Economics 690 – Spring 2020

Continuous-time methods in economic theory

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Class meetings	MW, 10:05 - 11:20 AM, SocSci 113
Office hours	Tuesdays 2-4 PM & by appointment
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Course description

In economics and finance, continuous-time techniques have found wide application beyond the celebrated Black-Scholes option pricing formula. They are used in dynamic models of learning and experimentation, contracting, reputation building, capital structure, financial frictions and macroeconomic crises, trade and information disclosure, among others. The main goal of this course is to prepare students to effectively follow and interact with the economic theory literature that makes use of continuous-time techniques. The course is organized in two modules. The first module will present useful concepts and tools in stochastic calculus and stochastic control, with a heavy emphasis on the Brownian motion. The second module will discuss in depth an array of applications from microeconomic theory, macroeconomics, and corporate finance. The choice of applications will also take into account the interests of the enrolled students.

Textbook / references:

- *Brownian Models of Performance and Control*, J. Michael Harrison, Cambridge University Press, 2013. (*)
- *The Economics of Inaction: Stochastic Control Models with Fixed Costs*, Nancy L. Stokey, Princeton University Press, 2009. (*)
- *The Art of Smooth Pasting*, Avinash Dixit, Routledge: Taylor & Francis Group, 2001.
- *Continuous-Time Models in Corporate Finance, Banking, and Insurance: A User's Guide*, Santiago Moreno-Bromberg and Jean-Charles Rochet, Princeton University Press, 2018. (*)
- *Elementary Stochastic Calculus with Finance in View*, Thomas Mikosch, World Scientific Publishing, 1998.

- *Stochastic Calculus for Finance II: Continuous-time Models*, Steven E. Shreve, Springer, 2004.
- *Stochastic Differential Equations: An Introduction with Applications*, Bernt Øksendal, Springer, 2003.

Note: (*) indicates main textbooks.

Prerequisites

The course material does not assume familiarity with measure theory, neither will it aim to cover any formally. It also does not attempt to replace a rigorous course in stochastic calculus. The course presupposes a first course in probability theory, analysis, and some degree of familiarity with differential equations. For a quick refresher, interested students are encouraged to read sections 1.1 and 1.4 of Mikosch (1998), chapter 1 in Shreve (2004), chapter 2 and appendices A and B in Øksendal (2003).

Grading policy

Evaluation is based on four biweekly written problem sets (40 %), a short in-class presentation on a student's topic of choice (30 %), and a take-home exam (30 %). Problem sets will be posted a week prior to their due date. Students are allowed (and indeed encouraged) to work in groups, but they are required to submit individual answers. Student presentations will be scheduled during the last two weeks of the semester. The take-home exam will be distributed on April 27 at 9AM and is due on April 28 at 12PM (noon).

Tentative schedule of topics

Week 1: Introduction

Jan 8: Basic notions: stochastic processes and equivalence notions, filtrations, martingales, stopping times.

Week 2: Brownian motion and related processes

Jan 13: Basic properties of the Brownian motion; quadratic variation; strong Markov property; occupancy measure and local time; Brownian martingales; the Innovation Theorem

Jan 15: Reflection principle and other properties of the Brownian; regular diffusions; Gaussian processes; geometric Brownian; Brownian bridge.

Jan 20: No class (MLK day).

Week 3: Stochastic integration and Itô processes

Jan 22: Itô integral for simple processes, approximation of general processes; properties of the general Itô integral

Jan 27: Five variations of Itô's lemma; Itô processes; area under a Brownian path

Assigment 1 due on January 27.

Week 4: SDEs and jump processes

Jan 29: Occupancy measure and local time revisited; Tanaka's formula; strong and weak solutions of SDEs

Feb 3: Introduction to Lévy processes; simple and compound Poisson processes and their properties; stochastic integral and Itô's lemma with jumps

Week 5

Feb 5: Optional Stopping Theorem; Martingale Convergence Theorem; change of measure and Girsanov's theorem; Black-Scholes option pricing

Feb 10: Useful solutions for Brownian motion (Stokey, chapter 5)

Assigment 2 due on Feb 10.

Week 6: Optimal control

Feb 12: Expected (discounted) local time; elements of a stochastic control problem; Markov control; dynamic programming and the HJB equation

Feb 17: Verification theorems; Merton's portfolio allocation problem

Week 7: Optimal stopping of a Brownian motion

Feb 19: Strulovici and Szydlowski (2015); verification in optimal stopping problems; value matching and smooth pasting

Feb 24: McDonald and Siegel (1986); Investment with costly waiting; Stokey's expected local time approach to stopping problems

Assigment 3 due on Feb 24.

Week 8: Impulse control

Feb 26: Menu cost pricing models; HJB approach to impulse control; random costless adjustment

March 2: Inventory models with holding and adjustment costs; long-run averages; difference between impulse control and instantaneous control

Week 9: Instantaneous control

March 4: Regulated Brownian motion; instantaneous control with discounting; inventory model with no fixed costs

March 16: Dumas (1991); super contact condition; investment with linear and convex adjustment costs

Assigment 4 due on March 16.

Spring recess

Week 10: Learning and experimentation

March 18: Brownian models of dynamic inference (Harrison, chapter 8)

March 23: Bolton and Harris (1999), Keller, Rady, Cripps (2005)

Week 11: Moral hazard and contracts

March 25: Bonnati and Hörner (2011)

March 30: Sannikov (2008)

Week 12: Irreversibility in firm decisions

April 1: Pindyck (1991), Pindyck (1988)

April 6: Bentolila and Bertola (1990), Dixit (1989)

Week 13: Debt value and optimal capital structure

April 8: Moreno-Bromberg and Rochet, chapter 1; Leland (1994)

April 13: Student presentations I

Week 14

April 15: Student presentations II

April 20: Student presentations III

Take-home exam due on April 28 at noon (12PM).

Additional topics, if time permits:

Continuous-time models of banking (Moreno-Bromberg and Rochet, chapter 4)

General equilibrium with financial frictions (Moreno-Bromberg and Rochet, chapter 8)

Continuous-time bargaining models (Perry and Reny (1993); Ortner (2018); Chaves (2018))

References

Ambrus, Attila and Shih En Lu. "A Continuous-Time Model of Multilateral Bargaining." *American Economic Journal: Microeconomics* 7(1): 208-49, 2015.

Bentolila, Samuel and Giuseppe Bertola. "Firing Costs and Labour Demand: How Bad is Eurosclerosis?" *The Review of Economic Studies* 57(3): 381-402, 1990.

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Bolton, Patrick and Christopher Harris. "Strategic Experimentation." *Econometrica* 67(2): 349-374, 1999.

Bonnati, Alessandro and Johannes Hörner. "Collaborating." *American Economic Review* 101: 632 - 663, 2011.

Brunnermeier, Markus K. and Yuliy Sannikov. "A Macroeconomic Model with a Financial Sector." *American Economic Review* 104(2): 379-421, 2014.

Chan, Jimmy, Alessandro Lizzeri, Wing Suen, and Leeat Yariv. "Deliberating Collective Decisions." *Review of Economic Studies* 85(2): 929-963, 2018.

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Cisternas, Gonzalez. "Two-sided learning and the ratchet principle." *The Review of Economic Studies*, 85(1): 307-351, 2018.

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Dixit, Avinash. "Entry and Exit Decisions under Uncertainty." *Journal of Political Economy* 97(3): 620-638, 1989.

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Fudenberg, Drew, Gleb Romanyuk, and Philipp Strack. "Active Learning with Misspecified Beliefs." *Theoretical Economics* 12(3): 1155-1189, 2017.

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Keller, Godfrey, and Sven Rady. "Strategic Experimentation with Poisson Bandits." *Theoretical Economics* 5: 275 - 311, 2010.

Kim, Kyungmin. "Information About Sellers' Past Behavior in the Market for Lemons." *Journal of Economic Theory*, 169: 365-399, 2017.

Leland, Hayne E. "Corporate Debt Value, Bond Covenants, and Optimal Capital Structure." *Journal of Finance* 49(4): 1213-1252, 1994.

McDonald, Robert and Daniel Siegel. "The Value of Waiting to Invest." *The Quarterly Journal of Economics* 101(4): 707-727, 1986.

Moscarini, Giuseppe and Lones Smith. "The Optimal Level of Experimentation." *Econometrica* 69(6): 1629 - 1644, 2001.

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