Answer key to problem set # 1

ECON 342

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Problem (4.4 – 1)**.** The 2SLS estimator for equation m,

$$y_{im} = \mathbf{z'_{im}}\delta_{\mathbf{m}} + e_{im}$$

using as instruments $\mathbf{x}_{im}$ can be written as,

$$\hat{\delta}_m = [S'_{\mathbf{x}_m\mathbf{z}_m} S^{-1}_{\mathbf{x}_m\mathbf{x}_m} S_{\mathbf{x}_m\mathbf{z}_m}]^{-1} S'_{\mathbf{x}_m\mathbf{z}_m} S^{-1}_{\mathbf{x}_m\mathbf{x}_m} s_{\mathbf{x}_m\mathbf{y}_m}$$

where,

$$S_{\mathbf{x}_m\mathbf{z}_m} = \frac{1}{n}\sum_i \mathbf{x}_{im}\mathbf{z}'_{im}$$

$$S_{\mathbf{x}_m\mathbf{x}_m} = \frac{1}{n}\sum_i \mathbf{x}_{im}\mathbf{x}'_{im}$$

$$s_{\mathbf{x}_m\mathbf{y}_m} = \frac{1}{n}\sum_i \mathbf{x}_{im}y_{im}$$

Stacking the coefficients of each equation we obtain,

$$\hat{\delta} = \begin{bmatrix} \hat{\delta}_1 \\ \vdots \\ \hat{\delta}_M \end{bmatrix} = \begin{bmatrix} [S'_{\mathbf{x}_1\mathbf{z}_1} S^{-1}_{\mathbf{x}_1\mathbf{x}_1} S_{\mathbf{x}_1\mathbf{z}_1}]^{-1} S'_{\mathbf{x}_1\mathbf{z}_1} S^{-1}_{\mathbf{x}_1\mathbf{x}_1} s_{\mathbf{x}_1\mathbf{y}_1} \\ \vdots \\ [S'_{\mathbf{x}_M\mathbf{z}_M} S^{-1}_{\mathbf{x}_M\mathbf{x}_M} S_{\mathbf{x}_M\mathbf{z}_M}]^{-1} S'_{\mathbf{x}_M\mathbf{z}_M} S^{-1}_{\mathbf{x}_M\mathbf{x}_M} s_{\mathbf{x}_M\mathbf{y}_M} \end{bmatrix}$$

This can be rewritten as,

$$\hat{\delta} = \left[\begin{bmatrix} S_{\mathbf{x}_1\mathbf{z}_1} & 0 & 0 \\ \vdots & \ddots & \vdots \\ 0 & 0 & S_{\mathbf{x}_M\mathbf{z}_M} \end{bmatrix}' \times \begin{bmatrix} S^{-1}_{\mathbf{x}_1\mathbf{x}_1} & 0 & 0 \\ \vdots & \ddots & \vdots \\ 0 & 0 & S^{-1}_{\mathbf{x}_M\mathbf{x}_M} \end{bmatrix} \times \begin{bmatrix} S_{\mathbf{x}_1\mathbf{z}_1} & 0 & 0 \\ \vdots & \ddots & \vdots \\ 0 & 0 & S_{\mathbf{x}_M\mathbf{z}_M} \end{bmatrix}\right]^{-1}$$
$$\begin{bmatrix} S_{\mathbf{x}_1\mathbf{z}_1} & 0 & 0 \\ \vdots & \ddots & \vdots \\ 0 & 0 & S_{\mathbf{x}_M\mathbf{z}_M} \end{bmatrix}' \times \begin{bmatrix} S^{-1}_{\mathbf{x}_1\mathbf{x}_1} & 0 & 0 \\ \vdots & \ddots & \vdots \\ 0 & 0 & S^{-1}_{\mathbf{x}_M\mathbf{x}_M} \end{bmatrix} \times \begin{bmatrix} s_{\mathbf{x}_1\mathbf{y}_1} \\ \vdots \\ s_{\mathbf{x}_M\mathbf{y}_M} \end{bmatrix}$$

where the (m, m)-element of the weighting matrix W is given by

$$S^{-1}_{\mathbf{x}_m\mathbf{x}_m} = \left[\frac{1}{n}\sum_i \mathbf{x}_{im}\mathbf{x}'_{im}\right]^{-1}.$$

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Problem (4.5 – 2)**.** Even if conditional homoskedasticity does not hold, the FIVE estimator is still consistent. For FIVE estimator to be consistent, as it is a GMM estimator, we only need that $\hat{W} \xrightarrow{p} W$ which is true. However, the weighting matrix is not the 'optimal weighting matrix', therefore, the FIVE estimator will not be efficient. The full multiple-equation GMM estimator without assuming conditional homoskedasticity will be more efficient.

Problem (4.5 – 5)**.** This is so because $\mathbf{x}_i$ is the union of all the regressors.

Problem (4.5 – 9)**.** We derived the efficient multiple equation GMM under conditional homoskedasticity at the TA session #1.

Problem (4.5 – 10)**.** In the multiple equation model $\mathbf{z}_{im} = \mathbf{x}_i$ that is the union of all regressors. In this case, the number of parameters to estimate equals the number of instruments, therefore, the model is exactly identified. Under this assumption the moment conditions hold exactly, therefore, the Sargan test that has the form

$$n(s_{\mathbf{zy}} - S_{\mathbf{zz}}\hat{\delta})[\Sigma \otimes S_{\mathbf{zz}}]^{-1}(s_{\mathbf{zy}} - S_{\mathbf{zz}}\hat{\delta}) = 0$$

since $s_{\mathbf{zy}} - S_{\mathbf{zz}}\hat{\delta} = 0$.

Problem (2)**.** **(a)** Using (4.2.4) we have,

$$\hat{\delta} = \left(S'_{\mathbf{xz}}S_{\mathbf{xz}}\right)^{-1} S'_{\mathbf{xz}}s_{\mathbf{xy}}$$

and you can show that each coefficient equals,

$$\hat{\delta}_m = \left(\frac{1}{n}\sum_{i=1}^{n}\mathbf{z}_{im}\mathbf{x}'_{im} \times \frac{1}{n}\sum_{i=1}^{n}\mathbf{x}_{im}\mathbf{z}'_{im}\right)^{-1}\left(\frac{1}{n}\sum_{i=1}^{n}\mathbf{z}_{im}\mathbf{x}'_{im} \times \frac{1}{n}\sum_{i=1}^{n}\mathbf{x}_{im}y_{im}\right)$$

which has the following sampling error,

$$\hat{\delta}_m - \delta_m = \left(\frac{1}{n}\sum_{i=1}^{n}\mathbf{z}_{im}\mathbf{x}'_{im} \times \frac{1}{n}\sum_{i=1}^{n}\mathbf{x}_{im}\mathbf{z}'_{im}\right)^{-1}\left(\frac{1}{n}\sum_{i=1}^{n}\mathbf{z}_{im}\mathbf{x}'_{im} \times \frac{1}{n}\sum_{i=1}^{n}\mathbf{x}_{im}\varepsilon_{im}\right)$$

which can be used to derive the asymptotic distribution.

(b) Since each equation is exactly identified, for any choice of W the estimator becomes the multiple equation IV estimator.

(c) We need conditional homoskedasticity.