Questions

1. Consider the following LIML model:

$$y_{i1} = Y_{i1}'\gamma + x_{i1}'\beta + u_{i1} \tag{1}$$
$$Y_{i1} = x_i'\Pi_1 + V_{i1} \tag{2}$$

where y_{i1} is a scale, and Y_{i1} is a vector with second equation characterizing the reduced form of the model. u_{i1} is a scalar random variable and V_{i1} is a vector, where the vector u_{i1}, V_{i1} is mean 0 with variance matrix

$$\Sigma = \begin{matrix} \sigma_1^2 & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{matrix}$$

Show the LIML estimator of γ is obtained by minimizing

$$\delta' W_1 \delta / \delta' W \delta$$

where $\delta = (1, -\gamma)$ with respect to γ. Also show that the minimization of $\delta' W_1 \delta - \delta' W \delta$ yields the 2SLS estimator of γ, where

$$W = [\mathbf{y}, \mathbf{Y}]' M [\mathbf{y}, \mathbf{Y}]$$

where the bold letters indicate stacking the individual elements of the dependent variables, and M is annihilator matrix of the matrix obtained by stacking x_i. Similarly,

$$W_1 = [\mathbf{y}, \mathbf{Y}]' M_1 [\mathbf{y}, \mathbf{Y}]$$

where M_1 is the annihilator matrix of the matrix obtained by stacking x_{i1}.

2. Consider the model

$$y_i = Y_i\gamma + x_i'\beta + u_i \tag{3}$$
$$Y_i = X_i\Pi + V_i = \bar{Y}_i + V_i \tag{4}$$

where y_i and Y_i are a scalar and vector respectively, with variance of the mean 0 vector (u_i, V_i) is

$$\Sigma = \begin{matrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{matrix}$$

Let α denote the stazcked parameter vector (γ, β).

Derive the asymptotic variance of the (infeasible) estimator obtained by regressing y_i on $\bar{Y}_i$, and compare this to the 2SLS estimator.

3. Consider the following 2 equation model:

$$\begin{aligned} y_{i1} &= \gamma_1 y_{i2} + u_{i1} & (5) \\ y_{i2} &= \gamma_2 y_{i1} + \beta_1 x_{i1} + \beta_2 x_{i2} + u_{2i} & (6) \end{aligned}$$

Compute the LIML and 2SLS estimates of γ_1 given the following moment conditions: $E[x_i x_i'] = I_2$,

$$E[y_i y_i'] = \begin{matrix} 7/2 & 2 \\ 2 & 3/2 \end{matrix}$$

$$E[x_i y_i] = \begin{matrix} 1 & 1 \\ 3/2 & 1/2 \end{matrix}$$

where $x_i = x_{i1}, x_{i2}$, $y_i = y_{i1}, y_{i2}$.

4. Let the sequence X_i be i.i.d., with bernouilli distribution with probability $X_i = 1$ denoted by p. Find the MLE of p and prove consistency and asymptotic normality of this estimator.

5. Let X_i be i.i.d with a uniform distribution between 0 and θ. Find the MLE of θ and establish consistency, by showing both the bias and the variance of the MLE converge to 0.

6. Consider the Logit model with $P(y_i = 1|x_i) = \Lambda(\beta_0 + \beta_1 x_i)$ where the scalar random variable x_i is binary, taking values 0 and 1.

   (a) Show this model can be written as a linear probability model, $P(y_i = 1|x_i) = \gamma_0 + \gamma_1 x_i$ and derive γ_0 and γ_1 as functions of β_0 and β_1.

   (b) Sow the MLE of γ_0 and γ_1 is equal to the least squares estimate of y_i on x_i with intercept.

7. For the MA(1) process defined by

$$y_t = \epsilon_t - \rho\epsilon_{t-1}$$

where ϵ_t is i.i.d. with mean 0 and variance σ^2. Define

$$y_t^* = \epsilon_t^* - \rho^{-1}\epsilon_{t-1}^*$$

where ϵ_t^* are i.i.d. with mean 0 and variance $\rho\sigma^2$. Compare the autcovariances for y_t and y_t^*.

8. Consider the AR(2) process:

$$y_t = \rho_1 y_{t-1} + \rho_2 y_{t-2} + \epsilon_t$$

where ϵ_t is i.i.d, mean 0, variance σ^2.

   (a) Determine if the stability (stationarity) condition is satisfied if

      i. $\rho_1 = \rho_2 = 1/2$

      ii. $\rho_1 = \rho_2 = -1/2$

   (b) Assuming ρ_1, ρ_2 satisfy the stability condition. Derive γ_0 and γ_1.