Econ 342 May 1, 2007

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Final Exam

180 mins / 180 points.

Points for each question are in parentheses.

This exam is closed book, but you may refer to a sheet of notes.

1. (Total 30) Suppose that each member of a population travels to work by one of two modes, automobile or public transit. Let

$$y_i = I[\text{person } i \text{ travels to work by automobile}]$$

Assume that every member of the population has access to transit service, but some people do not have access to a car. For each person, let

$$d_i = I[\text{person } i \text{ has access to an automobile}]$$

characterize their options. It is known that

$$d_i = I[\gamma_0 + v_i > 0]$$

where γ_0 is an (unknown) constant and v_i varies over the population. For individuals with $d_i = 1$, it is assumed that he or she chooses to travel by automobile if $x_i'\beta_0 + \epsilon_i > 0$, and selects public transit otherwise. Here, x_i is a k-vector, $k > 1$, that varies over the population, ϵ_i is an unobserved scalar random variable, and β_0 is a k-dimensional parameter vector. Thus we have:

$$y_i = I[x_i'\beta_0 + \epsilon_i > 0, \text{ and } d_i = 1]$$

The first component of x_i is a constant, and the remaining components are jointly continuously distributed. A random sample of n individuals are drawn from the population. Assume that v_i, ϵ_i, x_i are mutually independent for each individual.

You are told that v_i and ϵ_i have standard logistic distributions. Write down the likelihood function of this model as a function of the parameters $\theta_0 \equiv (\gamma_0, \beta_0')'$, and derive the asymptotic variance of the MLE.

2. (Total 36) A random sample $X_1, X_2, ...X_n$ is taken from a Poisson distribution with probability mass function:

$$P(X_i = x|\lambda) = \frac{e^{-\lambda}\lambda^x}{x!} \quad x = 0, 1, ...; \quad \lambda > 0;$$

It can be shown that $E[X_i] = \lambda$ and $Var(X_i) = \lambda$. Our aim is to estimate λ from the random sample.

   (a) Write down the log-likelihood function, defined as the log of the **joint probability** function. Recall that since the random variables in the random sample are mutually independent, the joint probability function is simply the product of the marginal probability functions.

   (b) Note by the definition of the variance, we have

   $$\lambda = E[X_i^2] - E[X_i]^2$$

   This suggests the "analogy" estimator:

   $$\hat{\lambda} = \frac{1}{n}\sum_{i=1}^{n} X_i^2 - (\frac{1}{n}\sum_{i=1}^{n} X_i)^2$$

      i. Prove that $\hat{\lambda}$ is consistent.

      ii. Establish the limiting distribution of this estimator.

3. (30) Consider the following $AR(1)$ process

$$y_t = c_0 + \phi_0 y_{t-1} + \epsilon_t \tag{1}$$

where $|\phi_0| < 1$, ϵ_t is an i.i.d. sequence with variance $E[\epsilon_t^2] = \sigma^2$.

Propose a root-n consistent and asymptotically normal estimator of ϕ_0 and derive its asymptotic covariance matrix.

4. (30) Consider the following Probit model:

$$y_i = I[\alpha_0 + x_{1i}\beta_0 + x_{2i}\beta_1 + \epsilon_i > 0]$$

where $\epsilon_i \sim N(0, 1)$ (where we have imposed the scale normalization in the model.

Consider testing the following hypothesis:

$$H_0 : \beta_0 * \beta_1 = 1$$

State the Wald test statistic (based on MLE) for this hypothesis, and establish its limiting distribution under the null.

5. (30) Consider the AR(2) process:

$$y_t = -0.5y_{t-1} + 0.25y_{t-2} + \epsilon_t$$

where ϵ_t is i.i.d, mean 0, variance σ^2.

(a) Determine if the stability (stationarity) condition is satisfied.

(b) Assuming $E[\epsilon_t^2] = 1$, establish the form of γ_0.

6. (24) Consider the following first order autoregressive model:

$$y_t = \rho y_{t-1} + \epsilon_t \tag{2}$$

where $\{\epsilon_t\}$ is independent white noise. For the OLS estimator of ρ,

$$\hat{\rho} = \frac{\sum_{t=1}^{T} y_t y_{t-1}}{\sum_{t=1}^{T} y_{t-1}^2} \tag{3}$$

Under the assumption that the process of y_t is $I(1)$, what is the probability limit of $\sqrt{T}(\hat{\rho} - \rho)$? Justify your answer.