Getting the Ball Rolling: Voluntary Contributions to a Large-Scale Public Project

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Abstract

This paper examines dynamic voluntary contributions to a large-scale project. In equilibrium, contributions are influenced by the interplay of two opposing incentives. While agents prefer to free ride on others for contributions, they also prefer to encourage others to contribute by increasing their own. Main findings of the paper are that (1) agents increase their contributions as the project moves forward; (2) as additional agents join the group, existing agents increase their contributions in the initial stages of the project while reducing them in the stages close to completion; (3) groups that are formed by more patient agents and that undertake larger projects tend to be larger; and (4) groups that rely on voluntary contributions tend to be too small compared to the social optimum. The empirical evidence on contributions to open-source software projects provides partial support for these findings.

1. Introduction

The successful completion of many large-scale projects depends on the continuous participation of multiple parties. Although binding contracts would be desirable in these situations, often they are not feasible. There are typically noncontractible uncertainties associated with the projects themselves or with the participants' economic and political environments. Moreover, in the case of international projects, contracts are notoriously hard to enforce due to the lack of a supranational authority. Binding contracts, however, are not

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always necessary for the successful completion of large-scale projects involving multiple parties.

There have been significant successes in the progress and completion of important projects that rely mostly on voluntary contributions. A notable example is the open-source software such as Apache, Linux, and GNOME. These large-scale programs are developed by programmers scattered around the world, who voluntarily and freely contribute their valuable time and skills. As of late 2000, the web server Apache was used by more than 59% of active servers across all domains, and Linux as a server-operating environment gained 17% market share (Johnson 2002, Lerner and Tirole 2001, 2002). Other notable examples are the international effort to restore the Earth's ozone layer (Murdoch and Sandler 1997), to clean up rivers crossing international boundaries (Sigman 2002), and to eradicate global terrorism.

What determines the successful completion of a large-scale project that relies on voluntary contributions? While a full answer to this question is obviously quite complex and context specific, in this paper I construct a fairly general model intended to capture the following salient features of such projects: (1) they require voluntary contributions by participants; (2) they are usually divided into smaller subprojects that are to be completed in a prespecified order;[1] (3) participants possess private information about their key characteristics such as current financial and political status, or simply their outside opportunities, which fluctuate over time; and (4) the benefits of a completed project are enjoyed for an extended period of time.

The formal model builds on the discrete public good framework of Palfrey and Rosenthal (1988, 1991). There are N risk-neutral agents who work on a joint project whose full return is realized only after a sequence of stages has been completed. In each period, the agents simultaneously decide whether or not to contribute to advancing the project after commonly observing the state of the project and privately observing their own cost of contributing. The costs of contributing vary across agents and time.

In general, contributions to a joint project suffer from the well-known free-rider problem. While agents benefit freely from others' contributions, they bear the full cost of their own efforts, leading them to rely "too much" on others. When projects require a sequence of contributions, however, a second effect, countervailing the free-riding incentive, emerges. To see this, suppose there is at first a single agent that is initially indifferent about contributing to a public project. Now, suppose this agent is informed that a second agent would be interested in contributing to the project if it were one step further along or sufficiently "mature." That is, the second agent's current valuation of the project is too small to attract his initial contribution. If the free-rider effect were the only force at play here, then the first agent would continue to remain indifferent about initiating the project. The presence of the second

[1]For instance, the open source programs are composed of modules of smaller programs.

agent, however, can in fact break the first agent's indifference and cause him to start the project in order to attract future contributions. In a sense, the first agent invests in the project in order to free ride on the second agent in the future. Using the terminology of Bolton and Harris (1999),[2] I call the latter effect the "encouragement" effect and examine how it interacts with the free-riding incentive throughout the relationship.

The organization of the paper and a brief preview of the findings are as follows. In Section 2, I present the basic model in which agents' contributions are perfect substitutes. In the unique symmetric Markov Perfect Equilibrium (MPE), I find that early in the project agents make concessions toward the completion of the project through increased efforts. In general, there are two opposing effects on an agent's effort choice arising from increased contribution by others: On the one hand, it makes his effort less pivotal thereby facilitating the free-rider effect. On the other hand, it brings the future returns generated by greater future contributions closer thereby facilitating the encouragement effect. The latter effect mitigates the former and thus in equilibrium each agent raises his effort level.

In Section 3, I consider group size. An increase in the group size has one direct and two strategic effects on agents' contribution decisions. The direct effect comes from the possible "congestion" in the utilization of the completed project such as clean international waters. The strategic effects are the encouragement and the well-known free-riding incentives. When the project is a noncongestible public good, for example, an open source program, I find that equilibrium contributions across states are not monotonically affected by the group size. As one more agent joins the group, the original members may increase their contributions in some states and reduce it in others. In general, the encouragement effect is stronger in a larger group in the initial stages of the project—simply because the gain from encouraging others is greater. However, as the project nears completion, the encouragement effect loses strength and the free-riding incentive becomes more dominant in a larger group, leading agents to lower their contributions. Despite this nonmonotonicity, agents strictly benefit from the inclusion of an additional group member. Interestingly, as the group size tends to infinity, the equilibrium contributions approach to the socially optimal level. This finding runs counter to that of Olson's (1965) intuition that as the group size increases, the inefficiency in public good provision also increases. Moreover, the empirical evidence on programmers' contributions to open-source software is consistent with my results as I discuss at length.

When the project is a congestible public good, the congestion effect counteracts the net positive dynamic effects. In such cases, I consider the

[2] Although my focus and formal model are considerably different from those in Bolton and Harris (1999), both papers highlight similar dynamic effects, as becomes clear in the analysis.

optimal group size that balances the congestion and the dynamic effects. I find that larger projects require a larger group, but—more interestingly—the optimal group size increases with agents' patience. This is because more patient agents are able to better control the free-riding incentive, which allows them to include more members to expedite the completion of the project. Nonetheless, in Section 4, I show that groups that rely on voluntary contributions tend to be too small with respect to the "social optimum" where they can coordinate their actions. Finally, in Section 5, I extend the model to incorporate complementaries in agents' efforts and, in Section 6, discuss possible extensions.

1.1. Related Work

My paper is closely related to two strands of the literature on the private provision of public goods. The first strand investigates the provision of discrete public goods in static models with incomplete information. See, for example, Menezes et al. (2001) and Palfrey and Rosenthal (1988, 1991).[3] My analysis however is intended to uncover the incentives in a dynamic setting. The second strand explores the dynamic contribution models with complete information. Most notably, papers by Admati and Perry (1991) and Fershtman and Nitzan (1991) consider models in which, like mine, contributions accumulate over time to provide a public good. Their main point is that the dynamics exacerbate the free-riding incentive, as agents view their contributions as a substitute not only to current contributions but also to the future ones. In particular, Admati and Perry find that many socially desirable projects are unlikely to be undertaken. Marx and Matthews (2000) analyze a similar framework but allow for trigger strategies so that if an agent fails to contribute his equilibrium amount, others punish him by delaying theirs. This implicit enforcement alleviates the free-rider incentive and leads to the positive results in Marx and Matthews.[4]

Unlike these papers, my model explores a setting in which agents' costs of contribution are private information and more importantly, they vary over time and across agents. I discover that even though agents use more "forgiving" strategies than the ones in Marx and Matthews, the dynamics help alleviate the free-riding incentives. In particular, while agents free ride on current contributions, they view future contributions as complementary in my model. This means that part of the reason for the negative result in Admati

[3]Bagnoli and Lipman (1989), and Palfrey and Rosenthal (1984) also study the private provision of a discrete public good in a static model but with complete information.

[4]Papers by Bac (1996), McMillan (1979), Palfrey and Rosenthal (1994), and Pecorino (1999) also explore the possibility of cooperation in public good provision in infinitely repeated game settings. However, unlike in my model, agents enjoy the full return in each period, and thus the incentives to complete a large-scale public project do not arise.

and Perry (1991), and Fershtman and Nitzan (1991) is the fact that agents' costs of contribution are common knowledge and stay the same over time.

2. The Model

Consider a dynamic extension of Palfrey and Rosenthal (1988, 1991). There are N agents who work on a joint project with H subprojects where H is a natural number. The progress of the project is observed by all agents. Let h be the state variable indicating that $(h-1)$ subprojects have been completed, and let $r(h)$ denote the corresponding per period return to each agent in state h. The project yields its full return, $v(N) > 0$, after all subprojects are completed. Namely,

ASSUMPTION 1:

$$r(h) = \begin{cases} v(N), & \text{for} \quad h > H \\ 0, & \text{for} \quad h \leq H, \end{cases}$$

where $v(N)$ can be decreasing, constant, or increasing in N, capturing different types of projects depending on how participants share the benefits of a completed project. I call the projects "congestible" if $v(N)$ is decreasing, and "noncongestible" if $v(N)$ is constant. I also allow for projects for which $v(N)$ is increasing, perhaps because there are positive externalities in utilizing the finished project.

Assumption 1 indicates that there are no immediate returns before completing the project. While certain projects might yield intermediate payoffs, as long as the cumulative payoff with the progress of the project weakly increases at a weakly increasing rate, the qualitative results in the paper will hold. Furthermore, by making each subproject ex ante identical, I abstract from the latter complication to better highlight the dynamics in the model. Zero return, however, is pure normalization.

The interaction among agents is modeled as an infinite horizon Markov game and the MPE is the solution concept.[5] Informally, a MPE induces a perfect equilibrium for all payoff-relevant histories described by the state h. The Markov property requires that agents base their decisions only on the progress of the project. This property also has intuitive appeal in my setting as I wish to investigate the effects of the project progress on agents' incentives to contribute.[6] In each period, agents observe the state of the project and

[5]See, for example, Fudenberg and Tirole (1991, ch. 13) for a review.

[6]In theory, agents can use less "forgiving" strategies for a deviation than the Markov ones. However, as with the international projects, each country will have difficulty punishing others if there is a defection, either because governments change or because it is hard to prove defection in the presence of incomplete information. Furthermore, focusing on Markov strategies allows me to determine the minimal level of cooperation agents can achieve.

simultaneously decide whether or not to contribute. Like in Rosenthal and Palfrey's setting, contribution is a 0–1 decision denoted by the indicator variable s.[7] In the basic model, I assume as long as one agent contributes, the project moves to the next state. Otherwise, it stalls one period, and the state does not change. The process repeats itself the next period with the updated state and with new cost draws for agents. This type of production function assumes agents' contributions are perfect substitutes. Nonetheless, I will show that agents' equilibrium contributions exhibit some intertemporal complementarity and discuss a more general production function in Section 5. The cost of contribution or effort comes from a twice continuously differentiable cumulative distribution, $F(c)$, which is independently and identically distributed over time and across agents.[8] The support of distribution is $[0, \bar{c}]$ with $\bar{c} > 0$ and $F'(c) = f(c) > 0$. Agents discount the future returns and costs by $\delta \in (0, 1)$.[9]

Let $W_i(h, c_i)$ be agent i's value function when his realized cost is c_i and the project is in state h. Suppose that $\lambda^*_{-i}(h)$ is the equilibrium probability that at least one agent other than i contributes. Agent i, then, solves the following dynamic program:[10]

$$W_i(h, c_i) = \max_{s_i \in \{0,1\}} \left\{ \begin{array}{c} r(h) + s_i[-c_i + \delta W_i(h+1)] \\ +(1 - s_i)\delta\left[\lambda^*_{-i}(h)\, W_i(h+1) + \left(1 - \lambda^*_{-i}(h)\right) W_i(h)\right] \end{array} \right\}, \tag{1}$$

where $W_i(h) \equiv E_c[W_i(h, c)]$ and E_c is the expectation operator with respect to c.[11]

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[7]In the working paper Yildirim (2005), I also study a continuous contribution extension and show that the same qualitative results as in the discrete case would arise.

[8]This assumption, aside from being realistic, rules out the strategic learning among agents in a public good context, which is the topic of Bliss and Nalebuff (1984) and Gradstein (1992), among others. In these papers, agents have private information about their costs of contribution, and these costs do not change over time. Thus, as time passes by, agents leak information to others often resulting in inefficient delays in contributions. Although this type of strategic learning is an important element of public good provision, here I abstract from this aspect to better focus on the effects of the progress of the project on agents' contribution pattern.

[9]Assuming that agents are ex ante symmetric clearly does not fit the examples in the Introduction, where it seems more reasonable to consider heterogenous cost distributions and discount factors. While I leave the generalization for future work, I believe the present model is rich enough to identify the main forces in the generalization.

[10]Note that agents incur the cost at the time they volunteer in this model. This might create ex post inefficiency in equilibrium as contributions in excess of one are wasted. However, following we will see that agents take this possibility into account when contributing.

[11]Since time enters only through discounting and the cost distribution is stationary over time, I drop the time index throughout the analysis. Also, following I show that the value functions exist and are well behaved.

According to (1), agent i decides whether or not to contribute to the project upon observing the state of the project and his own current cost. If he contributes, that is, $s_i = 1$, he bears the full cost of his effort in which case the project moves forward with certainty and he receives the discounted expected continuation value, $\delta W_i(h+1)$. However, if he decides not to, that is, $s_i = 0$, then he will have to rely on others for contribution. In this instance, he conjectures the expected probability that at least one other agent will contribute, which in turn determines his discounted future expected continuation value, $\delta[\lambda^*_{-i}(h) W_i(h+1) + (1-\lambda^*_{-i}(h)) W_i(h)]$. In each decision though, he receives the return, $r(h)$. Define the following cutpoint:

$$x_i(h) \equiv \left[1 - \lambda^*_{-i}(h)\right]\delta \Delta W_i(h), \tag{2}$$

where $\Delta W_i(h) \equiv W_i(h+1) - W_i(h)$.

Equation (1) reveals that agent i's equilibrium strategy simply has the following cutoff property:[12]

$$s_i^*(h, c) = \begin{cases} 1, & \text{if } c \leq x_i(h) \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

We say that as $x_i(h)$ increases, so does agent i's contribution or effort.[13,14] According to (2), this effort is monotonic in the likelihood of being the pivotal contributor and in the discounted net future gains.

As is common in sequential games, I characterize the equilibrium using backward induction, and for simplicity, focus on the symmetric one. Start with the completed project, $h > H$. Clearly, no further contribution is required in this state and hence $x_i(h) = 0$ for $h > H$. This implies that the expected discounted value of the completed project is $W_i(H+1) = \frac{v(N)}{1-\delta}$, or equivalently the average per period return is

$$\bar{W}_i(H+1) \equiv (1-\delta) W_i(H+1) = v(N). \tag{4}$$

Now, refer to Figure 1. Given $\bar{W}_i(H+1)$, we determine the cutoff cost level, $x(H)$, in state H in a symmetric equilibrium such that an agent would unilaterally complete the project if and only if his cost is below this cutoff. The expected completion time of subproject H and the cost incurred by each agent depend on $x(H)$ and in turn determine the average per period return in this state, $\bar{W}_i(H)$. Given this value, we then find the equilibrium cutoff, $x(H-1)$, and continue this induction process until all states are exhausted.

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[12]Since $c \in [0, \bar{c}]$, I adopt the convention that $x_i(h) = 0$ for $x_i(h) < 0$ and $x_i(h) = \bar{c}$ for $x_i(h) > \bar{c}$.

[13]This is in ex ante contribution sense. As the cutpoint increases, agent i becomes more likely to make a contribution.

[14]Agent i is indifferent about whether or not to contribute if he draws a cost $c = x_i(h)$. Given the cost distribution is continuous, the probability of this event is zero.

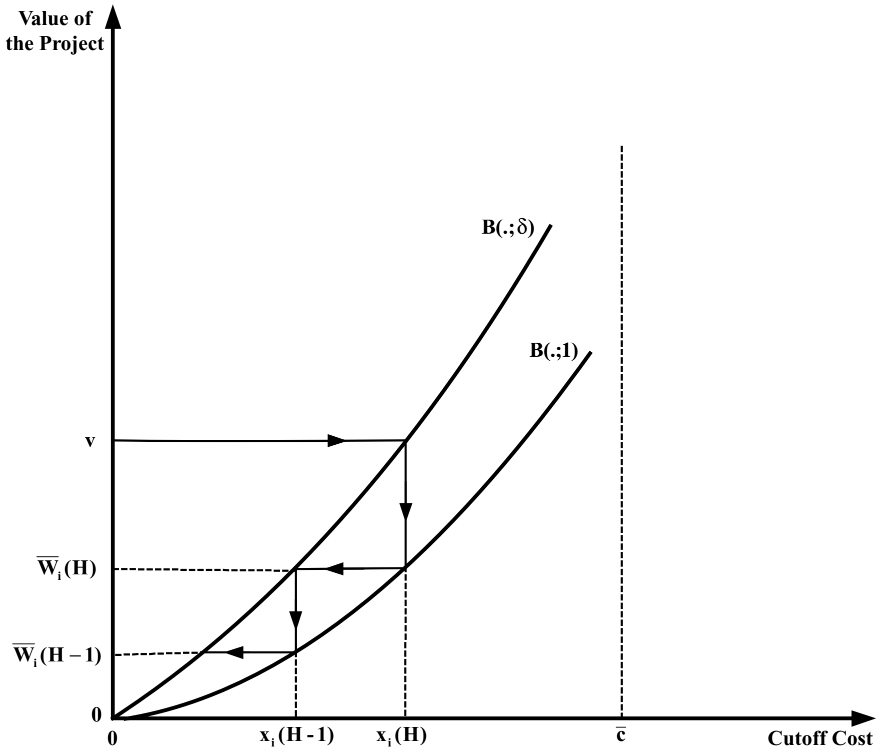

Figure 1: Backward induction with substitute contributions

More formally, upon choosing a symmetric cutoff, x, each agent's expected cost of completing a subproject is given by

$$B(x;\delta) \equiv \frac{1}{\delta}\frac{x}{[1-F(x)]^{N-1}} - \int_0^x [1-F(c)]\,dc. \tag{5}$$

Thus, the equilibrium cutoff, $x(H)$, has to balance the expected return and the expected cost, that is, $\bar{W}_i(H+1) = B(x(H);\delta)$. Note that $B(x(H);\delta)$ measures the expected cost in terms of tomorrow's dollars consistent with the return, $\bar{W}_i(H+1)$. Thus, the same cost in today's dollars, that is, $B(x(H);1)$, must be equal to the value of the project in state H. The following lemma records the properties of $B(\bullet)$ and shows this induction argument for the remaining states.

LEMMA 1:

(a) For $h \leq H$, $\bar{W}_i(h+1) = B(x(h);\delta)$, *and* $\bar{W}_i(h) = B(x(h);1)$, *where* $\bar{W}_i(h) = (1-\delta)\,W_i(h)$.

(b) $B'(x;\delta) > 0$ *for* $x \in (0,\bar{c}]$, *and* $B'(0;\delta) = \frac{1}{\delta} - 1$.

All proofs are contained in the Appendix, which, due to space constraints, is an abbreviated version of the complete Appendix provided in the working paper and available on request.

Armed with Lemma 1, the following proposition characterizes the equilibrium:

PROPOSITION 1: *There exists a unique symmetric MPE with the following properties. For* $h \leq H$:

(i) $x(h) > 0$ *for all* h.

(ii) $x(h)$ *is strictly increasing in* h.

(iii) $W_i(h)$ *is strictly increasing at an increasing rate in* h*; that is,* $\Delta W_i(h) > 0$ *and* $\Delta W_i(h) > \Delta W_i(h-1)$.

Part (i) implies that the project of any size is completed despite no immediate returns. In equilibrium, agents are willing to take current losses for future returns. Two assumptions derive this result: (1) The lower bound of cost distribution is zero, and (2) contributions are perfect substitutes.[15] In Section 5, I consider the case with complementary contributions and compare the equilibrium behaviors.

The intuition behind part (ii) is more involved. In equilibrium, the cutpoint for agent i given in (2) reduces to

$$x_i(h) = [1 - F(x(h))]^{N-1}\delta \Delta W_i(h). \tag{6}$$

Everyone else's increasing effort has two opposing effects on one's decision summarized on the right hand side of (6): On the one hand, it makes his effort less pivotal and thus facilitates the free-rider effect. Formally, as $x(h)$ increases $[1 - F(x(h))]^{N-1}$ becomes smaller. On the other hand, it brings future returns generated by greater future efforts closer and creates an encouragement effect given by $\delta \Delta W_i(h)$. As part (ii) of Proposition 1 indicates, the latter effect mitigates the former, and therefore each agent raises his effort as the project moves forward. Put differently, agents view their own efforts as strategic substitutes for others' current efforts and as strategic complements to the future efforts. This is so in equilibrium even though there are no complementaries in the production function.[16]

Since the present model is stationary, one can also determine the average waiting time in each state. Note that the equilibrium probability that

[15] A strictly positive lower bound on the cost distribution would imply that some large-scale projects may not take off. To make the analysis interesting, one would then restrict attention to the projects that would take off.

[16] The observation in Proposition 1 accords with Bolton and Harris (1999), who consider a continuous-time two-armed bandit model of strategic experimentation where agents privately choose whether to invest in a safe or risky asset each period. While they also identify free riding and encouragement effects in this social learning environment, my focus and model are significantly different from theirs. In particular, I consider the issue of private contributions to a large-scale public project and also deal with the optimal group size and complementary contributions.

the project will move from state h to $h+1$ is given by $\lambda^*(h) \equiv 1 - [1 - F(x(h))]^N$. Thus, the average waiting time in state h is $w(h) = \frac{1}{\lambda^*(h)}$. Part (ii) of Proposition 1 implies that this waiting time shrinks as the project moves forward.

The last part of Proposition 1 reveals that the value agents' attach to the project increases at an increasing rate over states.

3. The Effects of Group Size

In general, an increase in the group size has three effects on agents' contribution decisions: one direct effect due to the (possible) change in the payoff after the project is completed and two strategic effects due to the dynamics before the project is completed. To better understand these effects and distinguish between different types of projects, I first consider noncongestible projects for which the direct effect is not present and then consider other projects for which it is present.

3.1. Noncongestible Projects

Imagine that participants are contributing toward projects like open-source software or restoration of the ozone layer, where an additional user does not degrade one's final payoff, that is, $v(N) = v$ for all N. In this case, an increase in the group size has two opposing strategic effects on agents' contributions. First, it worsens the well-known free-rider incentive as there are now more agents to free ride on. Second, it also strengthens the encouragement incentive, as by increasing one's own contribution today, an agent can attract greater future contributions in a larger group. Furthermore, since there are more future contributions to be made in the initial stages of the project, it is intuitive that the latter incentive will be stronger in those stages while losing its steam with the progress of the project. Thus, it is reasonable to conjecture that an increase in the group size results in an increase in equilibrium contributions in the initial stages of the project and a decrease in the mature stages. I confirm this conjecture in

PROPOSITION 2: *Suppose $v(N) = v$ for all N. For $N \geq 1$,*

(i) $x(H; N+1) < x(H; N)$,

(ii) for sufficiently large H, there exists h_0 such that for $h < h_0$, $x(h; N+1) > x(h; N)$,

(iii) for $h \leq H$, $\lim_{N\to\infty} x(h; N) = 0$, and $\lim_{N\to\infty} \lambda^*(h; N) = 1$.

First two parts of Proposition 2 imply that agents' contributions are not uniformly affected by an increase in group size. Part (i) reveals that each agent reduces his equilibrium contribution to the last subproject in a larger group. Intuitively, since no future contributions are needed in the final stage of the project, the encouragement incentive vanishes and the free-rider effect

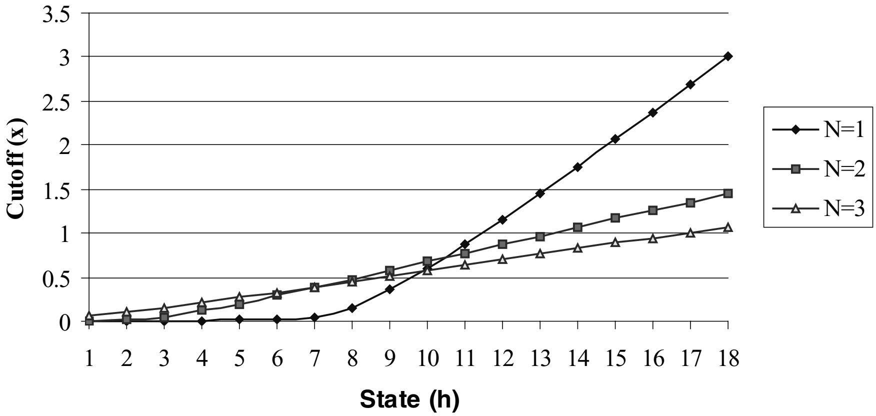

Figure 2: Nonmonotonicity of contributions

leads to a reduction in contributions. Part (ii) shows that when the project is sufficiently large, the encouragement incentive becomes sufficiently strong in the early stages of the project and dominates the free-rider incentive. To gain intuition about how contributions change in the intermediate states with the group size, I consider a numerical example depicted in Figure 2.

In the example, there are 18 subprojects, that is, $H = 18$.[17] The cost of effort is distributed uniformly in $[0, 3]$, and the project yields a value $v = 2$ to each agent upon completion. Also, agents discount future returns by $\delta = 0.9$.

Refer to Figure 2. When the group consists of a single agent, obviously there are no free-riding or encouragement incentives. In this case, notice that the agent would finish the project with certainty in the penultimate state, $h = 18$, that is, $x(18) = \bar{c} = 3$. However, the project virtually does not take off for the states below $h = 6$, that is, $x(h) \approx 0$ for $h \leq 6$. If there is one more agent in the group, agents encourage each other for the initial 10 states by choosing higher cutpoints than the single-agent case. Thus, the project is more likely to take off with a larger group in these states. However, for the remaining eight states, the free-rider effect becomes stronger resulting in lower effort levels. As the group size grows further to $N = 3$, the critical state below which agents put higher efforts in a larger group goes down. This is because with more agents, the project matures faster and the free-rider effect becomes more dominant starting in an earlier state. The example also shows that agents' effort levels stay relatively steady in a larger group.

According to parts (i) and (ii) of Proposition 2 and the example in Figure 2, it is clear that each agent's contribution in a given state is ambiguous in the group size. Even so, as the group size tends to infinity, each

[17]Figure 2 shows only states 1 through 18 since we already know that $x(h) = 0$ for $h > 18$.

agent expects that someone in the group will draw a cost close to zero and thus chooses a cutpoint close to zero as a result.[18] Interestingly, this holds in all states as recorded in part (iii). This does not mean, however, the project would never move forward. In contrast, I show in the Appendix that the equilibrium probability that the project moves from h to $h+1$, that is, $\lambda^*(h; N)$, converges to 1. The increase in the number of agents more than offsets the reduction in the cutpoint.[19] In fact, in Section 4, I demonstrate that as $N \to \infty$, agents' welfare approaches to the socially efficient level.

The predictions in Proposition 2 are widely consistent with the empirical studies of voluntary contributions by programmers to open-source software projects (see Hertel et al. [2003] for the Linux project; Koch and Schneider [2002] for GNOME project; and Mockus et al. [2000] for the Apache project). These studies report an increase in the average contributions with the number of programmers, especially in the early stages of the projects, and a decline in the mature stages.[20] The fact that more programmers need not reduce the pace of a software project was first argued by Raymond (1999), who made the assertion known as Linus' Law: "Given enough eyeballs, all bugs are shallow." This is indeed surprising, because previously it was believed that Brooks' Law (Brooks 1995) that states "... adding people to a late project makes it later ... the fact that a woman can have a baby in nine months does not imply that nine women can have a baby in one month" holds. Though the two assertions seem conflicting, they are not. In his book, Brooks also suggested an exception to his law by pointing out "... if a project's tasks are partitionable, you can divide them further and assign them to ... people who are added late to the project," which is also the building block of Raymond's argument.

Despite the ambiguous effects of group size on agents' effort levels, agents do strictly benefit from an additional group member as I note in

PROPOSITION 3: *Suppose $v(N) = v$ for all N. Then, for $h \leq H$, $W_i(h, N+1) > W_i(h, N)$.*

Proposition 3 shows that an increase in the number of agents is a Pareto improvement in the sense that each agent has a higher welfare in each state. This seems intuitive for the states in which agents increase their effort levels

[18]Again, the fact that the lower bound of cost distribution is zero is important here.

[19]This result might seem puzzling in light of Mailath and Postlewaite (1990), who, in a one-period mechanism design setting, conclude that as the group size tends to infinity, the probability of supplying the public good converges to zero, even though it should be provided with probability 1. While both Mailath and Postlewaite's and my model predict that as $N \to \infty$, agents would feel less pivotal and reduce their contributions, the difference stems from the fact that Mailath and Postlewaite assume the exogenous cost of providing the public good increases in group size faster than the free-riding incentive.

[20]Note that the average contribution is $F(x(h))$ in my model.

as this improves the pace of the project resulting in a higher value.[21] However, when the project matures, an additional agent exacerbates the free-rider incentive and might even slow down the progress of the project. Even so, the cost saving associated with one additional agent more than offsets this reduced pace. Thus, Proposition 3 implies that the optimal group size for noncongestible projects is infinity. Once again this is consistent with the fact that open source projects accept contributions from programmers all over the world rather than restricting the group to just a few.

In light of Proposition 3, I should also note that the qualitative results pertaining to congestible projects trivially hold when there are positive externalities in the utilization of the project, that is, $v(N)$ increases in N. In particular, both the direct and the net dynamic effects are positive. However, this is not the case in what follows with congestible projects.

3.2. Congestible Projects and the Optimal Group Size

When the completed project is a congestible public good, the direct congestion effect of the group size counteracts the net positive effect identified in Proposition 3. How the group balances the two effects depends on its ability to restrict entry into the group. It is thus important to ask what the optimal group size is. To put the analysis into perspective, I assume that the congestion is sufficiently severe for large groups.

ASSUMPTION 2:

$$\lim_{N\to\infty} v(N) = 0.$$

From Assumption 2, it easily follows that when the entry into the group cannot be restricted, the payoff will be driven down to zero after the project is completed. In such situations, the project will not be undertaken. This is the extreme case of congestion. However, suppose that the access can be restricted or simply that the project is an excludable public good. In these cases, I will assume that the optimal group size is the one that maximizes each member's expected net present value of the project, $\bar{W}_i(1, N \mid \delta, H)$, before the project starts.[22] Let $N^*(H, \delta)$ be the optimal group size for a project with H subprojects and an individual discount factor of δ. More formally, since from Lemma 1, $\bar{W}_i(1, N \mid \delta, H) = B(x^*(N, \delta); 1 \mid N, H)$, we have

[21] This is clearly seen from Lemma 1 where $B(x; 1)$ is strictly increasing in x.

[22] While I am not explicitly modeling how the group actually forms, one can imagine that one agent offers memberships to other ex ante identical agents, each of whom then either accepts or declines the offer. There are however no side payments. In this case, it is clear that the number of offers the first agent makes will maximize his expected discounted value of the project. Notice also that N^* maximizes the ex ante average per period payoff rather than just the discounted payoff. This allows me to isolate the strategic effect of discounting.

$$N^*(H, \delta) = \arg\max_N B(x^*(N, \delta); 1 \mid N, H). \quad (7)$$

The result in Proposition 3 for noncongestible projects along with Assumption 2 imply that $N^*(H, \delta)$ exists and finite. In what follows, I assume it is also unique. The following result records how the group size changes with the project size and the agents' discounting.

PROPOSITION 4: *Suppose that the project is an excludable public good and Assumption 2 holds. Then, the optimal group size (weakly) increases with the project size and with agents' patience level.*

To see the intuition behind the link between the group size and project scale, note that having the same number of participants that is optimal for a smaller project cannot be excessive for a larger project. Given the same group size, the return from a completed project remains the same across different size projects. But this means agents would prefer more entry into the group in order to finish a larger project in a shorter time (Proposition 3).

The intuition behind the second part of the proposition is more involved. For a fixed group size, an increase in the discount factor has three effects on agent i's contribution: First, since he now cares more about the future, he is willing to take larger current losses and thus increases his cutpoint in (6). Second, given that others contribute more in response to a higher δ, agent i's contribution becomes less pivotal, that is, $[1 - F(x(h; \delta))]^{N-1}$ is smaller, and thus he tends to free ride and decrease his contribution. Third, others' increasing contributions brings the future returns generated by future contributions closer and thus encourages agent i. Overall, agent i's contribution increases with δ. This implies that more patient agents are able to control the free-riding incentive more effectively, which in turn allows them to expand the group and expedite the completion of the project.[23] Next, I investigate whether or not the group tends to be too large or too small with respect to the social optimum.

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[23]The fact that more patient agents can control free-riding incentives more effectively is true despite there being no reputational concerns in my setting, which is in contrast to several other dynamic models of public good provision, for example, Bac (1996), Marx and Matthews (2000), McMillan (1979), and Pecorino (1999), where agents adopt trigger strategies. In such settings, as the discount factor becomes larger and agents become more patient, the punishment becomes more severe for any deviation, which helps sustain the cooperative outcome. Here, I focus on the Markov property that excludes such trigger strategies. Nonetheless, the relationship grows to be more cooperative in response to a higher discount factor.

4. Benchmark: Social Optimum

There are two sources of inefficiencies in the noncooperative contribution game I have analyzed: (1) Agents simultaneously choose their cutoff points, and (2) agents have incomplete information about others' costs of contribution. The first best would occur if neither of these problems existed. The optimal strategy in this case would be to assign the project to the lowest cost agent in each state unless this cost is too high. However, given the informational constraints, this solution is infeasible in the present setting, which relies on voluntary contributions. Therefore, the second-best situation where the social planner coordinates agents' cutpoints before the costs of contribution are realized seems more appropriate as a benchmark.

Suppose a social planner maximizes the total welfare, or equivalently the average welfare per agent in the group by choosing a cutpoint, $x_i^{**}(h)$, for agent i.[24] Let $W^{**}(h)$ be the optimal average payoff per agent in state h. It is clear that once the project is completed, the social planner will choose $x_i^{**}(h) = 0$ for $h > H$, and thus have no agent contribute. This implies the boundary condition as in (4): For $h > H$, $W^{**}(h) = \frac{v(N)}{1-\delta}$. For $h \leq H$, however, $W^{**}(h)$ satisfies the following dynamic program:

$$W^{**}(h) = \frac{1}{1-\delta} \max_{\{x_i(h)\}} \left\{ -\frac{1}{N} \sum_{i=1}^{N} \int_0^{x_i(h)} c\, dF(c) + \left[1 - \prod_{i=1}^{N} (1 - F(x_i(h)))\right] \delta \Delta W^{**}(h) \right\}, \quad (8)$$

where the first term inside maximization is the average cost, and the second term is the probability that the project will move to the next state times the expected increase in the value of the project. The first-order condition for $x_i(h)$ implies that

$$x_i^{**}(h) = \left[1 - \lambda_{-i}^{**}(h)\right] N \delta \Delta W^{**}(h), \quad (9)$$

where again $\lambda_{-i}^{**}(h)$ represents the optimal probability that at least one agent other than i will contribute.[25]

Comparing (9) and (2), we see that when determining the cutoff for agent i, the social planner takes the effect of i's contribution on the total welfare rather than just on i's own welfare. As before, I will assume that the social planner optimally assigns symmetric cutoff points to all agents, that is, $x_i^{**}(h) = x^{**}(h)$. Now define the following function analog of (5):

[24] Alternatively, suppose agents are able to write a one-period contract on their cutoffs before costs of contribution are realized.

[25] It is easy to verify that the maximized function is strictly quasiconcave.

$$B^{**}(x) = \frac{x}{\delta N[1-F(x)]^{N-1}} - \int_0^x [1-F(c)]\,dc + \left(1 - \frac{1}{N}\right) x[1-F(x)]. \tag{10}$$

A variant of Lemma 1 records how the social planner uses backward induction to find the optimal cutoff points.

LEMMA 2:

(a) *For* $h \leq H$, $\bar{W}^{**}(h+1) = B^{**}(x^{**}(h);\delta)$, *and* $\bar{W}^{**}(h) = B^{**}(x^{**}(h);1)$ *where* $\bar{W}^{**}(h) = (1-\delta)W^{**}(h)$.

(b) $B^{**\prime}(x;\delta) > 0$ *for* $x \in (0, \bar{c}]$, *and* $B^{**\prime}(0;\delta) = \frac{1}{N}(\frac{1}{\delta} - 1)$.

Given that the qualitative properties of $B^{**}(x;\delta)$ coincides with that of $B(x;\delta)$, the existence of a unique social optimum easily follows by applying the proof of Proposition 1. Here we are interested in knowing whether the coordination of efforts strictly benefit agents in all states, and more importantly whether the project progresses at the socially optimal rate. The following proposition answers these questions.

PROPOSITION 5: *For* $N > 1$, *and* $h \leq H$*:*

(i) $W_i(h) < W^{**}(h)$.

(ii) $x(h) < x^{**}(h)$.

(iii) $\lim_{\delta \to 1} x(h) = x_1 < \lim_{\delta \to 1} x^{**}(h) = x_1^{**}$.

According to part (i) of Proposition 5, agents strictly benefit from coordinating their efforts. This is because the equilibrium cutpoints are in the social planner's choice set. Part (ii) implies that agents contribute too infrequently to the project and thus the project progresses too slowly from the social standpoint. This means the positive encouragement effect cannot fully mitigate the negative free-rider effect in our model. This inefficiency remains even when the discounting is negligible as recorded in part (iii).

Next, I investigate the degree of inefficiencies caused by an increase in the group size when the project is a noncongestible public good and compare the optimal group size with the benchmark when it is a congestible public good. Let $N^{**}(H,\delta)$ be the socially optimal group size for a congestible project.

PROPOSITION 6:

(i) *If the project is a noncongestible public good, that is,* $v(N) = v$, *then* $\lim_{N\to\infty} W_i(h) = \lim_{N\to\infty} W^{**}(h) = \frac{\delta^{H+1-h}}{1-\delta} v$.

(ii) *If the project is congestible and Assumption 2 holds, then* $N^*(H;\delta) \leq N^{**}(H,\delta)$.

The first part of Proposition 6 reveals that as the group size grows without bound for a noncongestible project, the inefficiencies disappear and agents' expected payoffs converge to the socially optimal one. Together with part (iii) of Proposition 2, this result goes against our intuition from Olson (1965), who asserts "... the larger the group, the farther it will fall short of providing an optimal amount of collective good"(p. 35). The main difference comes from the fact that the technology in my model is that of a "best shot" one, which requires only one contribution in each stage. Thus, in both the incomplete information and the second-best cases, as the group size becomes sufficiently large, the contribution is made by the lowest cost agent and the expected probability of having such an agent converges to one in the limit.

The second part implies that groups that rely on voluntary contributions tend to be too small with respect to the social optimum. The intuition is clear: In the social optimum, agents are able to coordinate their strategies so that they can control the free-riding incentive more easily. This allows the group to include more members in order to finish the project in a shorter time. Conversely, when the coordination of strategies is not feasible, the group controls the free-riding incentive by being small.

5. Complementary Contributions

Until now, I have assumed that agents' contributions are perfect substitutes in that in a given period if at least one of them contributes, the project moves forward. Although this is the case for open-source software, or projects that require only the financial contributions, for example, other projects might require certain degree of complementarity in agents' efforts. To capture this possibility, suppose that at least $m \in \{1, \ldots, N\}$ agents need to contribute in a given period for the project to move forward. We say that a higher m corresponds to a higher degree of complementarity in production with $m = 1$ and $m = N$ being the two polar cases representing perfect substitutes and perfect complements, respectively. An increase in the degree of complementarity has two opposing effects on the group interaction: (1) It reduces the free-rider problem by making agents' contributions less substitutable, and (2) it exacerbates the coordination problem by requiring others' contributions to make one's own worthwhile.

Let $\lambda^*_{-i}(m, h)$ be the equilibrium probability that at least m agents other than i contribute in state h. Like in perfect substitute case, agent i solves the following dynamic program to decide whether or not to contribute in state h:

$$W_i(h, c_i) = \max_{s_i \in \{0,1\}} \Big\{ r(h) + s_i\big[-c_i + \delta\big[\lambda^*_{-i}(m-1, h)\, W_i(h+1) + \big(1 - \lambda^*_{-i}(m-1, h)\big) W_i(h)\big]\big] + (1 - s_i)\delta\big[\lambda^*_{-i}(m, h)\, W_i(h+1) + \big(1 - \lambda^*_{-i}(m, h)\big) W_i(h)\Big\}. \tag{11}$$

It is clear from (11) that agent i adopts a similar cutoff strategy to (2) with the following cutpoint:

$$x_i(h) = \left[\lambda^*_{-i}(m-1, h) - \lambda^*_{-i}(m, h)\right]\delta\Delta W_i(h). \tag{12}$$

Note that the expression, $\lambda^*_{-i}(m-1, h) - \lambda^*_{-i}(m, h)$, is the probability that agent i's contribution is pivotal. Thus, the intuition behind (12) is the same as in the substitute case. In equilibrium, agents optimally choose zero contributions once the project is completed, that is, $x_i(h) = 0$ for $h > H$. For the intermediate states, the zero contribution continues to be an equilibrium representing a complete coordination failure whenever there is complementarity, $m \geq 2$. However, there might be other equilibria in which all agents do contribute and which are clearly Pareto improving over the zero-contribution equilibrium. I will assume that agents coordinate on the most favorable equilibrium in each state.[26] Below, I demonstrate that despite agents' best efforts to overcome coordination problem, some projects may never start. Since the coordination problem gets worse in a larger group, I focus on noncongestible projects to abstract from further negative effect due to congestion. But I should note that the following qualitative results are strengthened by congestion. Furthermore, to simplify the formal exposition, I only consider the perfect complement case where the project requires full participation of agents in each stage and assume $F(c) = (\frac{c}{\bar{c}})^\alpha, \alpha > 0$. However, the essential intuition for the general case remains the same.[27] To determine the symmetric equilibrium, define the following function analog of (5):

$$B^C(x;\delta) = \frac{1-\delta}{\delta}\frac{x}{F(x)^{N-1}} + \int_0^x F(c)\,dc.$$

LEMMA 3: *For $h \leq H : \bar{W}_i(h+1) = B^C(x(h);\delta)$, and $\bar{W}_i(h) = B^C(x(h);1)$, where $\bar{W}_i(h) = (1-\delta)W_i(h)$.*

The following proposition characterizes the equilibrium.

PROPOSITION 7: *There exists a unique symmetric MPE with the following properties:*

(i) If $1 - \alpha(N-1) \leq 0$, then sufficiently large projects or projects with a sufficiently low final value, $v < v_{\min}$, where $v_{\min} \equiv B^C(x_{\min}; \delta)$ and

[26] Palfrey and Rosenthal (1988, 1991) also consider complementary contributions case and note the multiplicity of equilibria. They use the notion of an equilibrium being globally expectationally stable, which can be used in my setting to eliminate the zero-contribution equilibrium whenever another equilibrium exists.

[27] In particular, for a given degree of complementarity, the project may not start, depending crucially on the cost distribution and group size.

$x_{\min} = \{\frac{1-\delta}{\delta}[\alpha(N-1)-1]\}^{\frac{1}{\alpha N}}\bar{c}$ *never start and all other projects start with positive probability.*

(ii) If $1 - \alpha(N-1) > 0$*, then all projects start with positive probability.*

(iii) The projects that start have the following properties: For $h \leq H : x(h)$ *is strictly increasing in h, and* $W_i(h)$ *is strictly increasing at an increasing rate in* h *.*

Part (i) of Proposition 7 implies that when agents' contributions are complementary, large-scale projects may not start at all. This is unlike the substitute case where all projects take off (Part (i) of Proposition 1). The fact that agent *i*'s contribution is valuable only when other agents contribute creates a coordination problem and raises his "effective" cost of contribution. This cost may prove to be too high compared to the expected value of the project. Like Figure 1, Figure 3 demonstrates the backward induction described in Lemma 3 when there are complementaries in agents' inputs. Note, however, that unlike in Figure 1, there are no positive $x(h)$'s for projects longer than three stages. This means projects larger than three stages fail to take off.

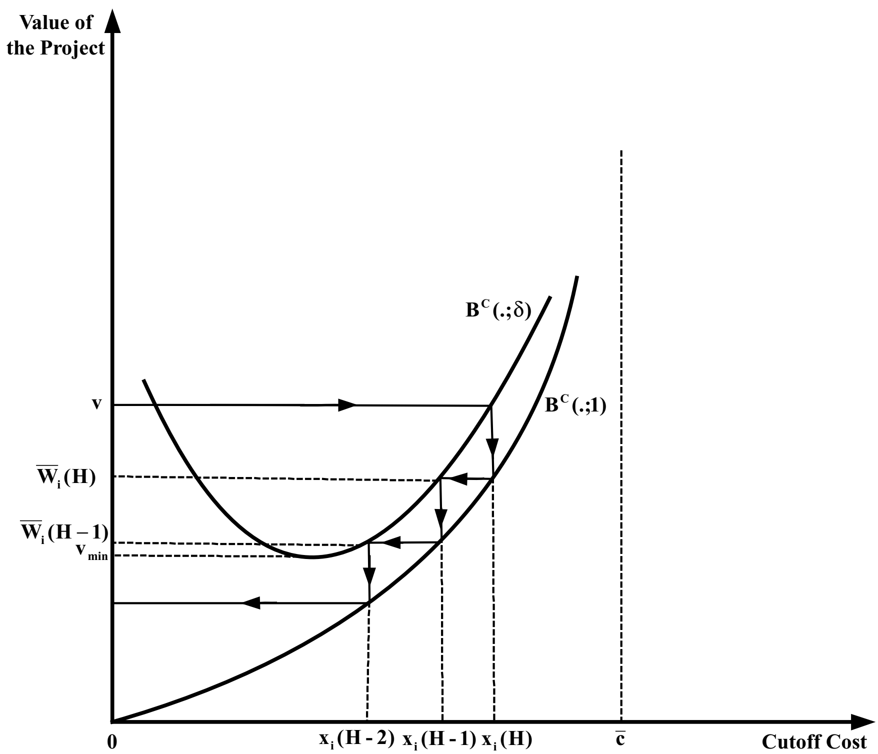

Figure 3: Backward induction with complementary contributions

The conditions in (i) and (ii) provide intuition regarding the roles of the discount factor, group size, and the cost distribution in coordination failure. As δ increases, it is more likely that the project of any size will start. In fact, as $\delta \to 1$, all projects start. The coordination failure worsens as group size increases and high costs become more likely. For a given distribution, the maximum group size that would satisfy the condition in part (ii) is $\bar{N} = \frac{1}{\alpha}$. For instance, when $\alpha = \frac{1}{2}$, a two-agent group will start any project. If the group size exceeds 2, some projects may not take off. This suggests that projects with complementary efforts might require outside help for a number of stages to insure that the rest will be finished through voluntary contributions.

This finding has similar flavor to that of Andreoni (1998), who considers the role of fundraising activities in inducing subsequent voluntary contributions. Within a one-shot contribution game with complete information, Andreoni assumes that contributions need to exceed some exogenous threshold level to yield any return. He notes that when contributions are perfect substitutes, the zero-contribution equilibrium exists if this threshold is too high. Therefore, an outside help, for example, an initial pledge or a government grant is needed to break this rather inefficient equilibrium and generate future voluntary contributions. The nonconvexity in my model is due to the complementaries in agents' contributions creating a complete coordination failure when agents' values of the project are very low.

Part (iii) indicates that once the project starts, the relationship among agents grows like in the substitute case.

6. Concluding Remarks

I have considered in some detail how participants voluntarily contribute to a large-scale project that consists of a sequence of subprojects. While the analysis has produced novel insights, it has also maintained restrictive assumptions. For one, the project is assumed to move only one step at a time regardless of the number of contributions. A more general approach would allow for a "smoother" production function and facilitate a performance comparison across projects with different technologies. Second, a number of richer strategies is ruled out by the MPE concept. For instance, if the project is an excludable public good, agents may adopt a time-dependent strategy that excludes partners who failed to contribute for a period of time. It would be interesting to see what the equilibrium "grace period" would be and how it would compare to the socially optimal one. Third, the analysis takes the division of the project as exogenous. An important extension would study the optimal division of the project and see how the division would be affected by the number of participants and the cost distribution. Finally, agents may be heterogenous in their key variables such as their cost distributions and discount factors. For instance, one country's financial fluctuation reflected by its cost distribution or political stability summarized by its discount factor can be different from another. An extension that allows for such heterogeneity can shed light on its role on cooperation.

Appendix

Proof of Lemma 1: Using (1) and (3) in the text, take the following expectation:

$$\begin{aligned} W_i(h) &= E_c[W_i(h, c)] \\ &= \int_0^{x_i(h)} [-c + \delta W_i(h+1)]\, dF(c) \\ &\quad + \int_{x_i(h)}^{\bar{c}} \delta\big[\lambda^*_{-i}(h)\, W_i(h+1) + \big(1 - \lambda^*_{-i}(h)\big) W_i(h)\big] dF(c). \end{aligned} \tag{A1}$$

Integrating the first integral by parts yields

$$\begin{aligned} W_i(h) = {} & -x_i(h)F(x_i(h)) + \delta W_i(h+1)F(x_i(h)) + \int_0^{x_i(h)} F(c)\, dc \\ & + \delta\big[\lambda^*_{-i}(h)\, W_i(h+1) + \big(1 - \lambda^*_{-i}(h)\big) W_i(h)\big][1 - F(x_i(h))]. \end{aligned} \tag{A2}$$

Now, recall from (2) that

$$x_i(h) = \big[1 - \lambda^*_{-i}(h)\big]\delta \Delta W_i(h). \tag{A3}$$

Using (A3) successively, (A2) reduces to

$$W_i(h) = \delta W_i(h+1) - \int_0^{x_i(h)} [1 - F(c)]\, dc. \tag{A4}$$

Moreover, (A3) implies that

$$W_i(h) = W_i(h+1) - \frac{x_i(h)}{\delta\big[1 - \lambda^*_{-i}(h)\big]}. \tag{A5}$$

Together with (A5), (A4) reveals that

$$\begin{aligned} \bar{W}_i(h+1) &= (1-\delta)\, W_i(h+1) \\ &= B(x(h); \delta), \end{aligned} \tag{A6}$$

where I make use of the symmetric equilibrium assumption so that $x_i(h) = x(h)$ for all i in equilibrium and

$$B(x; \delta) \equiv \frac{1}{\delta} \frac{x}{[1 - F(x)]^{N-1}} - \int_0^x [1 - F(c)]\, dc, \tag{A7}$$

as defined in (5) in the text.

Substituting for $W_i(h+1)$ from (A5) into (A4) also implies that

$$\begin{aligned}\bar{W}_i(h) &= (1-\delta)\,W_i(h) \\ &= B(x(h);1),\end{aligned} \tag{A8}$$

completing the proof of part (a) of Lemma 1.

Now differentiate $B(x;\delta)$ with respect to x to find

$$B'(x;\delta) = \frac{1}{\delta}\left[\frac{1}{(1-F(x))^{N-1}} + \frac{(N-1)xf(x)}{(1-F(x))^{N}}\right] - (1-F(x)). \tag{A9}$$

Since for $x \in [0, \bar{c}]$, $\frac{1}{(1-F(x))^{N-1}} - (1-F(x)) \geq 0$, we have $B'(x;\delta) > 0$. Also, (A9) implies that $B'(0;\delta) = \frac{1}{\delta} - 1$. ■

Proof of Proposition 1: I use backward induction. Note first that since the payoff does not change for states $h > H$, we have $\Delta W_i(h) = 0$. Then, (A3) implies that the unique equilibrium is $x(h) = 0$ for $h > H$, yielding the boundary condition in (4): For $h > H$,

$$\bar{W}_i(h) = v(N). \tag{A10}$$

Consider $h = H$. From (A6), $x(H)$ solves the following equation:

$$v(N) = B(x(H);\delta). \tag{A11}$$

For $N = 1$, if $v(N) \geq B(\bar{c};\delta)$, then $x(H) = \bar{c}$. However, if $v(N) < B(\bar{c};\delta)$, then since $v(N) > 0$ and $B(0;\delta) = 0$, the Intermediate Value Theorem implies there exists $x(H) \in (0, \bar{c})$ that solves (A11). For $N \geq 2$, since $v(N) > 0$, $B(0;\delta) = 0$, and $B(\bar{c};\delta) = \infty$, there exists $x(H) \in (0, \bar{c})$ that solves (A11). Moreover, $x(H)$ is unique due to $B'(x;\delta) > 0$. Also from (A8),

$$\begin{aligned}\bar{W}_i(H) &= B(x(H);1) \\ &< B(x(H);\delta) \leq v(N).\end{aligned} \tag{A12}$$

Thus, we have $\bar{W}_i(H) < \bar{W}_i(H+1)$.

Now suppose for some $j \geq 0$, there exists a unique $x(H-j) \in (0, \bar{c})$ and $\bar{W}_i(H-j) < \bar{W}_i(H-j+1)$. Again, from (A6), $x(H-j-1)$ solves

$$\bar{W}_i(H-j) = B(x(H-j-1);\delta). \tag{A13}$$

Since $\bar{W}_i(H-j) > 0$, using a similar argument above, there exists a unique $x(H-j-1) \in (0, \bar{c})$ that solves (A13). Furthermore,

$$\begin{aligned}\bar{W}_i(H-j-1) &= B(x(H-j-1);1) \\ &< B(x(H-j-1);\delta) = \bar{W}_i(H-j).\end{aligned} \tag{A14}$$

Since $x(H-j)$ uniquely solves $\bar{W}_i(H-j+1) = B(x(H-j);\delta)$ and $x(H-j-1)$ solves (A13), given that $\bar{W}_i(H-j) < \bar{W}_i(H-j+1)$ by the induction hypothesis and $B'(x;\delta) > 0$, we have

$$x(H-j-1) < x(H-j). \tag{A15}$$

Hence, there exists a unique symmetric MPE where for $h \leq H$, $x(h)$ is strictly positive and increasing in h.

To prove the last part of Proposition 1, note that in equilibrium (A3) can be written as

$$x(h) = [1 - F(x(h)]^{N-1}\delta\Delta W_i(h). \tag{A16}$$

Since $x(h)$ is strictly increasing for $h \leq H$, we must have

$$\Delta W_i(h-1) < \Delta W_i(h). \tag{A17}$$

■

LEMMA A1: *For sufficiently large H, there exists h_0 such that for $h < h_0$, $x(h)$ is arbitrarily close to* 0.

Proof of A1: Recall from Proposition 1 that the sequence $\{x(h)\}_{h=1}^{h=H}$ is strictly increasing or by reordering, the sequence $\{x(h)\}_{h=H}^{h=1}$ is strictly decreasing. Since the latter sequence is bounded below by 0, it converges to some $x_l \geq 0$; and so does its subsequence $\{x(h+1)\}_{h=H}^{h=1}$. Using Lemma 1, this implies that there exists h_0 such that for $h < h_0$, $\lim_{H\to\infty} B(x(h+1); 1) = \lim_{H\to\infty} B(x(h); \delta)$. Since $B(\bullet)$ is continuous in x, this further implies that $B(x_l; 1) = B(x_l; \delta)$, whose unique solution is $x_l = 0$. ■

Proofs of Proposition 2 and 3: Suppose $v(N) = v$ for all N. I first show by induction that for any $N \geq 1$, $W_i(h, N+1) > W_i(h, N)$ for any $h \leq H$.

Suppose in equilibrium that $W_i(H, N+1) \leq W_i(H, N)$. Since $W_i(H+1, N+1) = W_i(H+1, N) = \frac{v}{1-\delta}$, we have $\Delta W_i(H, N+1) \geq \Delta W_i(H, N)$. Then, (A16) implies $x(H, N+1) \geq x(H, N)$. Since $B(x; 1)$ is strictly increasing in x and N, (A8) reveals that $W_i(H, N+1) > W_i(H, N)$, which is a contradiction. Hence, $W_i(H, N+1) > W_i(H, N)$.

To complete the induction argument, suppose $W_i(h, N+1) > W_i(h, N)$ for some $h \leq H$. Suppose, on the contrary, that $W_i(h-1, N+1) \leq W_i(h-1, N)$. Since $W_i(h, N)$ is strictly increasing in h for any given N, we have

$$W_i(h-1, N+1) \leq W_i(h-1, N) < W_i(h, N) < W_i(h, N+1). \tag{A18}$$

Equation (A18) implies that $\Delta W_i(h-1, N+1) > \Delta W_i(h-1, N)$. Again, (A16) implies that $x(h-1, N+1) \geq x(h-1, N)$, which, in turn, implies $W_i(h-1, N+1) > W_i(h-1, N)$, yielding a contradiction. Hence, $W_i(h-1, N+1) > W_i(h-1, N)$. This completes the proof of Proposition 3.

Now I prove part (i) of Proposition 2. Suppose, by way of contradiction, that $x(H, N+1) \geq x(H, N)$. From above, we know that $W_i(H, N+1) > W_i(H, N)$. Again, given that $W_i(H+1, N+1) = W_i(H+1, N) = \frac{v}{1-\delta}$, we have $\Delta W_i(H, N+1) < \Delta W_i(H, N)$. From here, (A16) implies $x(H, N+1) < x(H, N)$, a contradiction. Hence, $x(H, N+1) < x(H, N)$.

To prove the last part of Proposition 2, note the following first-order Taylor expansion for x sufficiently close to 0:

$$B(x;\delta) \approx B(0;\delta) + B'(0;\delta)\,x$$
$$\approx \left(\frac{1}{\delta} - 1\right) x. \tag{A19}$$

Since, from Lemma A1, we know that for a sufficiently large H, there exists h_0 such that for $h < h_0$, $x(h, N)$ is arbitrarily close to 0, (A6) and (A19) imply that

$$\bar{W}_i(h, N+1) \approx \left(\frac{1}{\delta} - 1\right) x(h, N+1)$$
$$\bar{W}_i(h, N) \approx \left(\frac{1}{\delta} - 1\right) x(h, N).$$

Furthermore, since $\bar{W}_i(h, N+1) > \bar{W}_i(h, N)$ from Proposition 3, we must have $x(h, N+1) > x(h, N)$.

To prove part (iii), first recall from part (i) that $\{x(H; N)\}_{N=1}^{N=\infty}$ is a strictly decreasing sequence. Since it is bounded below by 0, it must converge to some $\alpha \geq 0$. Suppose $\alpha > 0$. For any N, $x(H; N)$ uniquely solves the equation $B(x(H; N)) = v$. Given $\lim_{N\to\infty} x(H; N) = \alpha > 0$, we have $\lim_{N\to\infty} B(x(H; N)) = \infty \neq v$. Hence, $\alpha = 0$.

Now take any $h < H$. Since $x(h; N)$ is increasing in h, we have

$$0 \leq x(h; N) \leq x(H; N).$$

This implies

$$0 \leq \lim_{N\to\infty} x(h; N) \leq \lim_{N\to\infty} x(H; N) = 0.$$

Thus, $\lim_{N\to\infty} x(h; N) = 0$.

To complete the proof, suppose $\lim_{N\to\infty}[1 - F(x(h; N)]^N > 0$. But given that $\lim_{N\to\infty} x(h; N) = 0$, we have $\lim_{N\to\infty} B(x(h; N)) = 0 \neq v > 0$. Thus, it must be that $\lim_{N\to\infty}[1 - F(x(h; N)]^N = 0$. ■

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