Econ 342 March 4, 2009

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Midterm Exam

75 mins / 75 points.

Points for each question are in parentheses.

This exam is closed book, but you may refer to one sheet of notes.

1. (Total 21 points) Believe it or not, this question relates to panel data:

   Suppose (X_i, Y_i) is a serially independent sample from a sequence of jointly normal distributions with $E[X_i] = E[Y_i] = \mu_i$, $\text{Var}[X_i] = \text{Var}[Y_i] = \sigma^2$, and $\text{Cov}[X_i, Y_i] = 0$ (i.e. X_i and Y_i are independent with common but varying means and constant common variance).

   (a) Derive the MLE of σ^2.

   (b) Evaluate the plim of the above estimator.

   (c) Propose an estimator that is consistent.

   Recall: density function for a normal distribution with mean μ, variance σ^2 is

   $$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp(-\frac{(x-\mu)^2}{\sigma^2})$$

2. (Total 28 points) Consider the following Probit model:

   $$y_i = I[\alpha_0 + x_{1i}\beta_0 + \epsilon_i > 0]$$

   where $\epsilon_i \sim N(0,1)$ (where we have imposed the scale normalization in the model.)

   Consider testing the following hypothesis:

   $$H_0 : \beta_0 * \alpha_0 = 1$$

   State the Wald test statistic (based on MLE) for this hypothesis, and establish its limiting distribution under the null.

3. (Total 26) Consider the following system of equations:

$$q_i = \alpha_0 + \alpha_1 p_i + \alpha_2 x_i + u_i \tag{1}$$

$$q_i = \beta_0 + \beta_1 p_i + \beta_2 z_i + v_i \tag{2}$$

(a) Determine if the rank and order conditions are satisfies for each equation, assuming $E[u_i] = E[v_i] = E[x_i u_i] = E[x_i v_i] = E[z_i u_i] = E[z_i v_i] = 0$

(b) Still assuming the following orthogonality conditions

$$E[u_i] = E[v_i] = E[x_i u_i] = E[x_i v_i] = E[z_i u_i] = E[z_i v_i] = 0$$

Write down the form of the multiple equation GMM estimator of the vector of parameters $(\alpha_0, \alpha_1, \alpha_2, \beta_0, \beta_1, \beta_2)$ assuming homoskedasticity of the error terms.

(c) Following the above question, what is the form of the optimal weighting matrix?