Assignment 1
International Macroeconomics
Fall 2007
Professor M. Uribe
Duke University

Consider an economy populated by a continuum of households indexed by $j \in [0,1]$ with preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t e^{z_t} [\ln(x_t^j) - \phi h_t^j],$$

where E_t denotes the expectations operator conditional on information available in period t, $\beta \in (0,1)$ is a subjective discount factor, and z_t is an aggregate, exogenous, and stochastic preference shock with law of motion given by

$$z_t = \rho z_{t-1} + \epsilon_t,$$

where ρ lies in the interval $(0,1)$ and ϵ_t is an iid disturbance with mean zero and standard deviation σ. The variable x_t^j denotes a composite measure of habit-adjusted consumption given by

$$x_t^j = \left[\int_0^1 (c_{it}^j - \theta \tilde{c}_{it-1})^{1-1/\eta} \right]^{1/(1-1/\eta)},$$

where c_{it}^j denotes consumption of variety $i \in [0,1]$ by consumer j in period t. The variable $\tilde{c}_{it-1} \equiv \int_0^1 c_{it-1}^j dj$ denotes its cross sectional average of consumption of variety i in $t-1$. Thus, the above aggregator function introduces external deep habits. The variable h_t^j denotes labor effort supplied by household j in period t. The parameters, ϕ, θ, and η govern, respectively, the marginal disutility of effort, the intensity of deep habit formation, and the intratemporal elasticity of substitution across habit-adjusted consumption of differentiated goods.

Households have access to a complete set of financial markets. Their budget constraint is given by

$$\int_0^1 c_{it}^j p_{it} di + E_t r_{t,t+1} a_{t+1}^j = a_t^j + w_t h_t^j + \gamma_t^j,$$

where p_{it} denotes the price of good i in terms of some numeraire, $r_{t,t+1}$ is a pricing kernel such that the period-t price of a random payment a_{t+1}^j is given by $E_t r_{t,t+1} a_{t+1}^j$, w_t denotes the real wage rate, and γ_t^j denotes profits received by household j from his ownership of firms. Households own all firms in equal shares.

Assume that each good $i \in [0,1]$ is produced by a monopolist who maximizes profits using the stochastic factor $r_{0,t}$ to discount future payments. Good i is produced with the linear technology

$$y_{it} = h_{it},$$

where y_{it} and h_{it} denote, respectively, output of good i in period t and employment in industry i in period t.

1. Derive the set of equations that constitute a symmetric equilibrium for this economy.

2. Compute the steady-state markup of prices over marginal costs.

3. Derive a log-linear approximation to the equilibrium conditions. In performing this step, define $\hat{x}_t = \ln(x_t/x)$, where x denotes the deterministic steady-state value of x_t.

4. Write a reduced form of the linearized equilibrium conditions as

$$A\hat{c}_t + B\hat{c}_{t-1} + CE_t\hat{c}_{t+1} + Dz_t = 0$$

Give expressions for A, B, C, and D in terms of the structural parameters of the model.

5. Postulate a solution for the stochastic difference equation of the previous item of the form

$$\hat{c}_t = \alpha \hat{c}_{t-1} + \delta z_t.$$

provide all the solutions for α and δ in terms of A, B, C, D, and ρ. What condition should the stationary rational expectations solution satisfy?

6. Set $\beta = 0.99$, $\theta = 0.4$, $\eta = 5$, and $\rho = 0.9$. Compute the values of α and δ associated with a stationary rational expectations equilibrium.

7. Compute the correlations of $\hat{c}_t$ with z_t up to order 5.

8. Let μ_t denote the gross markup. Compute the correlations of $\hat{\mu}_t$ with z_t up to order 5.

9. Plot the impulse response functions of $\hat{c}_t$ and $\hat{\mu}_t$ to a unit increase in z_t.