Technical Handout On Rule of Thumb Consumers

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The material in this documents draws heavily from Galí et al. (2007).

Optimizing Households

Optimizing households maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln c_{ot} - \frac{h_{ot}^{1+\varphi}}{1+\varphi} \right]$$

subject to

$$c_{ot} + i_{ot} + \frac{E_t r_{t,t+1} A_{t+1}}{P_t} + \tau_{ot} = \frac{A_t}{P_t} + w_t h_{ot} + r^k k_{ot} + \Phi_t$$

$$k_{ot+1} = (1-\delta) k_{ot} + \phi\left(\frac{i_{ot}}{k_{ot}}\right) k_{ot}$$

and to a no-Ponzi-game constraint of the form $\lim_{t\to\infty} E_t r_{t,t+j} A_{t+j} \geq 0$, where $r_{t,t+j}$ is a stochastic pricing factor such tha tthe period-t value of a stochastic nominal payment x_{t+1} in $t+j$ is given by $E_t r_{t,t+j} x_{t+j}$. Thte adjustment cost function ϕ satisfies $\phi(\delta) = \delta$, $\phi'(\delta) = 1$, and $\phi''(\delta) < 1$. We will specialize the adjustment cost function to

$$\phi(x) = \delta + \ (x - \delta) - \frac{\phi}{2}(x-\delta)^2$$

On the right side of this equation, $\phi > 0$ is a parameter. The optimality conditions are the two constraints and

$$\frac{1}{c_{ot}} = \lambda_t$$

$$c_{ot} h_{ot}^{\varphi} = w_t$$

$$\frac{\lambda_t}{P_t} r_{t,t+1} = \beta \frac{\lambda_{t+1}}{P_{t+1}}$$

$$q_t = \frac{1}{\phi'\left(\frac{i_{ot}}{k_{ot}}\right)}$$

$$\lambda_t q_t = \beta E_t \left\{ \lambda_{t+1} \left[r^k_{ot+1} + q_{t+1} \left(1 - \delta + \phi_{t+1} - \frac{i_{ot+1}}{k_{ot+1}} \phi'_{t+1} \right) \right] \right\}$$

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Rule-of-Thumb Consumers

Rule-of-thumb consumers maximize

$$\ln c_{rt} - \frac{h_{rt}^{1+\varphi}}{1+\varphi}$$

subject to

$$c_{rt} = w_t h_{rt} - \tau_{rt}.$$

The optimality conditions associated with this problem are the constraint and

$$c_{rt} h_{rt}^{\varphi} = w_t$$

Firms Producing Final Goods

The final good, y_t, is produced with a continuum of intermediate goods, y_{it}, $i \in [0,1]$, with the technology

$$y_t = \left[\int_0^1 y_{it}^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}}$$

Firms in this market operate under perfectly compatitive conditions. Profits are given by

$$P_t y_t - \int_0^1 P_{it} y_{it} di$$

Firms maximize profits subject to the above production technology. The implied demand functions for intermediate goods are

$$y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\epsilon} y_t.$$

Perfect competition drives profits to zero. As a consequence, the price level is given by

$$P_t = \left[\int_0^1 P_{it}^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}.$$

Firms Producing Intermediate Goods

Intermediate good i is produced with capital and labor services with a Cobb-Douglas technology. Formally,

$$y_{it} = k_{it}^{\alpha} h_{it}^{1-\alpha}$$

Given the output level y_{it} chosen in period t, firm i hires capital and labor services to minimize total cost, given by

$$r_t^k k_{it} + w_t h_{it}$$

subject to the produciton technology. The optimality consitions of this problem are the technological costraint and

$$\frac{k_{it}}{h_{it}} = \frac{\alpha}{1-\alpha} \frac{w_t}{r_t^k}$$

The associated marginal cost is given by

$$mc_{it} = r_t^{k\,\alpha} w_t^{1-\alpha} \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)}$$

We assume that prices are sticky a la Calvo (1983) and Yun (1996). Each period, firm i has the opportunity to adjust prices with probability $1-\theta$. Suppose firm i has the chance to adjust the price in period t. Let P_{it}^* be the chosen price. Then, P_{it}^* is set so as to maximize

$$E_t \sum_{j=0}^{\infty} \theta^j r_{t,t+j} y_{it+j} [P_{it}^* - mc_{it+j} P_{t+j}]$$

subject to the demand function conditional on P_{it} being equal to P_{it}^*

$$y_{it+j} = \left(\frac{P_{it}^*}{P_{t+j}}\right)^{-\epsilon} y_{t+j}.$$

$$E_t \sum_{j=0}^{\infty} \theta^j r_{t,t+j} \frac{P_{t+j}}{P_t} y_{it+j} \left[\frac{P_{it}^*}{P_{t+j}} - \mu mc_{it+j}\right],$$

where $\mu \equiv \epsilon/(\epsilon-1)$. Note that $r_{t,t+1} P_{t+j}/P_t = \lambda_{t+1}/\lambda_t$. Let $p_{it}^* \equiv P_{it}^*/P_t$

$$x_t^1 \equiv \mu E_t \sum_{j=0}^{\infty} (\theta\beta)^j \frac{\lambda_{t+j}}{\lambda_t} y_{it+j} mc_{it+j},$$

and

$$x_t^2 \equiv E_t \sum_{j=0}^{\infty} (\theta\beta)^j \frac{\lambda_{t+j}}{\lambda_t} y_{it+j} p_{it}^* \frac{P_t}{P_{t+j}}$$

Then, we can write x_t^1 and x_t^2 recursively as

$$x_t^1 = \mu y_{it} mc_{it} + \theta\beta E_t \frac{\lambda_{t+1}}{\lambda_t} x_{t+1}^1$$

$$x_t^2 = y_{it} p_{it}^* + \theta\beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{p_{it}^*}{p_{it+1}^*} \frac{P_t}{P_{t+1}} x_{t+1}^2$$

$$x_t^1 = x_t^2$$

Symmetric Equilibrium and Aggregation

We assume that all firms adjusting prices in period t set the same price, or $P_{it}^* = P_{jt}^*$ for all i, j. We can then write the price level as

$$\begin{aligned} P_t^{1-\epsilon} &= \int_0^1 P_{it}^{1-\epsilon} di \\ &= (1-\theta) P_t^{*1-\epsilon} + \theta P_{t-1}^{1-\epsilon}. \end{aligned}$$

Thus, we can write

$$1 = (1-\theta) p_t^{*1-\epsilon} + \theta \pi_t^{\epsilon-1},$$

where $\pi_t \equiv P_t/P_{t-1}$ denotes the gross rate of inflation between $t-1$ and t and $p_t^* \equiv P_t^*/P_t$ denotes the relative price of the varieties whose price are adjusted in t relative to the final good. Let γ

denote the fraction of rule-of-thumb households. Then, letting c_t, i_t, k_t, and τ_t denote aggregate consumption, investment, capital, and taxes, respectively, we have

$$c_t = \gamma c_{rt} + (1-\gamma)c_{ot}$$

$$i_t = (1-\gamma)i_{ot}$$

$$k_t = (1-\gamma)k_{ot}$$

$$\tau_t = \gamma \tau_{rt} + (1-\gamma)\tau_{ot}$$

Letting g_t denote government spending, we have

$$y_t = c_t + i_t + g_t$$

We now need a relationship linking y_t to production. The usual relation $y_t = k_t^{\alpha} h_t^{1-\alpha}$ does not hold in general because of inefficiencies in the allocaiton of output across varieties due to sticky prices. To obtain the correct relationship, start with the constraint

$$k_{it}^{\alpha} h_{it}^{1-\alpha} = \left(\frac{P_{it}}{P_t}\right)^{-\epsilon} y_t$$

Noting that the capital labor ratio is comon across firms, and letting $k_t = \int_0^1 k_{it} di$ and $h_t = \int_0^1 h_{it} di$.

$$h_{it}\left(\frac{k_t}{h_t}\right)^{\alpha} = \left(\frac{P_{it}}{P_t}\right)^{-\epsilon} y_t$$

Integrating,

$$h_t\left(\frac{k_t}{h_t}\right)^{\alpha} = y_t s_t$$

where $s_t \equiv \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\epsilon} di$. We can write s_t recursively as

$$\begin{aligned} s_t &= \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\epsilon} di \\ &= (1-\theta)p_t^{*-\epsilon} + \theta \pi_t^{\epsilon} s_{t-1} \end{aligned}$$

Monetary and Fiscal Policy

Taking conditional expectations on the expression $\frac{\lambda_t}{P_t} r_{t,t+1} = \beta \frac{\lambda_{t+1}}{P_{t+1}}$, we obtain

$$\lambda_t = R_t \beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}},$$

where R_t denotes the gross nominal interest rate. Monetary policy takes the form of a simple Taylor rule

$$R_t - R = \phi_\pi(\pi_t - \pi),$$

where π is an inflation target pursued by the monetary authority and R is the associated steady-state value of the nominal interest rate.

The government budget constraint is given by

$$\frac{B_{t+1}}{R_t} - B_t + P_t\tau_t = P_t g_t,$$

where B_t denotes nominally risk-free bonds issue in period $t-1$. Let $b_t \equiv B_{t-1}/P_t$. Then,

$$\frac{b_{t+1}}{R_t} - \frac{b_t}{\pi_t} + \tau_t = g_t,$$

Fiscal policy is given by

$$\frac{\tau_t - \tau}{y} = \phi_b\left(\frac{b_t - b}{y}\right) + \phi_g\left(\frac{g_t - g}{y}\right),$$

where g, y, and b denote the steady-state values of g_t, y_t, and b_t, respectively. Finally, we impose:[1]

$$\frac{\tau_{ot} - \tau_o}{y} = \frac{\tau_{rt} - \tau_r}{y}$$

Government spending is assumed to follow an exogenous AR(1) process of the form

$$\frac{g_t - g}{y} = \rho_g\left(\frac{g_{t-1} - g}{y}\right) + \epsilon_t^g,$$

where ϵ_t^g is an i.i.d. shock with mean zero and variance $\sigma^2_{\epsilon^g}$

Complete Set of Equilibrium Conditions

A rational expectations equilibrium is a set of processes k_{ot}, k_t, i_{ot}, i_t, c_{ot}, c_{rt}, c_t, h_{ot}, h_{rt}, h_t, τ_{ot}, τ_{rt}, τ_t, p_t^*, π_t, R_t, b_t, g_t, y_t, λ_t, x_t^1, x_t^2, mc_t, s_t, q_t, w_t, and r_t^k satisfying

$$k_{ot+1} = (1-\delta)k_{ot} + \phi\left(\frac{i_{ot}}{k_{ot}}\right)k_{ot}$$

$$\frac{1}{c_{ot}} = \lambda_t$$

$$c_{ot}h_{ot}^{\varphi} = w_t$$

$$q_t = \frac{1}{\phi'\left(\frac{i_{ot}}{k_{ot}}\right)}$$

$$\lambda_t q_t = \beta E_t\left\{\lambda_{t+1}\left[r^k_{ot+1} + q_{t+1}\left(1 - \delta + \phi_{t+1} - \frac{i_{ot+1}}{k_{ot+1}}\phi'_{t+1}\right)\right]\right\}$$

$$c_{rt} = w_t h_{rt} - \tau_{rt}.$$

$$c_{rt}h_{rt}^{\varphi} = w_t$$

$$\frac{k_t}{h_t} = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k}$$

$$mc_t = r_t^{k\,\alpha} w_t^{1-\alpha}\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}$$

[1] This expression is not stated explicitly in Galí et al. (2007), but appears to be implicit in the derivation of their equaiton (C.6). (Particularly in the last step of the derivaiton.)

$$x_t^1 = \mu p_t^{*-\epsilon} y_t mc_t + \theta\beta E_t \frac{\lambda_{t+1}}{\lambda_t} x_{t+1}^1$$

$$x_t^2 = p_t^{*1-\epsilon} y_t + \theta\beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{p_t^*}{p_{t+1}^*} \pi_{t+1}^{-1} x_{t+1}^2$$

$$x_t^1 = x_t^2$$

$$1 = (1-\theta) p_t^{*1-\epsilon} + \theta \pi_t^{\epsilon-1},$$

$$c_t = \gamma c_{rt} + (1-\gamma) c_{ot}$$

$$h_t = \gamma h_{rt} + (1-\gamma) h_{ot}$$

$$i_t = (1-\gamma) i_{ot}$$

$$k_t = (1-\gamma) k_{ot}$$

$$\tau_t = \gamma \tau_{rt} + (1-\gamma) \tau_{ot}$$

$$y_t = c_t + i_t + g_t$$

$$k_t^\alpha h_t^{1-\alpha} = s_t y_t$$

$$s_t = (1-\theta) p_t^{*-\epsilon} + \theta \pi_t^\epsilon s_{t-1}$$

$$\lambda_t = R_t \beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}},$$

$$R_t - R = \phi_\pi (\pi_t - \pi).$$

$$\frac{b_{t+1}}{R_t} - \frac{b_t}{\pi_t} + \tau_t = g_t,$$

$$\frac{\tau_t - \tau}{y} = \phi_b \left(\frac{b_t - b}{y} \right) + \phi_g \left(\frac{g_t - g}{y} \right),$$

$$\frac{\tau_{ot} - \tau_o}{y} = \frac{\tau_{rt} - \tau_r}{y}$$

$$\frac{g_t - g}{y} = \rho_g \left(\frac{g_{t-1} - g}{y} \right) + \epsilon_t^g$$

References

Galí, Jordi, David López-Salido, and Javier Vallés, "Understanding the Effects of Government Spending on Consumption," *Journal of the European Economic Association 5,* March 2007,