for buyer 2 and seller 2. To see how we arrive at these formulas, suppose for example that  $v_{11} + v_{22} \geq v_{12} + v_{21}$ , so that in the efficient allocation buyer 1 purchases from seller 1. When  $q = 0$ , the buyer pays the maximum price. The price vector is  $\mathbf{p} = (v_{11}, v_{22})$  unless at these prices some buyer would prefer the other seller's good. This happens if either  $v_{12} > v_{22}$  (so buyer 1 would prefer to buy seller 2's good) or  $v_{21} > v_{11}$  (so buyer 2 would prefer to buy seller 1's good). These two inequalities do not simultaneously hold. If  $v_{12} > v_{22}$ , the maximum price vector is  $\mathbf{p} = (v_{11} - (v_{12} - v_{22}), v_{22})$ . Buyer 1's revenue is  $v_{12} - v_{22}$  in agreement with the formula for  $R_1^b(\mathbf{G})$ . Seller 1's revenue is  $p_1 = v_{11} - (v_{12} - v_{22})$  in agreement with the formula for  $R_1^s(\mathbf{G})$ . If  $v_{21} > v_{11}$ , the maximum price vector is  $\mathbf{p} = (v_{11}, v_{22} - (v_{21} - v_{11}))$ . Buyer 1's revenue is 0. This is in agreement with the formula for  $R_1^b(\mathbf{G})$ , because the limits of integration impose the condition that  $v_{11} > v_{21}$ . Seller 1's revenue is  $p_1 = v_{11}$  in agreement with the formula for  $R_1^s(\mathbf{G})$ . If both inequalities fail, so that  $v_{12} \leq v_{22}$  and  $v_{21} \leq v_{11}$ , then the maximum price vector is  $\mathbf{p} = (v_{11}, v_{22})$ . Buyer 1's revenue is 0 and Seller 1's revenue is  $v_{11}$ . These revenues are in agreement with the formulas above. An analogous argument shows that the revenue formulas are correct when  $v_{11} + v_{22} \leq v_{12} + v_{21}$  so that buyer 1(2) purchases from seller 2 (1).

We note that a succinct way to express the maximum price vector when  $v_{11} + v_{22} \geq v_{12} + v_{21}$  is  $p_1 = v_{11} - \max\{v_{12} - v_{22}, 0\}$  and  $p_2 = v_{22} - \max\{v_{21} - v_{11}, 0\}$ . When  $v_{11} + v_{22} \leq v_{12} + v_{21}$ ,  $p_1 = v_{21} - \max\{v_{22} - v_{12}, 0\}$  and  $p_2 = v_{12} - \max\{v_{11} - v_{21}, 0\}$ .

**Proof of Lemma 1.**

In this lemma, we show that the second-stage investment game is supermodular for each of the three ownership structures. We first assume that  $q = 1$  so that buyers have all the bargaining power and prices are given by minimum prices. At the end of the proof, we extend the argument to hold for  $q$  sufficiently close to 1. The proof for  $q = 1$  involves using integration by parts to take derivatives of profit functions. It is long, but straightforward. In our notation, we will sometimes

suppress one or both arguments of the distribution functions  $F_{ij}(v_{ij}; g_{ij})$  and  $f_{ij}(v_{ij}; g_{ij})$ .

**Benchmark Case:**  $q = 1$ .

**No firms are merged.**

The second-stage investment game is supermodular if: (i) buyer  $i$ 's investments ( $g_{ii}$  and  $-g_{ij}$ ) are strategic complements with buyer  $j$ 's investments and (ii) buyer  $i$ 's investments ( $g_{ii}$  and  $-g_{ij}$ ) are strategic complements with each other. (For a discussion of this definition of supermodularity, see Milgrom and Roberts (1990, page 1255).) Following Theorem 4 in Milgrom and Roberts (1990), these conditions can be expressed in terms of cross partials of profit functions. For buyer 1, supermodularity requires: (i)  $\frac{\partial^2 \Pi_1^b}{\partial g_{11} \partial g_{22}} \geq 0$ ,  $\frac{\partial^2 \Pi_1^b}{\partial g_{11} \partial (-g_{21})} \geq 0$ ,  $\frac{\partial^2 \Pi_1^b}{\partial (-g_{12}) \partial (-g_{21})} \geq 0$ ,  $\frac{\partial^2 \Pi_1^b}{\partial g_{(-g_{12})} \partial g_{22}} \geq 0$ , and (ii)  $\frac{\partial^2 \Pi_1^b}{\partial g_{11} \partial (-g_{12})} \geq 0$ . Analogous conditions must hold for buyer 2.

We show here that  $g_{11}$  and  $g_{22}$  are strategic complements for buyer 1:  $\frac{\partial^2 \Pi_1^b(\mathbf{G})}{\partial g_{11} \partial g_{22}} \geq 0$ . All other cross partial derivatives of the buyers' profit functions can be obtained similarly. (Complete computations for this and other proofs are available from the authors on request.)

Buyer 1's profits are  $\Pi_1^b(\mathbf{G}) = R_1^b(\mathbf{G}) - C(\mathbf{g}_1)$ . Since  $\frac{\partial^2 C(\mathbf{g}_1)}{\partial g_{11} \partial g_{22}} = 0$ , the cross partial derivative of the profits is simply the cross partial derivative of the buyer's revenue:  $\frac{\partial^2 \Pi_1^b(\mathbf{G})}{\partial g_{11} \partial g_{22}} = \frac{\partial^2 R_1^b(\mathbf{G})}{\partial g_{11} \partial g_{22}}$ .

We first evaluate  $\frac{\partial R_1^b(\mathbf{G})}{\partial g_{11}}$ . Differentiating the expression for  $R_1^b(\mathbf{G})$  for  $q = 1$ , we have

$$\frac{\partial R_1^b(\mathbf{G})}{\partial g_{11}} = \int_0^\infty \int_0^\infty \int_0^\infty \left[ \int_0^\infty (\max\{v_{11} + v_{22}, v_{12} + v_{21}\} - \max\{v_{21}, v_{22}\}) \frac{\partial f_{11}}{\partial g_{11}} dv_{11} \right] f_{12} f_{21} f_{22} dv_{12} dv_{21} dv_{22}$$

We evaluate the bracketed expression using integration by parts. Let

$$x(v_{11}, v_{12}, v_{21}, v_{22}) = \max\{v_{11} + v_{22}, v_{12} + v_{21}\} - \max\{v_{21}, v_{22}\}$$

and  $dy = \frac{\partial f_{11}}{\partial g_{11}} dv_{11}$ . We may apply the integration by parts formula  $\int x dy = xy - \int y dx$ , where  $y = \frac{\partial F_{11}}{\partial g_{11}}$  and  $dx = \frac{\partial x}{\partial v_{11}} dv_{11}$ . (To obtain  $y$  from  $dy$ , we integrate the function  $\frac{\partial f_{11}(v_{11}; g_{11})}{\partial g_{11}}$  with respect to  $v_{11}$ , *not* with respect to  $g_{11}$ .) We have

$$\int_0^\infty x \frac{\partial f_{11}}{\partial g_{11}} dv_{11} = x \frac{\partial F_{11}}{\partial g_{11}} \Big|_{v_{11}=0}^\infty - \int_0^\infty \frac{\partial x}{\partial v_{11}} \frac{\partial F_{11}}{\partial g_{11}} dv_{11}$$

The first term is equal to zero: By our assumption on the support of  $F_{ij}$ , for all  $g_{ij}$ ,  $F_{ij}(0; g_{ij}) = 0$

and  $F(v; g_{ij}) = 1$  for  $v \geq \bar{v}$ . Hence,  $\frac{\partial F_{11}(v_{11}; g_{11})}{\partial g_{11}} = 0$  at both  $v_{11} = 0$  and  $v_{11} = \infty$ . Consider the second term. For  $v_{11} \geq v_{12} + v_{21} - v_{22}$ ,  $\frac{\partial x}{\partial v_{11}} = 1$ . Otherwise,  $\frac{\partial x}{\partial v_{11}} = 0$ . We now have

$$\frac{\partial R_1^b(\mathbf{G})}{\partial g_{11}} = - \int_0^\infty \int_0^\infty \int_0^\infty \tilde{x}(v_{12}, v_{21}, v_{22}) f_{12} f_{21} f_{22} dv_{12} dv_{21} dv_{22}$$

where

$$\tilde{x}(v_{12}, v_{21}, v_{22}) = \int_{\max\{0, v_{12} + v_{21} - v_{22}\}}^\infty \frac{\partial F_{11}}{\partial g_{11}} dv_{11}.$$

We next derive  $\frac{\partial^2 R_1^b(\mathbf{G})}{\partial g_{11} \partial g_{22}}$ . Differentiating with respect to  $g_{22}$ , we have

$$\frac{\partial^2 R_1^b(\mathbf{G})}{\partial g_{11} \partial g_{22}} = - \int_0^\infty \int_0^\infty \left[ \int_0^\infty \tilde{x}(v_{12}, v_{21}, v_{22}) \frac{\partial f_{22}}{\partial g_{22}} dv_{22} \right] f_{12} f_{21} dv_{12} dv_{21}.$$

We evaluate the bracketed expression using integration by parts to obtain

$$\int_0^\infty \tilde{x}(v_{12}, v_{21}, v_{22}) \frac{\partial f_{22}}{\partial g_{22}} dv_{22} = \tilde{x} \frac{\partial F_{22}}{\partial g_{22}} \Big|_{v_{22}=0}^\infty - \int_0^\infty \frac{\partial \tilde{x}}{\partial v_{22}} \frac{\partial F_{22}}{\partial g_{22}} dv_{22}$$

The first term is equal to zero because  $\frac{\partial F_{22}(v_{12}; g_{22})}{\partial g_{22}} = 0$  at  $v_{12} = 0$  and  $v_{12} = \infty$ . We will show that the integrand of the second term,  $\frac{\partial \tilde{x}}{\partial v_{22}} \frac{\partial F_{22}}{\partial g_{22}}$ , is negative.

Consider the function  $\frac{\partial \tilde{x}}{\partial v_{22}}$ . We have  $\frac{\partial \tilde{x}}{\partial v_{22}} = \frac{\partial F_{11}(v_{12} + v_{21} - v_{22}; g_{11})}{\partial g_{11}} \leq 0$  if  $v_{12} + v_{21} \geq v_{22}$  and  $\frac{\partial \tilde{x}}{\partial v_{22}} = 0$  otherwise. Therefore,  $\frac{\partial \tilde{x}}{\partial v_{22}} \leq 0$ . By our assumption of first order stochastic dominance of  $F_{22}$  with respect to  $g_{22}$ , we have  $\frac{\partial F_{22}}{\partial g_{22}} \leq 0$ . We conclude that  $\frac{\partial \tilde{x}}{\partial v_{12}} \frac{\partial F_{12}}{\partial g_{12}}$  is positive. It follows that the expression  $\int_0^\infty \tilde{x} \frac{\partial f_{22}}{\partial g_{22}} dv_{22}$  is negative, and  $\frac{\partial^2 R_1^b(\mathbf{G})}{\partial g_{11} \partial g_{22}} \geq 0$ . Then we have  $\frac{\partial^2 R_1^b(\mathbf{G})}{\partial g_{11} \partial g_{22}} = \frac{\partial^2 \Pi_1^b}{\partial g_{11} \partial g_{22}} \geq 0$  as desired. We have also derived the expression

$$\frac{\partial^2 \Pi_1^b(\mathbf{G})}{\partial g_{11} \partial g_{22}} = \int_0^\infty \int_0^\infty \int_0^{v_{12} + v_{21}} \frac{\partial F_{11}(v_{12} + v_{21} - v_{22}; g_{11})}{\partial g_{11}} \frac{\partial F_{22}(v_{22}; g_{22})}{\partial g_{22}} dv_{22} f_{12} f_{21} dv_{12} dv_{21}.$$

A similar integration by parts argument proves that  $\frac{\partial^2 \Pi_1^b}{\partial g_{12} \partial g_{11}} \leq 0$  and  $\frac{\partial^2 \Pi_1^b}{\partial g_{21} \partial g_{11}} \leq 0$ , using the fact that  $\frac{\partial \tilde{x}}{\partial v_{12}} \geq 0$  and  $\frac{\partial \tilde{x}}{\partial v_{21}} \geq 0$ .

### Some firms are merged.

When one or both pairs of firms are merged, the second-stage investment game is supermodular if the cross partial derivatives of the merged firm's profits satisfies the conditions for super-

modularity. (We have just shown that an unmerged buyer's profits satisfy the supermodularity conditions.) For a merged firm 1, it is sufficient that  $\frac{\partial^2 \Pi_1^M}{\partial g_{11} \partial g_{22}} \geq 0$ ,  $\frac{\partial^2 \Pi_1^M}{\partial g_{11} \partial (-g_{21})} \geq 0$ ,  $\frac{\partial^2 \Pi_1^M}{\partial (-g_{12}) \partial (-g_{21})} \geq 0$ ,  $\frac{\partial^2 \Pi_1^M}{\partial (-g_{12}) \partial g_{22}} \geq 0$ , and  $\frac{\partial^2 \Pi_1^M}{\partial g_{11} \partial (-g_{12})} \geq 0$  where  $\Pi_1^M = \Pi_1^b(\mathbf{G}) + \Pi_1^s(\mathbf{G})$ . Analogous conditions must hold for a merged firm 2.

We show here that  $\frac{\partial^2 [\Pi_1^b(\mathbf{G}) + \Pi_1^s(\mathbf{G})]}{\partial g_{11} \partial g_{22}} \geq 0$ . All other cross partial derivatives of a merged firm can be obtained similarly.

For the buyer, we have already derived

$$\frac{\partial^2 \Pi_1^b(\mathbf{G})}{\partial g_{11} \partial g_{22}} = \int_0^\infty \int_0^\infty \int_0^{v_{12}+v_{21}} \frac{\partial F_{11}(v_{12} + v_{21} - v_{22}; g_{11})}{\partial g_{11}} \frac{\partial F_{22}(v_{22}; g_{22})}{\partial g_{22}} dv_{22} f_{12} f_{21} dv_{12} dv_{21}.$$

We consider seller 1's profits

$$\Pi_1^s(\mathbf{G}) = R_1^s(\mathbf{G}) = \int_0^\infty \int_{v_{22}}^\infty \int_0^\infty \int_{v_{12}}^\infty \min\{v_{21} - v_{22}, v_{11} - v_{12}\} f_{11} f_{12} f_{21} f_{22} dv_{11} dv_{12} dv_{21} dv_{22}$$

Differentiating, we have

$$\frac{\partial \Pi_1^s}{\partial g_{11}} = \int_0^\infty \int_{v_{22}}^\infty \int_0^\infty \int_{v_{12}}^\infty \min\{v_{21} - v_{22}, v_{11} - v_{12}\} \frac{\partial f_{11}}{\partial g_{11}} dv_{11} f_{12} f_{21} f_{22} dv_{12} dv_{21} dv_{22}$$

Using integration by parts, we derive the expression

$$\frac{\partial \Pi_1^s}{\partial g_{11}} = \int_0^\infty \int_{v_{22}}^\infty \int_0^\infty x(v_{12}, v_{21}, v_{22}) f_{12} f_{21} f_{22} dv_{12} dv_{21} dv_{22}$$

where

$$x(v_{12}, v_{21}, v_{22}) = - \int_{v_{12}}^{v_{12}+v_{21}-v_{22}} \frac{\partial F_{11}}{\partial g_{11}} dv_{11} \text{ and } v_{21} \geq v_{22}.$$

and we can now see that  $\frac{\partial \Pi_1^s}{\partial g_{11}} \geq 0$  by our assumption that  $\frac{\partial F_{11}}{\partial g_{11}} \leq 0$ . Switching the order of integration in  $\frac{\partial \Pi_1^s}{\partial g_{11}}$ , we next derive the expression

$$\frac{\partial^2 \Pi_1^s}{\partial g_{11} \partial g_{22}} = \int_0^\infty \int_0^\infty \int_0^{v_{21}} x(v_{12}, v_{21}, v_{22}) \frac{\partial f_{22}}{\partial g_{22}} dv_{22} f_{12} f_{21} dv_{12} dv_{21}$$

The derivative of  $x$  with respect to  $v_{22}$  is  $\frac{\partial x}{\partial v_{22}} = \frac{\partial F_{11}(v_{12}+v_{21}-v_{22}; g_{11})}{\partial g_{11}}$ . Using integration by parts

where  $y = \frac{\partial F_{22}(v_{22}; g_{22})}{\partial g_{22}}$  and  $dx = \frac{\partial x}{\partial v_{22}} dv_{22}$  and noting that  $\frac{\partial F_{22}(v_{22}; g_{22})}{\partial g_{22}} = 0$  at  $v_{22} = 0$  and that  $x(v_{12}, v_{21}, v_{22}) = 0$  at  $v_{22} = v_{21}$ , we derive the expression:

$$\frac{\partial^2 \Pi_1^s}{\partial g_{11} \partial g_{22}} = - \int_0^\infty \int_0^\infty \int_0^{v_{21}} \frac{\partial F_{11}(v_{12} + v_{21} - v_{22}; g_{11})}{\partial g_{11}} \frac{\partial F_{22}(v_{22}; g_{22})}{\partial g_{22}} dv_{22} f_{12} f_{21} dv_{12} dv_{21}$$

We combine the cross partials for the buyer and seller

$$\frac{\partial^2 \Pi_1^b}{\partial g_{11} \partial g_{22}} + \frac{\partial^2 \Pi_1^s}{\partial g_{11} \partial g_{22}} = \int_0^\infty \int_0^\infty \int_{v_{21}}^{v_{12} + v_{21}} \frac{\partial F_{11}(v_{12} + v_{21} - v_{22}; g_{11})}{\partial g_{11}} \frac{\partial F_{22}(v_{22}; g_{22})}{\partial g_{22}} dv_{22} f_{12} f_{21} dv_{12} dv_{21}$$

Given our assumption that  $\frac{\partial F_{ij}(v_{ij}; g_{ij})}{\partial g_{ij}} \leq 0$  for all values of  $v_{ij}$ , it follows immediately that the cross partial for the merged firm is positive:  $\frac{\partial^2 \Pi_1^M(\mathbf{G})}{\partial g_{11} \partial g_{22}} \geq 0$ .

### Extension to $q$ close to 1.

We show that there is a  $\bar{q} < 1$  such that the investment game is supermodular when  $q > \bar{q}$  for each ownership structure. To do this, we show that the supermodularity conditions for the cross partials all hold with *strict* inequalities (e.g.  $\frac{\partial^2 \Pi_1^b(\mathbf{G})}{\partial g_{11} \partial g_{22}} > 0$ ) when  $q = 1$  and for any  $\mathbf{G}$ . The cross partials of the profit functions are continuous functions of  $q$  and  $\mathbf{G}$ . Because the set of investment patterns is compact ( $g_{ij} \in [0, \bar{g}]$  for all  $i, j$ ), we can find a  $\bar{q} < 1$  such that when  $q > \bar{q}$  we have  $\frac{\partial^2 \Pi_1^b(\mathbf{G})}{\partial g_{11} \partial g_{22}} > 0$  for all  $\mathbf{G}$ .

To finish the proof, we need to show that all the supermodularity conditions on the cross partials hold with strict inequality when  $q = 1$  and for any  $\mathbf{G}$ . For example, for buyer 1, we need to show that  $\frac{\partial^2 \Pi_1^b(\mathbf{G})}{\partial g_{11} \partial g_{22}} > 0$ ,  $\frac{\partial^2 \Pi_1^b(\mathbf{G})}{\partial g_{11} \partial (-g_{21})} > 0$ ,  $\frac{\partial^2 \Pi_1^b(\mathbf{G})}{\partial (-g_{12}) \partial (-g_{21})} > 0$ ,  $\frac{\partial^2 \Pi_1^b(\mathbf{G})}{\partial g(-g_{12}) \partial g_{22}} > 0$ , and  $\frac{\partial^2 \Pi_1^b(\mathbf{G})}{\partial g_{11} \partial (-g_{12})} > 0$ . We argue here that  $\frac{\partial^2 \Pi_1^b(\mathbf{G})}{\partial g_{11} \partial g_{22}} > 0$ . From above, we have

$$\frac{\partial^2 \Pi_1^b(\mathbf{G})}{\partial g_{11} \partial g_{22}} = \int_0^\infty \int_0^\infty \left[ \int_0^{v_{12} + v_{21}} \frac{\partial F_{11}(v_{12} + v_{21} - v_{22}; g_{11})}{\partial g_{11}} \frac{\partial F_{22}(v_{22}; g_{22})}{\partial g_{22}} dv_{22} \right] f_{12} f_{21} dv_{12} dv_{21} \geq 0.$$

The strict inequality,  $\frac{\partial^2 \Pi_1^b(\mathbf{G})}{\partial g_{11} \partial g_{22}} > 0$ , follows from our assumptions on the distributions:  $f_{ij}(v_{ij}; g_{ij}) > 0$  for  $v_{ij} \in [0, \bar{v}]$ , and for each value of  $g_{ij}$ , there is an open interval in  $[0, \bar{v}]$  on which  $\partial F_{ij}(v_{ij}; g_{ij}) / \partial g_{ij} < 0$ . To see this, suppose that  $\frac{\partial F_{11}}{\partial g_{11}} < 0$  when  $v_{11} \in (a, b)$  and that  $\frac{\partial F_{22}}{\partial g_{22}} < 0$  when  $v_{22} \in (c, d)$  (these intervals will depend on the values of  $g_{ij}$ ). When  $v_{12} = a$  and  $v_{21} = d$ , the bracketed expression above reduces to  $\int_0^{a+d} \frac{\partial F_{11}(a+d-v_{22}; g_{11})}{\partial g_{11}} \frac{\partial F_{22}(v_{22}; g_{22})}{\partial g_{22}} dv_{22}$ . Next, we can find

an  $\varepsilon > 0$  such that  $\frac{\partial F_{22}}{\partial g_{22}} < 0$  on the interval  $v_{22} \in (d - \varepsilon, d)$  and  $\frac{\partial F_{11}}{\partial g_{11}} < 0$  on the interval  $(a, a + \varepsilon)$ . It follows that the integrand  $\frac{\partial F_{11}(a+d-v_{22};g_{11})}{\partial g_{11}} \frac{\partial F_{22}(v_{22};g_{22})}{\partial g_{22}}$  is strictly positive when  $v_{22} \in (d - \varepsilon, d)$ . The integral is then also strictly positive. By continuity, the bracketed expression  $\int_0^{v_{12}+v_{21}} \frac{\partial F_{11}(a+d-v_{22};g_{11})}{\partial g_{11}} \frac{\partial F_{22}(v_{22};g_{22})}{\partial g_{22}} dv_{22}$  is strictly positive on an open neighborhood of  $v_{12} = a$  and  $v_{21} = d$ , so that the integral  $\frac{\partial^2 \Pi_1^b}{\partial g_{11} \partial g_{22}}$  is strictly positive.

**Proof of Proposition 2.**

The result follows directly from Milgrom and Roberts (1990), Theorem 5. This theorem shows that for supermodular games, there exist smallest and largest serially undominated strategies. These strategies are pure strategies and constitute pure Nash equilibria. A corollary to the Theorem is that there exists a smallest and largest pure strategy Nash equilibrium. In our framework, the smallest and largest equilibria correspond to the least and most skewed equilibrium for an ownership structure.

**Proof of Lemma 2.**

**Benchmark Case:**  $q = 1$ . The marginal return to firm  $i$ 's investment in  $g_{ii}$  is  $\frac{\partial \Pi_i^b}{\partial g_{ii}}$  if buyer  $i$  is not merged and  $\frac{\partial \Pi_i^b}{\partial g_{ii}} + \frac{\partial \Pi_i^s}{\partial g_{ii}}$  if buyer  $i$  is merged. Therefore, if  $\frac{\partial \Pi_i^s}{\partial g_{ii}} \geq 0$ , the marginal return to the merged firm  $i$ 's investment in  $g_{ii}$  is larger than the return to the independent buyer  $i$ . Similarly, if  $\frac{\partial \Pi_i^s}{\partial g_{ij}} \leq 0$  the marginal return to  $-g_{ij}$  is larger for the merged firm. When  $t = P$ , buyer 1 and seller 1 are merged and payoffs include both buyer and seller revenues. If seller 1's payoffs satisfy  $\frac{\partial \Pi_1^s}{\partial g_{11}} \geq 0$  and  $\frac{\partial \Pi_1^s}{\partial g_{12}} \leq 0$ , then the marginal return to  $g_{11}$  and to  $-g_{12}$  have both increased. For the unmerged buyer 2, when buyer 1 merges, the marginal returns to  $g_{22}$  and  $-g_{21}$  do not change (holding other investments fixed). Hence, we can say that these marginal returns weakly increase. When  $t = F$ , buyer 2 and seller 2 are also merged. As above, if  $\frac{\partial \Pi_2^s}{\partial g_{22}} \geq 0$  and  $\frac{\partial \Pi_2^s}{\partial g_{21}} \leq 0$ , then the marginal return to the merged buyer 2's investments in  $g_{22}$  and  $-g_{21}$  have both increased. For the other merged buyer 1, when buyer 2 merges, the marginal returns to  $g_{11}$  and  $-g_{12}$  do not change (holding other investments fixed).

In the proof of Lemma 1, we showed that  $\frac{\partial \Pi_1^s}{\partial g_{11}} \geq 0$ . The other partial derivatives ( $\frac{\partial \Pi_1^s}{\partial g_{12}} \leq 0$ ,  $\frac{\partial \Pi_2^s}{\partial g_{22}} \geq 0$ , and  $\frac{\partial \Pi_2^s}{\partial g_{21}} \leq 0$ ) can be obtained similarly.

**Extension to  $q$  close to 1.** We extend the lemma's result to  $q > \bar{q}$  for some  $\bar{q} < 1$ . To do this, we show that at  $q = 1$  and at any investment pattern  $\mathbf{G}$ , the following inequalities are strict:

$\frac{\partial \Pi_1^s(\mathbf{G})}{\partial g_{11}} > 0$ ,  $\frac{\partial \Pi_1^s(\mathbf{G})}{\partial g_{12}} < 0$ ,  $\frac{\partial \Pi_2^s(\mathbf{G})}{\partial g_{22}} > 0$  and  $\frac{\partial \Pi_2^s(\mathbf{G})}{\partial g_{21}} < 0$ . These derivative functions are continuous functions of  $q$  and  $\mathbf{G}$ . Because the set of investment patterns is compact, we can find a  $\bar{q} < 1$  such that the inequalities continue to hold for all  $\mathbf{G}$  when  $q > \bar{q}$ .

We show that  $\frac{\partial \Pi_1^s}{\partial g_{11}} > 0$  when  $q = 1$ . In the proof of Lemma 1, we derived the expression

$$\frac{\partial \Pi_1^s}{\partial g_{11}} = - \int_0^\infty \int_{v_{22}}^\infty \int_0^\infty \left[ \int_{v_{12}}^{(v_{21}-v_{22})+v_{12}} \frac{\partial F_{11}}{\partial g_{11}} dv_{11} \right] f_{12} f_{21} f_{22} dv_{12} dv_{21} dv_{22}$$

This expression showed that  $\frac{\partial \Pi_1^s}{\partial g_{11}}$  is weakly positive because  $\frac{\partial F_{11}}{\partial g_{11}} \leq 0$ . The strict inequality,  $\frac{\partial \Pi_1^s}{\partial g_{11}} > 0$ , now follows from our assumptions on the distributions:  $f_{ij}(v_{ij}; g_{ij}) > 0$  for  $v_{ij} \in [0, \bar{v}]$ , and for each value of  $g_{ij}$ , there is an open interval in  $[0, \bar{v}]$  on which  $\partial F_{ij}(v_{ij}; g_{ij}) / \partial g_{ij} < 0$ . To see this, let  $\frac{\partial F_{11}}{\partial g_{11}} < 0$  on the interval  $(a, b)$ . When  $v_{12} = a$  and  $v_{21} - v_{22} = b - a$ , the bracketed expression above reduces to  $\left[ \int_a^b \frac{\partial F_{11}}{\partial g_{11}} dv_{11} \right] < 0$ . By continuity, the bracketed expression is strictly negative on an open neighborhood of  $v_{11} = b$  and  $v_{21} - v_{22} = b - a$ , so that the integral  $\frac{\partial \Pi_1^s}{\partial g_{11}}$  is strictly positive.

The other inequalities,  $\frac{\partial \Pi_1^s}{\partial g_{12}} < 0$ ,  $\frac{\partial \Pi_2^s}{\partial g_{22}} > 0$  and  $\frac{\partial \Pi_2^s}{\partial g_{21}} < 0$ , are shown similarly.

### Proof of Proposition 2.

The result follows directly from Milgrom and Roberts (1990), Theorem 6. This theorem shows that for supermodular games, the smallest and largest serially undominated strategies are nondecreasing functions of a parameter that corresponds to our parameter  $t$ . By Lemma 1 and Lemma 2, our game is supermodular and satisfies the conditions of Theorem 6 and its corollaries.

### Proof of Proposition 5.

**Benchmark Case:**  $q = 1$ .

The merged firm 1 is hurt when buyer 2 skews its investments provided that  $\frac{\partial \Pi_1^M}{\partial g_{22}} \leq 0$  and  $\frac{\partial \Pi_1^M}{\partial g_{21}} \geq 0$ , where  $\Pi_1^M = \Pi_1^b + \Pi_1^s$ . Similarly, the merged firm 2 is hurt when buyer 1 skews its investments provided that  $\frac{\partial \Pi_2^M}{\partial g_{11}} \leq 0$  and  $\frac{\partial \Pi_2^M}{\partial g_{12}} \geq 0$ , where  $\Pi_2^M = \Pi_2^b + \Pi_2^s$ . We show here that  $\frac{\partial \Pi_1^M}{\partial g_{22}} \leq 0$ . The other conditions are shown similarly.

We first derive  $\frac{\partial \Pi_1^b}{\partial g_{22}}$ . This is equal to  $\frac{\partial R_1^b(\mathbf{G})}{\partial g_{22}}$ . Differentiating  $R_1^b(\mathbf{G})$ , we have

$$\frac{\partial \Pi_1^b}{\partial g_{22}} = \int_0^\infty \int_0^\infty \int_0^\infty \left[ \int_0^\infty x \frac{\partial f_{22}}{\partial g_{22}} dv_{22} \right] f_{11} f_{12} f_{21} dv_{11} dv_{12} dv_{21}.$$

where  $x(v_{11}, v_{12}, v_{21}, v_{22}) = \max\{v_{11} + v_{22}, v_{12} + v_{21}\} - \max\{v_{21}, v_{22}\}$ . We use integration by parts to obtain

$$\int_0^\infty x \frac{\partial f_{22}}{\partial g_{22}} dv_{22} = x \frac{\partial F_{22}}{\partial g_{22}} \Big|_{v_{22}=0}^\infty - \int_0^\infty \frac{\partial x}{\partial v_{22}} \frac{\partial F_{22}}{\partial g_{22}} dv_{22}$$

The first term is zero, because  $\frac{\partial F_{22}(v_{22}; g_{22})}{\partial g_{22}} = 0$  when  $v_{22} = 0$  and  $v_{22} = \infty$ . To evaluate the second term, consider the function  $\frac{\partial x}{\partial v_{22}}$ . We have that  $\frac{\partial x}{\partial v_{22}} = 1$  if  $v_{11} + v_{22} > v_{12} + v_{21}$  and  $v_{21} > v_{22}$ . And  $\frac{\partial x}{\partial v_{22}} = -1$  if  $v_{11} + v_{22} < v_{12} + v_{21}$  and  $v_{21} < v_{22}$ . Otherwise  $\frac{\partial x}{\partial v_{22}} = 0$ . We have

$$\begin{aligned} \frac{\partial \Pi_1^b}{\partial g_{22}} &= - \int_0^\infty \int_0^\infty \int_0^\infty \int_{\max\{v_{21} - (v_{11} - v_{12}), 0\}}^{v_{21}} \frac{\partial F_{22}}{\partial g_{22}} dv_{22} f_{11} f_{12} f_{21} dv_{11} dv_{12} dv_{21} \\ &\quad + \int_0^\infty \int_0^\infty \int_0^{v_{12}} \int_{v_{21}}^{v_{21} + (v_{12} - v_{11})} \frac{\partial F_{22}}{\partial g_{22}} dv_{22} f_{11} f_{12} f_{21} dv_{11} dv_{12} dv_{21} \end{aligned}$$

Next, consider  $\frac{\partial \Pi_1^s}{\partial g_{22}}$ . Differentiating  $R_1^s(\mathbf{G})$ , we have

$$\frac{\partial \Pi_1^s}{\partial g_{22}} = \int_0^\infty \int_0^\infty \int_{v_{12}}^\infty \int_0^{v_{21}} \tilde{x} \frac{\partial f_{22}}{\partial g_{22}} dv_{22} f_{11} f_{12} f_{21} dv_{11} dv_{12} dv_{21}$$

where  $\tilde{x}(v_{11}, v_{12}, v_{21}, v_{22}) = \min\{v_{21} - v_{22}, v_{11} - v_{12}\}$ . We integrate by parts to obtain

$$\int_0^{v_{21}} \tilde{x} \frac{\partial f_{22}}{\partial g_{22}} dv_{22} = \tilde{x} \frac{\partial F_{22}}{\partial g_{22}} \Big|_{v_{22}=0}^{v_{21}} - \int_0^{v_{21}} \frac{\partial \tilde{x}}{\partial v_{22}} \frac{\partial F_{22}}{\partial g_{22}} dv_{22}$$

The first term is zero, because  $\frac{\partial F_{22}(v_{22}; g_{22})}{\partial g_{22}} = 0$  at  $v_{22} = 0$  and  $\tilde{x} = 0$  at  $v_{22} = v_{21}$  (note that  $v_{11} > v_{12}$ ). To evaluate the second term, consider the function  $\frac{\partial \tilde{x}}{\partial v_{22}}$ . We have  $\frac{\partial \tilde{x}}{\partial v_{22}} = -1$  if  $v_{11} + v_{22} > v_{21} + v_{12}$ . Otherwise  $\frac{\partial \tilde{x}}{\partial v_{22}} = 0$ . We have

$$\frac{\partial \Pi_1^s}{\partial g_{22}} = \int_0^\infty \int_0^\infty \int_{v_{12}}^\infty \int_{\max\{v_{21} - (v_{11} - v_{12}), 0\}}^{v_{21}} \frac{\partial F_{22}}{\partial g_{22}} dv_{22} f_{11} f_{12} f_{21} dv_{11} dv_{12} dv_{21}$$

Combining  $\frac{\partial \Pi_1^b}{\partial g_{22}}$  and  $\frac{\partial \Pi_1^s}{\partial g_{22}}$ , we have

$$\frac{\partial(\Pi_1^b + \Pi_1^s)}{\partial g_{22}} = \int_0^\infty \int_0^\infty \int_0^{v_{12}} \int_{v_{21}}^{v_{21} + (v_{12} - v_{11})} \frac{\partial F_{22}}{\partial g_{22}} dv_{22} f_{11} f_{12} f_{21} dv_{11} dv_{12} dv_{21}.$$

By assumption  $\frac{\partial F_{22}}{\partial g_{22}} \leq 0$ , and we conclude that  $\frac{\partial \Pi_1^M}{\partial g_{22}} \leq 0$ .

**Extension to  $q$  close to 1.** As in earlier lemmas, we can extend this result to  $q > \bar{q}$  for some  $\bar{q} < 1$ . To do this, we note that at  $q = 1$  and for any investment pattern  $\mathbf{G}$ , the following inequalities are strict:  $\frac{\partial \Pi_1^M(\mathbf{G})}{\partial g_{22}} < 0$ ,  $\frac{\partial \Pi_1^M(\mathbf{G})}{\partial g_{21}} > 0$ ,  $\frac{\partial \Pi_2^M(\mathbf{G})}{\partial g_{11}} < 0$  and  $\frac{\partial \Pi_2^M(\mathbf{G})}{\partial g_{12}} > 0$ . (The argument is the same as in the proof of lemma 2.) These derivative functions are continuous functions of  $q$  and  $\mathbf{G}$ . Because the set of investment patterns is compact, we can find a  $\bar{q} < 1$  such that the inequalities hold for all  $\mathbf{G}$  when  $q > \bar{q}$ .

### Proof of Proposition 3.

We show that when  $q = 1$ , buyer 1's investment incentives are aligned with welfare. A similar argument shows that buyer 2's investment incentives are also aligned with welfare. For buyer 1, we show that

$$\frac{\partial \Pi_1^b(\mathbf{G})}{\partial g_{11}} = \frac{\partial W(\mathbf{G})}{\partial g_{11}} \quad \text{and} \quad \frac{\partial \Pi_1^b(\mathbf{G})}{\partial g_{12}} = \frac{\partial W(\mathbf{G})}{\partial g_{12}}.$$

We have  $\Pi_1^b(\mathbf{G}) = R_1^b(\mathbf{G}) - C(\mathbf{g}_1)$  and  $W(\mathbf{G}) = H(\mathbf{G}) - C(\mathbf{g}_1) - C(\mathbf{g}_2)$ . Taking derivatives, we have

$$\begin{aligned} \frac{\partial \Pi_1^b(\mathbf{G})}{\partial g_{11}} &= \frac{\partial R_1^b(\mathbf{G})}{\partial g_{11}} - \frac{\partial C(\mathbf{g}_1)}{\partial g_{11}} \\ \frac{\partial W(\mathbf{G})}{\partial g_{11}} &= \frac{\partial H(\mathbf{G})}{\partial g_{11}} - \frac{\partial C(\mathbf{g}_1)}{\partial g_{11}} \end{aligned}$$

We will be done if we show that  $\frac{\partial R_1^b(\mathbf{G})}{\partial g_{11}} = \frac{\partial H(\mathbf{G})}{\partial g_{11}}$ . Recall that

$$\begin{aligned} R_1^b(\mathbf{G}) &= \iiint\limits_{\mathbf{v} \geq \mathbf{0}} [\max\{v_{11} + v_{22}, v_{12} + v_{21}\} - \max\{v_{21}, v_{22}\}] f_{11} f_{12} f_{21} f_{22} dv_{11} dv_{12} dv_{21} dv_{22} \\ H(\mathbf{G}) &= \iiint\limits_{\mathbf{v} \geq \mathbf{0}} \max\{v_{11} + v_{22}, v_{12} + v_{21}\} f_{11} f_{12} f_{21} f_{22} dv_{11} dv_{12} dv_{21} dv_{22} \end{aligned}$$

The conclusion now follows immediately, once we note that the term  $\max\{v_{21}, v_{22}\}$  in the integrand of  $R_1^b(\mathbf{G})$  does not depend on the investment  $g_{11}$ . A similar argument shows that  $\frac{\partial \Pi_1^b(\mathbf{G})}{\partial g_{12}} = \frac{\partial W(\mathbf{G})}{\partial g_{12}}$ .

### Proof of Proposition 4.

From Proposition 3, we know that efficient investments are an equilibrium of the second-stage

investment game when no firms are merged and  $q = 1$ . By assumption, the efficient investments are not corner solutions. It follows that  $\frac{\partial \Pi_1^b}{\partial g_{1j}} = 0$  for  $j = 1, 2$  when investments are at efficient levels. In the proof of lemma 2, we showed that  $\frac{\partial \Pi_1^s}{\partial g_{11}} > 0$  when  $q = 1$ . Then, the merged firm 1 has  $\frac{\partial \Pi_1^M}{\partial g_{11}} = \frac{\partial \Pi_1^b}{\partial g_{11}} + \frac{\partial \Pi_1^s}{\partial g_{11}} > 0$ . So the merged firm 1 could strictly increase profits by decreasing  $g_{11}$ , and this is not an equilibrium.

**Proof of Corollary 1** We need to impose some standard regularity conditions. We assume that there is a  $\bar{q} < 1$  such that whenever  $q > \bar{q}$ , the investment game has a finite number of equilibria under each ownership structure. We also assume that each equilibrium can be locally expressed as a continuously differentiable function of  $q$  for  $q > \bar{q}$ . Sufficient conditions for this are given for example in Garcia and Zangwill (1981, Chapter 1).

Then the welfare associated with each equilibrium is a continuous, well-defined function of  $q$ . Let  $\mathbf{G}^*$  be an efficient investment pattern. When  $q = 1$ ,  $\mathbf{G}^*$  is an equilibrium of the no merger investment game, and all equilibria of the partial merger and full merger investment games are inefficient. It follows that there is some  $\varepsilon > 0$  such that  $W(\mathbf{G}^*) - W(G) > \varepsilon$  for any  $G$  that is an equilibrium under partial merger or full merger. The corollary then follows by continuity of the welfare function in  $q$  and  $\mathbf{G}$  and by the local continuity of each equilibrium in the parameter  $q$ . Formally, we can find a  $\bar{q}$  such that for  $q > \bar{q}$ , there is an equilibrium under no merger with welfare within  $\frac{\varepsilon}{2}$  of  $W(\mathbf{G}^*)$ , and such that the most efficient equilibrium under either partial or full merger has welfare within  $\frac{\varepsilon}{2}$  of  $W(G)$ .

**Proof of Proposition 6.**

The result generalizes Proposition 3 to the case of  $B$  buyers and  $S$  sellers. It follows from the fact that when  $q = 1$ , a buyer earns the full marginal surplus from exchange. The general result is proved in Kranton and Minehart (2001, Proposition 1, page 49). We show there that when prices are given by the minimum prices in the core of an assignment game (such as the one considered here) that a buyer's payoffs satisfy the Vickrey property. That is: let  $\mathbf{G}$  be any investment pattern and let  $\mathbf{G}'$  be any other investment pattern that differs only on the investments of buyer  $i$ . Then  $\Pi_1^b(\mathbf{G}) - \Pi_1^b(\mathbf{G}') = W(\mathbf{G}) - W(\mathbf{G}')$ . (Kranton and Minehart (2001) show that this relationship between profits and welfare holds ex post for every realization of  $\mathbf{v}$  and hence also holds in expectation.) It follows that  $\frac{\partial \Pi_1^b(\mathbf{G})}{\partial g_{ik}} = \frac{\partial W(\mathbf{G})}{\partial g_{ik}}$  where  $g_{ik}$  is any investment of buyer

*i.* That is, each buyer will make its investments efficiently given the investments of other buyers.

**Proof of Proposition 7.**

At the efficient investments, we have  $\frac{\partial \Pi_1^b}{\partial g_{11}} = 0$  because the efficient investments are an interior equilibrium when no firms are merged. We will show that when  $q = 1$ , we have  $\frac{\partial \Pi_1^s}{\partial g_{11}} > 0$ . Then  $\frac{\partial \Pi_1^b}{\partial g_{11}} + \frac{\partial \Pi_1^s}{\partial g_{11}} > 0$ , so that the efficient investment pattern is not an equilibrium when buyer 1 and seller 1 are merged.

The seller's profit function is

$$\Pi_1^s(\mathbf{G}) = \int \cdots \int_{\mathbf{v}} p_1(\mathbf{v}) f_{11} \cdots f_{BS} dv_{11} \cdots dv_{BS}$$

where  $\mathbf{v} = (v_{11}, \dots, v_{BS})$  is a realization of buyers valuations and  $p_1(\mathbf{v})$  is the minimum price vector. Differentiating, we have

$$\frac{\partial \Pi_1^s}{\partial g_{1k}} = \int \cdots \int_{v_{ij} \neq v_{1k}} \left[ \int_0^\infty p_1(\mathbf{v}) \frac{\partial f_{1k}}{\partial g_{1k}} dv_{1k} \right] f_{11} \cdots f_{BS} dv_{11} \cdots dv_{BS}$$

Using integration by parts, we obtain:

$$\frac{\partial \Pi_1^s}{\partial g_{1k}} = \int \cdots \int_{v_{ij} \neq v_{1k}} \left[ p_1(\mathbf{v}) \frac{\partial F_{1k}(v_{1k}, g_{1k})}{\partial g_{1k}} \Big|_0^\infty - \int_0^\infty \frac{\partial p_1(\mathbf{v})}{\partial v_{1k}} \frac{\partial F_{1k}}{\partial g_{1k}} dv_{1k} \right] f_{11} \cdots f_{BS} dv_{11} \cdots dv_{BS}$$

The first term in the bracketed expression above is zero. We have

$$\frac{\partial \Pi_1^s}{\partial g_{1k}} = \int \cdots \int_{v_{ij} \neq v_{1k}} \left[ - \int_0^\infty \frac{\partial p_1(\mathbf{v})}{\partial v_{1k}} \frac{\partial F_{1k}}{\partial g_{1k}} dv_{1k} \right] f_{11} \cdots f_{BS} dv_{11} \cdots dv_{BS}$$

By assumption, we have  $\frac{\partial F_{1k}}{\partial g_{1k}} \leq 0$ . The sign of  $\frac{\partial \Pi_1^s}{\partial g_{1k}}$  is therefore determined by the sign of  $\frac{\partial p_1(\mathbf{v})}{\partial v_{1k}}$ .

We will first show that  $\frac{\partial p_1(\mathbf{v})}{\partial v_{11}} \geq 0$  and hence we conclude that  $\frac{\partial \Pi_1^s}{\partial g_{11}} \geq 0$ . (At the end of the proof, we argue that the inequality is strict:  $\frac{\partial \Pi_1^s}{\partial g_{11}} > 0$ .) To analyze the term  $\frac{\partial p_1(\mathbf{v})}{\partial v_{11}}$ , we will use the fact that the minimum price  $p_1(\mathbf{v})$  satisfies the Vickrey property that a buyer's payoff is the buyer's marginal contribution to social welfare (see the proof of Proposition 6 for a definition of the Vickrey property.) From this, we will find a useful expression for  $p_1$ .

Fix a realization of buyers valuations  $\mathbf{v}$ , and consider an efficient allocation.  $A^*(\mathbf{v})$ . The

welfare of the allocation,  $w(A^*(\mathbf{v}))$  is a sum of buyers' valuations,  $\sum v_{ij}$ , where each buyer  $i$  obtains a good from seller  $j$ . If seller 1 sells an input, then some buyer  $i$  obtains seller 1's input and  $w(A^*(\mathbf{v})) = v_{i1} + \sum v_{hl}$ , where each buyer  $h$  obtains a good from a seller  $l \neq 1$ . To compute buyer  $i$ 's marginal contribution to surplus, we next assume that buyer  $i$  does not purchase an input, and given this, consider an allocation  $A^*_{-i}(\mathbf{v})$  that maximizes welfare. If seller 1 sells an input to some buyer  $j$ , then  $w(A^*_{-i}(\mathbf{v})) = v_{j1} + \sum v_{mn}$  where each buyer  $m$  obtains a good from a seller  $n \neq 1$ . (If seller 1 does not sell an input in  $A^*_{-i}(\mathbf{v})$ , then  $p_1(\mathbf{v}) = 0$  because there is no competition for seller 1's input.) Using the Vickrey property of buyers' payoffs, we write the buyer  $i$ 's payoff as

$$v_{i1} - p_1 = w(A^*(\mathbf{v})) - w(A^*_{-i}(\mathbf{v})) = (v_{i1} + \sum v_{hl}) - (v_{j1} + \sum v_{mn}).$$

Solving for  $p_1$ , we have

$$p_1(\mathbf{v}) = v_{j1} + \sum v_{mn} - \sum v_{hl} \text{ where } n, l \neq 1$$

We want to find  $\frac{\partial p_1(\mathbf{v})}{\partial v_{11}}$ . If we differentiate the above expression, we obtain  $\frac{\partial p_1}{\partial v_{11}} = 0$  if  $j \neq 1$  and  $\frac{\partial p_1}{\partial v_{11}} = 1$  if  $j = 1$ . This suggests that  $\frac{\partial p_1}{\partial v_{11}}$  is weakly positive. There is a caveat, however. The allocations  $A^*(\mathbf{v})$  and  $A^*_{-i}(\mathbf{v})$  that underlie the expression for  $p_1(\mathbf{v})$  may change when  $v_{11}$  changes. However, the space of valuations  $\mathbf{v}$  is comprised of regions, such that on each region the allocations  $A^*(\mathbf{v})$  and  $A^*_{-i}(\mathbf{v})$  are constant. We have showed that  $\frac{\partial p_1(\mathbf{v})}{\partial v_{11}}$  is a weakly positive constant on each region where seller 1 sells an input. If there are any additional regions where seller 1 does not sell an input, then  $p_1(\mathbf{v}) = 0$  and  $\frac{\partial p_1(\mathbf{v})}{\partial v_{11}} = 0$  on those regions. We have shown that  $\frac{\partial p_1(\mathbf{v})}{\partial v_{11}} \geq 0$  for all  $\mathbf{v}$  such that  $p_1(\mathbf{v})$  is differentiable.

To finish the proof, we must show the strict inequality:  $\frac{\partial \Pi_1^s}{\partial g_{11}} > 0$ . We have

$$\frac{\partial \Pi_1^s}{\partial g_{11}} = \int \dots \int_{v_{ij} \neq v_{11}} \left[ - \int_0^\infty \frac{\partial p_1(\mathbf{v})}{\partial v_{11}} \frac{\partial F_{11}}{\partial g_{11}} dv_{11} \right] f_{12} \dots f_{BS} dv_{12} \dots dv_{BS}$$

where  $\frac{\partial p_1(\mathbf{v})}{\partial v_{11}} \in \{0, 1\}$ . We will be done if we can show that  $\frac{\partial p_1(\mathbf{v})}{\partial v_{11}} = 1$  and  $\frac{\partial F_{11}(v_{11}, g_{11})}{\partial g_{11}} > 0$  on a set of  $\mathbf{v}$  with positive measure. This happens, for example, when some buyer  $k$  obtains seller 1's

good in the efficient allocation, but if buyer  $k$  were not allowed to obtain a good, then it would be efficient for buyer 1 to obtain the good. It is straightforward to construct a set of such  $\mathbf{v}$  with positive measure. We assume values of  $v_{11}$  in the open interval where  $\frac{\partial F_{11}(v_{11}, g_{11})}{\partial g_{11}} > 0$  and then construct open intervals of the other values  $v_{ij}$  consistent with such allocations. Because  $f_{ij} > 0$  for all  $v_{ij} \in [0, \bar{v}]$ , the set will have positive measure.

**Proof of Corollary 2** The proof is the same as the proof of Corollary 1.

**Note on Differentiation and Integration by Parts** We can (1) take derivatives under integral signs and (2) apply the integration by parts formula  $\int xdy = xy | - \int ydx$  provided the following conditions are met. The conditions are defined for integrals over compact intervals. Our integrals are all of this form, because of our assumption that the support of the distributions of the buyers' valuations is bounded. For notational simplicity, however, we will usually write the upper bound of the integrals as  $\infty$ .

(1) Consider an integral  $I(g) = \int_a^b h(v, g)dv$  where  $h(v, g)$  is a measurable function on  $R = \{(v, g) | a \leq v \leq b, c \leq g \leq d\}$ . If the integral  $I(g)$  exists for all  $g \in [c, d]$ , and if the partial derivative  $\frac{\partial h(v, g)}{\partial g}$  is continuous on  $R$ , then by Theorem 9-37 of Apostol (p. 196, 1957), the derivative  $I'(g)$  exists for each  $g \in [c, d]$  and is given by  $I'(g) = \int_a^b \frac{\partial h(v, g)}{\partial g} dv$ .

(2) Consider measurable functions  $x(v)$  and  $y(v)$  defined on an interval  $a \leq v \leq b$ . If  $x$  is continuous on  $[a, b]$  and if  $y$  has bounded variation on  $[a, b]$ , then  $x$  is Riemann-integrable with respect to  $y$ , and  $y$  is Riemann-integrable with respect to  $x$ , and  $\int_a^b xdy + \int_a^b ydx = xy|_a^b$ . The result follows from Theorems 9-6 and 9-26 of Apostol (p. 194 and p. 211, 1957). A second sufficient condition for the integration by parts result is that  $y$  be continuous and  $x$  have bounded variation. Bounded variation is a mild condition. For example, piece-wise continuous functions have bounded variation.

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