Econ 342 March 5, 2008

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**Midterm Exam**

75 mins / 75 points.

Points for each question are in parentheses.

This exam is closed book, but you may refer to one sheet of notes.

1. (Total 30 points) Consider the following binary choice panel data model:

$$y_{it}^* = \alpha_i + x_{it}'\beta_0 + \epsilon_{it} \quad (1)$$

$$y_{it} = I[y_{it}^* > 0] \quad (2)$$

for $t = 1, 2$, $i = 1, 2, ...N$. The econometrician observes $y_{it}, x_{it}$, but not the "individual specific effect" $\alpha_i$, nor the disturbance term $\epsilon_{it}$, which we will assume is independent and identically distributed across $i$ and $t$, is distributed independently of $x_{i1}, x_{i2}, \alpha_i$, and has a **Logistic Distribution**:

$$P(\epsilon_{it} \leq x) = \frac{1}{1+e^{-x}} \quad (3)$$

The parameter of interest is $\beta_0$, which we will try to estimate in this question.

(a) Evaluate

$$P(y_{i2} = 1 | y_{i1} = 0, x_{i1}, x_{i2}, \alpha_i)$$

How does it depend on $\alpha_i$?

(b) Evaluate

$$P(y_{i2} = 0 | y_{i1} = 1, x_{i1}, x_{i2}, \alpha_i)$$

How does it depend on $\alpha_i$?

(c) Propose a maximum likelihood estimator for $\beta_0$, only using individuals $i$ where $y_{i1} \neq y_{i2}$.

2. (Total 30 points) Consider the AR(2) process:

$$y_t = \rho_1 y_{t-1} + \rho_2 y_{t-2} + \epsilon_t$$

where $\epsilon_t$ is i.i.d, mean 0, variance $\sigma^2$.

(a) Determine if the stability (stationarity) condition is satisfied if

i. $\rho_1 = \rho_2 = 1/2$

ii. $\rho_1 = \rho_2 = -1/2$

(b) Assuming $\rho_1, \rho_2$ satisfy the stability condition. Derive $\gamma_0$ and $\gamma_1$.

3. (Total 15 points) In this question we will explore a right censored regression model, with varying censoring points. Consider a random sample of size $n$ from the following right censored regression model:

$$y_i = d_i(x_i'\beta_0 + \epsilon_i) + (1 - d_i)k_i \quad i = 1, 2, ...n$$

Where $d_i = I[x_i'\beta_0 + \epsilon_i \leq k_i]$ is an observed censoring indicator; the censoring threshold is $k_i$ and varies across observations but is observed to the econometrician for all observations. Therefore, the observed random variables are $(y_i, d_i, x_i, k_i)$. Assume that $(k_i, x_i, \epsilon_i)$ are jointly i.i.d. as well as mutually independent, and $\epsilon_i$ is normally distributed with mean 0 and variance $\sigma^2$.

(a) Write down the log-likelihood for this model.

(b) Now assume that $k_i$ is observed, but only for censored observations, and again assume that $(k_i, x_i, \epsilon_i)$ are mutually independent. Write down the likelihood function for this model; how does it differ from what you got in the first question?