

### Problem Set 1

1. Hayashi, page 274, #1; page 283 #2; page 285 #5, # 9; page 286 # 10;
2. Consider the multiple equation GMM model we went over in class where each of the  $m$  equations is linear:

$$y_{im} = z'_{im} \delta_m + \epsilon_{im} \quad (1)$$

$m = 1, 2, \dots, M$   $n = 1, 2, \dots, n$ . As in class, let  $x_{im}$  denote the vector of instruments for the  $m$ th equation.

Let  $\hat{\delta}(\hat{W})$  denote the GMM estimator of the "stacked" vector of coefficients- i.e. the coefficients in each equation stacked on top of each other.

Consider the weighting matrix  $\hat{W}_1 = I_{\sum_{m=1}^M K_m}$  where  $I_{\sum_{m=1}^M K_m}$  is the  $\sum_{m=1}^M K_m \times \sum_{m=1}^M K_m$  identity matrix.

- (a) What is the asymptotic distribution of the multiple equation GMM estimator using this weighting matrix?
- (b) Suppose we now use the weighting matrix  $\hat{S}_{xx}$ , which is the  $\sum_{m=1}^M K_m \times \sum_{m=1}^M K_m$  matrix whose  $j, k$  block is of the form

$$\frac{1}{n} \sum_{i=1}^n x_{ij} x'_{ik}$$

If the system of equations is exactly identified (i.e.  $L_m = K_m$ ) for all  $m$ , under what conditions is the GMM estimator using  $\hat{S}_{xx}$  as the weighting matrix asymptotically equivalent to the GMM estimator using the optimal weighting matrix?

- (c) Suppose now the system is overidentified, and we use the weighting matrix  $\hat{S}_{xx}^{-1}$ . Under what conditions is this GMM estimator asymptotically equivalent to the GMM estimator using the optimal weighting matrix?