# Answer Key

1 a) let $v = 1, y_2, z_1, z_3$ $\quad x = 1, z_1, z_2, z_3$

$$E[xv'] = E\begin{bmatrix} 1 & y_2 & z_1 & z_3 \\ z_1 & z_1 y_2 & z_1^2 & z_1 z_3 \\ z_2 & z_2 y_2 & z_2 z_1 & z_2 z_3 \\ z_3 & z_3 y_2 & z_3 z_1 & z_3^2 \end{bmatrix}.$$

if full rank, rank condition satisfied

$K = 4 = L$, so order condition satisfied

second equation the same except now

$v = (1, y_1, z_1, z_3) \quad x = 1, z_1, z_2, z_3$

1b) reduced form

system of equations:

$$\begin{bmatrix} 0 & -\gamma_{12} - \gamma_{13} z_1 \\ -\gamma_{21} & 1 \end{bmatrix} \begin{matrix} y_1 \\ y_2 \end{matrix} + \begin{bmatrix} -\delta_{10} & -\delta_{11} & -\delta_{12} & 0 \\ -\delta_{20} & -\delta_{21} & 0 & -\delta_{23} \end{bmatrix} \begin{matrix} 1 \\ z_1 \\ z_2 \\ z_3 \end{matrix} = \begin{matrix} u_1 \\ u_2 \end{matrix}$$

call this $R$, check $R$ invertible

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + R^{-1} \begin{bmatrix} -\delta_{10} & -\delta_{11} & -\delta_{12} & 0 \\ -\delta_{20} & -\delta_{21} & 0 & -\delta_{23} \end{bmatrix} \begin{bmatrix} 1 \\ z_1 \\ z_2 \\ z_3 \end{bmatrix} = R^{-1} \begin{matrix} u_1 \\ u_2 \end{matrix}$$

solve for $y_1, y_2$,

2) a) $f(x|\alpha,B) = \frac{\alpha}{B}\left(\frac{x}{B}\right)^{\alpha-1} I[0 \le x \le B]$

$\therefore \ell_n(\alpha,B) = \log \prod_{i=1}^{n} \frac{\alpha}{B}\left(\frac{x_i}{B}\right)^{\alpha-1} I[0 \le x_i \le B]$

$= \left[ n \log\frac{\alpha}{B} - (\alpha-1)\log B + (\alpha-1)\sum \log x_i \right] I[x_{max} \le B]$

$\hat{B}_{MLE} = x_{max}$

$\hat{\alpha}_{MLE} = \frac{1}{\log x_{max} - \frac{\sum \log x_i}{n}}$

distribution, from 341, we know

$\hat{B} - B = O_p\left(\frac{1}{n}\right)$

for $\hat{\alpha}$, let $f(x) = \frac{1}{x}$; so lets look at distribution of $\frac{1}{\hat{\alpha}} = \log \hat{B}_n - \frac{\sum \log x_i}{n}$

$E[\log X_i] = \log B - \frac{1}{\alpha}$

$\therefore \frac{1}{\hat{\alpha}} - \frac{1}{\alpha} = \log \hat{B} - \frac{\sum \log x_i}{n} - \frac{1}{\alpha} = O_p\left(\frac{1}{\sqrt{n}}\right)$;

$\therefore \hat{\alpha} - \alpha = O_p\left(\frac{1}{\sqrt{n}}\right)$ by delta method;

3) a) $y_t = \delta_1 + y_{t-1} + \varepsilon_t - \varepsilon_{t-1}$

b) note the first difference

is $\delta_1 + \varepsilon_t - \varepsilon_{t-1}$,

r.h.s has mean 0, variance $2\sigma^2$ neither depend on $t$;

l.r.v of $\Delta y_t = \mathrm{var}(\sqrt{n}\,\Delta\bar{y}_t)$

$= n\,\mathrm{var}\left(\delta_1 + \frac{1}{n}\sum \varepsilon_t - \varepsilon_{t-1}\right]$

$\lim_{n\to\infty} \mathrm{var}(\sqrt{n}\,\Delta\bar{y}_t) = 0$, so

$y_t$ is <u>not</u> $I(1)$ since $\Delta y_t$ does <u>not</u>

have l.r.v. $= 0$;

not generally $I(0)$ either, since process were stationary

we get a contradiction when $\delta \neq 0$;

4)

4) so likelihood $= \prod_{i=1}^{n} \left( \phi\left(\frac{v_i - x_i'\beta}{\sigma}\right) \frac{1}{\sigma} \right)^{I[d_i=1]} \Phi\left(\frac{c_{1i} - x_i'\beta}{\sigma}\right)^{I[d_i=2]} S\left(\frac{c_{2i} - x_i'\beta}{\sigma}\right)^{I[d_i=3]}$

where $S(\,) = 1 - \Phi(\,)$

$\mathcal{L}$ = log of above function:

$$Q_n(B, \sigma) = \frac{1}{n} \sum I[d_i=1] \log\left(\phi\left(\frac{v_i - x_i'\beta}{\sigma}\right)\frac{1}{\sigma}\right) + I[d_i=0] \log\left(\Phi\left(\frac{c_{1i} - x_i'\beta}{\sigma}\right)\right) + I[d_i=3] \log S\left(\frac{c_{2i} - x_i'\beta}{\sigma}\right)$$

$$LM = n \frac{\partial Q_n(\tilde{\theta})}{\partial \theta} \tilde{\Sigma}^{-1} \frac{\partial Q_n(\tilde{\theta})}{\partial \theta}$$

where $\tilde{\theta}$ = constrained optimizer,

$$\tilde{\Sigma} = \frac{1}{n} \sum s(v_i, d_i, x_i, \tilde{\theta}) s(v_i, d_i, x_i, \tilde{\theta})'$$

$s(v_i, d_i, x_i, \tilde{\theta})$ : $\frac{\partial Q_n(\tilde{\theta})}{\partial \theta} = \frac{1}{n} \sum s(v_i, d_i, x_i, \tilde{\theta})$

$$CM \xrightarrow{d, H_0} \chi^2_1$$

5) a) subbing in restrictions to impose cov statements, we get:

$$\gamma_0 = \sigma^2 \frac{1-\phi_{02}}{(\phi_{02}+1)(\phi_{02}-1+\phi_{01})(\phi_{02}-1-\phi_{01})}$$

$$\gamma_1 = \sigma^2 \frac{\phi_{01}}{(\phi_{02}+1)(\phi_{02}-1+\phi_{01})(\phi_{02}-1-\phi_{01})}$$

$$\gamma_2 = \frac{\phi_{01}^2 + \phi_{02} - \phi_{02}^2}{(\phi_{02}+1)(\phi_{02}-1+\phi_{01})(\phi_{02}-1-\phi_{01})}$$

5) b) after lagging the given equation, and subbing we get $\psi_2 = \phi_{01}^2 + \phi_{02}$;

6) a) bias $= o\left(\frac{1}{\sqrt{T}}\right)$ VAR $= O\left(\frac{1}{T}\right)$

b) bias $= o\left(\frac{1}{T}\right)$ ~~VAR~~ VAR $= O\left(\frac{1}{T^2}\right)$