Econ 342 April 28, 2008
Shakeeb Khan Spring 2008

Final Exam

180 mins / 180 points.

Points for each question are in parentheses.

This exam is closed book, but you may refer to 2 sheets of notes.

1. (Total 36 points) Consider the following system of equations:

$$q_i = \alpha_0 + \alpha_1 p_i + \alpha_2 x_i + u_i \quad (1)$$
$$q_i = \beta_0 + \beta_1 p_i + \beta_2 z_i + v_i \quad (2)$$

   (a) (12) Determine if the rank and order conditions are satisfies for each equation, assuming $E[u_i] = E[v_i] = E[x_i u_i] = E[x_i v_i] = E[z_i u_i] = E[z_i v_i] = 0$

   (b) (12) If any of the equations is identified write down the form of the 2SLS estimator for that equation.

   (c) (12) Now assuming the following orthogonality conditions

   $$E[u_i] = E[v_i] = E[x_i u_i] = E[x_i v_i] = E[z_i u_i] = E[z_i v_i] = 0$$

   Write down the form of the multiple equation GMM estimator of the vector of parameters $(\alpha_0, \alpha_1, \alpha_2, \beta_0, \beta_1, \beta_2)$ assuming homoskedasticity of the error terms.

2. (Total 40 points) A random sample $X_1, X_2, ...X_n$ is taken from an exponential distribution with common probability density function:

$$f_X(x|\lambda) = \frac{1}{\lambda} e^{-x/\lambda} \quad x \geq 0 \quad \lambda > 0;$$

Our aim is to estimate λ from the random sample.

   (a) (15) Write down the log-likelihood function, defined as the log of the **joint probability** function. Recall that since the random variables in the random sample are mutually independent, the joint probability function is simply the product of the marginal probability functions.

(b) (25) Derive the form of the MLE for λ and establish its asymptotic distribution.

3. (Total 30 points) Consider the following bivariate I(1) system:

$$\Delta y_{1t} = \delta_1 + \epsilon_{1,t} - \epsilon_{1,t-1} \tag{3}$$

$$\Delta y_{2t} = \delta_2 + \epsilon_{2,t} \tag{4}$$

where δ_1, δ_2 are constants, and the 2×1 vector $\epsilon_t = (\epsilon_{1t}, \epsilon_{2t})$ is iid across t with mean 0 and variance matrix Ω which is assumed to be positive definite.

- (15) Express the above system in levels, instead of in differences, as it is written above
- (15) Determine if $y_{1t} + y_{2t}$ is $I(1)$ or $I(0)$. (Hint that may help, but not required to use: establish the *long run variance*, as defined in class, for $\{\Delta y_{1t} + \Delta y_{2t}\}$).

4. (Total 30 points) In this question we will explore a right censored regression model, with varying censoring points. Consider a random sample of size n from the following right censored regression model:

$$y_i = d_i(x_i'\beta_0 + \epsilon_i) + (1 - d_i)k_i \quad i = 1, 2, ...n$$

Where $d_i = I[x_i'\beta_0 + \epsilon_i \leq k_i]$ is an observed censoring indicator; the censoring threshold is k_i and varies across observations but is observed to the econometrician for all observations. Therefore, the observed random variables are (y_i, d_i, x_i, k_i). Assume that (k_i, x_i, ϵ_i) are jointly i.i.d. as well as mutually independent, and ϵ_i is normally distributed with mean 0 and variance σ^2.

(a) (30) Propose a Wald statistic for the null $H_0 : 1'\beta_0 = 0$ where 1 is a vector of ones the same dimension as x_i, so $1'x_i$ is a scalar. Establish the limiting distribution of your test statistic when the null is true.

5. (Total 24 points) Consider the AR(2) process:

$$y_t = -0.25y_{t-1} + 0.5y_{t-2} + \epsilon_t$$

where ϵ_t is i.i.d, mean 0, variance σ^2.

(a) (12) Determine if the stability (stationarity) condition is satisfied.

(b) (12) Assuming $E[\epsilon_t^2] = 1$, establish the form of γ_0.

6. (Total 20 points) Consider the following first order autoregressive model:

$$y_t = \rho y_{t-1} + \epsilon_t \tag{5}$$

where $\{\epsilon_t\}$ is independent white noise. For the OLS estimator of ρ,

$$\hat{\rho} = \frac{\sum_{t=1}^{T} y_t y_{t-1}}{\sum_{t=1}^{T} y_{t-1}^2} \tag{6}$$

Recall we define the t statistic as:

$$t = \frac{\hat{\rho} - 1}{s \div \sqrt{\sum_{t=1}^{T} y_{t-1}^2}}$$

where s is

$$s \equiv \sqrt{\frac{1}{T-1} \sum_{t=1}^{T} (y_t - \hat{\rho} y_{t-1})^2}$$

What is the rate of convergence of t to its limiting distribution in each of the following two cases:

(a) (10) $\rho \in (0, 1)$

(b) (10) $\rho = 1$