Economics 690 – Spring 2019
Continuous-time methods in economic theory

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Class meetings  WF, 3:05 - 4:20 PM, Allen 304I
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Course description

In economics and finance, continuous-time techniques have found wide application beyond the celebrated Black-Scholes option pricing formula. They are used in dynamic models of learning and experimentation, contracting, reputation building, capital structure, financial frictions and macroeconomic crises, trade and information disclosure, among others. The main goal of this course is to prepare students to effectively follow and interact with the economic theory literature that makes use of continuous-time techniques. The course is organized in two modules. The first module will present useful concepts and tools in stochastic calculus and stochastic control, with a heavy emphasis on the Brownian motion. The second module will discuss in depth an array of applications from microeconomic theory, macroeconomics, and corporate finance. The choice of applications will also take into account the interests of the enrolled students.

Textbook / references:

• Brownian Models of Performance and Control, J. Michael Harrison, Cambridge University Press, 2013. (*)

• The Economics of Inaction: Stochastic Control Models with Fixed Costs, Nancy L. Stokey, Princeton University Press, 2009. (*)


Note: (*) indicates main textbooks.

**Prerequisites**

The course material does not assume familiarity with measure theory, neither will it aim to cover any formally. It also does not attempt to replace a rigorous course in stochastic calculus. The course presupposes a first course in probability theory, analysis, and some degree of familiarity with differential equations. For a quick refresher, interested students are encouraged to read sections 1.1 and 1.4 of Mikosch (1998), chapter 1 in Shreve (2004), chapter 2 and appendices A and B in Øksendal (2003).

**Grading policy**

Evaluation is based on four biweekly written problem sets (40 %), a short in-class presentation on a student’s topic of choice (30 %), and a final written project (30 %). Problem sets will be posted a week prior to their due date. Students are allowed (and indeed encouraged) to work in groups, but they are required to submit individual answers. Student presentations will be scheduled during the second half of the course (weeks 7-14). The written project can take the form of a referee report or a research proposal. Students are expected to discuss in advance their choice of presentation topic and final project with the instructor.

**Tentative schedule of topics**

**Week 1:**
Jan 11: Basic notions: stochastic processes, filtrations, martingales, stopping times, limit of a random walk

**Week 2:** Brownian motion
Jan 16: Brownian motion and its properties, Harrison chapter 1.
Jan 18: Brownian motion and its properties, Harrison chapter 3; Brownian bridge

**Assignment 1 due on January 23.**

**Week 3:** Stochastic integration and Itô processes
Jan 23: Itô integral for simple processes and properties of the general Itô integral
Jan 25: Itô-Doeblin lemma and extensions; Black-Scholes formula; occupancy measure and local time

**Week 4:**
Jan 30: SDEs, strong and weak solutions, existence and uniqueness of strong solutions, Feynman-Kac formula
Feb 1: Jump processes and properties; Itô-Doeblin’s lemma for jump processes

**Assignment 2 due on Feb 6.**

**Week 5**
Feb 6: Martingales and stopping times, Dynkin’s formula, Martingale Representation Theorem, Optional Sampling Theorem (Stokey chapter 4);
Feb 8: Change of measure, Girsanov’s theorem; useful solutions for Brownian motion (Stokey, chapter 5)

**Week 6**
Feb 13: Dynamic programming principle, Hamiltoni-Jacobi-Bellman equation, verification theorem
Feb 15: Merton’s portfolio allocation problem

**Assignment 3 due on Feb 20.**

**Week 7:** Optimal stopping of Brownian motion
Feb 20: Optimal stopping of Brownian motion, Harrison chapter 5; McDonald and Siegel (1986)

**Week 8** Impulse control
Feb 27: Models with fixed and variable costs (Stokey, chapters 7-8)
March 1: Models with continuous control variables (Stokey, chapter 9); Impulse control with discounting (Harrison, 7.1-7.3)

**Assignment 4 due on March 6.**

**Week 9** Instantaneous control
March 6: Regulated Brownian motion (Stokey, chapter 10); Instantaneous control with discounting (Harrison 7.7)
March 8: Investment with linear and convex adjustment costs (Stokey, chapter 11)

**Spring recess**

**Week 10:** Learning and experimentation I
March 20: Brownian models of dynamic inference (Harrison, chapter 8)

**Week 11:** Learning and experimentation II
March 27: Exponential bandits (Keller, Rady, Cripps, 2005; Keller and Rady, 2010)
March 29: Strulovici (2010); Bonnati and Hörner (2011)

**Week 12:** Learning and experimentation III
April 3: Trade dynamics and information arrival: Daley and Green (2012), Kim (2017)

**Week 13:** Debt value and optimal capital structure
April 10: Moreno-Bromberg and Rochet, chapter 1
April 12: Tax subsidies on debt and liquidation costs: Leland (1994)

**Week 14:** Agency frictions and dynamic incentives
April 17: Moreno-Bromberg and Rochet, chapter 7; Sannikov 2008; Demarzo and Sannikov 2006

**Additional topics, if time permits:**
Continuous-time bargaining models (Perry and Reny (1993); Ambrus and Lu (2015); Chaves (2018))

General equilibrium with financial frictions (Moreno-Bromberg and Rochet, chapter 8; Brunnermeier and Sannikov, 2014)

Continuous-time models of banking (Moreno-Bromberg and Rochet, chapter 4)

**Final project due on April 29 at 2 PM.**

**Article readings**


