Econ 342 May 2, 2009

Shakeeb Khan Spring 2009

Final Exam

180 mins / 180 points.

Points for each question are in parentheses.

This exam is closed book, but you may refer to 2 sheets of notes.

1. (Total 34 points) Consider the following 2 equation model:

$$\begin{aligned} y_1 &= \delta_{10} + \gamma_{12} y_2 + \gamma_{13} y_2 z_1 + \delta_{11} z_1 + \delta_{12} z_2 + u_1 \\ y_2 &= \delta_{20} + \gamma_{21} y_1 + \delta_{21} z_1 + \delta_{23} z_3 + u_2 \end{aligned}$$

   (a) (17) Assume $\gamma_{13} = 0$ and determine whether or not we have identification for each equation. Justify your answers with rank and order conditions.

   (b) (17) For any value of γ_{13}, find the reduced form for y_1 in terms of z_1, z_2, z_3 and u_1, u_2.

2. (Total 40 points) A random sample $X_1, X_2, ...X_n$ is taken from a distribution with common distribution function:

$$\begin{aligned} F_X(x|\alpha, \beta) &= 0 \text{ if } x < 0 \\ &= (x/\beta)^\alpha \text{ if } 0 \leq x \leq \beta \\ &= 1 \text{ if } x > \beta \end{aligned}$$

Our aim is to estimate α, β from the random sample.

(a) (15) Write down the log-likelihood function, defined as the log of the **joint density** function. Recall that since the random variables in the random sample are mutually independent, the joint density function is simply the product of the marginal density functions.

(b) (25) Derive the form of the MLE for α, β and establish its asymptotic distribution.

3. (Total 20 points) Consider the following equation, expressed in first differences:

$$\Delta y_{1t} = \delta_1 + \epsilon_t - \epsilon_{t-1}$$

where δ_1 is a constant, and the scalar process ϵ_t is iid across t with mean 0 and variance σ^2 which is assumed to be positive.

(a) (5) Express the above system in levels, instead of in differences, as it is written above

(b) (15) Establish the long run variance of the above process and determine if y_{1t} is $I(1)$ or $I(0)$.

4. (Total 30 points) In this question we will explore a doubly censored regression model. Many data sets in both biostatistics and economics are subject to double (i.e. left and right) random censoring. Examples are when the dependent variable is the age of

the individual at which a particular event(e.g. cancerous tumor, change in employment status) occurs, and individuals are regularly and frequently surveyed or tested for an interval of time. If the occurrence of the event is detected on the first survey/test, the dependent variable (age) is left censored, as the recorded value is greater than the actual (latent) value. If no such events have occurred by the last survey/test, the dependent variable is right censored, as the recorded value is exceeded by the actual value.

So the econometrician does not always observe the dependent variable $y_i^* = x_i'\beta_0 + \epsilon_i$. Instead one observes the doubly censored sample, which we can express as the pair (v_i, d_i) where

$$d_i = I[c_{1i} < x_i'\beta_0 + \epsilon_i \leq c_{2i}] + 2 \cdot I[x_i'\beta_0 + \epsilon_i \leq c_{1i}] + 3 \cdot I[c_{2i} < x_i'$$

with $I[]$ denoting the usual indicator function, and

$$v_i = I[d_i = 1] \cdot (x_i'\beta_0 + \epsilon_i) + I[d_i = 2]c_{1i} + I[d_i = 3]c_{2i}$$

where c_{1i}, c_{2i} denote left and right censoring variables, whose distributions satisfy $P(c_{1i} < c_{2i}) = 1$. Assume ϵ_i is distributed normal, mean 0, variance σ^2, and is distributed independently of (x_i, c_{1i}, c_{2i}) and (c_{1i}, c_{2i}) is distributed independently of x_i. Consider a random sample of size n observations.

(a) (10) Write down the log-likelihood function for the model.

(b) (20) Propose a LM statistic for the null $H_0 : 1'\beta_0 = 0$ where 1 is a vector of ones the same dimension as x_i, so $1'x_i$ is a scalar. Establish the limiting distribution of your test statistic when the null is true.

5. (Total 36 points) Consider the AR(2) process:

$$y_t = \phi_{01}y_{t-1} + \phi_{02}y_{t-2} + \epsilon_t$$

where ϵ_t is i.i.d, mean 0, variance σ^2.

(a) (20) Assume the stability (stationarity) condition is satisfied. Establish the form of $\gamma_0, \gamma_1, \gamma_2$.

(b) (16) In the MA(∞) representation for the above process, establish ψ_2, i.e. the coefficient on ϵ_{t-2}.

6. (Total 20 points) Consider the following first order autoregressive model:

$$y_t = \rho y_{t-1} + \epsilon_t$$

where $\{\epsilon_t\}$ is independent white noise. For the OLS estimator of ρ,

$$\hat{\rho} = \frac{\Sigma_{t=1}^{T} y_t y_{t-1}}{\Sigma_{t=1}^{T} y_{t-1}^2}$$

As a function of T, at what rate does the bias,variance of $\hat{\rho}$ go to 0 in each of the cases:

(a) (10) $\rho \in (0, 1)$

(b) (10) $\rho = 1$