

Question 2: Note the infeasible estimator is an OLS estimator of the model:

$$y_i = \bar{Y}_i\gamma + V_i\gamma + x_i'\beta + u_i$$

combine the two error terms and let $\epsilon_i = V_i\gamma + u_i$ and so the model is

$$y_i = Z_i'\alpha + \epsilon_i$$

which from what we have done on day 1 is root- n consistent and asymptotically normal with variance covariance matrix:

$$E[Z_i Z_i']^{-1} E[\epsilon_i^2 Z_i Z_i'] E[Z_i Z_i']^{-1}$$

The feasible has a different distribution due to the correction term that arises from replacing Π with its estimator. Plugging in $\hat{\Pi}$ instead of Π we can write

$$y_i = \hat{g}_i'\alpha + u_i$$

where $\hat{g}_i = X_i'\hat{\Pi}$ which serves to estimate $g_i = X_i'\Pi$.

so we can derive a linear representation for:

$$\hat{\alpha} - \alpha_0 = \left(\frac{1}{n} \sum_{i=1}^n \hat{g}_i \hat{g}_i'\right)^{-1} \hat{g}_i (y_i - \hat{g}_i'\alpha_0)$$

We can now use the fact that $\hat{\Pi} - \Pi$ is $O_p(n^{-1/2})$ with the linear representation:

$$\hat{\Pi} - \Pi = \frac{1}{n} \sum_{i=1}^n \psi_i + o_p(n^{-1/2})$$

Therefore, going back to the right hand side of the linear representation, the first piece converges in probability to $E[g_i g_i']^{-1}$ and the numerator term can be expressed as

$$\frac{1}{n} \sum_{i=1}^n g_i u_i + E[g_i X_i'] \frac{1}{n} \sum_{i=1}^n \psi_i$$

the second piece corresponding to the correction term. Thus asymptotic normality follows from applying the CLT. (Of course the formula is in Hayashi, but I thought going through some of the steps would be good practice.)

Question 3: (Apologies for how I worded the question- should've phrased in terms of sample, and not population moments. Therefore, to keep notation as consistent as possible

with the text, instead of $E[x_i x_i']$, I mean the $X'X$. Anyhow, 2SLS is straightforward: $P = XX'$ and $Z'PZ = y_2'XX'y_2 = [1 \ 1/2]'[1 \ 1/2] = 5/4$. $Z'Py_1 = [1 \ 1/2][1 \ 3/2]' = 7/4$, so answer is $4/5 * 7/4$;

For LIML, let me first say I WILL NOT put a LIML questions like this on the final- it takes too much time to compute. I asked this because I think everyone should crank this out once in their life, and see how much a pain it is vis a vis 2SLS (and don't forget its based on the stronger normality assumption).

Anyhow,

$$W = \begin{bmatrix} 7/2 & 2 \\ 2 & 3/2 \end{bmatrix} - \begin{bmatrix} 1 & 3/2 \\ 1 & 1/2 \end{bmatrix} * \begin{bmatrix} 1 & 1 \\ 3/2 & 1/2 \end{bmatrix}$$

$$\text{and } W_1 = Y'Y = \begin{bmatrix} 7/2 & 2 \\ 2 & 3/2 \end{bmatrix}$$

so invert W and determine the smallest eigenvalue of W_1W^{-1} . Note the annihilator of the regressor matrix in this case is $I_n - XX'$, so the FIML is

$$(y_2'(I - \lambda M)y_2)^{-1}y_2'(I - \lambda M)y_1$$

Question 6

Note $P(y_i = 1|x_i = 0) = \Lambda(\beta_0)$, $P(y_i = 1|x_i = 1) = \Lambda(\beta_0 + \beta_1)$ so write: $P(y_i = 1|x_i) = \Lambda(\beta_0) + (\Lambda(\beta_0 + \beta_1) - \Lambda(\beta_0))x_i = \gamma_0 + \gamma_1x_i$

To see the equivalence, let $n_{00}, n_{01}, n_{11}, n_{10}$ denote the number of observations where $y_i = x_i = 0$, $y_i = 0, x_i = 1$, $y_i = x_i = 1$, $y_i = 1, x_i = 0$ respectively. Note when we write the objective function (MLE) or Least squares, the parameters must be constrained to ensure the probabilities are in between 0 and 1 in the linear probability framework (note this is not required in the Logit.) The likelihood and OLS objective functions can be written as:

$$MLE = n_{11} \log(\gamma_0 + \gamma_1) + n_{10} \log(\gamma_0) + n_{00} \log(1 - \gamma_0) + n_{01} \log(1 - \gamma_1 - \gamma_0) \quad (1)$$

$$LS = n_{11}(1 - \gamma_1 - \gamma_0)^2 + n_{10}(1 - \gamma_0)^2 + n_{00}\gamma_0^2 + n_{00}(\gamma_1 + \gamma_0)^2 \quad (2)$$

Then it is easy to see (that they are the same) by solving and comparing first order conditions for each estimator.

Question 8.

For the first part, the roots are (remember the quadratic formula, or better yet, put it on your cheat sheet): $-1/4 + / - \sqrt{7}/4i$ each satisfying the stability condition as the absolute value is $1/2$. In the second case, the roots are 1 and $-1/2$ so the stability condition is not satisfied. The autocovariances are:

$$\gamma_0 = \frac{\sigma^2(\rho_2 - 1)}{(1 + \rho_2)(\rho_1^2 - (1 - \rho_2)^2)}$$

$$\gamma_0 = \frac{-\sigma^2(\rho_1)}{(1 + \rho_2)(\rho_1^2 - (1 - \rho_2)^2)}$$