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## Why charities announce donations: a positive perspective

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### Abstract

Charities frequently announce contributions of donors as they accrue. Doing so induces donors to play a sequential-move rather than simultaneous-move game. We examine the conditions under which a charity prefers such sequential play. It is known that if donors only value contributions through their effect on the total provision of a public good, then the charity will not announce contributions sequentially. However, with more general utility functions that include additional effects such as warm-glow or snob appeal, the charity may benefit from announcing contributions. © 2001 Elsevier Science B.V. All rights reserved.

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### 1. Introduction

Fundraising by charities and other nonprofit institutions is commonly characterized by announcement of receipts as they accrue. Telethons continuously update their receipts. United Way campaigns post signs telling what fraction of their total goal has been reached. Universities frequently announce large contributions at the official commencement of their capital campaigns, and regularly provide updates. What is the role of announcement in the fund-raising process?

Most of the literature on voluntary provision of public goods presumes simultaneous moves in the contribution game. With announcements, however,

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donors would play sequentially. This suggests a potential explanation for announcement in fundraising: sequential equilibria might produce a higher total than results in the simultaneous-move alternative. In fact, Varian (1994) shows that the sequential-contribution game provides less of the public good in the ‘standard’ model in which agents care only about the total supply of the public good.<sup>1</sup>

Why then, do fund raisers commonly facilitate sequential play in the contribution game? One possibility is that the standard utility specification is wrong and that donors have additional motives such as warm-glow effects. The warm-glow utility specification introduces an agent’s own contribution into the utility function, so that an agent gets utility not only from the total provision but also from his own contribution. The warm-glow specification has reconciled two inconsistencies between the theoretical predictions of the standard model of voluntary contributions and empirical evidence on contributions. The standard model predicts small (arbitrary) income redistributions among contributors will be neutral, i.e. not change utility levels or total contributions.<sup>2</sup> Secondly, in sufficiently large economies, only the rich contribute to the provision of public goods.<sup>3</sup> These striking predictions, however, conflict with the empirical evidence on private charities. White (1989), for example, indicates that nine out of 10 Americans donate to charities. Steinberg (1989) provides empirical evidence that neutrality fails. With the warm-glow utility specification, income redistributions are not neutral and even in large economies poorer people with such preferences will make contributions.<sup>4</sup> Other recent models departing from the standard one suggest that enhancement of prestige and reputation may also be relevant in making charitable contributions.<sup>5</sup>

To consider whether these types of effects can make sequential-contribution equilibria better for the charity, we consider a two-agent model with a general utility function that includes the standard and warm-glow effect models as special cases, as well as allowing other motives for contributing. For the general utility function, sequential equilibria can produce a higher supply of the public good than can the simultaneous-move alternative. This can occur, for example, under the warm-glow specification. If a charity has the power to choose the order of agents’ contributions or if an order arises exogenously, the charity will sometimes prefer to announce contributions as they arise. We also show how the result generalizes to the  $n$ -player case.

The environment with announcements may better be modeled as allowing the order of contributions to arise endogenously. Here agents decide when they will

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<sup>1</sup>While not the focus of this paper, we provide here generalizations of Varian’s analysis to the  $n$ -agent case and to the case of endogenous timing of contributions.

<sup>2</sup>See Warr (1983), Bergstrom et al. (1986) and Bernheim (1986).

<sup>3</sup>See Andreoni (1988) and Fries et al. (1991).

<sup>4</sup>See Andreoni (1989, 1990), Cornes and Sandler (1984, 1994), Kingma (1989), McClelland (1989), Roberts (1987), Sandler and Posnett (1991) and Steinberg (1986, 1987).

<sup>5</sup>See Glazer and Konrad (1996) and Harbaugh (1998a,b). These papers are discussed further in the next footnote.

contribute in addition to the amount that they will contribute. We also show that making a commitment to announce contributions may also be preferred in several versions of the endogenous-timing game.

Our findings provide additional support for utility specifications that admit effects like warm glow. They also point toward the organizational role that a charity can play in the provision of public goods.<sup>6</sup>

Andreoni (1998) and Vesterlund (1998) also offer explanations for announcements of initial donations in voluntary contribution equilibria. Andreoni presumes expenditure must exceed a threshold for the public good to be of value, implying no contributions will frequently be a simultaneous-move equilibrium even when an equilibrium with positive provision also exists. By enticing one or more donors to contribute first and then announcing that total, the zero-contribution equilibrium can be eliminated.<sup>7</sup> Vesterlund presumes incomplete information among donors about the quality of the charity, which can be resolved by a donor's costly inspection. By committing to announce the first contribution, a high-quality charity can sometimes induce an inspection and subsequent signaling of quality via the initial contribution. Our analysis complements these alternative explanations. For example, after the threshold has been met in a model like Andreoni's, our analysis rationalizes continued announcement of contributions. While our analysis relies on motivations for giving like warm glow which these alternatives do not require, our results hold in the otherwise simplest setting with complete information and a concave provision technology.<sup>8</sup>

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<sup>6</sup>In addition to the papers discussed next in the text, Glazer and Konrad (1996), Harbaugh (1998a,b) and Bilodeau and Slivinski (1997) also analyze the behavior of charitable organizations. Konrad and Glazer characterize a charity as providing a means to signal wealth, which requires public disclosure of donations. Harbaugh demonstrates the value of categorical reporting of donations in raising funds, when a motivation for giving is prestige. Because donors' utility functions in Konrad and Glazer's and Harbaugh's models do not depend on other donors' contributions, there is no role for sequential announcements. Bilodeau and Slivinski show the value of specialization of charities when there are multiple public goods, as this allows donors better control of use of their contributions. They do not consider the possibility of sequential announcements. In a non-game theoretic model where a charity's administrator does not necessarily share the same ideology as donors, Rose-Ackerman (1981) demonstrates that government grants may increase private donations. This can occur, for example, if grants are given conditional on charity's adopting an ideology closer to donors', or grants give donors better information about the charity. In her 1987 paper, using a similar model, she also shows that a charity might change its ideology closer to donors' to rely more on private donations if government grants are cut back.

<sup>7</sup>A similar incentive for announcement arises if the public good is a discrete one, as in the models of Bagnoli and Lipman (1989, 1992) and Romano (1991). Here there is usually a continuum of equilibria with provision (as well as an equilibrium with no provision). Sequential play can resolve the coordination problem in finding an equilibrium.

<sup>8</sup>Glazer and Konrad (1999) examine contribution equilibria when there are multiple inputs to production of the public good, assuming donors have standard utility function. One of their results is that a charity can increase provision by committing before donors contribute to devote an inefficiently high level of the contributions to one input. The commitment has the effect of increasing the marginal productivity of dollar contributions.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 presents the main results. Section 4 considers extensions to the endogenous-timing game. Section 5 draws some conclusions. Appendix A contains most of the more technical analysis.

## 2. The model

Let there be two agents with utility functions:

$$U^i = U^i(x_i, Y, h^i(y_i, y_j)), \quad i, j = 1, 2, \quad i \neq j;$$

where  $y_i$  denotes agent  $i$ 's monetary contribution to the public good,  $Y = y_i + y_j$  denotes the total provision of the public good, and  $x_i$  is (monetary) private-good consumption. Assume that  $U^i$  is increasing strictly in  $x_i$  and  $Y$ , and weakly in  $h^i$ ; twice continuously differentiable; and strictly quasi-concave in  $(x_i, Y^i, h^i)$  on the interior of agent  $i$ 's constraint set (presented below). To insure interior solutions, we adopt the Inada conditions:  $\lim_{x_i \rightarrow 0} U_1^i(\cdot) = \infty$  and  $\lim_{x_i \rightarrow I_i} U_1^i(\cdot) = 0$ , where  $I_i$  is endowed income and subscripts stand for partial derivatives.<sup>9</sup> We also assume  $h^i$  is continuous, twice differentiable, and weakly monotonic in both arguments. At times, it will be useful to assume further that  $h^i$  is concave.

This form for the utility function is more general than the common specifications in the literature and incorporates them as special cases. If  $h^i(\cdot)$  is a constant, then the present model reduces to the standard model. If  $h^i(\cdot) \equiv y_i$ , then it reduces to the warm-glow model. The general form assumed here admits other possibilities in which agent  $i$  may be affected by the other agent's contribution not only through  $Y$ , but also through some other private interests captured in  $h^i(\cdot)$ . For instance, each agent may be concerned about his contribution relative to the other agent in as much as this affects social prestige and reputation. Suppose, for example, that two rival businessmen are invited to donate to support a public school. They may compete for potential customers' goodwill by means of their relative contributions. In this case,  $h_2^i < 0$  due to a snob effect. In other cases,  $h_2^i > 0$  is plausible due to a *bandwagon effect*. For football fans, their team's victory is a public good. One fan's cheering at the stadium may increase others' utilities both by increasing total support, which may help the team win, and also by directly making it more fun to attend and cheer.

Agent  $i$  allocates his endowment,  $I_i$ , between  $x_i$  and  $y_i$ . As is common in the literature, suppose that both private good and individual contributions are normal

<sup>9</sup>Bergstrom et al. (1986) extensively analyze the impacts of income redistributions on the private provision of a public good without assuming interior solutions in a standard model. Our focus is different from theirs, thus analysis around boundaries is less interesting and would unduly complicate our paper.

in income (but not luxuries), i.e.,  $0 < \partial x_i^d / \partial I_i < 1$  and  $0 < \partial y_i^d / \partial I_i < 1$ , where  $x_i^d(I_i, y_j)$  and  $y_i^d(I_i, y_j)$  are the ordinary demand functions.

Throughout the analysis, we assume preferences and incomes are common knowledge among the agents. Assume, for now, that the charity shares the same information. Moreover, for now, also let the charity have the power to ask for contributions in a particular order and assume that each agent can contribute only once. We examine more limited control settings below, including with multiple contributions. In a fund-raiser, a charity may ask for contributions in a particular order and reveal the contributed amounts as they arise, thereby inducing agents to play a sequential game. Alternatively, the charity may induce simultaneous play by not revealing contributions until all have been made regardless of whether they arrive sequentially. The timing of the contribution game is exogenous from the agents' perspective. Our general results also can be applied to compare cases in which the 'natural play' of the contribution game is sequential or simultaneous-move. Varian (1994) provides interesting examples of such games.<sup>10</sup>

To keep the analysis simple, we assume that both the sequential- and simultaneous-move games have a unique and interior (subgame-perfect) pure-strategy Nash equilibrium.<sup>11</sup> Let  $G_0$  and  $G_k$ ,  $k = 1, 2$ , denote, respectively, the simultaneous- and sequential-move games where agent  $k$  moves first in the latter cases. Let  $Y(G_k)$  and  $y_i(G_k)$  denote, respectively, the equilibrium total provision of the public good and the equilibrium contribution of agent  $i$  under game  $G_k$ ,  $k = 0, 1, 2$ .

Solve (P1) below to find agent  $i$ 's best-reply function:  $y_i = f^i(y_j)$ .

$$\begin{aligned}
 & \text{Max}_{x_i, y_i} U^i(x_i, Y, h^i(y_i, y_j)) \\
 & \text{s.t. } x_i + y_i = I_i \\
 & \quad y_i + y_j = Y \\
 & \quad x_i, y_i \geq 0.
 \end{aligned} \tag{P1}$$

Substitute for  $x_i$  and  $Y$ , and rewrite (P1):

$$\text{Max}_{0 \leq y_i \leq I_i} U^i(I_i - y_i, y_i + y_j, h^i(y_i, y_j)).$$

Using that the solution is interior, the first-order condition is:

<sup>10</sup>For example, Varian considers a variation on the Samaritan's dilemma game played between the older and the younger generations. This game is obviously inherently sequential.

<sup>11</sup>Sufficient conditions for these results are most readily and meaningfully established in the particular applications. For the sequential model, concavity of  $h^i(y_i, f^j(y_j))$  in  $y_i$ , where  $f^j$  is agent  $j$ 's best-reply function, along with our other assumptions, constitutes one set of sufficient conditions. In the simultaneous case, uniqueness is the key requirement (given our other assumptions), and, of course, assumptions that insure a unique intersection of best-reply functions are required. The examples in Section 3 demonstrate the ease in applying our results.

$$-U_1^i + U_2^i + U_3^i h_1^i = 0. \tag{1}$$

Our assumptions above imply that second-order condition is satisfied,<sup>12</sup> i.e. that

$$K_i \equiv U_{11}^i + U_{22}^i - 2U_{12}^i + 2h_1^i(-U_{13}^i + U_{23}^i + U_{33}^i h_1^i/2) + U_{33}^i h_{11}^i \leq 0. \tag{2}$$

From (1), the slope of agent *i*'s best-reply function is:

$$\frac{\partial f^i}{\partial y_j} = \frac{U_{12}^i - U_{22}^i - U_{32}^i h_1^i + U_{13}^i h_2^i - U_{23}^i h_2^i - U_{33}^i h_1^i h_2^i - U_{33}^i h_{12}^i}{K_i}. \tag{3}$$

The ‘total effect’ of agent *j*'s contribution on *i*'s utility is:

$$\frac{dU^i}{dy_j} = U_2^i + U_3^i h_2^i. \tag{4}$$

Interpretations of (3) and (4) will be provided in the next section.

The simultaneous-move equilibrium of  $G_0$  occurs at the intersection of  $y_i = f^i(y_j)$  and  $y_j = f^j(y_i)$ . The subgame-perfect equilibrium of the sequential-move game  $G_i$  occurs at the value of  $y_i$  on  $y_j = f^j(y_i)$  that maximizes  $U^i$ . The latter is found by solving the program:

$$\text{Max}_{0 \leq y_i \leq I_i} U^i(I_i - y_i, y_i + f^j(y_i), h^i(y_i, f^j(y_i))). \tag{P2}$$

We assume that (P2) is a quasi-concave programming problem. Given our other assumptions, one set of sufficient conditions is that  $f^j(y_i)$  is concave (convex) in  $y_i$  for  $U^i(x_i, y_i + y_j, h^i(y_i, y_j))$  increasing (decreasing) in  $y_j$ .

### 3. Results

Proposition 1 contains the main results of this paper.

**Proposition 1.** *The signs of  $(dU^i/dy_j \partial f^j/\partial y_i)|_{G_0}$  and  $(y_i(G_i) - y_i(G_0))$  are the same. Furthermore,  $(dU^i/dy_j \partial f^j/\partial y_i)|_{G_0}$  and  $(Y(G_i) - Y(G_0))$  have the same (opposite) sign whenever  $(1 + \partial f^j/\partial y_i)$  is positive (negative) for  $y_j$  between and including  $y_j(G_0)$  and  $y_j(G_i)$ .<sup>13</sup>*

The proof is presented in Appendix A. Here we provide an intuitive discussion. Figs. 1–3 illustrate several possibilities regarding the slopes of best-reply functions

<sup>12</sup>Using concavity of  $h^i(y_i, y_j)$ , it is straightforward to write (P1) as a quasi-concave programming problem.

<sup>13</sup>If  $dU^i/dy_j = 0$ , then ‘whenever  $(1 + \partial f^j/\partial y_i) \dots$ ’ is not needed. If  $\partial f^j/\partial y_i = -1$  over the given range, then  $Y(G_i) = Y(G_0)$ .

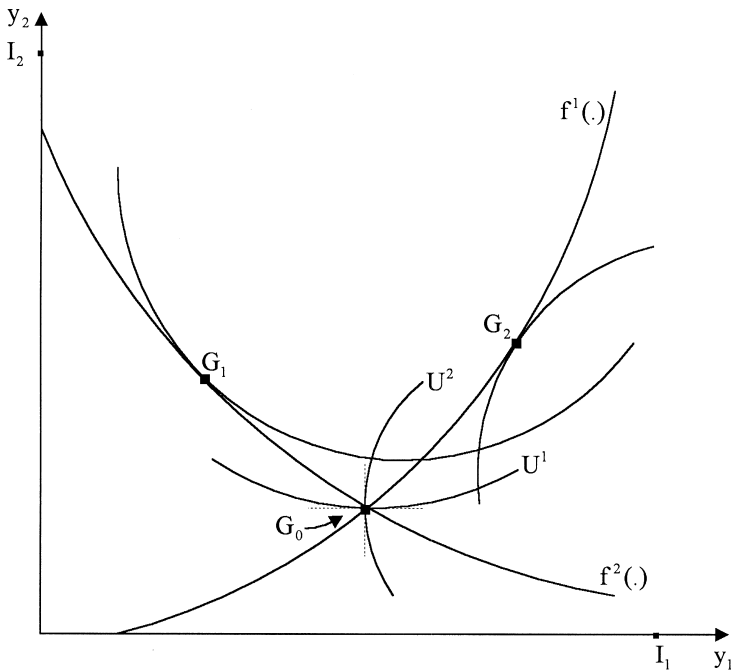


Fig. 1. Example One.

and the total effect of agent  $i$ 's contribution on agent  $j$ 's utility. In Fig. 1 for example, both agent 1's best-reply function and utility are increasing in agent 2's contribution. The latter and quasi-concavity of utility in  $(y_1, y_2)$  imply that agent 1's indifference curves open upward with utility increasing as  $y_2$  increases. Agent 2 obtains utility from agent 1's contribution, but his best-reply function decreases in  $y_1$ . The two Stackelberg equilibria and the simultaneous-move equilibrium are shown in each figure. We refer to these figures and rationalize the cases at various points below.

Whether sequential or simultaneous play yields a larger contribution by agent  $i$  depends on whether  $dU^i/dy_j$  and  $\partial f^j/\partial y_i$  have the same or opposite signs. By switching  $i$  and  $j$  in (3) and comparing (3) with (4), one can see that any pattern of signs of these expressions is possible since each expression has terms not in the other. See (4) and consider  $dU^1/dy_2$ . If agent 2 increases his contribution by one dollar, agent 1 gets extra utility  $U_2^1$  due to the one dollar increase in total supply of the public good and extra utility (or disutility)  $U_3^1 h_2^1$  from the other private interests discussed above. Although  $U_2^1$  and  $U_3^1$  are always non-negative, the sign of  $h_2^1$  can vary across plausible settings. When the effect of an increase in  $y_2$  on the private interests of agent 1 is negative,  $dU^1/dy_2$  indicates the degree of relative importance between this and the positive effect on the total supply of the public

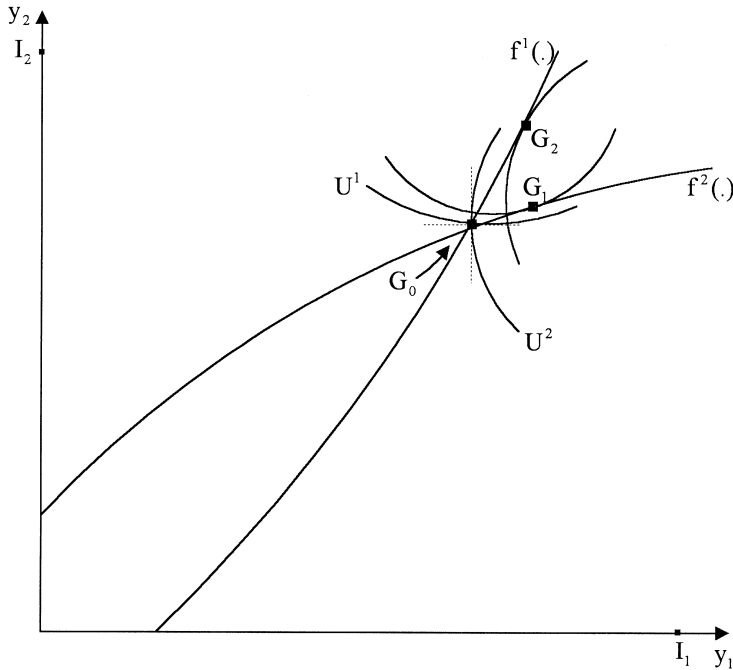


Fig. 2. Example Two.

good. One might say that a dominating public-good effect implies that the net effect is positive.

Now consider the example in Fig. 1 where  $dU^1/dy_2 > 0$  and  $\partial f^2/\partial y_1 < 0$ . If agent 1 moves first, he knows he can engender the simultaneous-move outcome by committing to  $y_1(G_0)$ . However, if agent 1 contributes less, agent 2 would respond to this by increasing her contribution, which, in turn, increases agent 1's utility. Agent 1's incentive at the margin is then to decrease his contribution. This incentive carries over globally under our concavity assumptions. Whether total provision of the public good increases relative to the simultaneous case depends on whether agent 2 raises her contribution more than agent 1 decreases his, which depends on the sign of  $(1 + \partial f^2/\partial y_1)$ . If the latter expression is negative between  $y_1(G_0)$  and  $y_1(G_1)$ , then  $Y(G_1) > Y(G_0)$ .

In general, normality assumptions about private good consumption and contributions have some relation to the slopes of the best-reply functions. To see this, use (1) to calculate:

$$\frac{\partial y_i^d}{\partial I_i} = \frac{U_{11}^i - U_{21}^i - U_{31}^i h_1^i}{K_i} \tag{6}$$

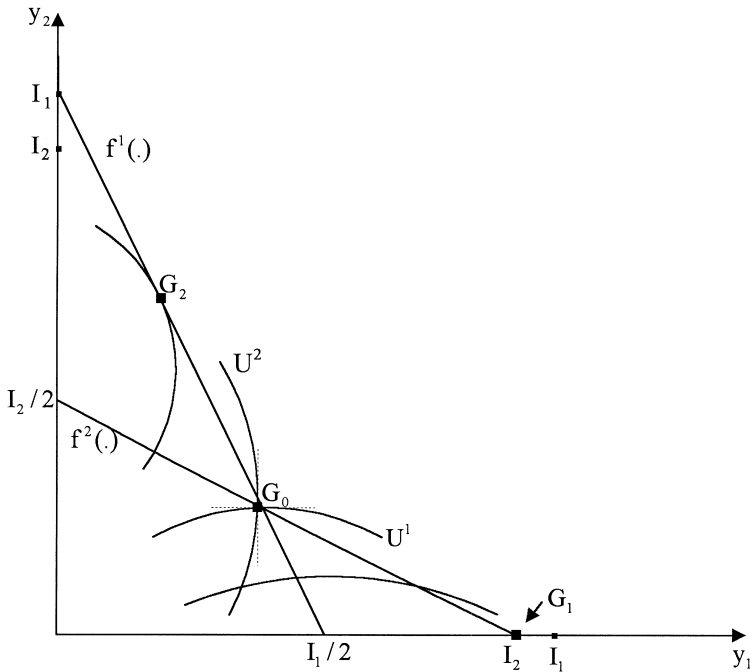


Fig. 3. Example Three.

and

$$\begin{aligned} \frac{\partial x_i^d}{\partial I_i} &= 1 - \frac{\partial y_i^d}{\partial I_i} \\ &= \frac{-U_{12}^i + U_{22}^i + U_{32}^i h_1^i + U_{13}^i h_1^i + U_{23}^i h_1^i + U_{33}^i (h_1^i)^2 + U_3^i h_{11}^i}{K_i}. \end{aligned} \quad (7)$$

Substituting (7) into (3) yields:

$$\frac{\partial f^i}{\partial y_j} = -\frac{\partial x_i^d}{\partial I_i} + \frac{(h_1^i - h_2^i)(-U_{13}^i + U_{23}^i + U_{33}^i h_1^i) + (h_{11}^i - h_{12}^i)U_3^i}{K_i}. \quad (8)$$

Eq. (8) shows how agent  $i$  would respond to a change in agent  $j$ 's contribution. For example, suppose agent  $j$  increases his contribution by one dollar. This affects  $i$ 's behavior through several channels. To distill these effects, it is useful to think of agent  $i$  as initially reducing his contribution by one dollar. Then agent  $j$ 's increased contribution corresponds to an increase in agent  $i$ 's income of one dollar. Due to the normality assumptions on demands, agent  $i$  will spend a fraction of the one dollar increase on the private good. This income effect implies a net decrease

in agent  $i$ 's contribution to the public good which is reflected in the first term of (8). If agents' preferences obey the standard model (i.e.  $h^i$  is constant), then the latter effect is the sole determinant of the slope of the best-reply function, and this is why the slope lies between  $-1$  and  $0$  in the standard model. As we will see, this is the main intuition behind the first part of Corollary 1 below.

With general utility functions, however, there are other effects reflected in the remaining terms in (8). To clarify these terms, it is useful to note that  $dU_3^i/dy_i = -U_{13}^i + U_{23}^i + U_{33}^i h_1^i$ , which is the total effect of agent  $i$ 's own contribution on his marginal utility from the additional motives. For example, in the warm-glow model which has  $h_1^i = 1$  and  $h_2^i = h_{11}^i = h_{12}^i = 0$ , the slope simplifies to  $\partial f^i/\partial y_j = -\partial x_i^d/\partial I_i + 1/K_i dU_3^i/dy_i$ . Here,  $dU_3^i/dy_i$  derives solely from the warm-glow effect. Agent  $i$ 's increased expenditure on the private good and associated reduction in own contribution affects his marginal value of making contributions, as captured by this term. If, for example,  $U_{33}^i$  is negative and dominates the other terms in  $dU_3^i/dy_i$ , then  $\partial f^i/\partial y_j > -\partial x_i^d/\partial I_i$  (using  $K_i \leq 0$ ). In any warm-glow specification having  $dU_3^i/dy_i < 0$ , agent  $i$  will either not decrease his contribution as much as dictated by the income effect or will increase his contribution. The warm-glow case is analyzed further in the next subsection. In the model with general  $h^i$ , the more direct effects of the change in  $y_j$  will come into play as well as effects operating through concavity properties of  $h^i$ .

Proposition 1 can be related to the familiar notions of strategic substitutes and complements (Bulow et al., 1985). To agent  $i$ ,  $y_i$  and  $y_j$  are strategic substitutes (complements) if the sign of  $dU^i/dy_j \cdot \partial f^j/\partial y_i > (<) 0$ . However, because Bulow et al. (1985) define strategic variables so that  $dU^i/dy_j < 0$  (with no loss of generality!), one needs to exercise care in using these constructs to restate our results. For example, one can say that agent  $i$ 's sequential leadership contribution exceeds his simultaneous choice if  $y_i$  and  $y_j$  are for him strategic substitutes (complements) and  $dU^i/dy_j < (>) 0$ . The point is that for our application the sign of  $dU^i/dy_j$  is directly relevant.

### 3.1. The warm-glow and standard models

To get further insight, it is useful to consider the implications of Proposition 1 in two important special cases, i.e. the standard and warm-glow models.

#### Corollary 1.

(A) (Varian) If donors care only about the total supply of the public good, then the simultaneous-move game yields the highest amount of the public good, and agent  $i$  contributes less under  $G_i$  than under  $G_0$ .

(B) In the warm-glow utility model in which  $h^i(y_i, y_j) \equiv y_i$ , the signs of  $\partial f^j/\partial y_i$  and  $(y_i(G_i) - y_i(G_0))$  are the same. Moreover,  $\partial f^j/\partial y_i$  and  $(Y(G_i) - Y(G_0))$  have the same (opposite) sign whenever  $(1 + \partial f^j/\partial y_i)$  is positive (negative).

Again, proof is in Appendix A. Part A of Corollary 1 coincides with Varian’s (1994) result. The important point is that in the standard model, the highest provision of a public good is secured under the simultaneous-move game. If a charity has the power to arrange the timing of agents’ contributions and the standard model holds, then the charity wants to induce donors to contribute simultaneously. The charity may ensure simultaneous contributions simply by not revealing any information about contributed amounts as they arise. However, this does not rationalize the common observation that some charities do reveal information about contributed amounts.

Part B of Corollary 1 indicates that when agents have warm-glow preferences, it may be possible to engender higher total contributions when donors move sequentially than when they move simultaneously. In addition, the ranking of total contributions is determined completely by the slope of the second mover’s best-reply function, since the first mover always prefers a higher contribution by the second mover. A sufficient condition here for a higher total under sequential play is that the second mover’s best-reply function is upward sloping. Equilibria in this case are illustrated in Fig. 2.<sup>14</sup> Vesterlund observes that a property of sequential equilibrium in the standard model of utility is that the first mover would contribute more if given the chance. Hence, contributing twice must not be feasible if we are to take the sequential equilibria seriously. Note that this caveat is unnecessary when warm-glow agents have best-reply functions that are upward sloping. In the sequential equilibrium here, the leader makes a choice above his best-reply function so would not want to contribute more if given the opportunity. In Section 4 we analyze such games.

Best-reply functions in the warm-glow model have some relatively simple interpretations. Again, substituting for  $h_1^i = 1$ ,  $h_2^i = h_{11}^i = h_{12}^i = 0$  and utilizing Eq. (3), one can see that  $\partial f^i / \partial y_j$  is positive if and only if  $dU_2^i / dy_i = -U_{12}^i + U_{22}^i + U_{23}^i$  is positive. Expression  $dU_2^i / dy_i$  is the total effect of agent  $i$ ’s own contribution on his marginal utility of public good. Hence, any utility function  $U^i(x_i, Y, y_i)$  that is additively separable in the numeraire ( $U_{12}^i = 0$ ) and for which  $U_{22}^i + U_{23}^i > 0$  implies a higher total with sequential contributions. In words, the result holds if utility from the warm-glow effect is sufficiently complementary with total provision. This is satisfied, for example, if the utility function is:

$$U^i = x_i^\alpha + y_i^\beta Y^{1-\beta}, \quad \alpha \text{ and } \beta \in (0, 1). \tag{9}$$

The case of (9) having  $I_1 = I_2$  is depicted in Fig. 2, where it is also assumed that  $I_i > \alpha^{1/1-\alpha}$ , implying interior equilibria.<sup>15</sup>

<sup>14</sup>The agents in Fig. 1 have asymmetric preferences but could both be warm-glow types. The sequential equilibrium where agent 2 moves first clearly has a higher total than in the simultaneous equilibrium. As discussed above, the total if agent 1 moves first may be higher.

<sup>15</sup>It is straightforward but tedious to show that this case is ‘well behaved’, with unique equilibria, etc. (details available on request). While Fig. 2 assumes  $I_1 = I_2$ , the results apply too if incomes differ.

While it is obviously unnecessary to have  $U_{12}^i = 0$  to obtain these results, note that utility functions that are ordinarily equivalent to being additively separable in  $x_i$ ,  $y_i$ , and  $Y$  do not yield upward sloping best replies. Most prominently, a CES utility function in  $(x_i, y_i, Y)$  does not admit the necessary complementarity.

Though empirical research on complementarity of private contributions is scant, it is quite supportive of the hypothesis that best-reply functions slope upward. Andreoni and Scholz (1998) estimate the elasticity of an individual's overall charitable contributions with respect to the average charitable contribution of the individual's reference group to be between 0.2 and 0.3.<sup>16</sup> In many instances of fund raising, e.g. for one's university, we think it more than plausible that the set of contributors will be members of the same reference group. Hence, we find their estimates to be quite supportive of the 'upward-sloping hypothesis'. Using individual contributions to public radio stations as the dependent variable, Manzoor et al. (1997) also find statistically significant evidence supporting this hypothesis. A much larger empirical literature on complementarity or substitutability of private contributions and government support of the public good has inconsistent findings (see Khanna et al. (1995) for references and their results). However, Manzoor et al. (1997) find that government expenditures have the opposite effect on an individual's contribution from that of other individuals' contributions, and it is self evident that the government might be viewed as a different type of player (e.g. in which of Andreoni and Scholz's reference groups does the government belong?). Our interpretation of the empirical evidence as supporting the warm-glow model with upward-sloping best replies is with the caveat that one must assume others' contributions are a good, a point we return to in the Conclusion.

With warm-glow utility functions and upward-sloping best replies, announcement by a charity leads to a Pareto improvement over the simultaneous-contribution equilibrium (see Fig. 2). Ideally, a charitable organization would dictate a particular order of moves. In the radio-station example, granting the station such knowledge and control may be unrealistic. However, our results imply gains from announcing donations as they arise in any exogenously determined order as long as both warm-glow players have upward sloping best-reply functions.

### 3.2. *Additional findings*

So far we have shown gains might result with announcement only for the case of two players. We show in Appendix A that, with  $n$  warm-glow players that have upward-sloping, best-reply functions, strict gains arise simply from announcing the first donation. We have also developed somewhat stronger conditions sufficient for

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<sup>16</sup>Andreoni and Scholz find that income level per se is not a good measure of an individual's reference group. This may explain why Feldstein and Clotfelter (1976) failed to find such complementarities in their earlier empirical study that identified reference groups based on income.

higher contributions as the frequency of announcement rises (Romano and Yildirim, 1999).<sup>17</sup>

We close this section with a two-player example that posits utility functions for the two agents of the form:

$$U^i(x_i, Y, h^i(y_i, y_j)) = x_i Y \frac{1}{y_j} = x_i \left( 1 + \frac{y_i}{y_j} \right). \tag{10}$$

This specification embodies a snob effect or reputation motive. Holding  $Y$  constant,  $i$ 's utility declines as  $j$ 's contribution rises, associated with the implied decline in  $i$ 's relative contribution. Such a case might arise where rival businessmen are competing for goodwill as discussed above. Best-reply functions are given by:

$$f^i = \text{MAX} \left[ \frac{I_i - y_j}{2}, 0 \right], \tag{11}$$

as depicted in Fig. 3. In Fig. 3, we assume  $I_1 > I_2$ , but that the incomes are close, which leads to somewhat different Stackelberg outcomes.

Without announcement, i.e. in the simultaneous-move equilibrium, it is interesting that the outcome is precisely as if the agents had standard Cobb–Douglas preferences:  $U^i = x_i Y$ . By failing to announce, the charity would not allow the snob effect to come into play.

The sequential equilibria are depicted in Fig. 3, and the total is higher in each case than in the simultaneous-move equilibrium. Relative to the simultaneous-move choices, the incentive of the leader is to increase his contribution to induce a reduction in the other agent's contribution. Because the follower is sluggish to reduce her contribution, the total rises. If the lower-income agent leads, then the Stackelberg equilibrium is interior ( $G_2$ ). If the higher-income agent leads, then the Stackelberg equilibrium is at the 'corner' where the follower just ceases to contribute.<sup>18</sup> In both cases, there is here a preference for leading over following, as leading permits one to win the prestige game. The increase in total contributions in the Stackelberg games is in stark contrast to the contribution game with the above

<sup>17</sup>More specifically, we show when total contributions rise in the subset of games where announcement begins after  $m < n$  contributions have been made and then continues after every subsequent contribution. That is, we show total contributions rise as  $m$  declines. This implies, for example, always announcing is better than never announcing when the conditions are met.

<sup>18</sup>This example does not conform perfectly to all the assumptions we made in the general analysis (e.g.  $U^i(I_i - y_i, y_i + y_j, h^i(y_i, y_j))$  is not everywhere quasi-concave). But it is well behaved enough that the results apply. If one is bothered by the infinite utility that agent 1 obtains in  $G_1$ , then modify utility to have  $y_j + k$  in the denominator of the middle expression of (10). For  $k$  small, the results are virtually the same. We have not done this in the text just to keep the analysis simpler.

standard utility function (depending only on the total contribution). In the standard case, the leader prefers higher contribution by the follower (the indifference curves open away from a player's contribution axis), and the leader reduces his own contribution to get the follower to contribute more.

#### 4. Endogenous timing

The ideal charity would have the power and information about agents that permit it to set the order of contributions that raises the highest total. Gathering this information and engendering such an outcome might be a key function of fund raisers. In this section, we relax the assumption that the charity has the power to choose a particular order of contributions while maintaining the assumption of complete information. It may be difficult or inefficient for even a fully informed charity to induce a particular order of moves. The charity may have difficulty contacting donors in the preferred order. With many small donors, it may be relatively efficient to solicit donations from everyone at once, as in the public radio and TV examples. Hence, we assume a period of time over which donations may be made and let the agents choose the timing and amounts of their contributions. Again, the question is whether the charity can make use of being able to announce the contributed amounts as they arise in this setting. Endogenous-timing games have been analyzed by Hamilton and Slutsky (1990, 1993) and Van Damme and Hurkens (1996), and the results can be applied to our problem.

Specifically, assume a charity collects contributions from two agents over two periods. We assume first that agents may contribute only once, but in the period they choose. It may be impractical, i.e. prohibitively costly, to contribute twice. Allowing contributions in both periods is also analyzed below, and our main points continue to hold. We assume the charity reports the magnitude of any donations that arise in period 1, before period 2 donations if relevant. We ask whether the charity gains from such reporting. For simplicity, we abstract from discounting as is realistic if the periods are short. Let  $y_i(t)$ ,  $t = 1, 2$ , denote agent  $i$ 's contribution in period  $t$ . The pure strategy set of agent  $i$  consists of: (a) all strategies  $y_i(1) \in (0, I_i]$  and  $y_i(2) = 0$ ; and (b) all strategies  $y_i(1) = 0$  and  $y_i(2) = y_i^2(y_j(1))$ , with  $y_i^2(\cdot)$  any function from  $[0, I_j] \rightarrow [0, I_i]$ . We require subgame perfection in equilibrium. This game corresponds to the action-commitment game of Hamilton and Slutsky (1990). Proposition 2 reports properties of the equilibrium set.

**Proposition 2.** (Hamilton and Slutsky) *In the endogenous-timing contribution game, exactly three subgame-perfect Nash equilibria exist: an equilibrium where both donors contribute the one-period, simultaneous-move equilibrium amounts in*

period 1; and both Stackelberg equilibria, with the leader contributing the one-period leader's amount in the first period and the follower contributing the one-period follower's amount in the second period.<sup>19</sup>

The arguments can be easily summarized.<sup>20</sup> If donor  $i$  expects  $j$  to make his Stackelberg-leadership contribution in the first period, then  $i$  can do no better than to wait and respond accordingly in the second period. In turn,  $j$ 's leadership strategy is optimal given  $i$  will wait to contribute. If both donors expect the other to make their one-period, simultaneous-move equilibrium contribution in the first period, then each can do no better than to contribute in the first period. That is, there is nothing to be gained by waiting, given the equilibrium expectation. Both donors waiting to contribute is not an equilibrium because either donor would be better off first playing his Stackelberg leadership contribution in the first period given the expectation that the other donor will wait.

The implications for the desirability of announcement are somewhat obscured by the presence of the multiplicity of equilibria.<sup>21</sup> Suppose that both Stackelberg equilibria yield greater total contributions than the simultaneous-move equilibrium as with warm-glow players and upward-sloping best-reply functions. If the agents can coordinate on an equilibrium under announcements, then the charity has nothing to lose from announcing and gains if the resulting equilibrium is one of the Stackelberg equilibria. Note, too, that a failure of the agents to coordinate on an equilibrium would never lead to a lower total in this case. That is, in any case where the agents make first-period choices consistent with alternative equilibria, the total will never be below the simultaneous-move total. If either plays their Stackelberg leadership amount in the first period, then the total will be higher. If they both wait attempting to be followers, then they will end up playing the simultaneous-move total in the second period.

A charity may also be able to facilitate sequential play in an endogenous-timing environment. If contributors attempt to give simultaneously, the charity acts to delay one contribution. If the phones in a telethon ring simultaneously, one contribution is taken first, while the phone operator stalls the contribution of the

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<sup>19</sup>Van Damme and Hurkens (1996) show that the 'action-commitment game' we are studying has no mixed-strategy equilibria unless no player prefers his Stackelberg leadership payoff to the payoff in a mixed-strategy equilibrium of the one-period game. Under our assumptions, no mixed-strategy equilibria exist in the one-period game, so no mixed-strategy equilibria exist in the endogenous-timing game.

<sup>20</sup>For proof, see Hamilton and Slutsky (1990, Theorem VII).

<sup>21</sup>Van Damme and Hurkens (1996) show that further refinements of equilibrium fail to eliminate any of the equilibria so long as either player strictly prefers leadership to simultaneous moves, which will be so except in contrived cases of our problem.

other, of course reporting the first contribution when completed. If hands of contributors are raised simultaneously at a live fund raiser, then the M.C., perhaps ceremoniously, flips a coin to see who gets to contribute first. In Romano and Yildirim (1999), we analyze this problem and show our results apply with unique equilibria.

Another variation of endogenous timing permits agents to contribute more than once. More specifically, suppose agent  $i$  selects a non-negative contribution  $y_i^t$ ,  $t = 1, 2$ , in each of two periods (again with no discounting). Agent  $i$ 's aggregate contribution continues to be denoted by  $y_i \equiv y_i^1 + y_i^2$ , and the agent totals  $(y_1, y_2)$  continue to enter the utility functions. With no announcements after first-period contributions, it is easy to see that equilibrium has  $y_i = y_i(G_0)$ .

If the charity announces first-period contributions, then agent  $i$ 's strategy set consists of:  $y_i^1 \in [0, I_i]$  and any function  $y_i^2(y_i^1, y_j^1)$  from  $[0, I_i] \times [0, I_j] \rightarrow [0, I_i - y_i^1]$ . The sets of subgame perfect equilibria for the three cases depicted in Figs. 1–3 are reported in Proposition 3. We say that equilibrium is equivalent to a Stackelberg equilibrium if both agents' aggregate contributions conform to the Stackelberg levels, and likewise about the simultaneous-move equilibrium.

### Proposition 3.

(A) *If both agents have upward sloping best-reply functions and utilities that increase in the other agent's contribution (e.g. see Fig. 2), then the equilibrium set consists of one or both of the two Stackelberg equivalents.*

(B) *If agent  $i$  has an upward sloping best-reply function, agent  $j$  has a downward sloping best reply function, and both agents' utilities increase in the other's contribution (e.g. see Fig. 1), then equilibrium must be equivalent to the Stackelberg equilibrium in which agent  $i$  leads (e.g.  $G_2$  in Fig. 1).*

(C) *(Saloner) If both agents have downward sloping best-reply functions with utilities that decrease in the other's contribution (e.g. see Fig. 3), then the set of equilibria consist of all contribution pairs on the outer envelope of the best-reply functions between and including the Stackelberg equilibria.*

The proof and more detail about the equilibrium strategies is provided in Appendix A. While the results are somewhat different than in the endogenous-timing game where agents can contribute only once, the main point that announcement leads to equilibria with a greater total holds in important cases. In case A of Proposition 3 which we have been emphasizing, for example, both equilibria have a greater total. In fact the only difference in this case from the first endogenous-timing game is that the set of equilibria now does not include the simultaneous-move equivalent.

Equilibrium has a unique total in case B. Case C is analytically equivalent to a

standard Nash–Cournot output game. Saloner (1987) has analyzed the two-period output game, and has shown the continuum of equilibria described in Proposition 3C. We have restricted attention here to these three cases to save space and because we feel they are adequate to convey the results.<sup>22</sup>

## 5. Conclusion

Charities frequently announce donations as they accrue. In this paper, we view these announcements as a means of inducing a sequential game among donors as an alternative to having them contribute simultaneously. Such a rationalization for announcement by charities fails if agents have the standard utility function since then contributions would decline. We have shown that with more general utility functions including the warm-glow one, however, it is possible that a charity increases total contributions by announcing the magnitudes of realized contributions. This provides another argument in favor of more general utility functions when analyzing voluntary contributions. Moreover, it supports the idea that adopting a simultaneous-move assumption in such analyses should not be taken for granted. This paper also points to the proactive role that a charity might play in collecting donations which is increasingly being recognized in the literature.

Perhaps the strongest assumption maintained in our analysis is complete information about preferences among donors. Investigating the incentives to announce contributions when all parties lack information about donor preferences is a topic for more study. This research also further motivates empirical efforts to understand what drives donors to contribute. We have shown that the linkage between donor preferences and the game form that maximizes contributions admits a variety of possibilities. We know, for example, that if agents have warm-glow utility functions and upward-sloping best-reply functions, then Stackelberg equilibria produce higher contributions than in the simultaneous-move equilibrium. But it is not enough to know *only* that best-reply functions are upward sloping. There may be cases where donors have upward-sloping best-reply functions due purely to competition for prestige, e.g. if  $U^i = x_i \cdot (k + y_i - y_j)$ . Then the Stackelberg equilibria are associated with a lower total. Hence, understanding well the form of the utility function can be important to making basic predictions about the nature of contribution equilibria.

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<sup>22</sup>Ten distinct cases can be identified as delineated by the slopes of best-reply functions and how utility depends on the other agent's contribution. The techniques employed in Appendix A can be used to analyze each case.

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## Appendix A

### A.1. Proof of Proposition 1

Assume  $(dU^i/dy_j \partial f^j/\partial y_i)|_{G_0} > 0$ . In the equilibrium for  $G_0$ , both agents are on their best-reply functions so that (1) holds for both of them. In the sequential game  $G_i$ ,  $j$  is on his best-reply function so that (1) still holds for  $j$ . Here, agent  $i$  satisfies the following first-order condition, derived from (P2):

$$[-U_1^i + U_2^i + U_3^i h_1^i] + \frac{dU^i}{dy_j} \frac{\partial f^j}{\partial y_i} = 0. \quad (\text{A.1})$$

If we evaluate (A.1) at  $G_0$ , then the bracketed term vanishes by (1). Hence, quasi-concavity of  $U_i(I_i - y_i, y_i + f^j(y_i), h^i(y_i, f^j(y_i)))$  in  $y_i$  and  $(dU^i/dy_j \partial f^j/\partial y_i)|_{G_0} > 0$  imply  $y_i(G_i) > y_i(G_0)$ .

Now consider total provision of the public good.  $Y = y_i + y_j = y_i + f^j(y_i)$  in both equilibria. Then,  $\partial Y/\partial y_i = 1 + \partial f^j/\partial y_i$ . Thus, if  $\partial f^j/\partial y_i > -1$ , then  $\partial Y/\partial y_i > 0$ . Since  $y_i(G_i) > y_i(G_0)$ ,  $Y(G_i) > Y(G_0)$ . Analogously, if  $\partial f^j/\partial y_i < -1$ , then  $\partial Y/\partial y_i < 0$ . In this case,  $y_i(G_i) > y_i(G_0)$  implies  $Y(G_i) < Y(G_0)$ . Of course, if  $\partial f^j/\partial y_i = -1$ , then  $Y(G_i) = Y(G_0)$  independent of the assumption made at the beginning of the proof.

By using a similar argument to that above, the results for  $(dU^i/dy_j \partial f^j/\partial y_i)|_{G_0} < 0$  easily follow. Finally, assuming  $(dU^i/dy_j \partial f^j/\partial y_i)|_{G_0} = 0$  and comparing (1) and (A.1) cover the final possibility, completing the proof.  $\square$

### A.2. Proof of Corollary 1

(A) Under the hypothesis of Corollary 1,  $h_1^i = h_2^i = h_{12}^i = h_{11}^i = 0$ . From (8),  $\partial f^j/\partial y_i = -\partial x_j^d/\partial I_j$ , which implies  $-1 < \partial f^j/\partial y_i < 0$  due to the normality assumptions on demands. From (4),  $dU^i/dy_j = U_2^i > 0$ . Thus,  $dU^i/dy_j \partial f^j/\partial y_i < 0$ .

$\partial y_i < 0$ . Utilizing Proposition 1,  $y_i(G_i) < y_i(G_0)$  and  $Y(G_i) < Y(G_0)$  easily follow.

(B) By assumption,  $dU^i/dy_j = U_2^i > 0$ . By applying Proposition 1, the results follow.  $\square$

*A.3. Generalization with n warm-glow players and upward sloping best-reply functions*

Assume there are  $n \geq 2$  warm-glow players. Let  $y_i^R(Y_{-i})$  denote agent  $i$ 's best-reply function,  $i = 1, 2, \dots, n$ , where  $Y_{-i} \equiv \sum_{j \neq i} y_j$ , denotes other agents' total contribution. Observe that, for warm-glow utility, the generalization of the total effect in (4) satisfies:

$$\frac{dU^i}{dY_{-i}} = \frac{\partial U^i}{\partial Y_{-i}} > 0.$$

Assume:

$$y_i^R(Y_{-i}) \text{ is increasing, continuous, and twice differentiable;} \tag{a-1}$$

and

simultaneous-move equilibrium among any subset of the  $n$  agents (with other agents'  $y_i$ 's exogenous) exists and is unique.  $\tag{a-2}$

Refer to the quasi-sequential game where agent  $i$  publicly selects  $y_i$  first, followed by simultaneous choices of the remaining agents as the ' $i$ -leader game'.

**Proposition A.1.** *Total contributions in the  $i$ -leader game exceed those in the  $n$ -agent, simultaneous-move game for any  $i$ .*

**Proof.** Let  $R_{-i}$  denote the collective best-reply function of the  $n - 1$  agents other than  $i$ . It satisfies by definition:

$$R_{-i}(y_i) \equiv \sum_{j \neq i} y_j^e(y_i); \tag{A.2}$$

where  $y_j^e(y_i)$  is agent  $j$ 's equilibrium choice of  $y_j$  in the simultaneous-move stage of the game. It must be that:

$$y_j^e(y_i) = y_j^R(R_{-i} - y_j^e + y_i), \tag{A.3}$$

as  $y_j^e$  is the best reply to  $R_{-i} - y_j^e + y_i$ . Differentiate (A.3) and rearrange:

$$y_j^{e'} = \frac{(R_{-i}' + 1)y_j^{R'}}{1 + y_j^{R'}}. \tag{A.4}$$

Now, (A.2), (A.4), and (a-1) imply  $R'_{-i} \in [-1, 0]$  is impossible. If  $R'_{-i} \in (-1, 0]$ , then (A.4) and (a-1) imply  $y_j^{e'} > 0$ , and (A.2) implies  $R'_{-i} > 0$ , a contradiction. If  $R'_{-i} = -1$ , then (A.4) implies  $y_j^{e'} = 0$ , again contradicting (A.2). Hence, the collective best-reply function has slope everywhere positive or everywhere less than minus one. (This holds everywhere or  $R'_{-i}$  would have to jump, violating (a-1).)

The result follows then by application of Proposition 1. This follows since the  $i$ -leader game and  $n$ -agent simultaneous-move game are equivalent to the two-agent games of Proposition 1 with ‘one agent’ described by the best-reply function  $R'_{-i}(y_i)$ .  $\square$

**Remark.** A very similar argument can be made that establishes, for the standard model of utility, that any  $i$ -leader Stackelberg equilibrium has lower total contributions than in the simultaneous-move equilibrium. To show this, show that  $R'_{-i}(y_i) \in (-1, 0)$  given  $y_i^{R'} \in (-1, 0)$ . Using (A.2) and (A.4) a contradiction to any  $R'_{-i}(y_i) \notin (-1, 0)$  is easily generated. Then apply Proposition 1. Hence, Varian’s result can be so generalized.

#### A.4. Proof of Proposition 3

We first provide three simple lemmas that severely restrict the candidate set of equilibrium points, in some cases reducing that set to a singleton. Throughout the presentation,  $i, j = 1, 2$  and  $i \neq j$ .

**Lemma B1.** In equilibrium,  $y_i^2 = \max\{0, f^i(y_j) - y_i^1\}$ .

**Proof.** If  $f^i(y_j) - y_i^1 \geq 0$ , then  $y_i = f^i(y_j)$  is feasible and obviously best for agent  $i$ . If, however,  $f^i(y_j) - y_i^1 < 0$ , then  $y_i^2 = 0$  is best for agent  $i$ , which follows from quasi-concavity of  $U^i(I_i - y_i, y_i + y_j, h^i(y_i, y_j))$  in  $(y_i, y_j)$ .  $\square$

**Corollary B1.** In equilibrium,  $y_i \geq f^i(y_j)$ .

**Proof.** This immediately follows from Lemma B1.  $\square$

**Lemma B2.** In equilibrium,  $y_i = f^i(y_j)$  for at least one agent.

**Proof.** Suppose not. Then  $y_i > f^i(y_j)$  for both agents and  $y_i^2 = y_j^2 = 0$ , which follow from Lemma B1 and Corollary B1. Thus  $y_i^1 > f^i(y_j^1)$  for each agent. If the continuation equilibrium leads to  $(y_i, y_j)$  on at least one best-reply function, then

the result follows. Suppose no such continuation equilibria exist. By Corollary B1, then the continuation equilibrium must lead to  $y_i > f^i(y_j)$  for both agents and thus  $y_i^2 = y_j^2 = 0$ . But then agent  $i$  would be better off reducing his first-period contribution marginally by quasi-concavity of  $U^i$  in  $(y_i, y_j)$  (and because agent  $j$  would continue to prefer  $y_j^2 = 0$  given a marginal reduction in  $y_i^1$ ). In this case we then have a contradiction.  $\square$

**Lemma B3.** *If, in equilibrium, agent  $i$  has  $y_i \neq f^i(y_j)$ , then  $y_i \leq y_i(G_i)$ . (Recall that  $y_i(G_i)$  is agent  $i$ 's Stackelberg leadership contribution.)*

**Proof.** Given,  $y_i \neq f^i(y_j)$ , it must be that  $y_i > f^i(y_j)$  from the above Corollary B1. Applying Lemma B1, it must also be that  $y_i^2 = 0$  and thus  $y_i^1 = y_i$ . Since  $y_i > f^i(y_j)$ , we have  $y_j = f^j(y_i)$  from Lemma B2, implying that  $y_j^1 \in [0, f^j(y_i^1)]$ . Now we argue that given  $y_i^1 > y_i(G_i)$ , agent  $i$  could increase his utility by marginally reducing  $y_i^1$ .

If  $y_j^1$  is such that following the marginal reduction in  $y_i^1$ , agent  $j$  can choose  $y_j^2$  such that  $y_j = f^j(y_i)$  (i.e. if the non-negativity constraint on  $y_j^2$  does not bind), then agent  $i$  is better off due to quasi-concavity of  $U^i(I_i - y_i, y_i + f^j(y_i), h^i(y_i, f^j(y_i)))$  in  $y_i$ . If, however,  $y_j^1$  is such that following the marginal reduction in  $y_i^1$ , agent  $j$  cannot choose  $y_j^2$  such that  $y_j = f^j(y_i)$  (i.e. the non-negativity constraint on  $y_j^2$  binds), then  $y_j$  would be unchanged. In the latter case, agent  $i$  is better off since  $y_i^1 > f^i(y_j)$  and  $U^i(\cdot)$  is quasi-concave in  $(y_i, y_j)$ .  $\square$

**Proof of Proposition 3.**

(A) Define the following sets:

$$F_1 = \{(y_1, y_2) \in [0, I_1] \times [0, I_2] \text{ s.t. } y_1 = f^1(y_2) \text{ and } y_2(G_0) \leq y_2 \leq y_2(G_2)\};$$

and

$$F_2 = \{(y_1, y_2) \in [0, I_1] \times [0, I_2] \text{ s.t. } y_2 = f^2(y_1) \text{ and } y_1(G_0) \leq y_1 \leq y_1(G_1)\}.$$

Let  $F = F_1 \cup F_2$ . Applying Lemmas B1–B3 and Corollary B1, the elements of  $F$  are the only equilibrium candidates in this case.

We next further restrict the candidate points for equilibrium to the two Stackelberg equivalents. Take any  $(y_1, y_2)$  in  $F_2$  such that  $y_1 < y_1(G_1)$ . Since  $y_2 = f^2(y_1)$ , we have  $y_2^1 \leq f^2(y_1)$  by definition. But then agent 1 could engender  $(y_1(G_1), y_2(G_1))$  as the equilibrium outcome. Suppose agent 1 chooses  $y_1^2 = y_1(G_1)$ . Then the equilibrium in period 2 has  $y_1^2 = 0$  and  $y_2^2 = y_2(G_1) - y_2^1$ . Since  $(y_1(G_1), y_2(G_1))$  is a strictly better outcome for agent 1,  $(y_1, y_2)$  cannot be an equilibrium outcome. This argument rules out all points but

that with  $y_1 = y_1(G_1)$  in  $F_2$ . A similar argument reduces  $F_1$  only to the point having  $y_2 = y_2(G_2)$ . In some (asymmetric) cases, one Stackelberg equivalent can be further eliminated from the candidate set. Without loss of generality, if at  $G_2$  both agents have lower contributions and are worse off than at  $G_1$ , then the Stackelberg equivalent at  $G_2$  cannot be an equilibrium outcome. In such cases the possibility of an equilibrium at  $G_2$  can be rejected by an argument like that just made, an argument that shows agent 1 could engender his preferred outcome at  $G_1$ .

Next we show a Stackelberg equivalent(s) is an equilibrium, using  $G_1$  as the generic example. Consider the second-period strategies:

$$y_i^2(y_i^1, y_j^1) = \begin{cases} 0 & \text{if } y_i^1 \geq f^i(y_j^1) \text{ and } y_j^1 \geq f^j(y_i^1), \\ y_i(G_0) - y_i^1 & \text{if } y_i^1 \leq y_i(G_0) \text{ and } y_j^1 \leq y_j(G_0), \\ 0 & \text{if } y_i^1 \geq y_j(G_0) \text{ and } y_j^1 \leq f^j(y_i^1), \\ f^i(y_j^1) - y_i^1 & \text{if } y_j^1 \leq y_j(G_0) \text{ and } y_i^1 \leq f^i(y_j^1), \end{cases}$$

and the first-period strategies:  $y_1^1 = y_1(G_1)$  and  $y_2^1 = 0$ . To verify that the second-period strategies are subgame perfect, first, suppose  $y_i^1 \geq f^i(y_j^1)$  and  $y_j^1 \geq f^j(y_i^1)$ . Given that  $y_j^2 = 0$ ,  $y_i^2 = 0$  since  $U^i(\cdot)$  is quasi-concave in  $(y_i, y_j)$ . Now suppose  $y_i^1 \leq y_i(G_0)$ . Further suppose  $y_j^2 = y_j(G_0) - y_j^1$  is given. This implies  $y_j = y_j(G_0)$ . Since  $y_i(G_0) - y_i^1 \geq 0$ , and by definition agent  $i$ 's best response to  $y_j(G_0)$  is  $y_i(G_0)$ , we have  $y_i^2 = y_i(G_0) - y_i^1$ .

In the third case, suppose  $y_i^1 \geq y_i(G_0)$  and  $y_j^1 \leq f^j(y_i^1)$ . Further suppose  $y_j^2 = f^j(y_i^1) - y_j^1$  is given, implying that  $y_j = f^j(y_i^1)$ . This implies that  $y_j^2 = 0$  using quasi-concavity of  $U^j(\cdot)$  in  $(y_i, y_j)$ . Given  $y_i^2 = 0$ , obviously  $y_j^2 = f^j(y_i^1) - y_j^1$  is the best response for agent  $j$ . Finally, consider the last possibility in the second period. Given  $y_j^2 = 0$ , since  $y_i^1 \leq f^i(y_j^1)$ ,  $y_i^2 = f^i(y_j^1) - y_i^1$  is the best response for agent  $i$ . Now given  $y_i^2 = f^i(y_j^1) - y_i^1$ , i.e.  $y_i = f^i(y_j^1)$ ,  $y_j^2 = 0$  is the best response for agent  $j$  due to quasi-concavity of  $U^j(\cdot)$  in  $(y_i, y_j)$ .

Thus the second-period strategies are equilibrium strategies for the agents.

Now consider the first-period strategies. Suppose  $y_1^1 = y_1(G_1)$  is given. In this case, it is straightforward to confirm that the best agent 2 can do is to reach the point  $G_1$  (see Fig. 2). Given the second-period strategies, agent 2 can achieve this goal by contributing nothing in the first period so that the second period strategies will dictate  $y_1^2 = 0$  and  $y_2^2 = y_2(G_1)$ . Thus,  $y_2^1 = 0$  is a best response for agent 2. Now suppose  $y_2^1 = 0$  is given. Again the best agent 1 can do is to reach  $G_1$ . Given the second-period strategies, he can do so by contributing  $y_1(G_1)$  in the first period. (Agent 2 can also contribute a positive amount in period 1, up to a threshold, with the equilibrium outcome still at  $G_1$ . This multiplicity is unimportant in that the agents' ultimate contributions and payoffs do not vary.)

Thus the above strategies constitute an equilibrium at point  $G_1$ . If  $G_2$  is also in the candidate set, then the same second-period strategies and  $y_1^1 = 0$ ,  $y_2^1 = y_2(G_2)$  along with an analogous argument proves that  $G_2$  is an equilibrium outcome as well.

(B) Applying Lemmas B1–B3 and Corollary B1, the set  $F_1$  defined in part A above contains the equilibrium candidates for this case (see Fig. 1). Using a similar argument to the proof of part A, we can rule out as candidates for equilibrium all the points but  $G_2$  in  $F_1$ . Also, the same second-period strategies above, and  $y_2^1 = y_2(G_2)$ ,  $y_1^1 = 0$ , along with similar arguments, prove that  $G_2$  is indeed the equilibrium outcome.

(C) Applying Lemmas B1–B3 and Corollary B1, the set  $F$  defined in part A contains the equilibrium candidates in this case (see Fig. 3). However, here any point can be supported as an equilibrium outcome.

Take any  $(y_1, y_2)$  in  $F$ . Let  $y_1^1 = y_1$ ,  $y_2^1 = y_2$  and let the second-period strategies remain the same as in part A. Similar arguments in part A will confirm that these are equilibrium strategies. Since this case is analytically equivalent to Saloner's (1987) Cournot duopolists with two production periods, the interested reader can refer to that paper for more details.  $\square$

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