

Assignment 6
International Macroeconomics
Fall 2007
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1. Consider a two-period economy. The representative household has preferences described by the utility function

$$\sum_{s=1}^3 \pi_i \ln(c_i),$$

where c_i denotes consumption in state i of period 2, and π_i denotes the probability of occurrence of state i in period 2, conditional on information available in period 1. The household derives no satisfaction from consuming in period 1. There are three possible states of nature in period 2. Each state occurs with probability $1/3$. In period 2, households receive a stochastic endowment y_i given by

$$y_i = 3 + \epsilon_i,$$

where ϵ_i denotes the endowment innovation in state i of period 2. The possible realizations of this innovation are $\epsilon_1 = -1$, $\epsilon_2 = 0$, and $\epsilon_3 = 1$. Endowment shocks represent the only source of uncertainty in this economy.

In period 1, households can buy insurance contracts from risk-neutral, perfectly competitive foreign insurance companies. These contracts take the form of mean-zero, state-contingent payments to the insurance company in period 2, denoted P_i . In case of default by the household, foreign insurance companies have the ability to cease a fraction $\eta > 0$ of the defaulter's endowment. Insurance companies do not default.

- (a) Derive the optimal insurance contract under the assumption of no default risk.
 - (b) Now assume that the household cannot commit to honoring its contractual obligations. Let η equal $1/5$. Derive the optimal, incentive-compatible contract.
 - (c) Answer the question in item 2 under the assumption that η equals $1/20$. (In this case, foreign insurance companies have a weaker punishment capacity).
 - (d) Finally, answer the question in item 2 under the assumption that η equals $1/2$. (In this case, foreign insurance companies have a strong punishment capacity).
2. Consider a small open, perfect-foresight, endowment economy with a representative agent described by the utility function

$$-\sum_{t=0}^{\infty} \frac{\beta^t}{c_t},$$

where c_t denotes consumption in period t and β denotes the subjective discount factor. The endowment sequence is given by

$$y_t = \begin{cases} 1 & t = 0, 2, 4, \dots \\ 1.2 & t = 1, 3, 5, \dots \end{cases}$$

Suppose that the gross world interest rate is constant and given by $R = 1.1$. The subjective discount factor is given by $\beta = 1/R$.

- (a) Consider first the case in which agents have the capacity to commit to honor their debt obligations. In this case, the budget constraint of the representative agent is given by

$$d_t = R d_{t-1} + c_t - y_t,$$

where d_t denotes the amount borrowed in period t . Assume that $d_{-1} = 0$. The representative agent is subject to the no-Ponzi-game constraint $\lim_{t \rightarrow \infty} d_t/R^t \leq 0$. Compute the equilibrium sequences of consumption and debt.

- (b) Now assume that agents cannot commit to honor their debts. Suppose that foreign lenders can ‘punish’ defaulters with perpetual exclusion from financial markets.
- i. Determine whether the equilibrium under commitment obtained in item 1 is sustainable.
 - ii. Consider a reputational equilibrium in which agents borrow x units in low-endowment periods and pay xR units in high-endowment periods. Calculate the optimal sustainable level of x .
- (c) Answer items 1 and 2 under the assumption that the endowment sequence is of the form

$$y_t = \begin{cases} 1.2 & t = 0, 2, 4, \dots \\ 1 & t = 1, 3, 5, \dots \end{cases}$$