1 a) first equation:

regressors $1, \ p_i, \ x_i$

instruments 1 (since $E[1 \cdot u_i] = E[u_i] = 0$,

$x_i, \ z_i$

let $\vec{x}_i$ denote regressors

$\vec{z}_i$ do instruments,

rank condition the 3x3 matrix $E[\vec{z}_i \vec{x}_i']$

is invertible, which it should be

generally speaking

order condition indeed satisfied

# of predetermined $= 3 =$ number of regressors,

b) 1 regress p on x, z; let $\hat{p}_i$ denote predicted values

let fitted regressors be denoted by $\hat{x}_i = 1, \hat{p}_i \ x_i$

let $\hat{\alpha}$ denote $\hat{\alpha}_0, \hat{\alpha}_1, \hat{\alpha}_2$

then

$$\hat{\alpha} = \left(\frac{1}{n} \sum \hat{x}_i \hat{x}_i'\right)^{-1} \left(\frac{1}{n} \sum \hat{x}_i q_i\right)$$

other equation (2) handeled analogously;

1c) set $\underset{6\times 6}{\hat{W}} = I$,

now using notation on page 265

$x_{i1} = 1, z_i, x_i \qquad z_{i1} = 1, p_i, x_i$

$x_{i2} = 1, x_i, z_i \qquad z_{i2} = 1, p_i, z_i$

$\delta = \alpha_0, \alpha_1, \alpha_2, \quad \beta_0, \beta_1, \beta_2$

$$g_n(\delta) = \begin{bmatrix} \frac{1}{n} \sum x_{i1} (q_i - \alpha_0 - \alpha_1 p_i - \alpha_2 x_i) \\ \frac{1}{n} \sum x_{i2} (q_i - \beta_0 - \beta_1 p_i - \beta_2 z_i) \end{bmatrix}$$

$$\hat{\delta} = (S_{xz}' S_{xz})^{-1} S_{xz}' s_{xy}$$

$$s_{xy} = \begin{pmatrix} \frac{1}{n} \sum x_{i1} q_i \\ \frac{1}{n} \sum x_{i2} q_i \end{pmatrix} \qquad S_{xz} = \begin{bmatrix} \frac{1}{n} \sum x_{i1} z_{i1}' & \\ & \frac{1}{n} \sum x_{i2} z_{i2}' \end{bmatrix}$$

2) $\log f = -\log\lambda - \frac{x}{\lambda}$

$$\frac{1}{n}\sum \log f(x_i,\lambda) = -\log\lambda - \frac{1}{n}\sum\frac{x_i}{\lambda}$$

F.O.C $\quad -\frac{1}{\lambda} + \frac{1}{n}\sum x_i \frac{1}{\lambda^2} = 0$

$$-1 + \bar{x}\frac{1}{\lambda} = 0$$

$$\bar{x}\frac{1}{\lambda} = 1$$

$$\Rightarrow \quad \bar{x} = \hat{\lambda}$$

$$\hat{\lambda} = \bar{x}$$

$$\Rightarrow \quad \sqrt{n}(\hat{\lambda} - \lambda) \longrightarrow N(0, V)$$

where $V = \text{VAR}(x_i) = \lambda^2$

3) $y_{1t} = y_{10} + \delta_1 t + \varepsilon_{1t} - \varepsilon_{1,0}$

$y_{2t} = y_{2,0} + \delta_2 t + \varepsilon_{2,1} + \varepsilon_{2,2} + \dots \varepsilon_{2,t}$

b) $\Delta y_{1t} + \Delta y_{2t} = \delta_1 + \delta_2 + \varepsilon_{1t} - \varepsilon_{1,t-1} + \varepsilon_{2t}$

$$\text{L.R.V} = \lim_{n\to\infty} \text{VAR}(\sqrt{n}\,\overline{\Delta y}) = \lim_{n\to\infty} \text{VAR}(\sqrt{n}(\delta_1 + \delta_2 + e_{1n} - e_{10} + \bar{\varepsilon}_{2})) > 0$$

$\therefore \quad I(1)$

4) let

$$\underset{p\times 1}{\hat{\beta}} = \arg\max \sum d_i \phi\left(\frac{y_i - x_i'\beta}{\sigma}\right) + (1-d_i)\, \Phi\left(\frac{y_i - x_i'\beta}{\sigma}\right)$$

wald stat:

let $\underset{1\times p}{A(\theta)} = 1\ 1\ 1\ 1$

wald: $n(1'\hat{\beta}) \left[\underset{1\times 1}{A(\beta_0)} \underset{p\times p}{\Sigma^{-1}} \underset{p\times 1}{A(\beta)'} \right] (1'\hat{\beta})$

where Σ^{-1} is inverse of outer score in likelihood prob

WALD $\sim$ χ^2_1 if null is true.

5) Solve: $z^2 + 0.25z - 0.5 = 0$,

get $z = \dfrac{-.25 \pm \sqrt{0.25^2 + 2}}{2}$

$= -\dfrac{.25}{2} \pm \dfrac{\sqrt{2 + 0.25^2}}{2}$

$= \frac{1}{2}(-0.25 \pm 1.43)$

$= -0.843,\ 0.59$

both inside unit circle so stability satisfied

6) square both sides:

$y_t^2 = 0.25^2 y_{t-1}^2 + 0.25 y_{t-2}^2 + \varepsilon_t^2 + 2\times -0.25 \times 0.5\, y_{t-2} y_{t-1} + 2\times \varepsilon_t \times -0.25 y_{t-1} + 2\times 0.5 y_{t-2} \varepsilon_t$ take expectations: $\Rightarrow$

$\gamma_0 = 0.25^2 \gamma_0 + 0.25 \gamma_0 + 1 - 0.25\, \gamma_1$

now multiply both sides of original equation by y_{t-1}

$$y_t y_{t-1} = -0.25\, y_{t-1}^2 + 0.5\, y_{t-2} y_{t-1} + \varepsilon_t y_{t-1}$$

take expectations

$$\gamma_1 = -0.25\,\gamma_0 + 0.5\,\gamma_1$$

So have 2 equations, 2 unknowns, (γ_0, γ_1)

solve for γ_0, γ_1.

6) mistake in question, t should be

$$\frac{\hat{p} - p}{s / \sqrt{\sum y_{t-1}^2}}$$

any how, in both cases

$$t = O_p(1),$$

now in case a) if they used $p = 1$ for t stat

when $p < 1$ so $p - 1 = -\sigma$

$$\frac{\hat{p} - 1}{s / \sqrt{\sum y_{t-1}^2}} = \frac{\hat{p} - p}{s / \sqrt{\sum y_{t-1}^2}} + \frac{p - 1}{s / \sqrt{\sum y_{t-1}^2}}$$

$$= O_p(1) + \frac{\sqrt{\sum y_{t-1}^2}}{s} \times (-\sigma)$$

$$= O_p(1) + \frac{\sqrt{\sum y_{t-1}^2}}{O_p(1)}$$

so t stat diverges to infinity