

Problem Set 2

1. For $T = 2$, consider the standard panel data model:

$$y_{it} = x_{it}\beta + \alpha_i + \epsilon_{it} \quad t = 1, 2$$

- (a) Numerically compare the fixed effect and first difference estimates.
- (b) Compare the error variance estimates from the two methods.

2. Consider the following panel data model:

$$y_{it} = \alpha_i + x_{it}\beta + z_i\gamma + \epsilon_{it}$$

Let $x_i = x_{i1}, x_{i2}, \dots, x_{iT}$, and assume $E[\epsilon_{it}|x_i, z_i, \alpha_i] = 0$. Let $\sigma_\alpha^2 = \text{Var}(\alpha_i)$ and $\sigma_\epsilon^2 = \text{Var}(\epsilon_{it})$.

- (a) Let $c_i = \alpha_i + z_i\gamma$. Find $\text{Var}(c_i)$ and compare it to σ_α^2 .
- (b) Compare the estimated variance of the unobserved effect when estimating the model by fixed effects to the estimated variance of the unobserved effect to if we estimated the model by random effects.

3. Hayashi- Analytical exercises starting on page 349: 1,2, and 4.

1. (Matlab exercise) Here we will get to know some of the most basic commands from Matlab and examine asymptotics at the same time. We have learned in class that the sample mean can be a "good" estimator of the population mean in the sense that it is often root- n consistent (i.e. $O_p(n^{-1/2})$). We will see if this can be verified by generating random numbers in Matlab.

- (a) Generate 100 i.i.d. draws from a standard normal distribution, and evaluate the sample mean. If the sample mean is good estimator it should be close to the population mean, in this case 0. Of course, the sample mean is a random variable, depending on what the draws are. Therefore, repeat the exercise 401 times so

you now have 401 sample means. Evaluate the root-mean squared error (RMSE), defined here as

$$\sqrt{\frac{1}{401} \sum_{i=1}^{401} \bar{X}_i^2}$$

where \bar{X}_i denotes the sample means you got.

- (b) If the sample mean is root- n consistent, what would the ratio of RMSE's be for $n = 100$ and $n = 400$? Verify this by performing the above exercise with 400 i.i.d. draws (still repeating this 401 times). Does the RMSE you get here agree with your answer?
- (c) Repeat the above 2 exercises now drawing from a cauchy distribution instead of a standard normal. How do your answers change?