Economics 690 – Spring 2019

Continuous-time methods in economic theory

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Class meetings	WF, 3:05 - 4:20 PM, Allen 304I
Office hours	Tuesdays 3-5 PM & by appointment
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Course description

In economics and finance, continuous-time techniques have found wide application beyond the celebrated Black-Scholes option pricing formula. They are used in dynamic models of learning and experimentation, contracting, reputation building, capital structure, financial frictions and macroeconomic crises, trade and information disclosure, among others. The main goal of this course is to prepare students to effectively follow and interact with the economic theory literature that makes use of continuous-time techniques. The course is organized in two modules. The first module will present useful concepts and tools in stochastic calculus and stochastic control, with a heavy emphasis on the Brownian motion. The second module will discuss in depth an array of applications from microeconomic theory, macroeconomics, and corporate finance. The choice of applications will also take into account the interests of the enrolled students.

Textbook / references:

- *Brownian Models of Performance and Control*, J. Michael Harrison, Cambridge University Press, 2013. (*)
- *The Economics of Inaction: Stochastic Control Models with Fixed Costs*, Nancy L. Stokey, Princeton University Press, 2009. (*)
- *The Art of Smooth Pasting*, Avinash Dixit, Routledge: Taylor & Francis Group, 2001.
- *Continuous-Time Models in Corporate Finance, Banking, and Insurance: A User's Guide*, Santiago Moreno-Bromberg and Jean-Charles Rochet, Princeton University Press, 2018. (*)
- *Elementary Stochastic Calculus with Finance in View*, Thomas Mikosch, World Scientific Publishing, 1998.

- *Stochastic Calculus for Finance II: Continuous-time Models*, Steven E. Shreve, Springer, 2004.
- *Stochastic Differential Equations: An Introduction with Applications*, Bernt Øksendal, Springer, 2003.

Note: (*) indicates main textbooks.

Prerequisites

The course material does not assume familiarity with measure theory, neither will it aim to cover any formally. It also does not attempt to replace a rigorous course in stochastic calculus. The course presupposes a first course in probability theory, analysis, and some degree of familiarity with differential equations. For a quick refresher, interested students are encouraged to read sections 1.1 and 1.4 of Mikosch (1998), chapter 1 in Shreve (2004), chapter 2 and appendices A and B in Øksendal (2003).

Grading policy

Evaluation is based on four biweekly written problem sets (40 %), a short in-class presentation on a student's topic of choice (30 %), and a final written project (30 %). Problem sets will be posted a week prior to their due date. Students are allowed (and indeed encouraged) to work in groups, but they are required to submit individual answers. Student presentations will be scheduled during the second half of the course (weeks 7-14). The written project can take the form of a referee report or a research proposal. Students are expected to discuss in advance their choice of presentation topic and final project with the instructor.

Tentative schedule of topics

Week 1:

Jan 11: Basic notions: stochastic processes, filtrations, martingales, stopping times, limit of a random walk

Week 2: Brownian motion

Jan 16: Brownian motion and its properties, Harrison chapter 1.

Jan 18: Brownian motion and its properties, Harrison chapter 3; Brownian bridge

Assigment 1 due on January 23.

Week 3: Stochastic integration and Itô processes

Jan 23: Itô integral for simple processes and properties of the general Itô integral

Jan 25: Itô-Doeblin lemma and extensions; Black-Scholes formula; occupancy measure and local time

Week 4:

Jan 30: SDEs, strong and weak solutions, existence and uniqueness of strong solutions, Feynman-Kac formula

Feb 1: Jump processes and properties; Itô-Doeblin's lemma for jump processes

Assigment 2 due on Feb 6.

Week 5

Feb 6: Martingales and stopping times, Dynkin's formula, Martingale Representation Theorem, Optional Sampling Theorem (Stokey chapter 4);

Feb 8: Change of measure, Girsanov's theorem; useful solutions for Brownian motion (Stokey, chapter 5)

Week 6

Feb 13: Dynamic programming principle, Hamiltoni-Jacobi-Bellman equation, verification theorem

Feb 15: Merton's portfolio allocation problem

Assigment 3 due on Feb 20.

Week 7: Optimal stopping of Brownian motion

Feb 20: Optimal stopping of Brownian motion, Harrison chapter 5; McDonald and Siegel (1986)

Feb 22: Exercising an option, Stokey chapter 6.

Week 8 Impulse control

Feb 27: Models with fixed and variable costs (Stokey, chapters 7-8)

March 1: Models with continuous control variables (Stokey, chapter 9); Impulse control with discounting (Harrison, 7.1-7.3)

Assigment 4 due on March 6.

Week 9 Instantaneous control

March 6: Regulated Brownian motion (Stokey, chapter 10); Instantaneous control with discounting (Harrison 7.7)

March 8: Investment with linear and convex adjustment costs (Stokey, chapter 11)

Spring recess

Week 10: Learning and experimentation I

March 20: Brownian models of dynamic inference (Harrison, chapter 8)

March 22: Bolton and Harris (1999), Moscarini and Smith (2001), Fudenberg, Romanyuk, and Strack (2017)

Week 11: Learning and experimentation II

March 27: Exponential bandits (Keller, Rady, Cripps, 2005; Keller and Rady, 2010)

March 29: Strulovici (2010); Bonnati and Hörner (2011)

Week 12: Learning and experimentation III

April 3: Trade dynamics and information arrival: Daley and Green (2012), Kim (2017)

April 5: Monitoring with two-sided learning: Prat and Jovanovic (2014), Cisternas (2017)

Week 13: Debt value and optimal capital structure

April 10: Moreno-Bromberg and Rochet, chapter 1

April 12: Tax subsidies on debt and liquiditation costs: Leland (1994)

Week 14: Agency frictions and dynamic incentives

April 17: Moreno-Bromberg and Rochet, chapter 7; Sannikov 2008; Demarzo and Sannikov 2006

Additional topics, if time permits:

Continuous-time bargaining models (Perry and Reny (1993); Ambrus and Lu (2015); Chaves (2018))

General equilibrium with financial frictions (Moreno-Bromberg and Rochet, chapter 8; Brunnermeier and Sannikov, 2014)

Continuous-time models of banking (Moreno-Bromberg and Rochet, chapter 4)

Final project due on April 29 at 2 PM.

Article readings

Ambrus, Attila and Shih En Lu. "A Continuous-Time Model of Multilateral Bargaining." *American Economic Journal: Microeconomics* 7(1): 208-49, 2015.

Bolton, Patrick and Christopher Harris. "Strategic Experimentation." *Econometrica* 67(2): 349-374, 1999.

Bonnati, Alessandro and Johannes Hörner. "Collaborating." *American Economic Review* 101: 632 - 663, 2011.

Brunnermeier, Markus K. and Yuliy Sannikov. "A Macroeconomic Model with a Financial Sector." *American Economic Review* 104(2): 379-421, 2014.

Chaves, Isaias. "Privacy in Bargaining: The Case of Endogenous Entry." Unpublished manuscript, 2018.

Cisternas, Gonzalez. "Two-sided learning and the ratchet principle." *The Review of Economic Studies*, 85(1): 307-351, 2018.

Daley, Brendan and Brett Green. "Waiting for News in the Market for Lemons." *Econometrica* 80(4): 1433-1504, 2012.

Demarzo, Peter M. and Yuliy Sannikov. "Optimal Security Design and Dynamic Capital Structure in a Continuous - Time Agency Model." *Journal of Finance*, 61(6): 2681- 2724, 2006.

Demarzo, Peter M. and Yuliy Sannikov. "Learning, Termination, and Payout Policy in Dynamic Incentive Contracts." *The Review of Economic Studies* 84 (1), 2017.

Fudenberg, Drew, Gleb Romanyuk, and Philipp Strack. "Active Learning with Misspecified Beliefs." *Theoretical Economics* 12(3): 1155-1189, 2017.

Keller, Godfrey, Sven Rady and Martin Cripps. "Strategic Experimentation with Exponential Bandits." *Econometrica* 74(1): 39-68, 2005.

Keller, Godfrey, and Sven Rady. "Strategic Experimentation with Poisson Bandits." *Theoretical Economics* 5: 275 - 311, 2010.

Kim, Kyungmin. "Information About Sellers' Past Behavior in the Market for Lemons." *Journal of Economic Theory*, 169: 365-399, 2017.

Leland, Hayne E. "Corporate Debt Value, Bond Covenants, and Optimal Capital Structure." *Journal of Finance* 49(4): 1213-1252, 1994.

McDonald, Robert and Daniel Siegel. "The Value of Waiting to Invest." *The Quarterly Journal of Economics* 101(4): 707-727, 1986.

Moscarini, Giuseppe and Lones Smith. "The Optimal Level of Experimentation." *Econometrica* 69(6): 1629 - 1644, 2001.

Perry, Motty and Phil Reny. "A Non - Cooperative Bargaining Model with Strategically Timed Offers." *Journal of Economic Theory* 59(1): 50-77, 1993.

Prat, Jullien and Boyan Jovanovic. "Dynamic Contracts When the Agent's Quality is Unknown." *Theoretical Economics* 9: 865-914, 2014.

Sannikov, Yuliy. "A Continuous-Time Version of the Principal-Agent Problem." *The Review of Economic Studies*, 75(3):957 - 984, 2008.

Strulovici, Bruno. "Learning While Voting: Determinants of Collective Experimentation." *Econometrica* 78(3): 933-971, 2010.